NLO QCD corrections to hadronic $\mathrm{WWb\bar{b}}$ production

Stefano Pozzorini
Zürich University

in collaboration with

A. Denner, S. Dittmaier and S. Kallweit

RADCOR 2011, Mamallapuram, September 26, 2011

Outline of the talk

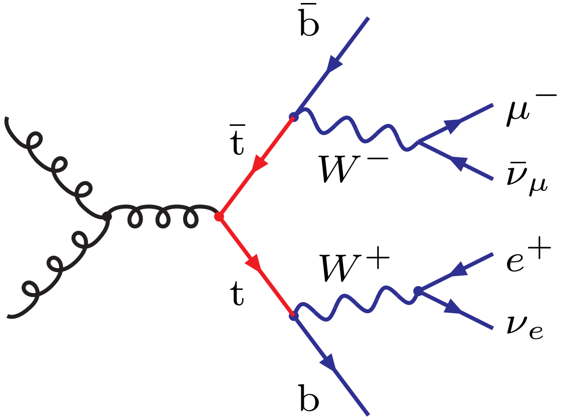
1. **Why** $pp \to t\bar{t} \to W^+W^-b\bar{b}$ **at NLO?**
2. **Technical aspects of the calculation**
3. **NLO predictions for Tevatron and LHC**

NLO priority list (Les Houches '05): completed $2 \to 4$ calculations

- **Two calculations for** $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ **with permille agreement**
  - arXiv:0905.0110 and arXiv:1001.4006 by Bredenstein, Denner, Dittmaier and S. P.
    Feynman diagrams and tensor integrals
  - arXiv:0907.4723 by Bevilacqua, Czakon, Papadopoulos, Pittau and Worek
    OPP reduction and HELAC
- **Two calculations for** $\mathrm{pp} \to Vjjj$
  - arXiv:0906.1445 ($Wjjj$) by Ellis, Melnikov and Zanderighi
    D-dimensional unitarity (leading colour)
  - arXiv:0907.1984 ($Wjjj$) and arXiv:1004.1659 ($Zjjj$) by Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower and Maitre
    generalized unitarity (full colour)
- **One calculation for** $\mathrm{pp} \to \mathrm{t\bar{t}}jj$
  - arXiv:1002.4009 and 1108.2851 by Bevilacqua, Czakon, Papadopoulos and Worek
    OPP reduction and HELAC

- **One calculation for** $\mathrm{pp} \to WWjj$
  - arXiv:1007.5313 and arXiv:1104.2327 by Melia, Melnikov, Rontsch and Zanderighi
    D-dimensional unitarity
- **First 7-leg results for** $\mathrm{pp} \to V + 4j$
  - arXiv:1009.2338 ($Wjjjj$) and arXiv:1108.2229 ($Zjjjj$) by Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower and Maitre
    generalized unitarity (leading colour)
- **One calculation for** $\mathrm{pp} \to \mathrm{b\bar{b}b\bar{b}}$
  - arXiv:1105.3624 by Greiner, Guffanti, Reuter and Reiter
    Feynman diagrams and OPP reduction (GOLEM–SAMURAI)
- **Two calculations for** $\mathrm{pp} \to W^+W^-\mathrm{b\bar{b}}$
  - arXiv:1012.3975 by Denner, Dittmaier, Kallweit and S. P.
    Feynman diagrams and tensor integrals
  - arXiv:1012.4230 by Bevilacqua, Czakon, van Hameren, Papadopoulos and Worek
    OPP reduction and HELAC

Why $W^+W^-b\bar{b}$ production at NLO?



Full description of $t\bar{t}$ prod×decay

- off-shell tops and non-resonant backgr.
- $W \to l\nu$ decays in spin-correlated NWA

Huge $t\bar{t}$ samples at hadron colliders

- Tevatron: few 10^4 events $\Rightarrow \frac{\delta\sigma}{\sigma} < 10\%$
- LHC at 7(14) TeV: $1.5(9) \times 10^5$ events per fb^{-1} $\Rightarrow \frac{\delta\sigma}{\sigma} = \text{few}\,\%$

Crucial measurements and tests

- precise studies of rich variety of (differential) observables
- checks and tuning of many theoretical/experimental tools
- $\delta m_t^{\mathrm{exp}} \sim 1\,\mathrm{GeV}$ measurements

Relevance for discoveries

- leptons + jets + missing E_T is a typical discovery signature (SUSY, $H \to W^+W^-, \ldots$)
- various BSM scenarios predict heavy resonances decaying into $t\bar{t}$

Precise predictions for hadronic $t\bar{t}$ production (and decay)

NLO QCD corrections

Beenakker, Dawson, Ellis, Frixione, Kuijf, Meng, Nason, van Neerven, Schuler, Smith

Electroweak NLO corrections

Beenakker, Bernreuther, Denner, Fücker, Hollik, Kao, Kollar, Kühn, Ladinsky, Mertig, Moretti, Nolten, Ross, Sack, Scharf, Si, Uwer, Wackeroth, Yuan

From LL to NNLL resummations

Ahrens, Beneke, Berger, Bonciani, Catani, Contopanagos, Czakon, Falgari, Ferroglia, Frixione, Kidonakis, Kiyo, Laenen, Mangano, Mitov, Moch, Nason, Neubert, Pecjak, Ridolfi, Schwinn, Sterman, Uwer, Vogt, Yang

Towards full NNLO predictions

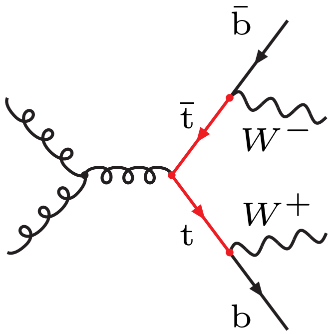
Anastasiou, Aybat, Bonciani, Czakon, Dittmaier, Ferroglia, Gehrmann, Gerhmann–De Ridder, Kniehl, Körner, Langenfeld, Maitre, Merebashvili, Mitov, Moch, Ritzmann, Rogal, Studerus, von Manteuffel, Uwer, Weinzierl

NLO $t\bar{t}$ production×decay in spin-correlated narrow-width approx.

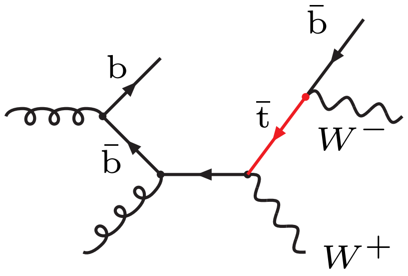
Bernreuther, Brandenburg, Melnikov, Schulze, Si, Uwer

Full $W^+W^-b\bar{b}$ description vs Narrow-Width Approximation in LO

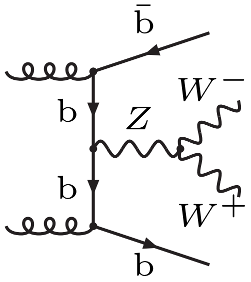
Doubly-Resonant (DR)



Singly-Resonant (SR)



Non-Resonant (NR)



Narrow-Width Approximation ($\Gamma_t \to 0$)

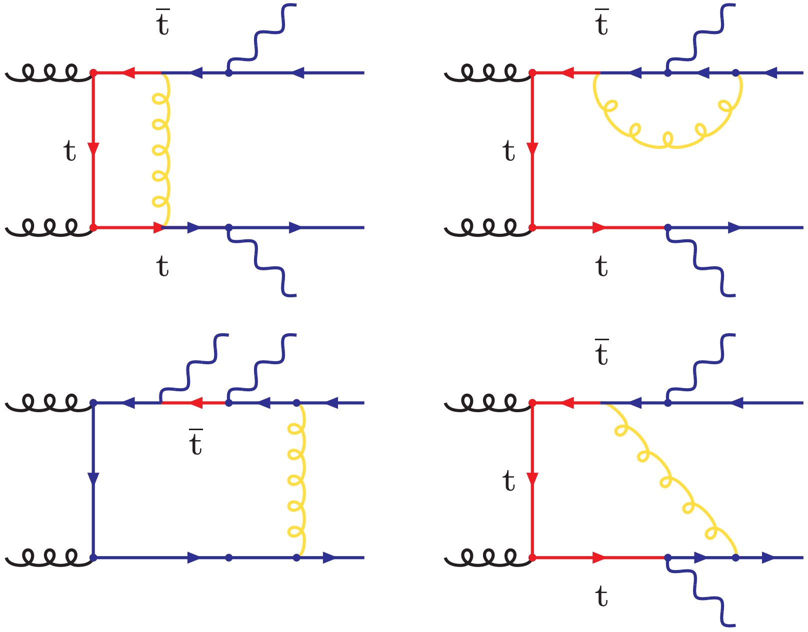
- only doubly-resonant channels
- narrow-with limit of Breit-Wigner top resonances

$$\lim_{\Gamma_t \to 0} \left| \frac{1}{p_t^2 - m_t^2 + i\Gamma_t m_t} \right|^2 = \frac{\pi}{\Gamma_t m_t} \delta(p_t^2 - m_t^2)$$

Finite-width contributions to $W^+W^-b\bar{b}$

- Off-shell corrections to doubly-resonant channels
- Singly + non-resonant channels and interferences
- finite-width corrections to *inclusive* observables of order $\Gamma_t/m_t \simeq 1\%$

Full $W^+W^-b\bar{b}$ description vs Narrow-Width Approximation in NLO



Narrow-Width Approximation ($\Gamma_t \to 0$)

- only factorisable corr. to DR channels
- huge technical simplification

Finite-width contributions to $W^+W^-b\bar{b}$

- non-factorisable and non-DR corrections
- pentagons and hexagons

In *inclusive* observables non-fact. virtual and real $\ln(\Gamma_t/m_t)$ corr. from soft gluons cancel, and finite-width effects remain $\mathcal{O}(\Gamma_t/m_t)$ suppressed [Fadin/Khoze/Martin '94].

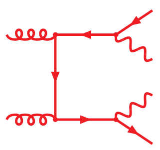
Finite-width effects can be important for

- percent-level precision in σ_{incl}
- shape of top resonance and related observables (m_t measurement)
- cuts suppressing on-shell $t\bar{t}$ background and enhancing off-shell $W^+W^-b\bar{b}$

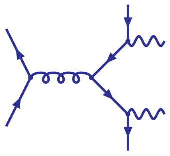
(2) Technical aspects of the calculation

Ingredients of $pp \to W^+W^-b\bar{b}$ at NLO

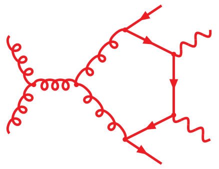
Partonic channels



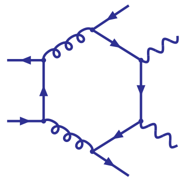
31 trees



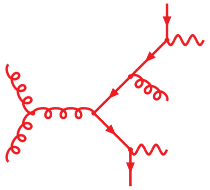
14 trees



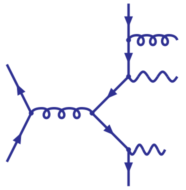
788 loops



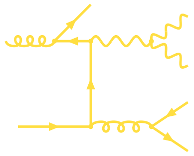
280 loops



222 NLO trees



90 NLO trees



90 NLO trees

Full calculation twice and independently

Generation of Feynman diagrams

- `FeynArts 1.0 / 3.2`

Algebraic reduction

- `MATHEMATICA` / `FormCalc` [Hahn]

Tensor integrals & numerics

- `Fortran77 / C++` executables: 0.25–1.2 GB

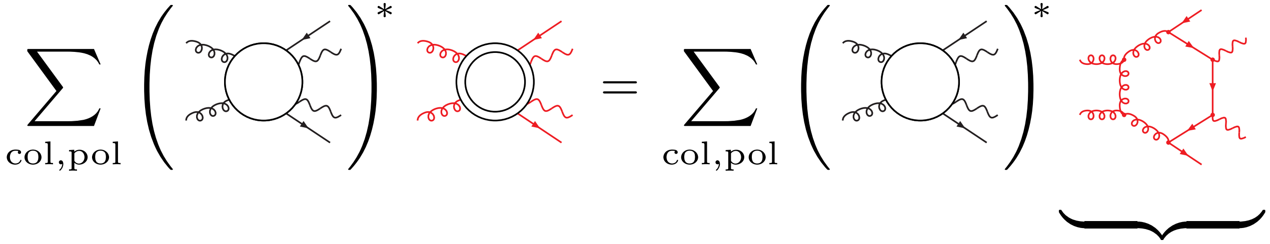
Real emission & IR Subtraction

- `Madgraph` & spinors
- Dipoles [Catani/Dittmaier/Seymour/Trócsányi ’97/’02] & `AutoDipole` [Hasegawa/Moch/Uwer ’09]

Integration over 11-dim PS

- adaptive multi-channel Monte Carlo with 250–650 mappings per partonic channel

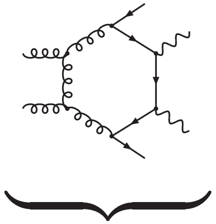
Feynman diagrams and tensor integrals



$$\sum_{\text{col,pol}} \left(\quad \right)^{*} \quad = \sum_{\text{col,pol}} \left(\quad \right)^{*} \underbrace{\qquad} + \mathcal{O}(1000) \text{ more diagrams}$$

Colour sums at zero cost

thanks to *colour factorisation*



$$\times\, f^{a_1bd} f^{a_2cd} \left(T^cT^b\right)_{i_5i_6}$$

$$\sum a_{i_1\ldots j_P} \underbrace{\epsilon_{\mu_1}\epsilon_{\mu_2}\epsilon_{\mu_3}\epsilon_{\mu_4}\left[\bar{v}_5\,\gamma_{\mu_5}\ldots\gamma_{\mu_k}\,u_6\right]}\{g\ldots p\}^{\mu_1\ldots\nu_P}_{i_1\ldots j_P} \underbrace{\int \mathrm{d}^Dq\,\frac{q_{\nu_1}\ldots q_{\nu_P}}{N_0\ldots N_{N-1}}}$$

Algebraic reduction of helicity structures

$\mathcal{O}(10^3$–$10^4)$ *compact spinor chains* → fast helicity sums

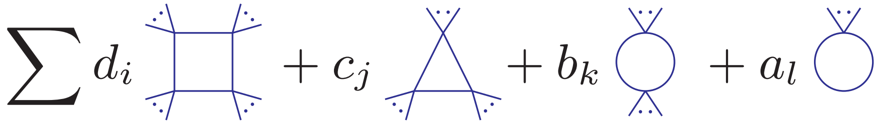
completely automatic!

$$\underbrace{\sum T^{(N)}_{j_1\ldots j_P}}\{g\ldots p\}^{\nu_1\ldots\nu_P}_{j_1\ldots j_P}$$

Numerical tensor-integral reduction

$e^+e^- \to 4f$ methods [Denner/Dittmaier'05]

completely general and numerically stable!



$$\sum d_i \quad + c_j \quad + b_k \quad + a_l$$

Very high CPU efficiency

First physical application up to tensor rank $P = 5$

- **CPU cost of colour/helicity summed** $\mathrm{gg} \to \mathrm{W^+W^-b\bar{b}}$ **loop amplitudes very low** (200-350 ms*) similarly as for $\mathrm{gg} \to \mathrm{t\bar{t}b\bar{b}}$ (180 ms) where $P = 4$
- σ_{NLO} with **statistical accuracy of** $\mathcal{O}(10^{-3})$ requires $\mathcal{O}(10^8)$ events obtained within **5–10 days on single CPU**
- Total CPU cost at LHC dominated by real and virtual gg-channel corrections

---

[*] 180 ms using ifort compiler on Intel Core i5-750 CPU (8M Cache, 2.66 GHz)

Main lessons on Tensor Integrals and Feynman diagrams

Big virtues

- high CPU efficiency
- numerical stability
- automation

Bottlenecks (in nontrivial 6-particle processes)

- $\mathcal{O}$(10 days) algebraic manipulations
- $\mathcal{O}$(100-1000 MB) numerical code

Solution (see talk by Philipp Maierhöfer)

- under development in Zürich (F. Cascioli/P. Maierhöfer/S. P.)
- avoids computer algebra with new numerical/recursive approach

Treatment of unstable particles

Regularisation of unstable-particle propagators via $\mathrm{Im}[\Sigma(M^2)] = M\Gamma$ resummation

$$\frac{1}{p^2 - M^2 + \mathrm{i}\epsilon} \to \frac{1}{p^2 - M^2 + \mathrm{i}M\Gamma + \mathrm{i}\epsilon}$$

can violate gauge invariance

Complex mass scheme at NLO (introduced for $\mathrm{e}^+\mathrm{e}^- \to 4f$ Denner/Dittmaier '05)

- Γ is absorbed into the renormalised pole mass $M^2 \to \mu^2 = M^2 - \mathrm{i}M\Gamma$ without modifying the bare Lagrangian
- Lagrangian symmetries require (in general) complex couplings

Technical aspects

- On-shell renormalisation with complex momenta: $\hat{\Sigma}(p^2) = 0$ at $p^2 = \mu^2$
- Scalar box integrals with complex masses (subtle analytic continuations!)
  - 't Hooft/Veltman approach: $24 \to 108$ Li_2 Nhung/Ninh '09; van Hameren '10
  - Denner/Niertse/Scharf approach: $16 \to 32$ Li_2 Denner/Dittmaier '10

(3.1) $W^+W^-b\bar{b}$ cross section at the Tevatron (1.96 TeV) and the LHC (7 TeV)

Setup and input parameters for Tevatron (LHC)

Particle masses and widths ($M_{\mathrm{H}} = \infty$, $m_{\mathrm{b}} = 0$)

$m_{\mathrm{t}} = 172.0\,\mathrm{GeV}$ $\quad M_{\mathrm{W}} = 80.399\,\mathrm{GeV}$ $\quad M_{\mathrm{Z}} = 91.1876\,\mathrm{GeV}$

$\Gamma_{\mathrm{t,LO}} = 1.4655\,\mathrm{GeV}$ $\quad \Gamma_{\mathrm{t,NLO}} = 1.3376\,\mathrm{GeV}$ $\quad \Gamma_{\mathrm{W,NLO}} = 2.0997\,\mathrm{GeV}$

G_μ-scheme couplings ($G_\mu = 1.16637 \times 10^{-5}\,\mathrm{GeV}^{-2}$)

$$\sin^2\theta_{\mathrm{w}} = 1 - M_{\mathrm{W}}^2/M_{\mathrm{Z}}^2, \qquad \alpha = \sqrt{2} G_\mu M_{\mathrm{W}}^2 \sin^2\theta_{\mathrm{w}}/\pi$$

PDFs and α_{S}: MSTW2008NLO(LO) with $1/2 \leq \mu_{\mathrm{R,F}}/m_{\mathrm{t}} \leq 2$ variations

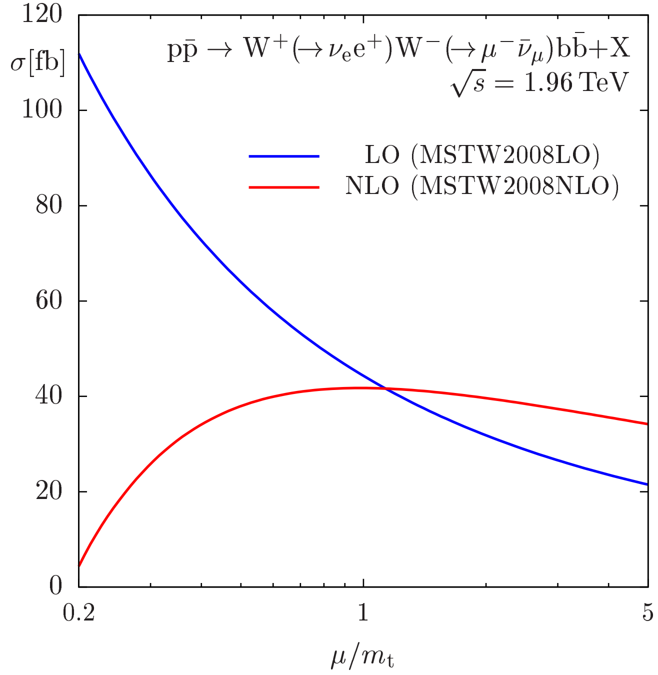
Anti-k_{T} Jet Algorithm

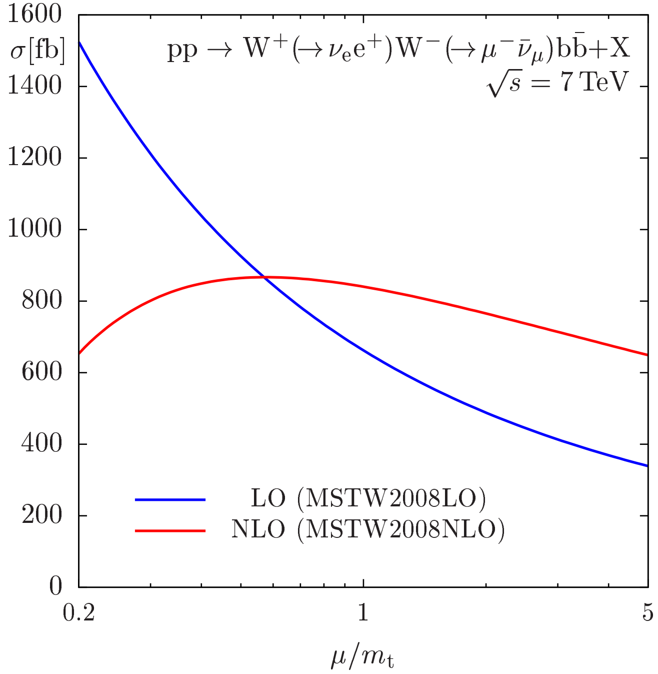
QCD partons with $|\eta| < 5$ $\quad\Rightarrow\quad$ jets with $\sqrt{\Delta\phi^2 + \Delta y^2} > R = 0.4\,(0.5)$

Typical Tevatron (LHC) cuts

b-jets: $\quad p_{\mathrm{T,b}} > 20\,(30)\,\mathrm{GeV}$ $\quad |\eta_{\mathrm{b}}| \leq 2.5$

leptons: $\quad p_{\mathrm{T},l} > 20\,\mathrm{GeV}$ $\quad |\eta_l| \leq 2.5$ $\quad p_{\mathrm{T,miss}} > 25\,(20)\,\mathrm{GeV}$





Integrated $e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}$ cross section

Predictions for $\mu_{R,F} = m_t$ and $m_t/2 \leq \mu_{R,F} \leq 2m_t$

σ	LO	NLO	NLO/LO
Tevatron	$44.31^{+19.68}_{-12.49}$ fb	$41.75^{+0.00}_{-3.79}$ fb	$0.942^{+0.000}_{-0.085}$
LHC	$662.4^{+263.4}_{-174.1}$ fb	840^{+27}_{-75} fb	$1.27^{+0.04}_{-0.11}$

Scale uncertainty at the Tevatron (LHC)

- 44% (40%) LO uncertainty is mostly due to $\frac{\Delta\sigma_{LO}}{\sigma_{LO}} \simeq \frac{\Delta\alpha_S^2(\mu)}{\alpha_S^2(\mu)}$ and reduces to 9%(9%) at NLO

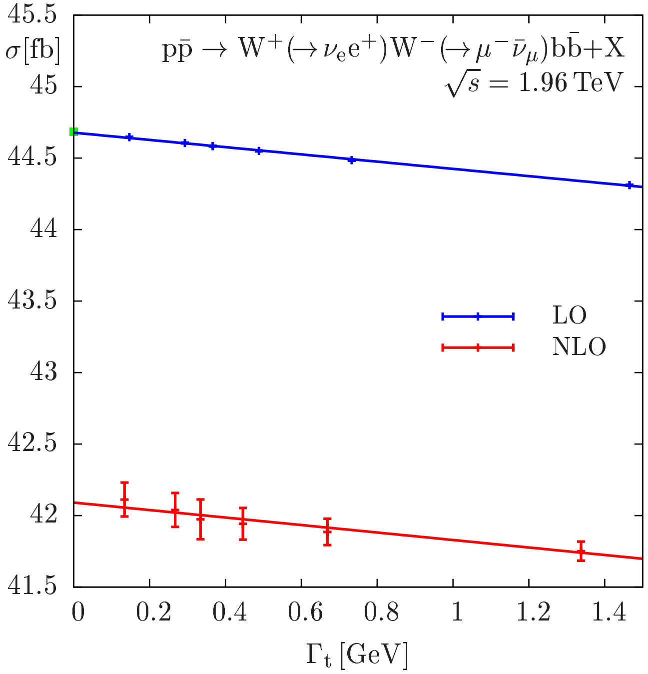
Moderate NLO corrections

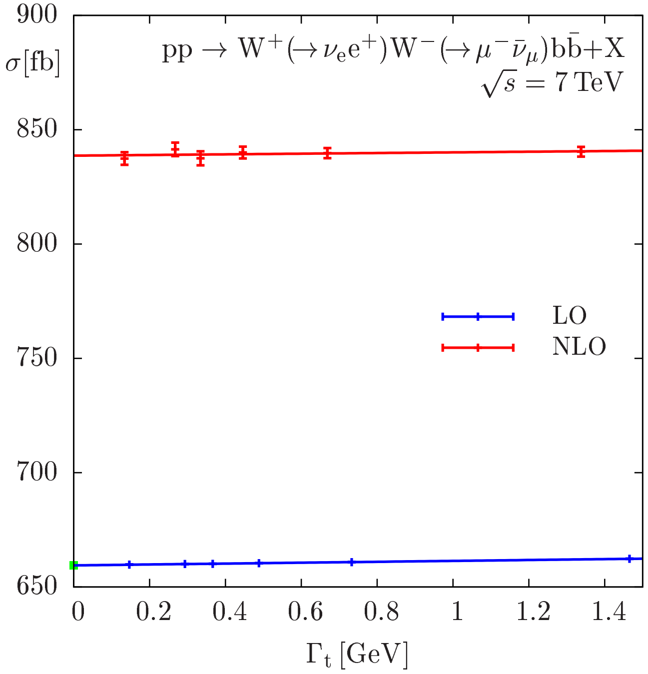
- $K_{\text{Tevatron}} \simeq 0.94$ and $K_{\text{LHC}} \simeq 1.27$

Agreement with HELAC–NLO [Bevilacqua et al. '10]

σ_{Tevatron}	LO	NLO
DDKP	44.310[3] fb	41.75[5] fb
BCHPW	44.32[3] fb	41.86[6] fb

Off-shell and non-resonant contributions to $\sigma_{\text{int.}}$





Assessment of finite-width effects $\sigma(\Gamma_t) - \sigma(0)$

- numerical extrapolation to $\Gamma \to 0$ using five rescaled values $\Gamma_t \to \xi\Gamma_t$ with $0.1 \lesssim \xi \leq 1$

Cancellation of soft-gluon $\ln(\Gamma_t/m_t)$ singularities

- dipole-subtracted virtual and real parts diverge logarithmically when $\Gamma \to 0$
- linear convergence of $\sigma(\Gamma_t) \to \sigma(0)$ provides non-trivial consistency and stability check

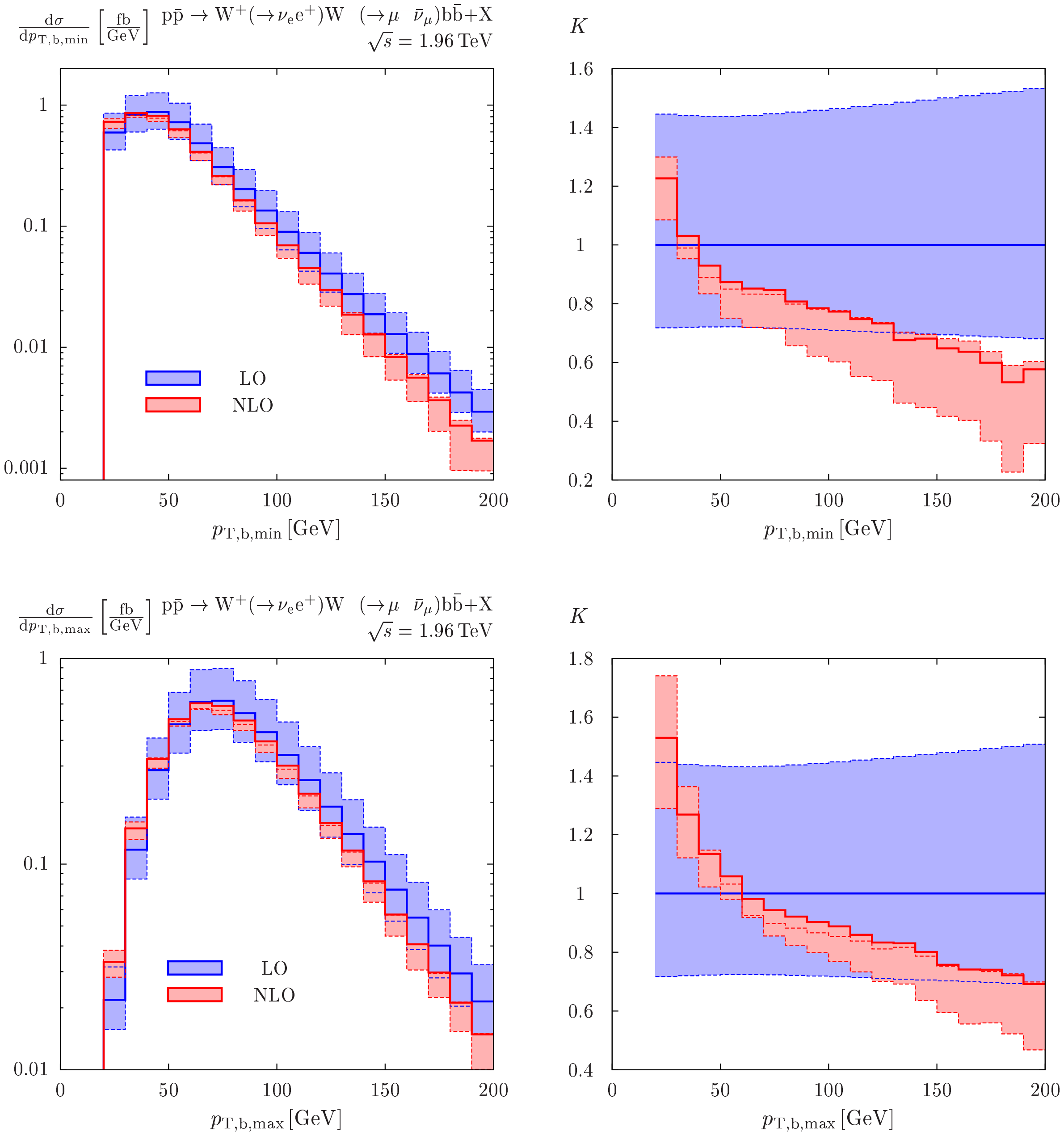
Finite-width effects comparable to $\Gamma_t/m_t \simeq 0.8\%$

	$\sigma_{\text{LO}}(\Gamma_t)/\sigma_{\text{LO}}(0) - 1$	$\sigma_{\text{NLO}}(\Gamma_t)/\sigma_{\text{NLO}}(0) - 1$
Tevatron	-0.8%	-0.9%
LHC	$+0.4\%$	$+0.2\%$

quantifies precision of NWA for $\sigma_{\text{inclusive}}$

(3.2) Differential $\mathrm{W^+W^-b\bar{b}}$ distributions at the Tevatron (1.96 TeV) and the LHC (7 TeV)

b-jet p_{T} at the Tevatron



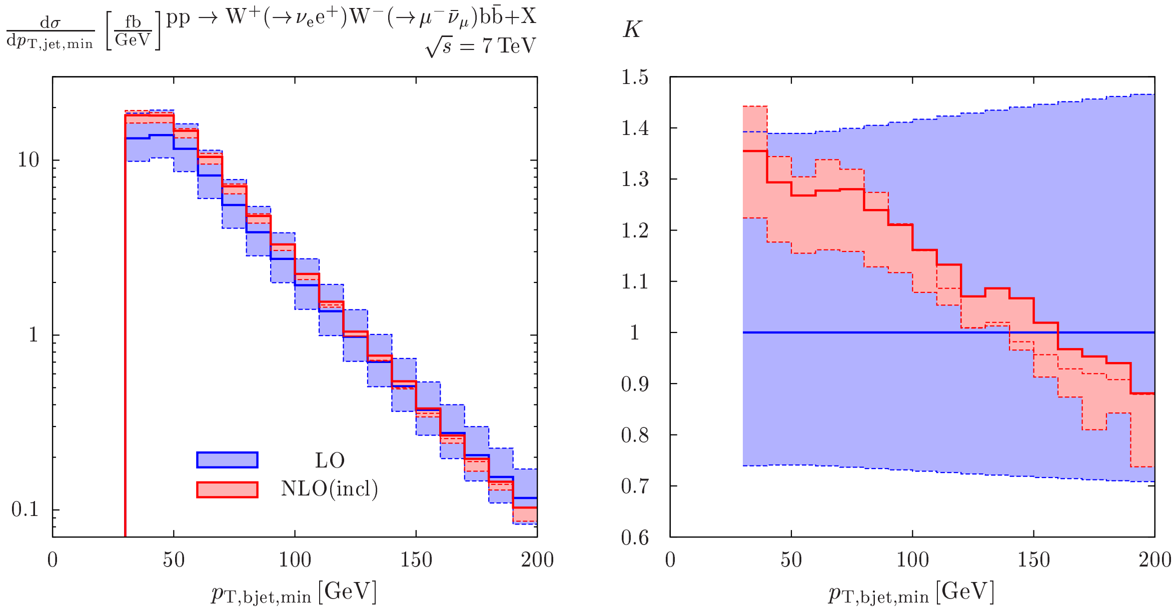
Soft b-jet (upper)

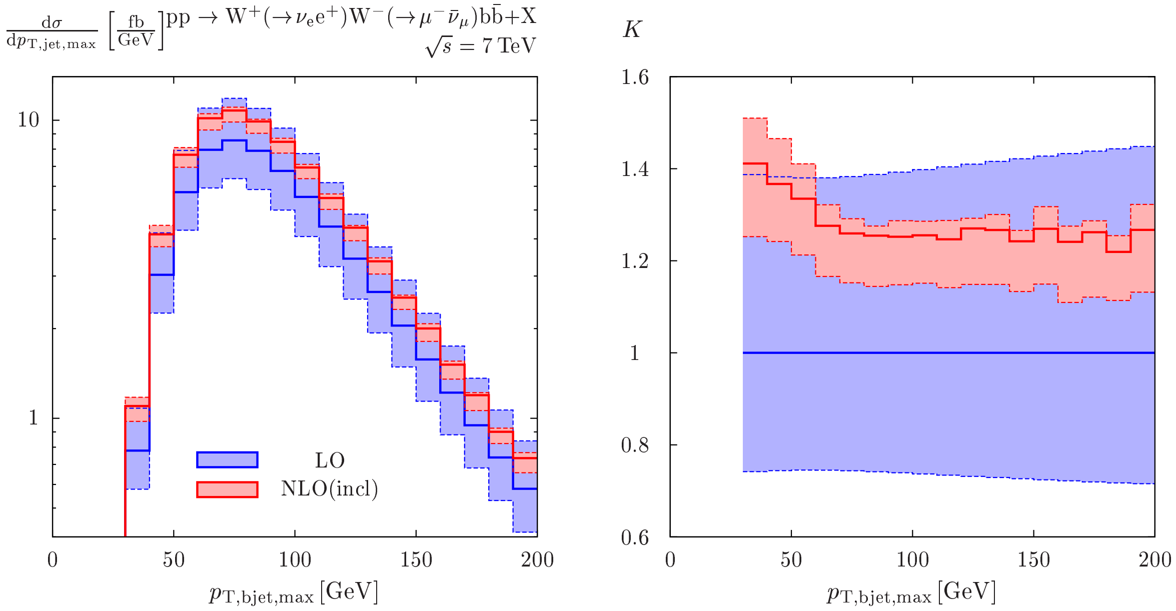
- saturates cut at 20 GeV
- +20% to −40% corrections
- strong shape distortions (relevant for acceptance)

Hard b-jet (lower)

- peaked around 80 GeV
- +50% to −30% corrections
- strong shape distortions

b-jet p_{T} at the LHC



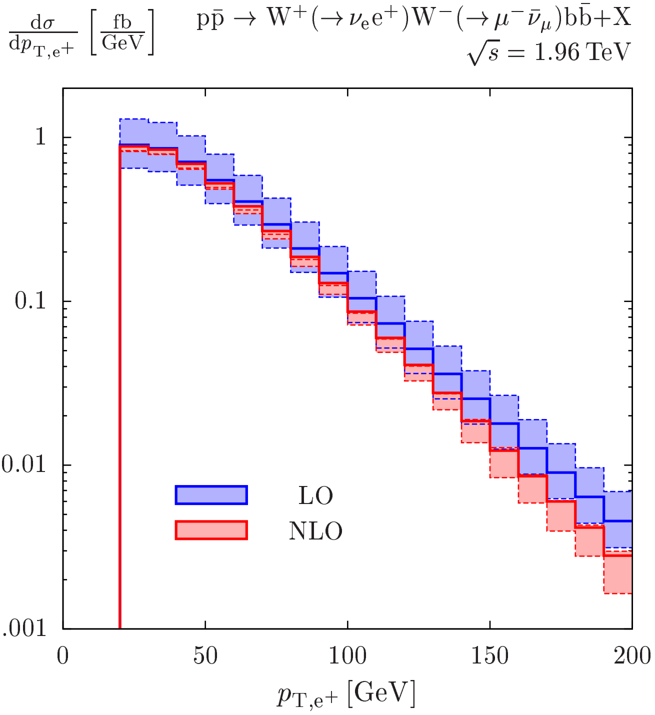


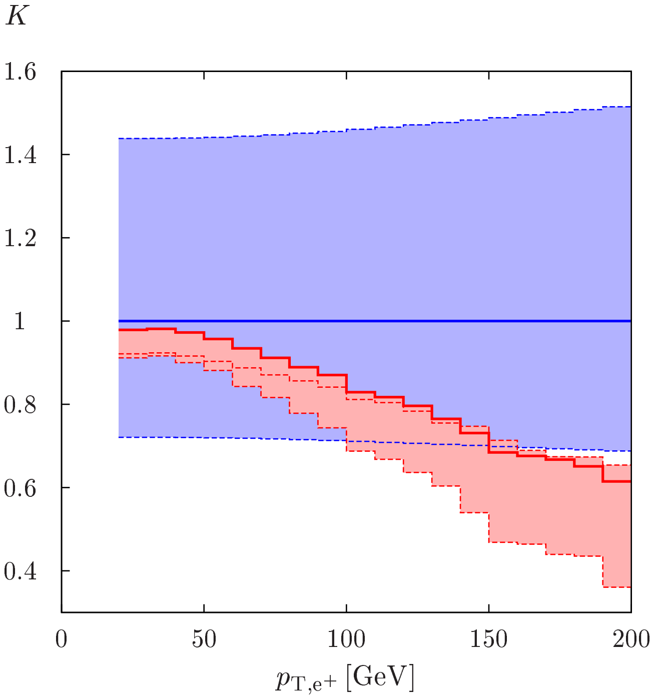
Soft b-jet (upper)

- saturates cut at 30 GeV
- +30% to −10% corrections
- strong shape distortions (relevant for acceptance)

Hard b-jet (lower)

- peaked around 80 GeV
- +40% to +20% corrections
- moderate shape distortions





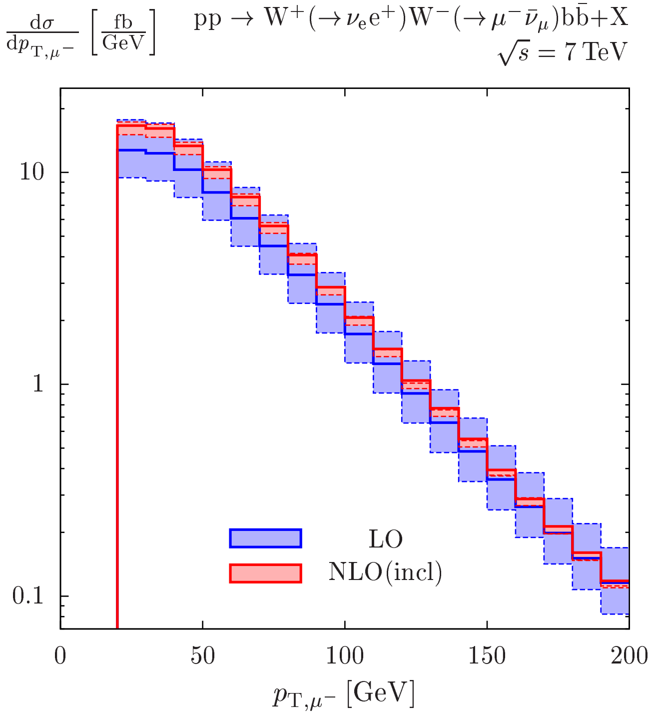
Lepton p_T at the Tevatron

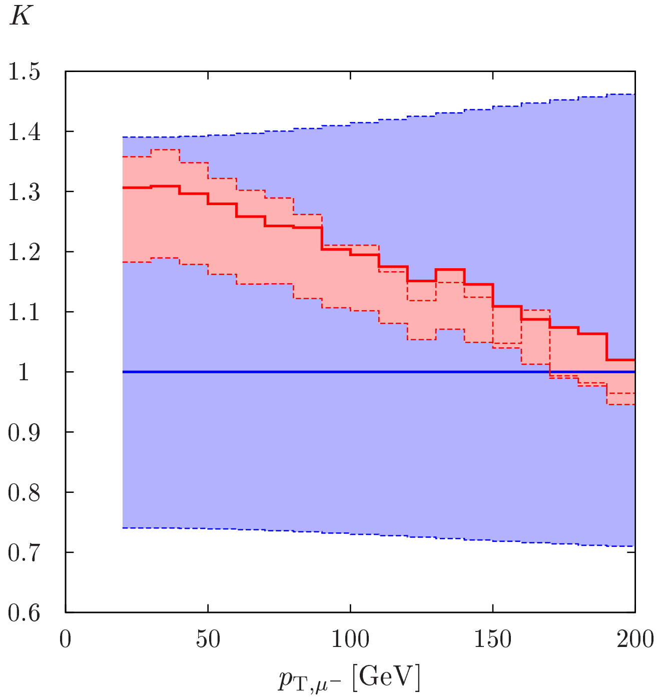
e^+ (μ^-) from W^+ (W^-) decay

- have typically $p_T \lesssim 100\,\mathrm{GeV}$ and tend to saturate the cut at 20 GeV
- corrections range from 0% to −40%

Shape distortion

- mild in the vicinity of the cut but fairly strong at high p_T
- relevant for boosted tops and NP searches
- when $p_T \gtrsim 100\,\mathrm{GeV}$ fixed $\mu = m_t$ should be replaced by dynamical QCD scale





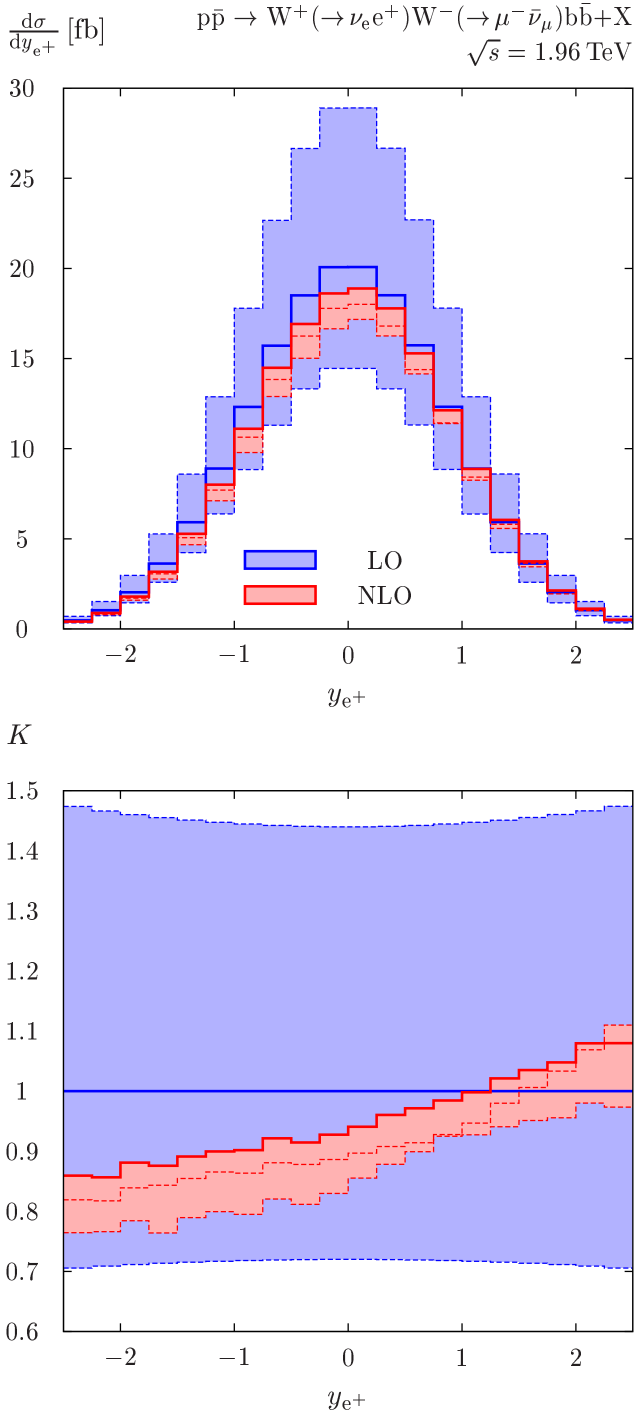
Lepton p_T at the LHC

e^+ (μ^-) from W^+ (W^-) decay

- have typically $p_T \lesssim 100\,\mathrm{GeV}$ and tend to saturate the cut at 20 GeV
- corrections range from +30% to 0%

Shape distortion

- mild in the vicinity of the cut but fairly strong at high p_T
- relevant for boosted tops and NP searches
- when $p_T \gtrsim 100\,\mathrm{GeV}$ fixed $\mu = m_t$ should be replaced by dynamical QCD scale



Charged-lepton rapidity at the Tevatron

LO y_{e^+} distribution

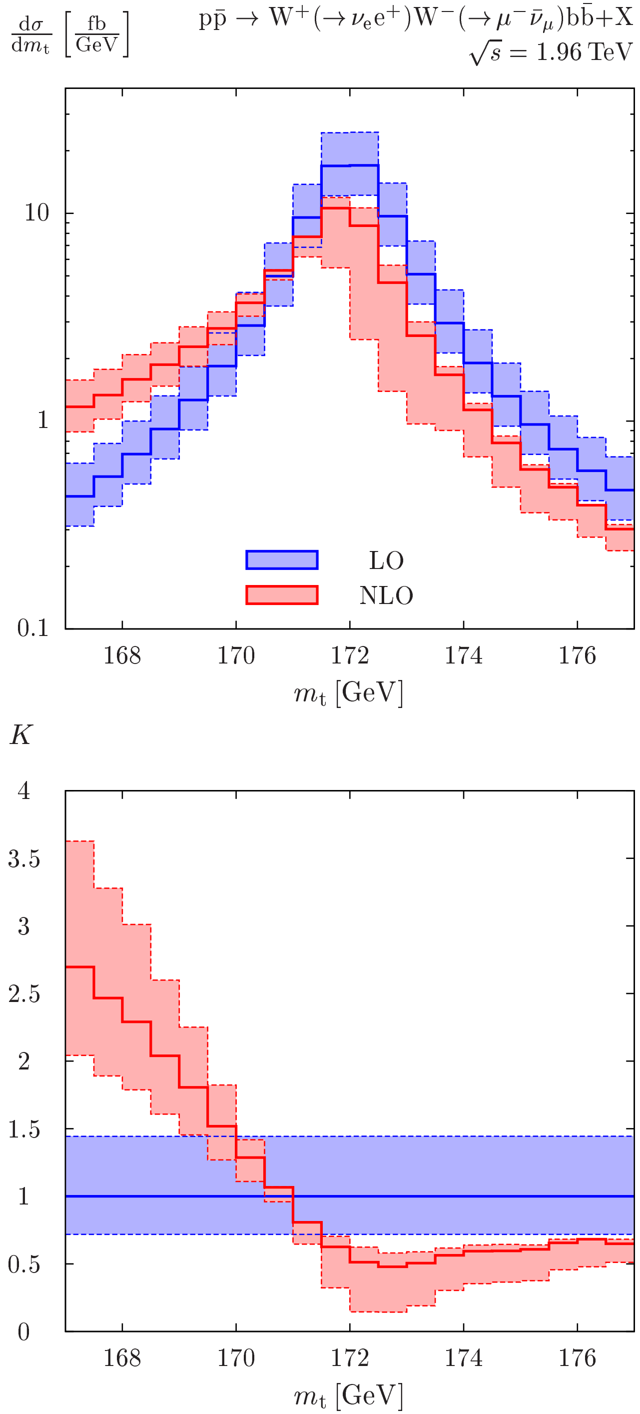
- e^+ populates central region
- almost exactly symmetric due to $t \leftrightarrow \bar{t}$ invariance of $q\bar{q}/gg \to t\bar{t}$

NLO charge and FB asymmetry

- IS–FS gluon exchange induces $t\bar{t}$ charge asymmetry
- reflected in y_{e^+} shape distortion (−15% to +10% corrections) and FB asymmetry

$$A_{\rm FB} = \frac{\sigma(y_{e^+} > 0) - \sigma(y_{e^+} < 0)}{\sigma(y_{e^+} > 0) + \sigma(y_{e^+} < 0)} = 0.035(2)$$

consistent with NWA [Bernreuther/Si ’10]

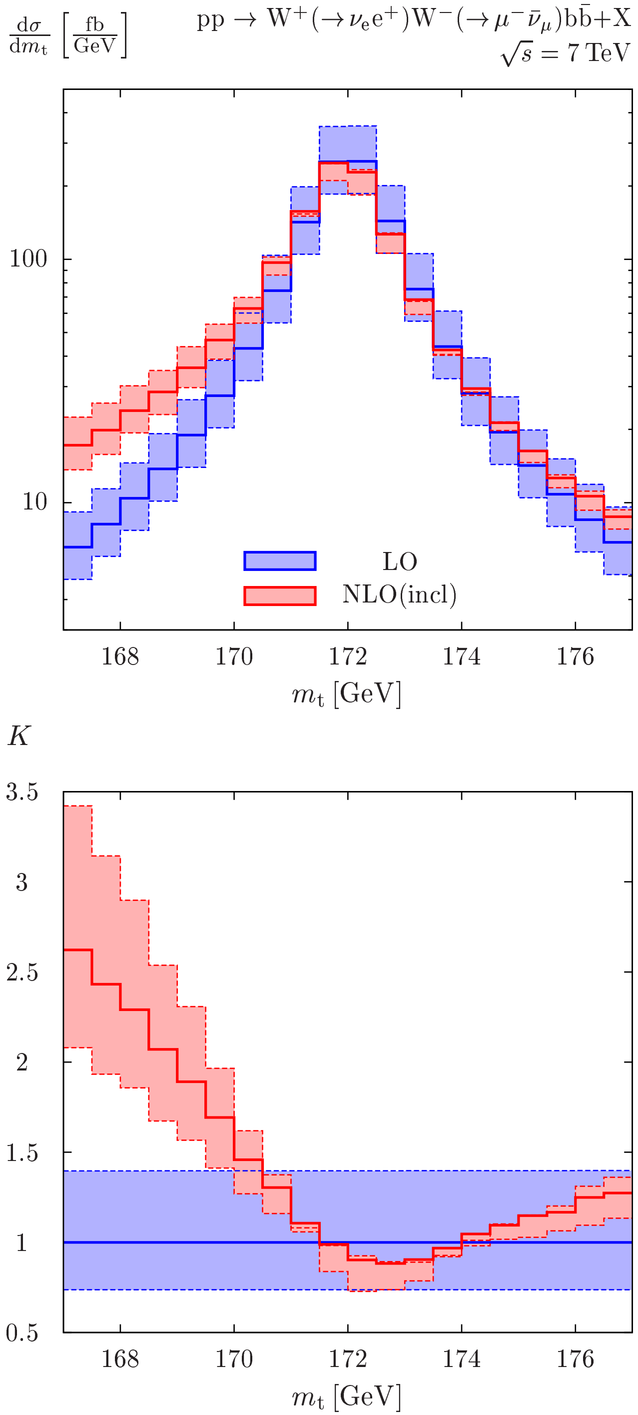


Top-quark invariant mass at the Tevatron

Although not observable $M_t = M_{be^+\nu_e}$ reflects off-shell nature of $2 \to 4$ calculation

- Breit–Wigner shape in the resonance region
- $\delta\Gamma_{NLO}/\Gamma_{LO} \simeq -9\%$ crucial for consistent normalisation of $\sigma_{incl.} \sim 1/\Gamma_t^2$
- Pole of top-quark progagator not shifted in on-shell scheme, but QCD radiation leads to invariant-mass shift $\lesssim 1\,\text{GeV}$
- m_t-shift depends on jet algorithm

NLO and Γ_t effects will improve description of observables used for m_t determination



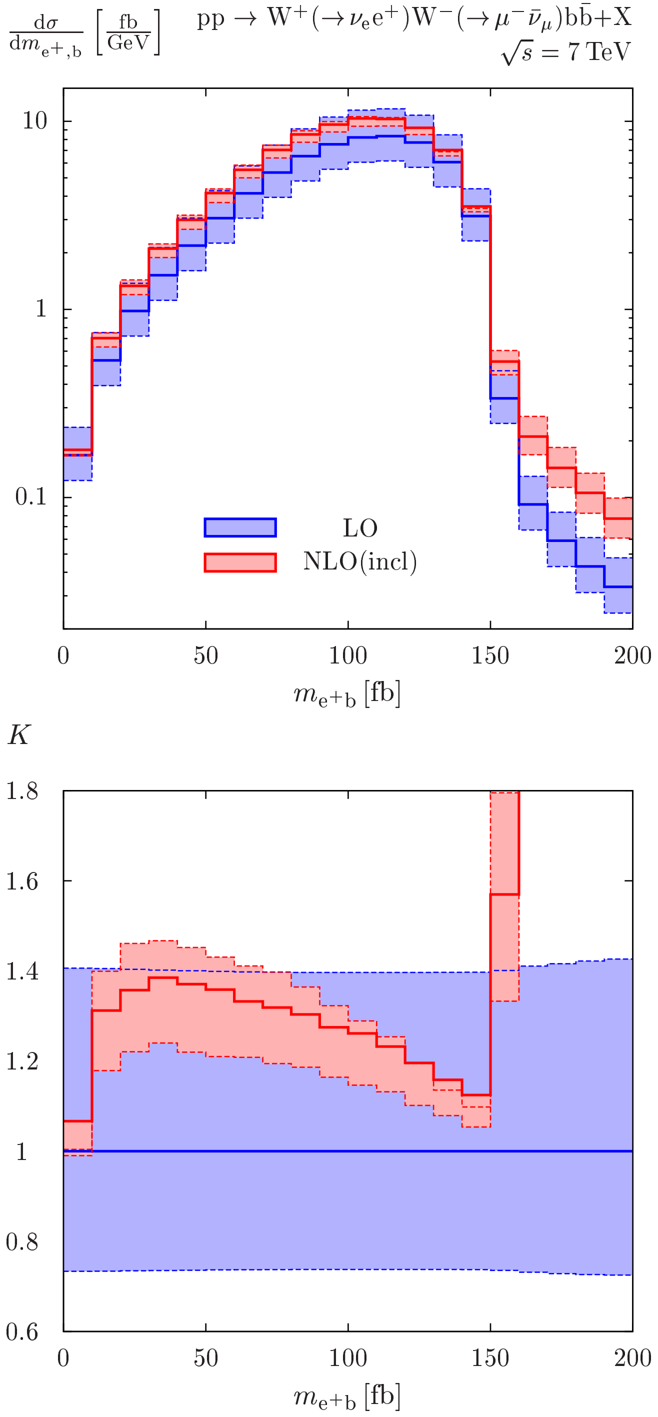
Top-quark invariant mass at the LHC

Although not observable $M_t = M_{be^+\nu_e}$ reflects off-shell nature of $2 \to 4$ calculation

- Breit–Wigner shape in the resonance region
- $\delta\Gamma_{NLO}/\Gamma_{LO} \simeq -9\%$ crucial for consistent normalisation of $\sigma_{incl.} \sim 1/\Gamma_t^2$
- Pole of top-quark progagator not shifted in on-shell scheme, but QCD radiation leads to invariant-mass shift $\lesssim 1\,\mathrm{GeV}$
- m_t-shift depends on jet algorithm

NLO and Γ_t effects will improve description of observables used for m_t determination

Invariant mass of b-jet–e^+ pair at the LHC



Observable related to m_t measurement

- visible decay products in $t \to bW^+ \to be^+\nu_e$ retain significant fraction of m_t
- good sensitivity to m_t via kinematic bound

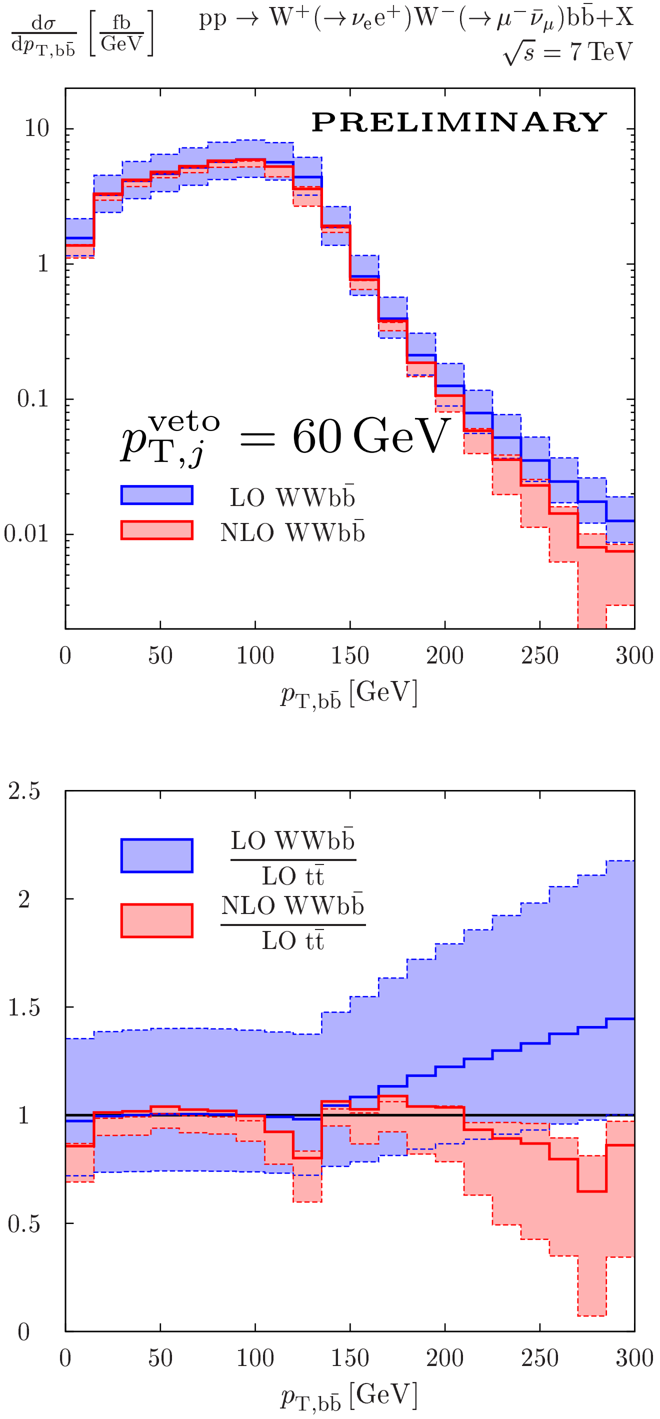
$$M_{be^+}^2 \leq m_t^2 - M_W^2 \simeq (152\,\mathrm{GeV})^2$$

in LO and narrow-width approximation

Off-shell and NLO corrections

- M_{be^+} bound violated by LO off-shell effects
- additional violation from NLO radiation
- strong NLO shape distortion below the bound: from +40% to +5% corrections

Large off-shell effects in $\mathrm{WWb\bar{b}}$ backg.



$\mathrm{pp} \to \mathrm{WH} \to \mathrm{Wb\bar{b}}$ search at the LHC

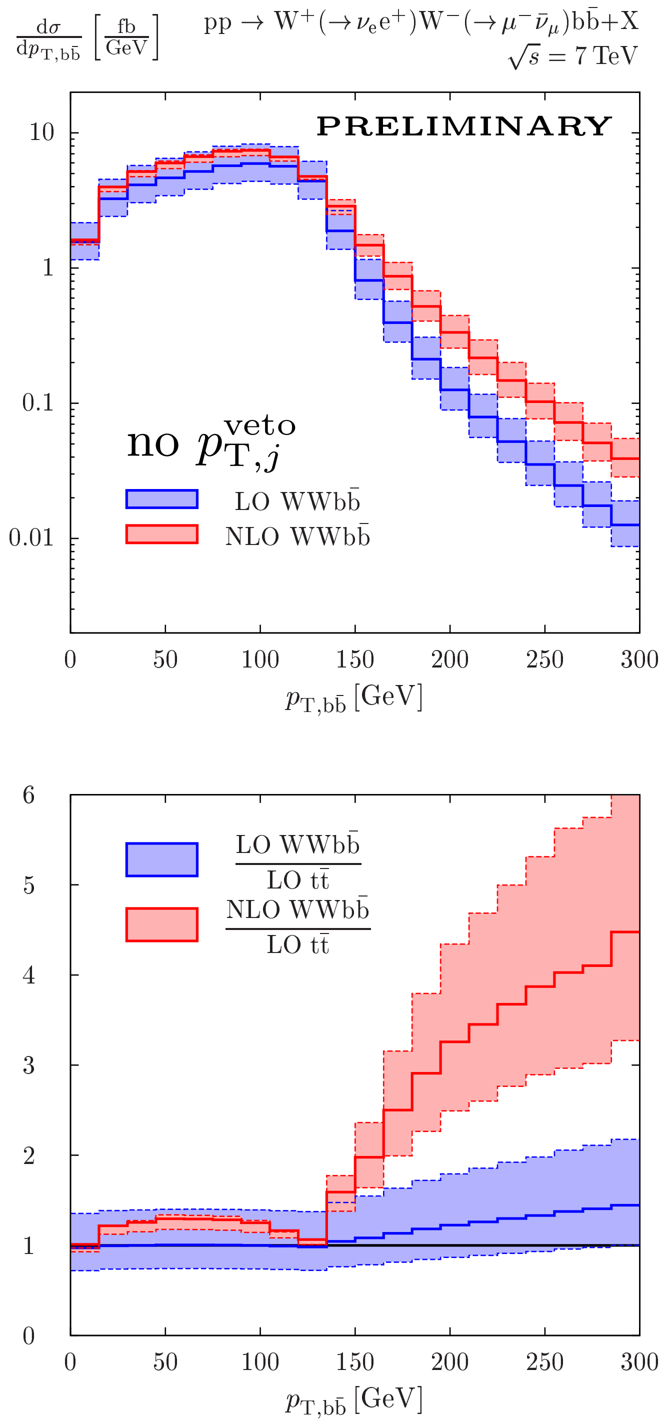
- huge QCD background suppressed with **boosted-Higgs strategy**
- $p_{\mathrm{T},b\bar{b}} > 200\,\mathrm{GeV}$ and $p_{\mathrm{T},j}^{\mathrm{veto}} = 30\,\mathrm{GeV}$ yield $S/B \sim 1$ and $S/\sqrt{B} \sim 3\sigma$ with $30\mathrm{fb}^{-1}$

Butterworth et. al. (2008)

Corrections to dominant $\mathrm{WWb\bar{b}}$ background

- **0.4% off-shell effects increase to $\gtrsim$ 30%**
- **strong $\mathrm{WWb\bar{b}j}$ NLO emission very sensitive to jet veto**
- **NLO unstable for $p_{\mathrm{T},j}^{\mathrm{veto}} < 60\,\mathrm{GeV}$**

Full $2 \to 4$ NLO crucial to control $\mathrm{WWb\bar{b}}$!



Large off-shell effects in $WWb\bar{b}$ backg.

$pp \to WH \to Wb\bar{b}$ **search at the LHC**

- huge QCD background suppressed with **boosted-Higgs strategy**
- $p_{T,b\bar{b}} > 200\,\text{GeV}$ and $p_{T,j}^{\text{veto}} = 30\,\text{GeV}$ yield $S/B \sim 1$ and $S/\sqrt{B} \sim 3\sigma$ with 30fb^{-1}

Butterworth et. al. (2008)

Corrections to dominant $WWb\bar{b}$ **background**

- **0.4% off-shell effects increase to** $\gtrsim$ **30%**
- **strong** $WWb\bar{b}j$ **NLO emission very sensitive to jet veto**
- **NLO unstable for** $p_{T,j}^{\text{veto}} < 60\,\text{GeV}$

Full $2 \to 4$ **NLO crucial to control** $WWb\bar{b}$ **!**

Conclusions

NLO QCD calculation for $\mathrm{W^+W^-b\bar{b}}$ production

- precise description of $\mathrm{t\bar{t}}$ production and decay
- including off-shell effects, non-resonant backgrounds and interferences

Inclusive cross section at the Tevatron (LHC)

- moderate corrections K=0.94 (1.27) and stable NLO predictions ($\delta\sigma/\sigma \simeq 9\%$)
- quantitative assessment of finite-width effects $\lesssim \Gamma_\mathrm{t}/m_\mathrm{t} = 0.8\%$

NLO corrections to differential distributions

- rich and non-trivial kinematic dependence
- potentially large impact on acceptances and shape-dependent precision measurements (like m_t)
- large off-shell effects in $\mathrm{t\bar{t}}$ background to $\mathrm{pp} \to \mathrm{WH}$ boosted-Higgs search

BACKUP SLIDES

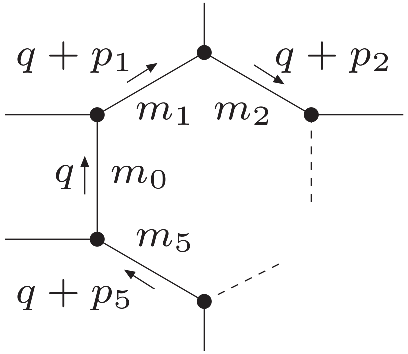
Reduction of tensor integrals – *collection of* $e^+e^- \to 4f$ *methods* [Denner/Dittmaier '05]

(A) Space-time 4-dim ($N \geq 5$ prop.)

simultaneous prop. & rank reduction

Melrose '65; Denner/Dittmaier '02&'05; Binoth et. al. '05

$$\begin{vmatrix} q^\mu & 2qp_1 & \dots & 2qp_5 \\ p_1^\mu & 2p_1p_1 & \dots & 2p_1p_5 \\ \vdots & \vdots & \ddots & \vdots \\ p_4^\mu & 2p_4p_1 & \dots & 2p_4p_5 \\ 0 & f_1 & \dots & f_5 \end{vmatrix} = \mathcal{O}(D-4)$$



(B) Lorentz invariance ($N \leq 4$ prop.)

reduction of rank (P)

Passarino/Veltman '79; Denner '93

$$2(D+P-N-1)\, T^{(P)}_{00i_3\dots i_P} = \sum_{k=1}^{N-1} f_k\, T^{(P-1)}_{ki_3\dots i_P} + 2m_0^2\, T^{(P-2)}_{i_3\dots i_P} + \text{lower-point}$$

$$\sum_{n=1}^{N-1} Z_{mn}\, T^{(P)}_{ni_2\dots i_P} = -2\sum_{r=2}^{P} \delta_{mi_r}\, T^{(P)}_{00i_2\dots\hat{i}_r\dots i_P} - f_m\, T^{(P-1)}_{i_2\dots i_P} + \text{lower-point}$$

inversion of Gram matrix $Z_{mn} = 2p_mp_n$ **unstable** when $\det(Z) \to 0$

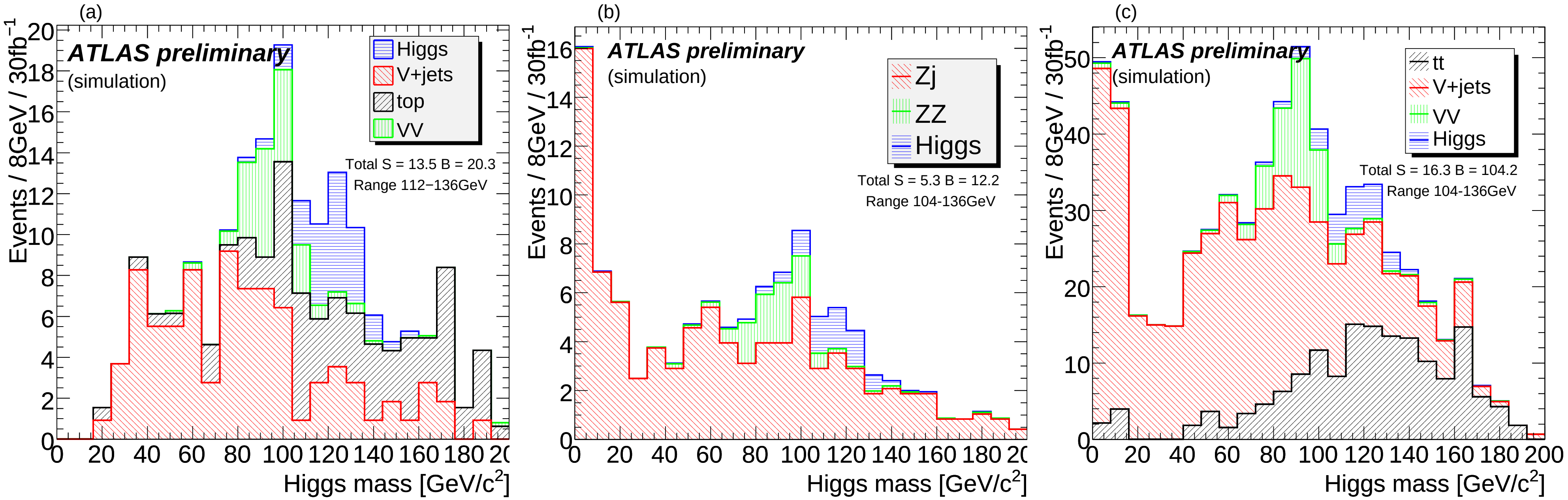
(C) **General and robust solution of instability problems**

iterative $\det(Z)$-expansion (and various alternative methods)

$$\tilde{X}_{0j}\, T^{(P)}_{i_1\ldots i_P} = \det(Z)\; T^{(P+1)}_{j i_1\ldots i_P} + 2\sum_{n=1}^{N-1}\tilde{Z}_{jn}\sum_{r=1}^{P}\delta_{n i_r}\, T^{(P+1)}_{00 i_1\ldots \hat{i}_r\ldots i_P} + \text{lower-point}$$

$$\begin{aligned}
2\tilde{Z}_{kl}\, T^{(P+1)}_{00 i_2\ldots i_P} = \Big\{ & -\det(Z)\; T^{(P+1)}_{kl i_2\ldots i_P} + 2m_0\tilde{Z}_{kl}\, T^{(P-1)}_{i_2\ldots i_P} + \sum_{n,m=1}^{N-1}\Big[f_n f_m T^{(P-1)}_{i_2\ldots i_P} + 2\sum_{r=2}^{P}\left(f_n\delta_{m i_r} + f_m\delta_{n i_r}\right) \\
& \times T^{(P)}_{00 i_2\ldots \hat{i}_r\ldots i_P} + 4\sum_{\substack{r,s=2\\ r\neq s}}^{P}\delta_{n i_r}\delta_{m i_s}\, T^{(P+1)}_{0000 i_2\ldots \hat{i}_r\ldots \hat{i}_s\ldots i_P}\Big]\tilde{\tilde{Z}}_{(kn)(lm)} + \text{lower-point}\Big\}\Big(D+1+P-N+\sum_{r=2}^{P}\bar{\delta}_{i_r 0}\Big)^{-1}
\end{aligned}$$

Boosted-Higgs search in $\mathrm{pp} \to V\mathrm{H}(\mathrm{H} \to \mathrm{b}\bar{\mathrm{b}})$



(a) $\mathrm{b}\bar{\mathrm{b}}l\nu$ channel

(b) $\mathrm{b}\bar{\mathrm{b}}l\bar{l}$ channel

(c) $\mathrm{b}\bar{\mathrm{b}}\nu\bar{\nu}$ channel

ATLAS note ATL-PHYS-PUB-2009-088 (cut-based analysis)

- $M_\mathrm{H} = 120\,\mathrm{GeV}$, $\sqrt{s} = 14\,\mathrm{TeV}$, $L = 30\mathrm{fb}^{-1}$
- $p^\mathrm{T}_{\mathrm{b}\bar{\mathrm{b}}}$, $p^\mathrm{T}_V > 200\,\mathrm{GeV} \Rightarrow 5\%$ signal
- $p^\mathrm{T}_{\mathrm{jet\ veto}} = 20\,\mathrm{GeV}$ in (a)
- $\mathrm{t}\bar{\mathrm{t}}$ simulated with HERWIG
- $(S/\sqrt{B})_a = 3.0$, $(S/B)_a \simeq 2/3$
- $(S/\sqrt{B})_{a+b+c} = 3.7$

Double suppression of finite W-width effects

Finite W-width corrections to (leading order) Top-quark decay

$$\Gamma_{t\to l\nu b} = \Gamma_{t\to Wb}\left(\frac{\Gamma_{W\to l\nu}}{\Gamma_W}\right)\left[1+\left(\frac{\Gamma_W}{M_W}\right)K^{\mathrm{FW}}_{t\to ff'b}\right]$$

$$\Gamma_t = \Gamma_{t\to Wb}\left[1+\left(\frac{\Gamma_W}{M_W}\right)K^{\mathrm{FW}}_{t\to ff'b}\right]$$

cancel in the branching ratios

$$BR(t\to l\nu b) = \frac{\Gamma_{t\to l\nu b}}{\Gamma_t} = \frac{\Gamma_{W\to l\nu}}{\Gamma_W}$$

Thus finite W-width corrections to $t\bar{t}$ production are $\mathcal{O}(\frac{\Gamma_W}{M_W}\frac{\Gamma_t}{M_t})$ suppressed

$$\sigma_{gg\to l\nu l\nu bb} = \sigma_{gg\to tt}\left(\frac{\Gamma_{t\to l\nu b}}{\Gamma_t}\right)^2\left\{1+\left(\frac{\Gamma_t}{M_t}\right)K^{\mathrm{FW}}_{gg\to WWbb}\left[1+\left(\frac{\Gamma_W}{M_W}\right)K^{\mathrm{FW}}_{gg\to l\nu l\nu bb}\right]\right\}$$