MASS EFFECT ON THE PHOTON STRUCTURE FUNCTIONS

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Plan of this talk

- Introduction
  - Kinematics
  - Previous works
- Our method of calculation for the photon structure functions in massive parton model(PM)
  - ⇨ LO calculation as an example
  - ⇨ NLO computation
- NLO results of the structure functions
- First moment sum rule for polarized photon structure func.
- Summary

Kinematics of two-photon process

- Two-photon processes are dominant in e+e- collision experiments at high energy! 1/s suppression is absent
- Photon structure functions can be studied

$$F_2^\gamma(x,Q^2,m^2) \quad F_L^\gamma(x,Q^2,m^2) \quad g_1^\gamma(x,Q^2,m^2) \quad W_3^\gamma(x,Q^2,m^2)$$

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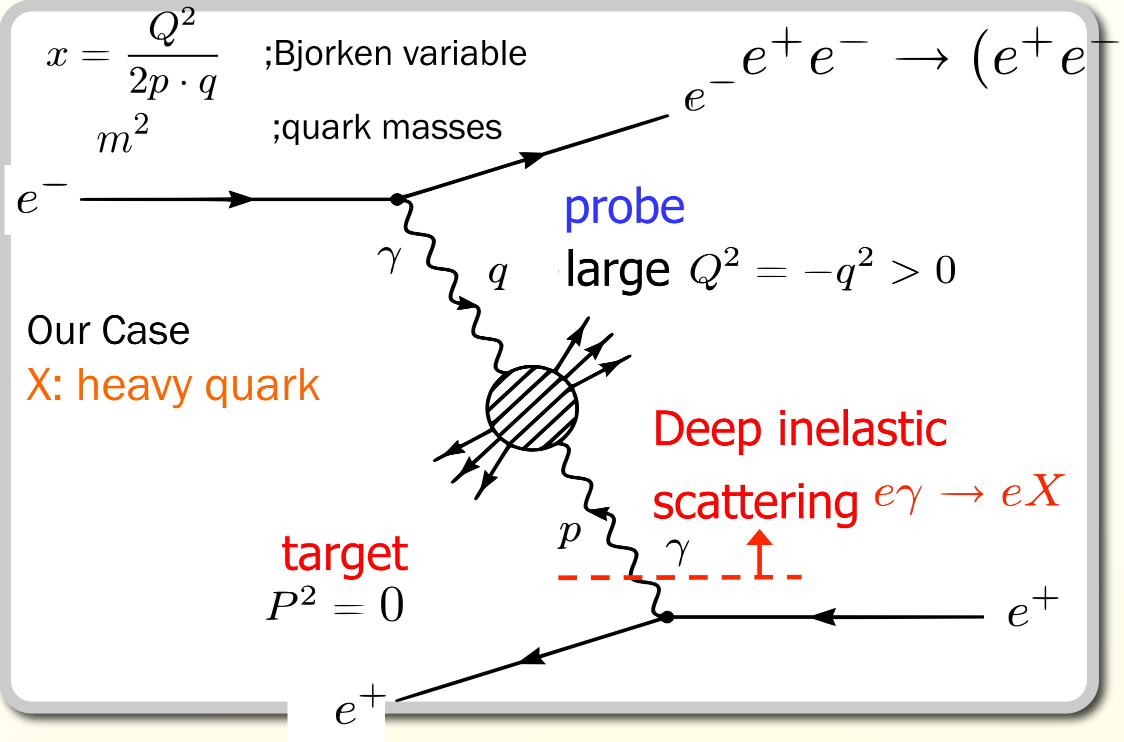
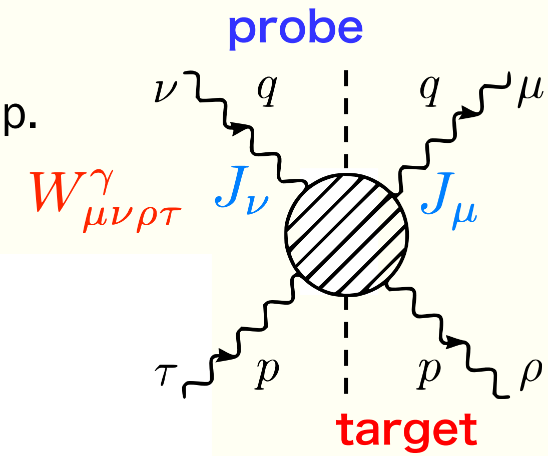
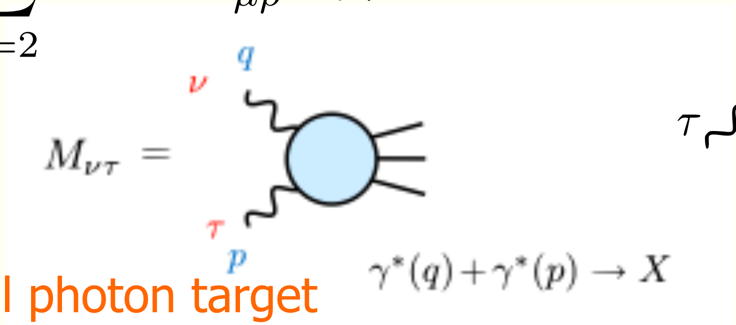


Photon Structure Functions



- Absorptive part of photon-photon forward scattering amp.

$$(4\pi\alpha)W_{\mu\nu\rho\tau} = \frac{1}{2\pi}\sum_{k=2}^{\infty} dPS^k M^*_{\mu\rho} M_{\nu\tau}$$



- 4 structure funcs. for the real photon target

$$W^{\mu\nu\rho\tau} = (T_{TT})^{\mu\nu\rho\tau}W_{TT} + (T^a_{TT})^{\mu\nu\rho\tau}W^a_{TT} + (T^\tau_{TT})^{\mu\nu\rho\tau}W^\tau_{TT} + (T_{LT})^{\mu\nu\rho\tau}W_{LT}$$

Budnev-Ginzburg-Meledin-Serbo('75)

In terms of s-channel helicity amp.

- $W_{TT} = W(1,1|1,1) + W(1,-1|1,-1) \quad W_{LT} = W(0,1|0,1)$

  $W^a_{TT} = W(1,1|1,1) - W(1,-1|1,-1) \quad W^\tau_{TT} = W(1,1|-1,-1)$

- Note; $F_2^\gamma = x(W_{TT}+W_{LT}),\ F_L\gamma = xW_{LT},\ g_1^\gamma = W^a_{TT},\ W_3^\gamma = \frac{1}{2}W^\tau_{TT}$

- The projection operators; $(P_{TT})^{\mu\nu\rho\tau}\,W_{\mu\nu\rho\tau} = W_{TT}$

e.g., $(P_{TT})^{\mu\nu\rho\tau} = \frac{3D-8}{2D(D-2)(D-3)}(T_{TT})^{\mu\nu\rho\tau} + \frac{D-4}{D(D-2)(D-3)}(T^\tau_{TT})^{\mu\nu\rho\tau}$

Heavy quark mass effects on the NLO photon structure functions

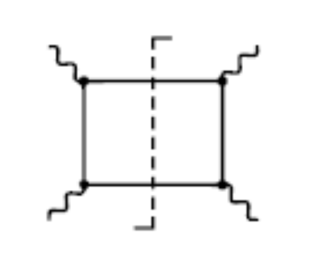
- QCD NLO corrections to $F_2^\gamma(x,Q^2)$ $F_L^\gamma(x,Q^2)$

  Laenen-Riemersma-Smith-van Neerven('94)

- PDF's in the virtual photon at NLO

  Kitadono-Sasaki-Ueda-Uematsu('09)

- In PM model



LO

Sasaki-Uematsu-Soffer('02)

Budnev-Ginxburg-Meledin('75)

Gluck-Reya-Schienbein('01)

- Here we complete the analysis of NLO heavy quark effects on the photon structure functions

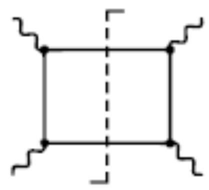
$g_1^\gamma(x,Q^2,m^2)$ $W_3^\gamma(x,Q^2,m^2)$ $F_L^\gamma(x,Q^2,m^2)$ $F_2^\gamma(x,Q^2,m^2)$

new

Re-analysis

How to calculate structure functions

- Using Cutkosky rule $(2\pi)\delta^{(+)}(k^2-m^2) = \frac{i}{k^2-m^2+i\epsilon} - \frac{i}{k^2-m^2-i\epsilon}$



  - δ-function in phase-space is replaced by imaginary part of propagator

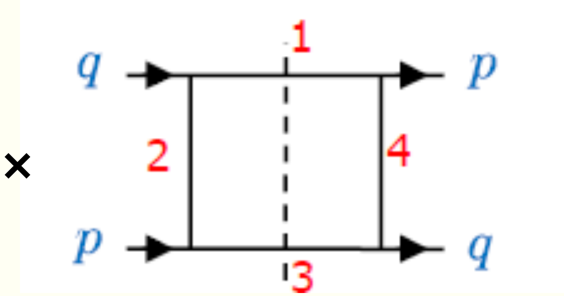
- Take a trace of diagrams and apply the projection operators
  The result is a linear combination of many scalar integrals

- The scalar integrals are expressed by
  fewer master integrals (Laporta reduction algorithm) Laporta('00)
  - Integration by parts relations used Chetrkin-Tkachov('81)

- Discontinuities of the master integrals are evaluated
  - Analytic forms as much as possible

Example(LO Calculation)

- Applying the projection operator $(P_{TT})^{\mu\nu\rho\tau}$ ×



$$(P_{TT})^{\mu\nu\rho\tau} A_{\mu\nu\rho\tau} = c_1 B(1, -2, 1, 0) + c_2 B(1, -1, 1, 0) + c_3 B(0, 1, 1, 1) + \cdots$$

Non-positive → irrelevant

$$B(\nu_1, \nu_2, \nu_3, \nu_4) = \int \frac{d^D k}{(2\pi)^D} \frac{1}{[k^2 - m^2]^{\nu_1}[(k-q)^2 - m^2]^{\nu_2}[(k-p-q)^2 - m^2]^{\nu_3}[(k-p)^2 - m^2]^{\nu_4}}$$

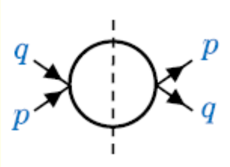
- Taking the disconitinuities

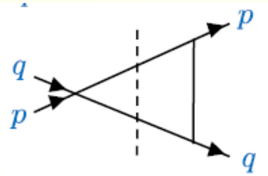
$$\mathrm{Disc}B(0, \times, \times, \times) = \mathrm{Disc}B(\times, \times, 0, \times) = 0$$

- Using reduction method,

We can express a scalar integral as a linear combination of master integrals

- Taking discontinuities of the master integrals





NLO calculation

- Taking trace, Applying the projection operators, Reduction method, Evaluation of master integrals…etc

  - 2-loop reduction

    Note; Choice of a set of the master integrals is not unique

- Mathematica and FORM are used

  - Mathematica ➡ Reduction, Evaluation of master integrals

    FIRE;Smirnov('08)

  - FORM ➡ Trace, Projection…etc

    http://www.nikhef.nl/~form/

Calculation of NLO corrections

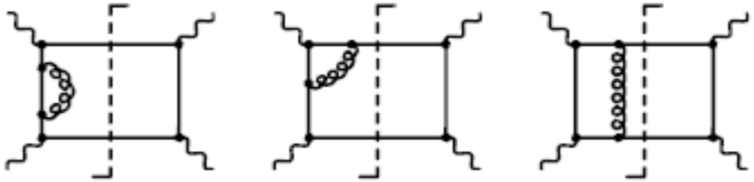
- Infrared and Ultraviolet divergences appear

  Dimensional regularization is used $(D = 4 - 2\epsilon)$

  - Collinear divergence does not appear due to quark mass

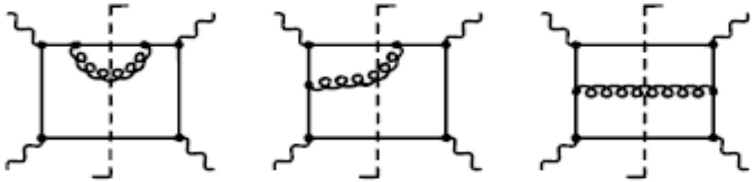
- Feynman diagrams

  - Virtual corrections



… → 2-body phase space

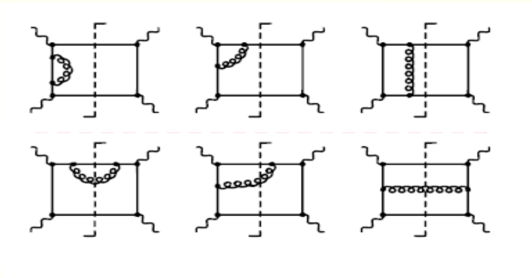
  - Real emissions



… → 3-body phase space

IR divergences are canceled between the two contributions

- UV divergences are renormalized(by wave function and mass renormalization)



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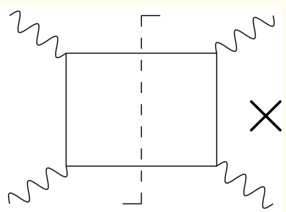
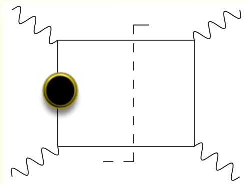


$\times Z_2 +$

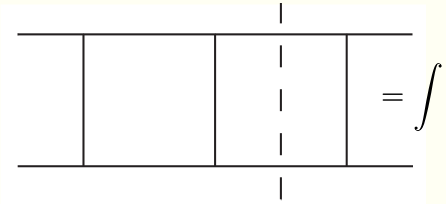


Mass Counter term

Calculation of virtual correction diagrams

- There are 26 master integrals in virtual correction diagrams

- Most of the master integrals can be integrated analytically



$$= \int dPS^{(2)} \frac{1}{(k-q)^2 - m^2} \int \frac{d^D l}{(2\pi)^D} \frac{1}{[l^2 - m^2 + i\epsilon][(l-q)^2 - m^2 + i\epsilon][(l-p-q)^2 - m^2 + i\epsilon](l-k)^2}$$

IR div

$$= iS^{2\epsilon}\left(\frac{1}{m^2(s-4m^2)}\right)^{\epsilon} \frac{1}{128\pi^3} \frac{-xs}{2Q^2} B(1-\epsilon, 1-\epsilon) \ln\left(\frac{1-\beta}{1+\beta}\right)\left\{\frac{1}{\epsilon}\ln\left(\frac{1-\beta}{1+\beta}\right) - 2\ln\left(\frac{1-\beta}{1+\beta}\right)\left[1 - \ln\left(\frac{m^2}{p\cdot q}\right)\right]\right.$$

$$-\frac{1}{2}\ln^2\left(\frac{1-\beta}{1+\beta}\right) - \mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right) - \ln^2\left(\frac{\alpha-1}{\alpha+1}\right) - \mathrm{Li}_2\left(\frac{2\beta}{(1+\beta)^2}\right)$$

$$\left.+2\mathrm{Li}_2\left(\frac{2(\alpha+\beta)}{(1+\beta)(\alpha+1)}\right) + 2\mathrm{Li}_2\left(\frac{2(\alpha-\beta)}{(1+\beta)(\alpha-1)}\right) + (\log^2(1+\beta) - \log^2(1-\beta))\right\}$$

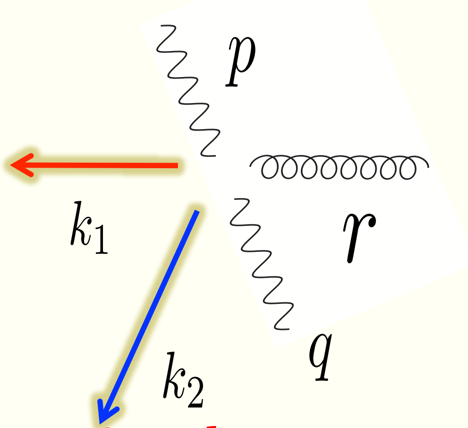
$$\alpha = \sqrt{1 + \frac{4m^2}{Q^2}}$$

$$\beta = \sqrt{1 - \frac{4m^2}{s}}$$

- For some of the mater integrals, it is difficult to perform the phase-space integral analytically, We perform numerical calculation for these integrals

Calculation of real emission diagrams

CM frame quark and gluon



$s_4 = 2r \cdot k_1$

gluon

$$\int d\,PS_3^{\gamma\gamma} = \frac{1}{(4\pi)^D}\frac{1}{\Gamma(D-3)}(s-q^2)^{3-D}\int_{-\frac{s-q^2}{2}(1+\beta)}^{-\frac{s-q^2}{2}(1-\beta)} dt_1 \int_{-s+q^2-t_1}^{\frac{q^2}{s-q^2}+q^2+\frac{m^2(s-q^2)}{t_1}} du_1$$

$$\times \int ds_4 \delta(s-q^2+t_1+u_1-s_4)\frac{s_4^{D-3}}{(s_4+m^2)^{\frac{D-2}{2}}}$$

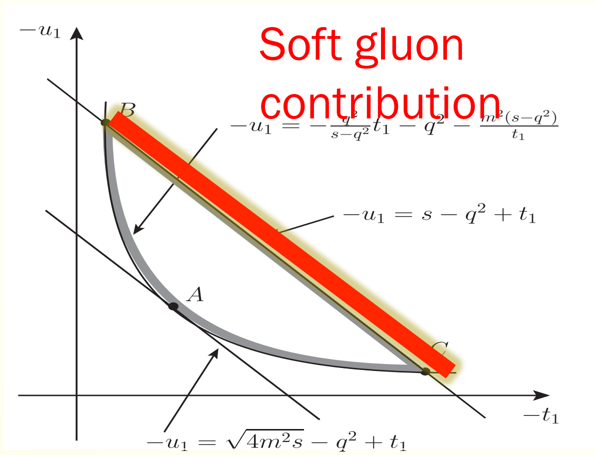
$$\int_0^\pi d\theta_1 \sin^{D-3}\theta_1 \int_0^\pi d\theta_2 \sin^{D-4}\theta_2 \left\{-m^2(s-q^2)^2 + st_1u_1 - t_1q^2(s-q^2+t_1+u_1)\right\}^{\frac{D-4}{2}}$$

$$= \left\{\int dPS_3^{\gamma\gamma} - \int^\Delta dPS_3^{\gamma\gamma}\right\} + \int^\Delta dPS_3^{\gamma\gamma}\Big|_{soft}$$

$$= \int_\Delta dPS_3^{\gamma\gamma} + \int^\Delta dPS_3^{\gamma\gamma}\Big|_{soft}$$

Hard contribution

Soft contribution



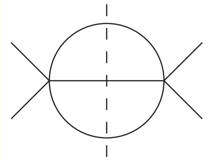
- To extract the soft gluon contribution, we set the cut-off scale
  - IR singularities are treated as follows

$$\int ds_4 \delta(s-q^2+t_1+u_1-s_4)\frac{s_4^{D-3}}{(s_4+m^2)^{\frac{D-2}{2}}} \to \delta(s-q^2+t_1+u_1)\int_0^\Delta \frac{s_4^{D-3}}{(s_4+m^2)^{\frac{D-2}{2}}}$$

Master Integrals of real emission diagrams

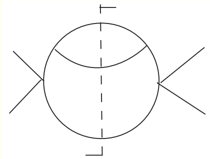
- If IR div. does not appear, only hard contribution remains

$D \to 4$



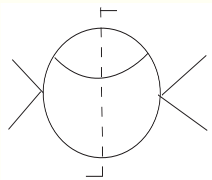
$$= \frac{4\pi s}{8(4\pi)^4}\left\{(\beta^4+2\beta^2-3)\frac{1}{2}\log\left(\frac{1+\beta}{1-\beta}\right)+\beta(3-\beta^2)\right\}$$

- If IR div. appears, integral is decomposed into hard and soft-part
  - Hard part
    - It is finite, since the soft part is subtracted $D \to 4$
    - Hard part has a cut-off scale



$$= \frac{1}{128\pi^3}\frac{\beta}{m^2}\left(\left(\frac{1}{\beta}+\frac{\beta}{2}\right)\log\left(\frac{1-\beta}{1+\beta}\right)+\beta\log\left(\frac{8\beta^2}{(1-\beta^2)^{3/2}}\right)-\log\left(\frac{\Delta}{m^2}\right)\right)$$

  - Soft part
    - Soft part also has a cut-off scale $D \to 4-2\epsilon$



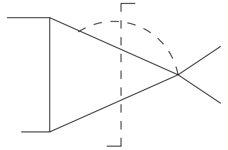
$$_{\text{Soft}} = \frac{1}{128\pi^3}\frac{\beta}{m^2}\left(\frac{-1}{2\epsilon}+\log\left(\frac{\Delta}{m^2}\right)-2\right)$$

Cut off scale is canceled between hard and soft part

- There are 35 master integrals in real gluon emission diagrams

Master Integrals of real emission diagrams

- Examples
  - Analytic form

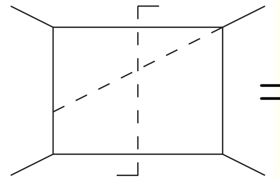


$$= \frac{1}{128\pi^3 (Q^2+s)} \Bigg[-\log(2)\log\left(1-\beta^2\right)\log\left(\frac{\beta+1}{1-\beta}\right) + \log^2(2)\log\left(\frac{\beta+1}{1-\beta}\right) - 2\mathrm{Li}_3\left(\frac{1-\beta}{2}\right) + 2\mathrm{Li}_3\left(\frac{\beta+1}{2}\right)$$
$$- \frac{1}{6}\left(\log^2\left(\frac{\beta+1}{1-\beta}\right) - 6\log(1-\beta)\log(\beta+1) + \pi^2\right)\log\left(\frac{\beta+1}{1-\beta}\right)\Bigg]$$

$$\alpha = \sqrt{1+\frac{4m^2}{Q^2}}$$

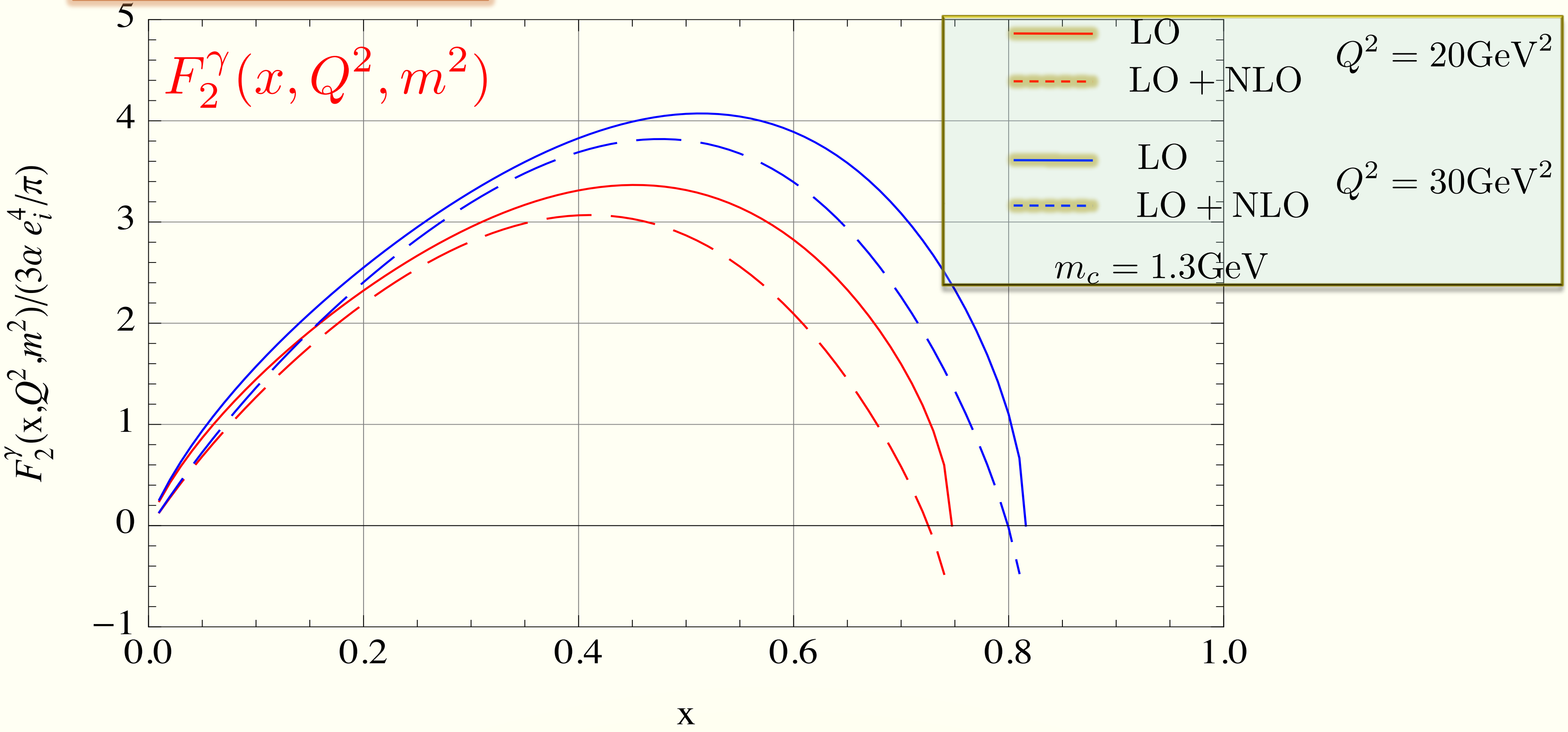
$$\beta = \sqrt{1-\frac{4m^2}{s}}$$

- It is difficult to calculate all of the integrals analytically, numerical computations are needed
  - Numerical computation

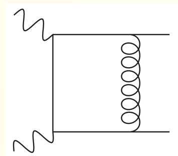


$$= \int_0^1 dy \Bigg[\frac{\beta\left(\beta^2-1\right)^2 \mathrm{Li}_2\left(\frac{2M^2\left(\beta^2-1\right)}{2(\beta^2+(2-4y)\beta-3)M^2+Q^2((2y-1)\beta+1)(\beta^2-1)}\right)}{32\pi^3\left((1-2y)^2\beta^2-1\right)\left(Q^2\left(\beta^2-1\right)-4M^2\right)^2}$$
$$- \frac{\beta\left(\beta^2-1\right)^2\left(\log\left(\frac{(2y-1)\beta^3+\left(-8y^2+8y-1\right)\beta^2-2y\beta+\beta+1}{((2y-1)\beta-1)(\beta^2-1)}\right)\log\left(\frac{\left(\beta^2-1\right)\left(Q^2\left((1-2y)^2\beta^2-1\right)-4M^2\right)}{((2y-1)\beta-1)(2(\beta^2-4y\beta+2\beta-3)M^2+Q^2(2y\beta-\beta+1)(\beta^2-1))}\right)\right.}{32\pi^3\left((1-2y)^2\beta^2-1\right)\left(Q^2\left(\beta^2-1\right)-4M^2\right)^2}\Bigg]$$

NLO Result



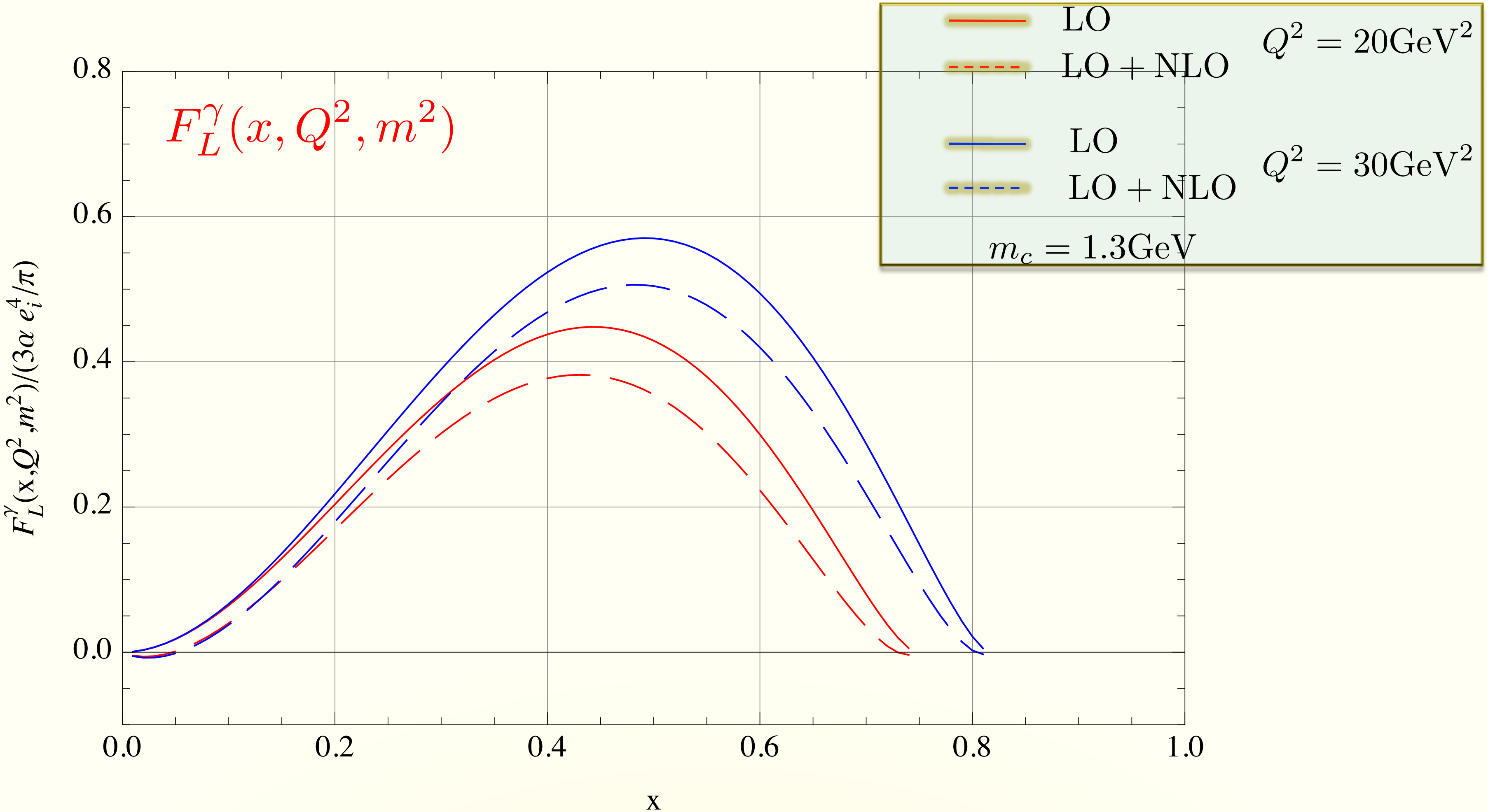
- At NLO, structure function does NOT vanish at threshold doe to Coulomb singularity



$\times \int PS^{(2)} = \text{finite}$

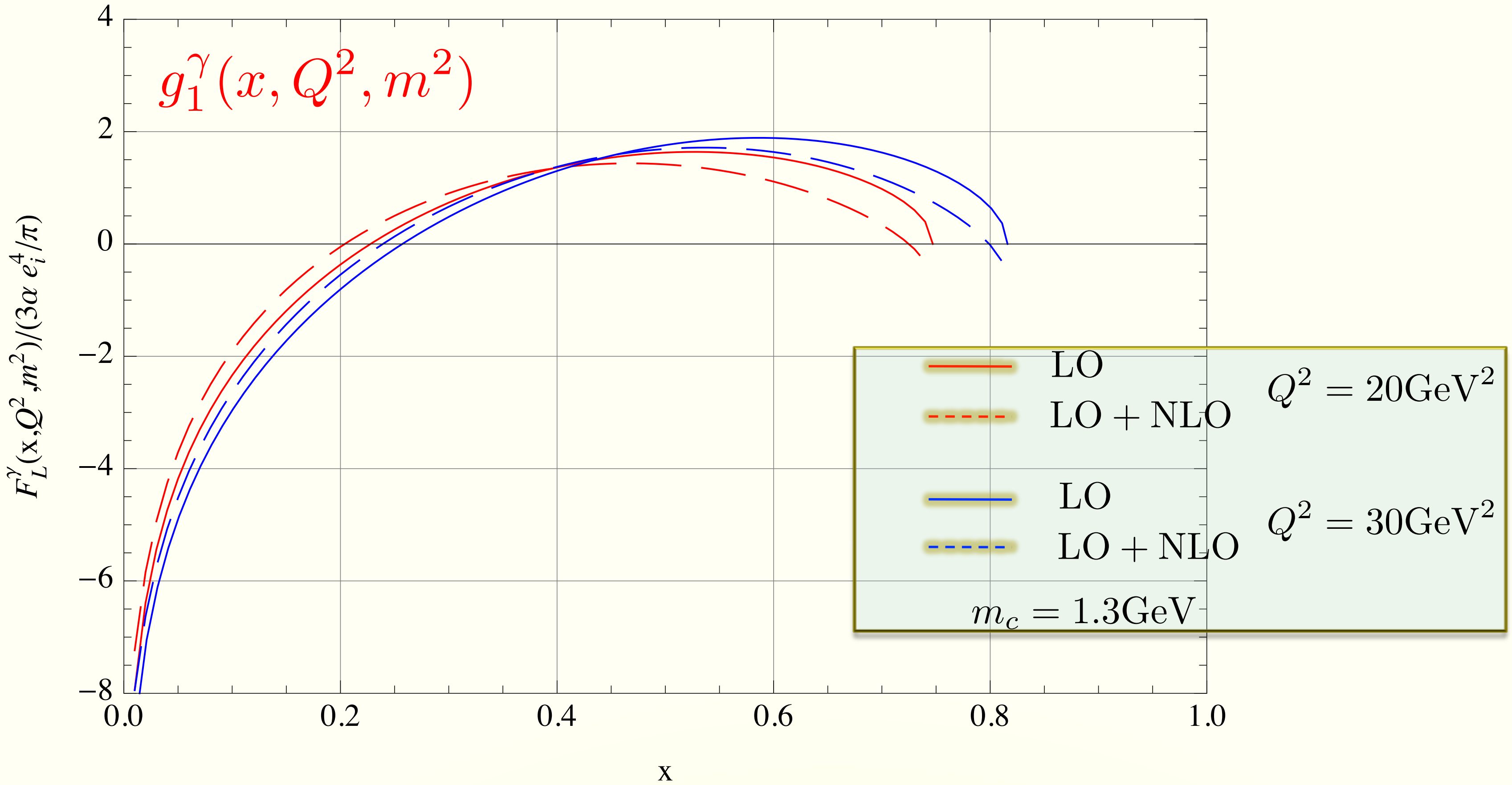
$\propto \frac{1}{\beta}$ $\propto \beta$

NLO Result



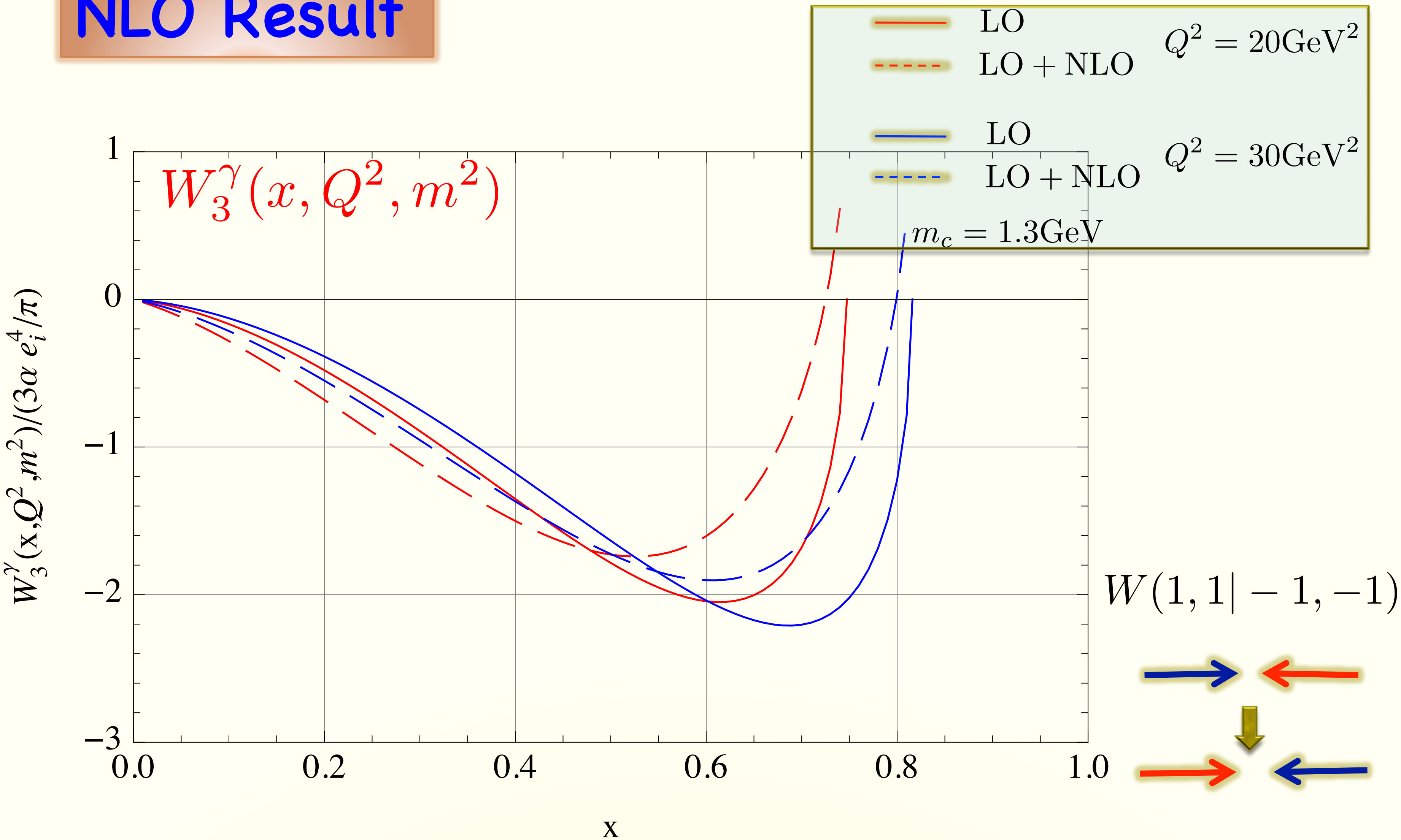
- At threshold, structure function vanishes
  - There is no s-wave contribution at the threshold $W(0,1|0,1)$

NLO Result



- It is polarized structure function
- At NLO, structure function does NOT vanish at threshold doe to Coulomb singularity

NLO Result



- This structure function is corresponding to the helicity-flip amplitude

The first moment sum rule for $g_1^\gamma(x, Q^2, m^2)$

- $$\int g_1^\gamma(x, Q^2, m^2)|_{\rm LO} = 0$$

  Efremov-Teraev('90)

  Bass ('92)

  Narison-Shore-Veniziano('93)

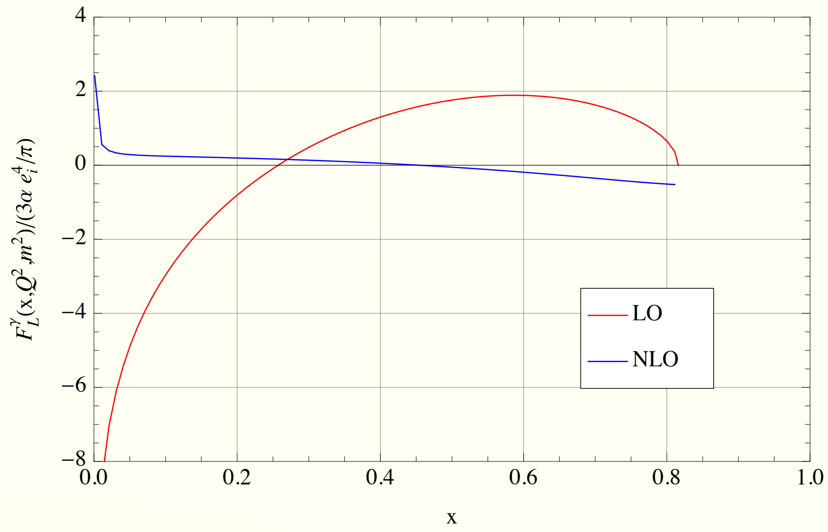
- $$\int_0^{x_{max}} dx g_1(x, Q^2, m^2) = 0$$

  Bass-Brodsky-Schimit ('98)

In the presence of QCD effects

- Our Result

$$\int g_1^\gamma(x, Q^2, m^2)|_{\rm NLO} = 0$$



We confirmed sum rule holds at NLO within the accuracy of our numerical computation. Does it hold to all orders in pQCD?

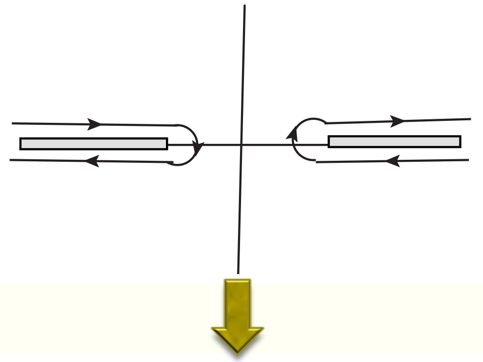
Violation of first moment sum rule (pQCD)

- Sum rule(analiticity and low-energy theorem)

$$\mathrm{Amp}(\nu=0) = \frac{2}{\pi}\int_{\nu_{th}}^{\infty}\frac{d\nu'}{\nu'}\mathrm{Im\ Amp}(\nu')$$

Drell-Hearn('66)

Gerasimov('65)

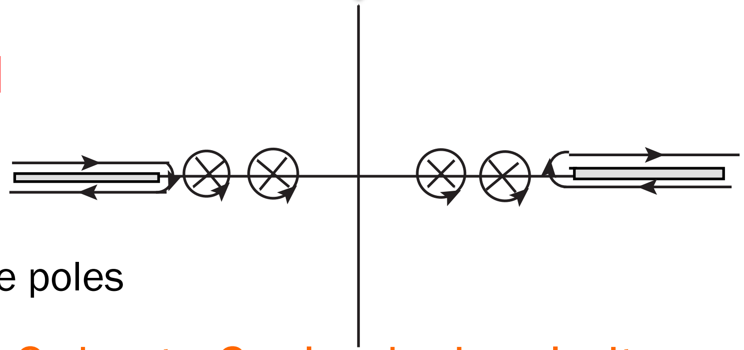


In pQCD

$$\frac{2}{\pi}\int_{\nu_{th}}^{\infty}\frac{d\nu'}{\nu'}\mathrm{Im\ Amp}^{(n)}(\nu') \to \infty \quad n \geq 2$$

Coulomb Singularity ->Resummation is needed

$$G^{\mathrm{resum}} = \cdots + \psi(1-\lambda(s))$$ qqbar Resonance poles



- New finding; Sum rule will break down at NNLO due to Coulomb singularity

$$\mathrm{Amp}(0) = \frac{2}{\pi}\int_{\nu_{th}}^{\infty}\frac{d\nu'}{\nu'}\mathrm{Im\ Amp}(\nu')$$

$$= \sum_i \oint \frac{d\nu'}{2\pi i\ \nu'}\mathrm{Amp}^{(i)}|_{\mathrm{resonance}} + \int_{\nu_{th}}^{\infty}\frac{d\nu'}{\nu'}\mathrm{Im\ Amp}(\nu')_{\mathrm{resum}}$$

Summary and Future

- The heavy quark mass effects on the photon structure functions are analyzed up to NLO

  $F_2^\gamma(x,Q^2,m^2)$ $F_L^\gamma(x,Q^2,m^2)$ $g_1^\gamma(x,Q^2,m^2)$ $W_3^\gamma(x,Q^2,m^2)$

- The first moment sum rule for polarized structure function was confirmed at NLO

  Sum rule will break down at NNLO due to coulomb singularity

- Further studies of NLO two-photon processes are ongoing ,,,
  - Heavy quark production via two-photon annilation at ILC
  - Positivity constraints at NLO