Photon Structure in Supersymmetric QCD

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in collaboration with

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Plan of the talk

1. Introduction and motivation
2. Squark contributions to virtual photon structure functions
3. Numerical analysis
4. Evolution equation and heavy mass effects
5. Summary and outlook

1. Introduction and Motivation

How SUSY affects photon structure?

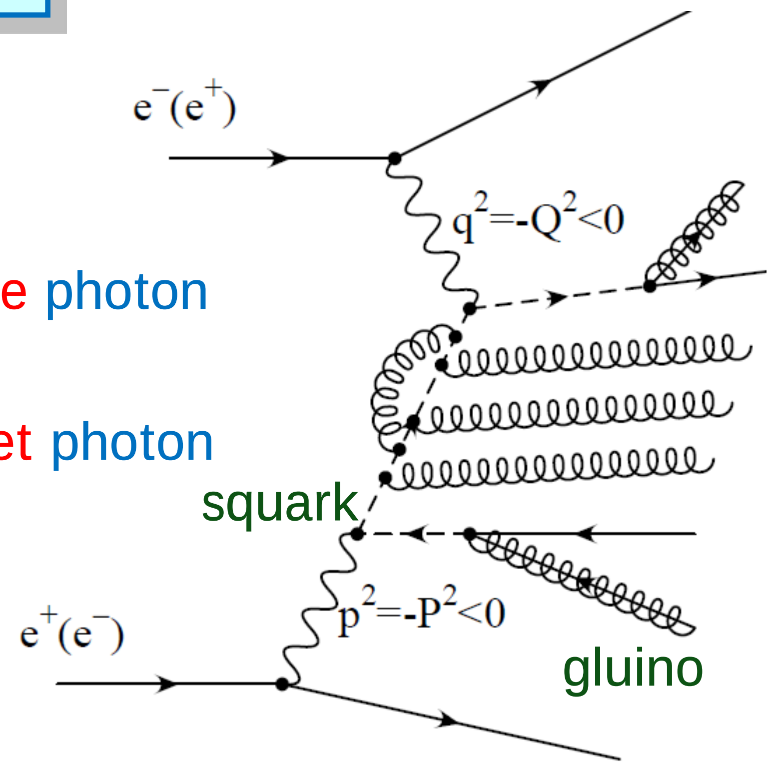
- Investigate photon structure functions in supersymmetric QCD where squarks and gluinos are present in addition to quarks and gluons
- Here we particularly interested in the heavy particle mass effects in the PDFs in SUSY QCD
- We first perform Box diagram calculation of squark contribution to the eight virtual photon structure functions without radiative corrections

Virtual-Photon Kinematics

$$x = \frac{Q^2}{2p \cdot q}$$: Bjorken variable

$$Q^2 = -q^2 > 0$$: Mass squared of probe photon

$$P^2 = -p^2 > 0$$: Mass squared of target photon



In the kinematic region:

$$\Lambda^2 \ll P^2 \ll Q^2$$

structure fns. F_2^γ and F_L^γ

perturbatively calculable !

e^+e^-collision

& SQCD interactions

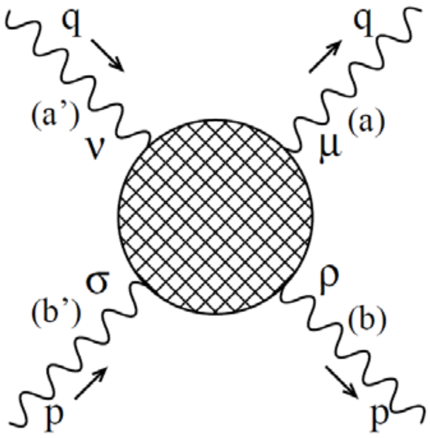
Photon structure in SUSY QCD so far studied

- Reya studied SUSY effects in photon structure function where squark mass is less than 40GeV (PLB 1983)
- Kounnas-Ross & Jones-Llewellyn-Smith computed the 1-loop splitting functions for SUSY QCD (NPB 1983)
- Scott-Stirling studied the longitudinal photon structure function in SUSY QCD
- Ross-Weston considered evolution and threshold effects for the virtual photon structure functions (EPJC 2001)
- We first perform Box diagram calculation of squark contribution to the eight virtual photon structure funs without radiative corrections (arXiv:1108.2847)

2. Squark contribution to the photon structure functions

Before studying the supersymmetric QCD radiative effects, it is worthwhile to investigate squark contributions to the photon structure functions through the pure QED interaction fully taking into account the squark mass effects.

Forward photon-photon amplitude:



Kitadono et al. arXiv:1108.2847, PRD to appear

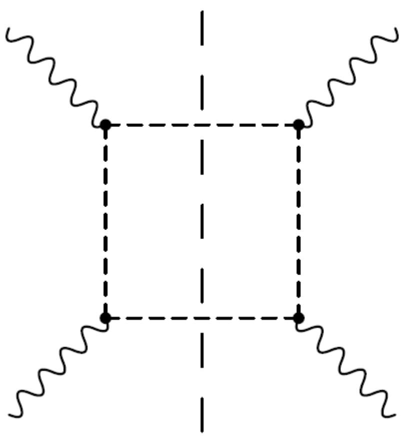
We compute the discontinuity of the forward photon-photon amplitudes

$$T_{\mu\nu\rho\sigma}(p,q) = i\int d^4x d^4y d^4z e^{iq\cdot x} e^{ip\cdot(y-z)} \langle 0|T(J_\mu(x)J_\nu(0)J_\rho(y)J_\sigma(z))|0\rangle$$

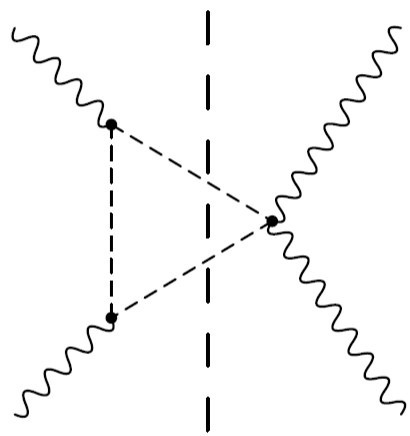
$$W_{\mu\nu\rho\sigma}(p,q) = \frac{1}{\pi}\mathrm{Im}T_{\mu\nu\rho\sigma}(p,q)$$

Or integrating the squared amplitudes over the two-body phase space

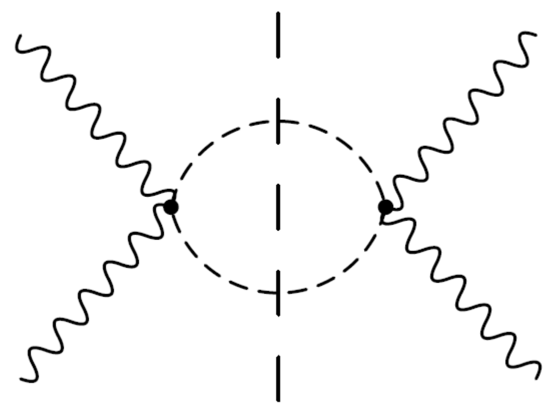
$$W_i = P_i^{\mu\nu\rho\sigma}\frac{1}{\pi}\mathrm{Im}T_{\mu\nu\rho\sigma} = \int dPS^{(2)} P_i^{\mu\nu\rho\sigma}\mathcal{M}^*_{\mu\rho}\mathcal{M}_{\nu\sigma}$$



Box



Triangle



Bubble

8 virtual photon structure functions

$$W_{TT} \quad W^a_{TT} \quad W^\tau_{TT} \quad W_{LT} \quad W_{TL} \quad W_{LL} \quad W^\tau_{TL} \quad W^{\tau a}_{TL}$$

T: transverse L: longitudinal τ : spin-flip a:$\mu\nu$ anti-sym.

$$\begin{aligned}
F_1^\gamma(x,Q^2,P^2) &= W_{TT} - \frac{1}{2}W_{TL}, \\
F_2^\gamma(x,Q^2,P^2) &= \frac{x}{\widetilde{\beta}^2}\left[W_{TT} + W_{LT} - \frac{1}{2}W_{LL} - \frac{1}{2}W_{TL}\right] \\
F_L^\gamma(x,Q^2,P^2) &= F_2^\gamma - xF_1^\gamma, \\
g_1^\gamma(x,Q^2,P^2) &= \frac{1}{\widetilde{\beta}^2}\left[W^a_{TT} - \sqrt{1-\widetilde{\beta}^2}W^{\tau a}_{TL}\right], \\
g_2^\gamma(x,Q^2,P^2) &= -\frac{1}{\widetilde{\beta}^2}\left[W^a_{TT} - \frac{1}{\sqrt{1-\widetilde{\beta}^2}}W^{\tau a}_{TL}\right],
\end{aligned}$$

$$\widetilde{\beta} = \sqrt{1 - \frac{4x^2P^2}{Q^2}}$$

Structure Tensors

$$\begin{aligned} W_{\mu\nu\rho\sigma} &= (T_{TT})_{\mu\nu\rho\sigma} W_{TT} + (T^a_{TT})_{\mu\nu\rho\sigma} W^a_{TT} + (T^\tau_{TT})_{\mu\nu\rho\sigma} W^\tau_{TT} + (T_{LT})_{\mu\nu\rho\sigma} W_{LT} \\ &\quad + (T_{TL})_{\mu\nu\rho\sigma} W_{TL} + (T_{LL})_{\mu\nu\rho\sigma} W_{LL} - (T^\tau_{TL})_{\mu\nu\rho\sigma} W^\tau_{TL} - (T^{\tau a}_{TL})_{\mu\nu\rho\sigma} W^{\tau a}_{TL} \end{aligned}$$

T_i's projection operators

$$(T_{TT})^{\mu\nu\rho\sigma} = R^{\mu\nu} R^{\rho\sigma},$$

$$(T_{TL})^{\mu\nu\rho\sigma} = R^{\mu\nu} k_2^\rho k_2^\sigma,$$

$$(T_{LT})^{\mu\nu\rho\sigma} = k_1^\mu k_1^\nu R^{\rho\sigma},$$

$$(T_{LL})^{\mu\nu\rho\sigma} = k_1^\mu k_1^\nu k_2^\rho k_2^\sigma,$$

$$(T^a_{TT})^{\mu\nu\rho\sigma} = R^{\mu\rho} R^{\nu\sigma} - R^{\mu\sigma} R^{\nu\rho},$$

$$(T^\tau_{TT})^{\mu\nu\rho\sigma} = \frac{1}{2}(R^{\mu\rho} R^{\nu\sigma} + R^{\mu\sigma} R^{\nu\rho} - R^{\mu\nu} R^{\rho\sigma}),$$

$$(T^\tau_{TL})^{\mu\nu\rho\sigma} = R^{\mu\rho} k_1^\nu k_2^\sigma + R^{\mu\sigma} k_1^\nu k_2^\rho + k_1^\mu k_2^\rho R^{\nu\sigma} + k_1^\mu k_2^\sigma R^{\nu\rho}$$

$$(T^{\tau a}_{TL})^{\mu\nu\rho\sigma} = R^{\mu\rho} k_1^\nu k_2^\sigma - R^{\mu\sigma} k_1^\nu k_2^\rho + k_1^\mu k_2^\rho R^{\nu\sigma} - k_1^\mu k_2^\sigma R^{\nu\rho}$$

$$R^{\mu\nu} = -g^{\mu\nu} + \frac{1}{X}\left[p\cdot q(q^\mu p^\nu + q^\nu p^\mu) - q^2 p^\mu p^\nu - p^2 q^\mu q^\nu\right]$$

$$k_1^\mu = \sqrt{\frac{-q^2}{X}}\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)$$

$$k_2^\mu = \sqrt{\frac{-p^2}{X}}\left(q^\mu - \frac{p\cdot q}{p^2} p^\mu\right)$$

$$X = (p\cdot q)^2 - p^2 q^2$$

photon helicity amplitudes

$$W(ab|a'b') = \epsilon^*_\mu(a)\epsilon^*_\rho(b) W^{\mu\nu\rho\sigma}\epsilon_\nu(a')\epsilon_\sigma(b')$$

$\epsilon_\mu(a)$ polarization vector $\quad a, a', b, b' = 0, \pm 1$

Due to the angular momentum conservation, parity conservation and time reversal invariance , we have 8 independent helicity amplitudes

$$W_{TT} = \frac{1}{2}[W(1,1|1,1) + W(1,-1|1,-1)] \ , \qquad W_{LT} = W(0,1|0,1) \ ,$$

$$W_{TL} = W(1,0|1,0) \ , \qquad W_{LL} = W(0,0|0,0) \ ,$$

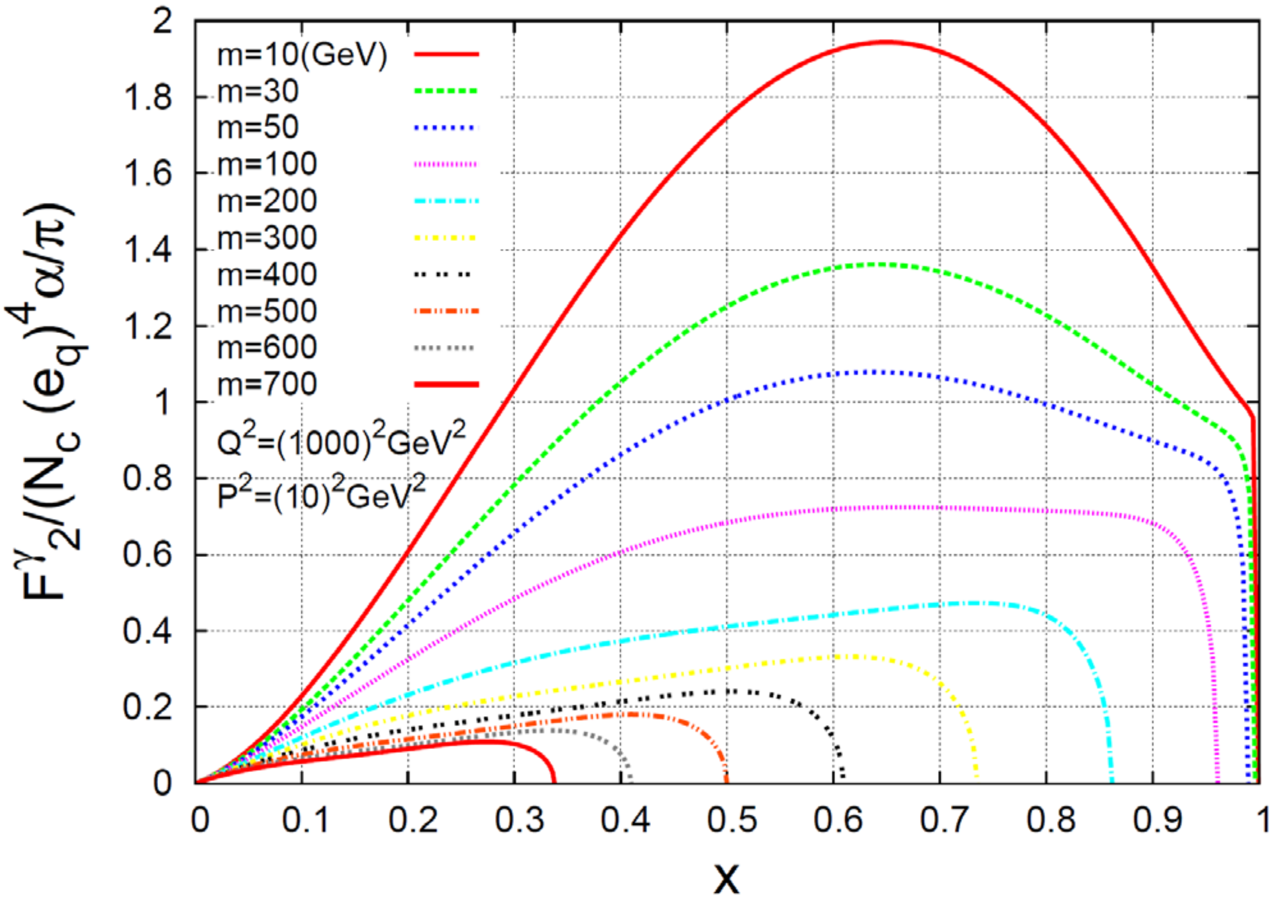
$$W^a_{TT} = \frac{1}{2}[W(1,1|1,1) - W(1,-1|1,-1)] \ , \qquad W^\tau_{TT} = W(1,1|-1,-1)$$

$$W^\tau_{TL} = \frac{1}{2}[W(1,1|0,0) - W(1,0|0,-1)]$$

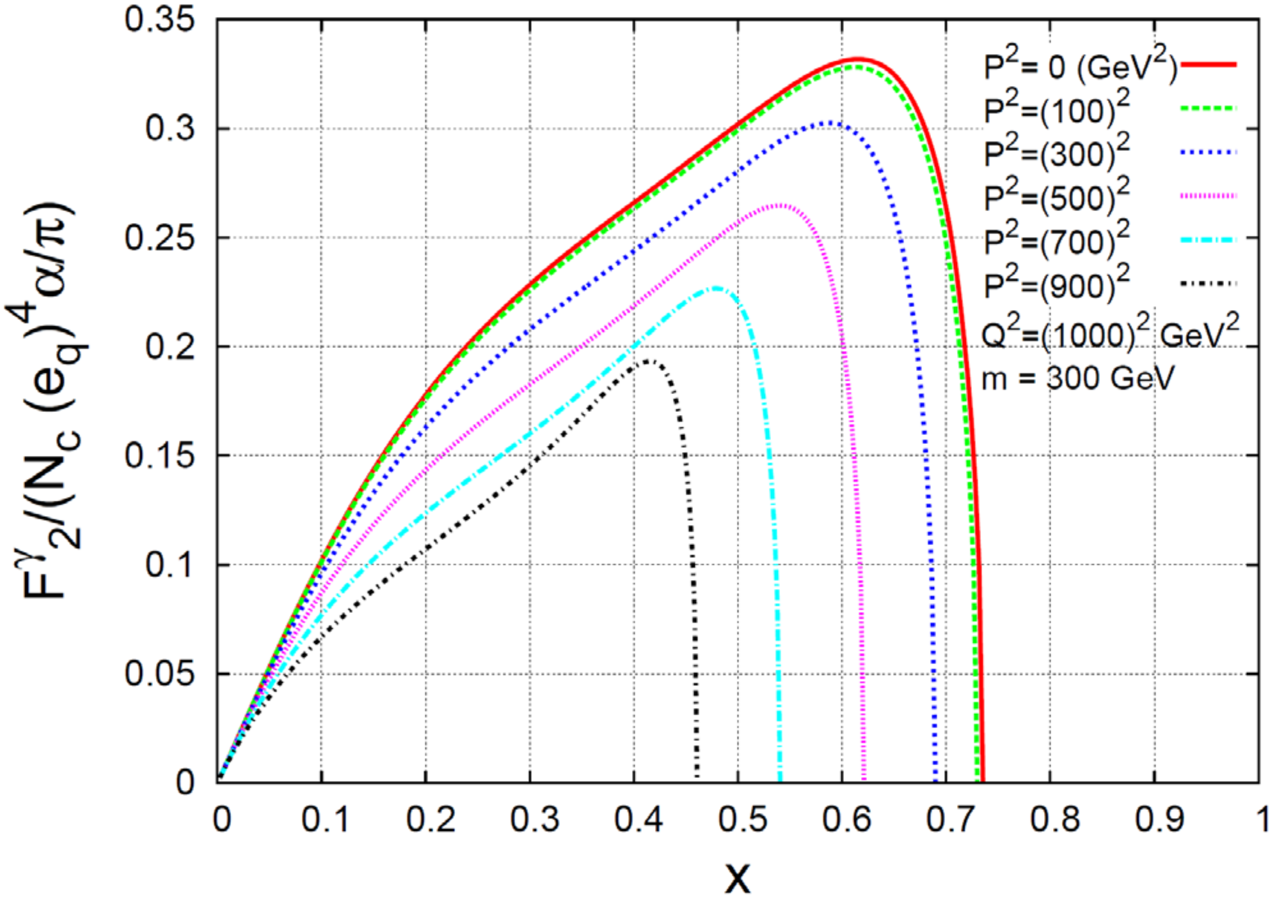
$$W^{\tau a}_{TL} = \frac{1}{2}[W(1,1|0,0) + W(1,0|0,-1)]$$

3. Numerical Analysis

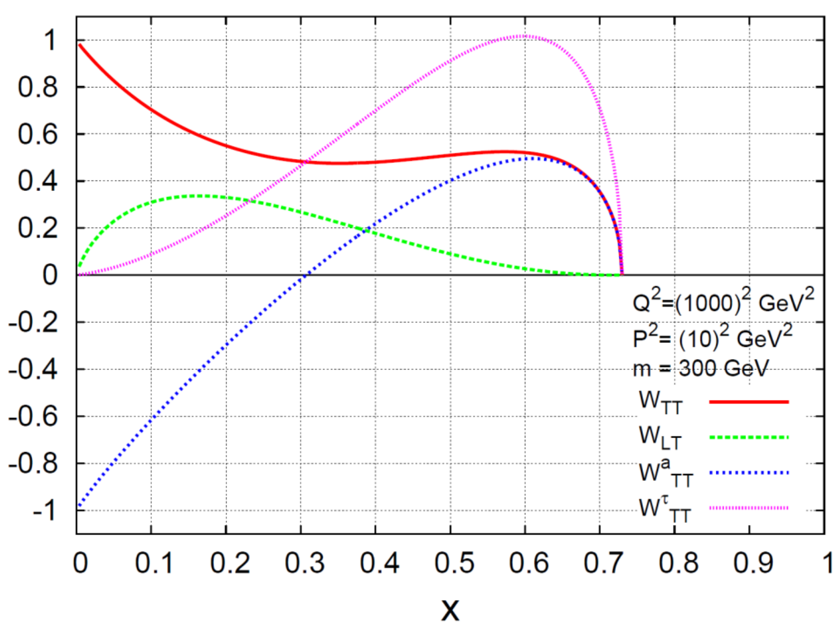
F_2 with various mass

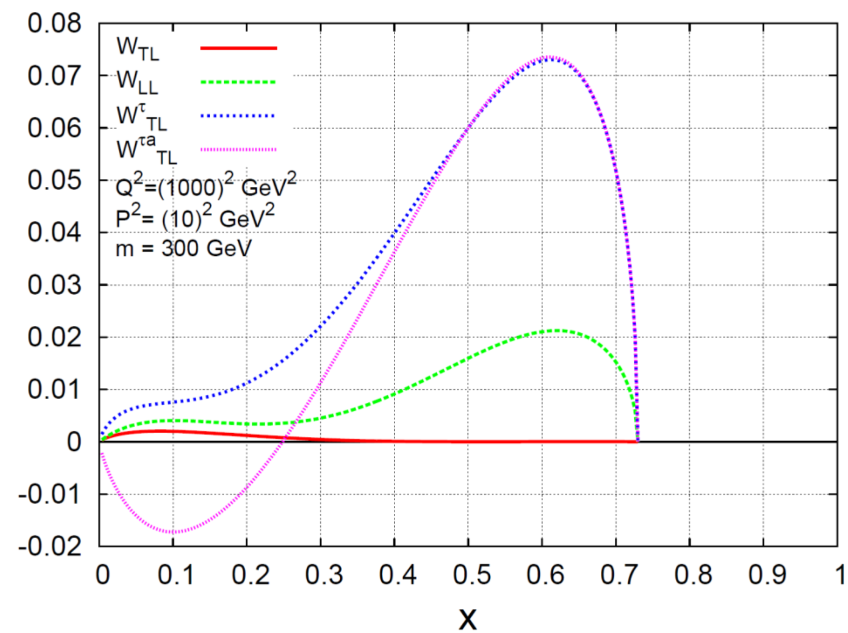


F_2 with various P^2



8 virtual photon structure functions





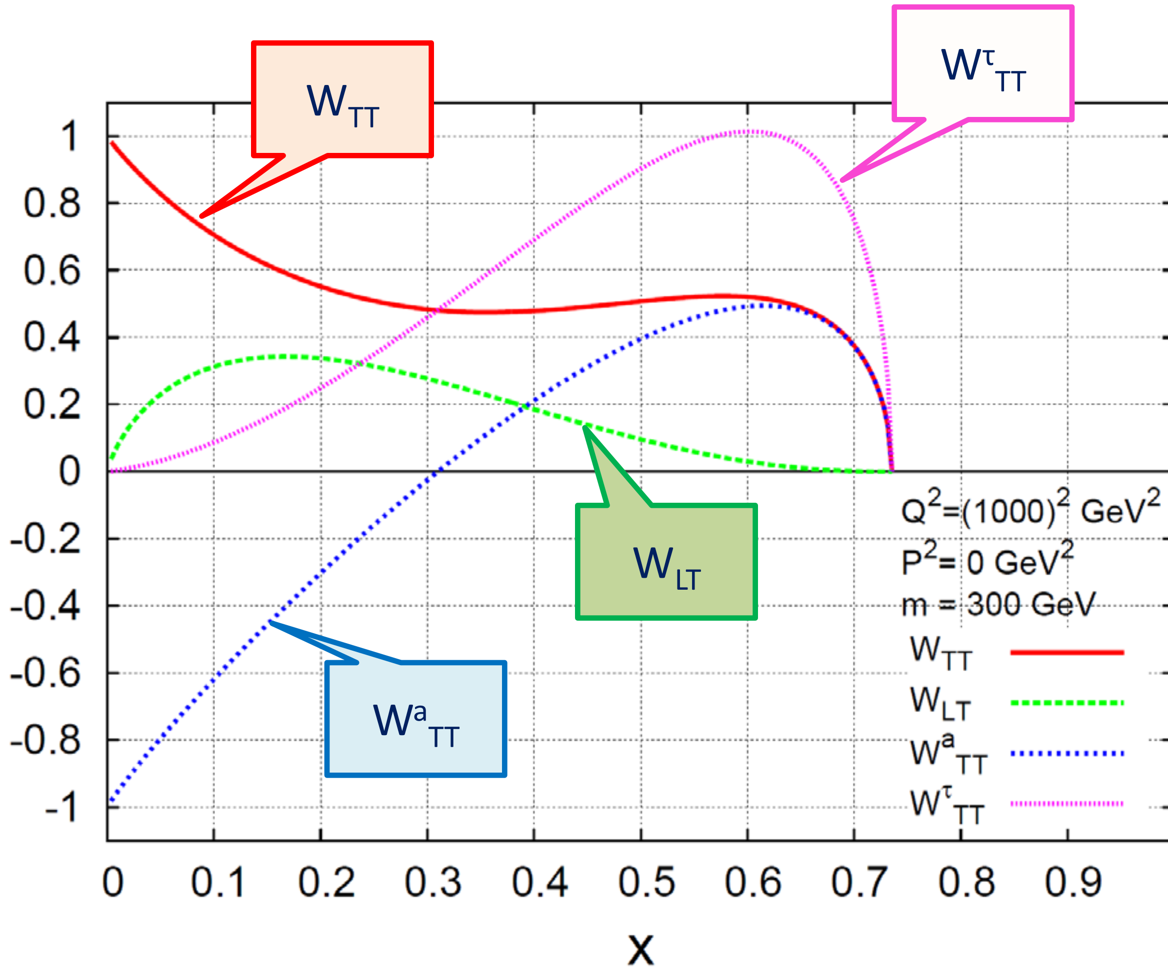
W_{TT} W^{a}_{TT} W^{τ}_{TT} W_{LT}

Both real and virtual photon target

W_{TL} W_{LL} W^{τ}_{TL} $W^{\tau a}_{TL}$

Only for virtual photon target

4 real photon structure functions



$$W^{\tau}_{TT} = W_{TT} + W^{a}_{TT}$$ Equality holds for both real & virtual photon

Positivity Constraints

Three model-independent positivity constraints:

$$\begin{aligned} \left|W^{\tau}_{TT}\right| &\le \left(W_{TT} + W^{a}_{TT}\right) \ , \\ \left|W^{\tau}_{TL} + W^{\tau a}_{TL}\right| &\le \sqrt{(W_{TT} + W^{a}_{TT})W_{LL}} \ , \\ \left|W^{\tau}_{TL} - W^{\tau a}_{TL}\right| &\le \sqrt{W_{TL}W_{LT}} \ . \end{aligned}$$

quark parton case: Sasaki, Soffer & TU(2002)

In squark parton case (LO QED), we found that first and third equality hold

$$W^{\tau}_{TT} = W_{TT} + W^{a}_{TT} \ ,$$

$$|W^{\tau}_{TL} - W^{\tau a}_{TL}| = \sqrt{W_{TL}W_{LT}}$$

$$W(1,1|-1,-1) = W(1,1|1,1)$$

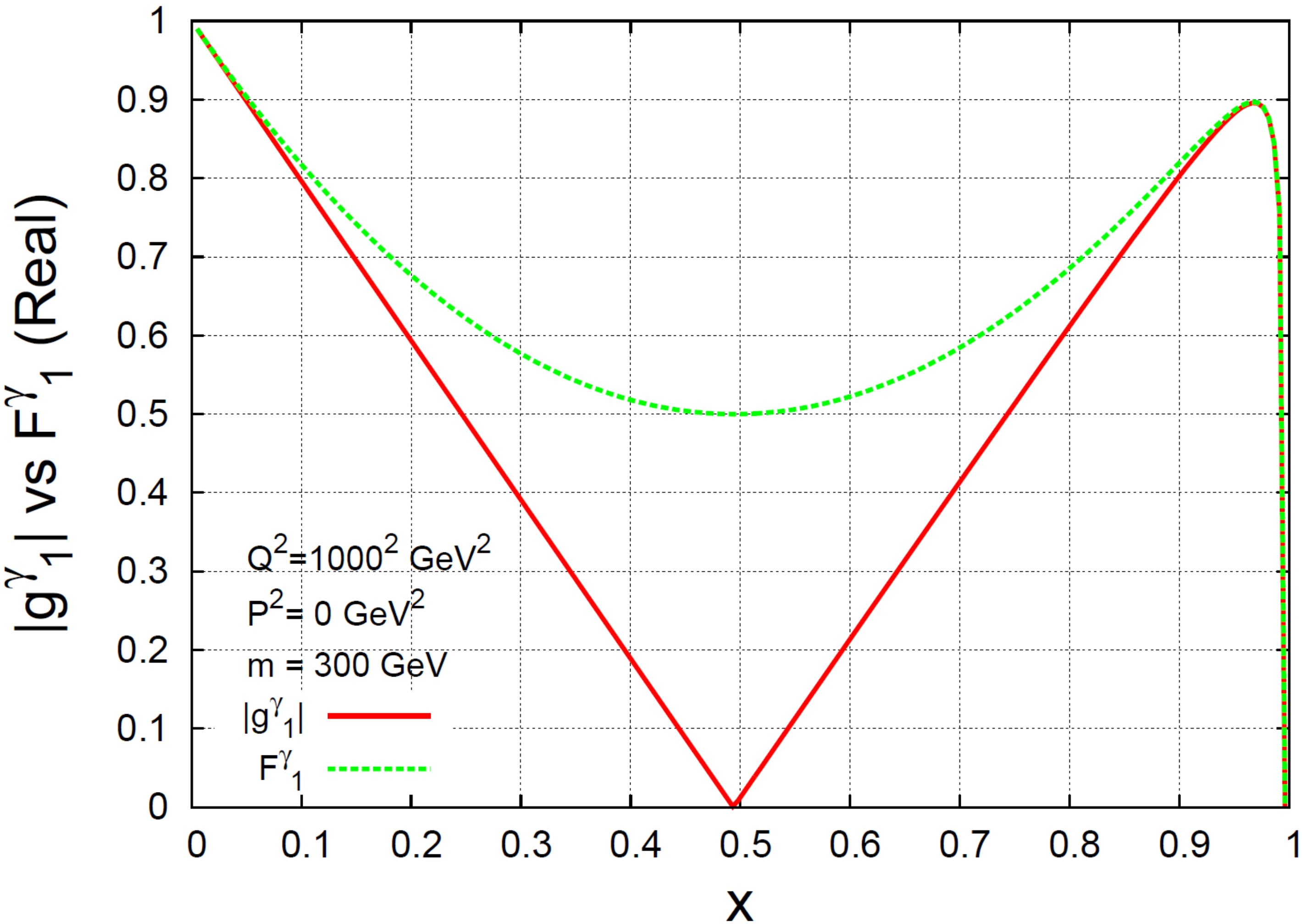
For the real photon

$$g_1^\gamma = W_{TT}^a = \frac{1}{2}\left[W(1,1|1,1) - W(1,-1|1,-1)\right]$$
$$F_1^\gamma = W_{TT} = \frac{1}{2}\left[W(1,1|1,1) + W(1,-1|1,-1)\right]$$

Helicity non-flip amplitudes is positive semi-definite.
From triangle inequality,

$$|g_1^\gamma| \le F_1^\gamma$$

For virtual photon, this inequality is not satisfied in the small-x region if P^2 is much bigger than m^2.



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
|g^γ_1| vs F^γ_1 (Real)
Q^2=1000^2 GeV^2
P^2= 0 GeV^2
m = 300 GeV
|g^γ_1|
F^γ_1

g_1 sum rule

In general, polarized photon structure function g_1^γ satisfies remarkable sum rule:

$$\int_0^{x_{\rm max}} g_1^\gamma(x,Q^2)dx = 0$$

Our result (Real photon):

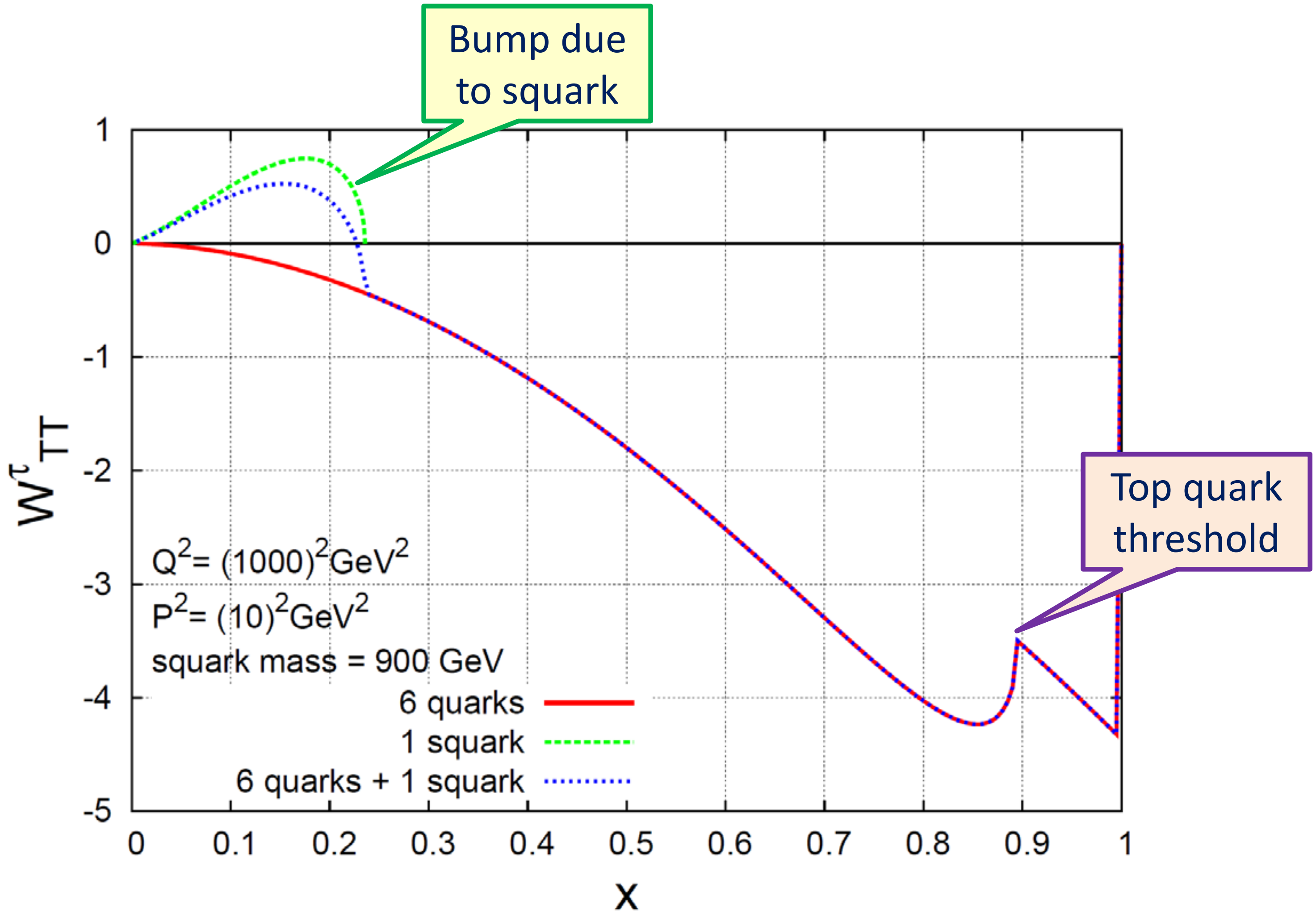
$$\int_0^{x_{\rm max}} g_1^\gamma(x,Q^2)dx = N_c\frac{\alpha}{\pi}e_q^4\left[\int_0^{x_{\rm max}}\tau x L dx + \int_0^{x_{\rm max}}\beta(2x-1)dx\right] = 0$$

Using integration by parts where $\boldsymbol{x_{\rm max}} = \dfrac{\mathbf{1}}{\mathbf{1}+\frac{\mathbf{4}\boldsymbol{m^2}}{\boldsymbol{Q^2}}}$

$$\int_0^{x_{\rm max}}\tau x L dx = -\int_0^{x_{\rm max}}\beta(2x-1)dx$$

$$= -\frac{\tau}{2(\tau+1)} + \frac{\tau(\tau+1)\log\left(\frac{\sqrt{\tau+1}+1}{\sqrt{\tau}}\right)}{2(\tau+1)^{3/2}}$$

Squark signature in W^{τ}_{TT}



$$d^6\sigma = d^6\sigma(ee \to eeX) = \frac{d^3p_1' \, d^3p_2'}{E_1' \, E_2'} \frac{\alpha^2}{16\pi^4 Q^2 P^2} \left[\frac{(p \cdot q)^2 - Q^2 P^2}{(p_1 \cdot p_2)^2 - m_e^2 m_e^2} \right]^{1/2}$$

$$\times \left(4\rho_1^{++}\rho_2^{++}\sigma_{TT} + 2|\rho_1^{+-}\rho_2^{+-}|\tau_{TT}\cos 2\bar{\phi} + 2\rho_1^{++}\rho_2^{00}\sigma_{TL}\right.$$

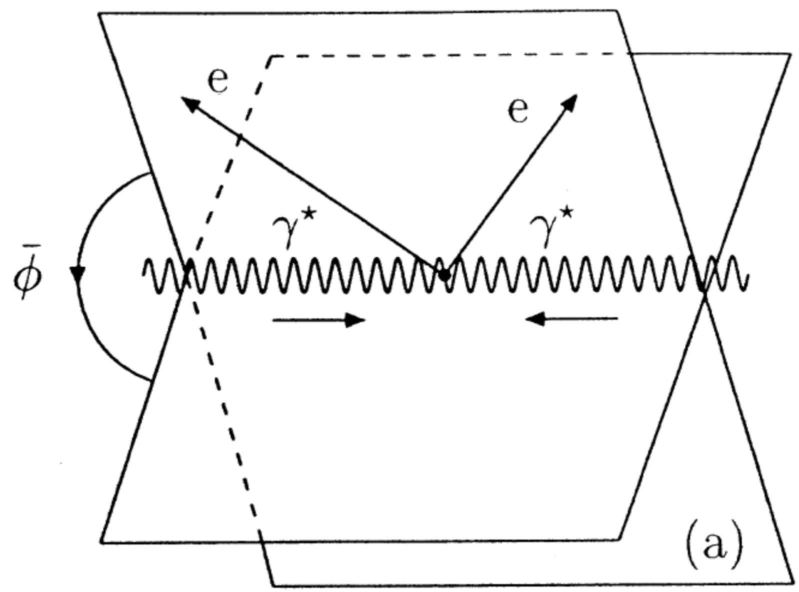
$$\left. + 2\rho_1^{00}\rho_2^{++}\sigma_{LT} + \rho_1^{00}\rho_2^{00}\sigma_{LL} - 8|\rho_1^{+0}\rho_2^{+0}|\tau_{TL}\cos\bar{\phi}\right) ,$$

$$2\rho_1^{++} = \frac{(2p_1 \cdot p - p \cdot q)^2}{(p \cdot q)^2 - Q^2P^2} + 1 - 4\frac{m_e^2}{Q^2} , \qquad 2\rho_2^{++} = \frac{(2p_2 \cdot q - p \cdot q)^2}{(p \cdot q)^2 - Q^2P^2} + 1 - 4\frac{m_e^2}{P^2}$$

$$\rho_1^{00} = 2\rho_1^{++} - 2 + 4\frac{m_e^2}{Q^2} , \qquad \rho_2^{00} = 2\rho_2^{++} - 2 + 4\frac{m^2}{P}$$

$$|\rho_i^{+-}| = \rho_i^{++} - 1 , \qquad |\rho_i^{+0}| = \sqrt{(\rho_i^{00} + 1)|\rho_i^{+-}|} .$$

$\bar{\phi}$ Angle between scattered planes of e^+ and e^-



Nisius, Phys.Rept.332 (2000)

4. Evolution equation and heavy mass effects

Evolution eq. for SUSY QCD

$$\frac{d\boldsymbol{q}^{\gamma}(t)}{dt} = \boldsymbol{q}^{\gamma}(t) \otimes P^{(0)} + \frac{\alpha}{\alpha_s(t)} \boldsymbol{k}^{(0)}$$

$$t = \frac{2}{\beta_0} \ln \frac{\alpha_s(P^2)}{\alpha_s(Q^2)}$$

$P^{(0)}$ parton—parton splitting function

$\boldsymbol{k}^{(0)}$ photon—parton splitting function

$$\beta_0 = 9 - n_f$$

n_f # of active flavors

$$\boldsymbol{q}^{\gamma}(t) = (G, \lambda, q_i, s_i) \quad (i = 1, \cdots, n_f)$$

$q_i(x, Q^2, P^2)$ quark PDFs

$s_i(x, Q^2, P^2)$ squark PDFs

$G(x, Q^2, P^2)$ gluon PDF

$\lambda(x, Q^2, P^2)$ gluino PDF

SUSY relation for splitting fns.

$$P_{qq} + P_{sq} = P_{qs} + P_{ss} \equiv P_{\phi\phi}$$
$$P_{qG} + P_{sG} = P_{q\lambda} + P_{s\lambda} \equiv P_{\phi V}$$
$$P_{Gq} + P_{\lambda q} = P_{Gs} + P_{\lambda s} \equiv P_{V\phi}$$
$$P_{GG} + P_{\lambda G} = P_{G\lambda} + P_{\lambda\lambda} \equiv P_{VV}$$

then we take $\phi \equiv \Sigma + S, \quad V \equiv G + \lambda$ singlet

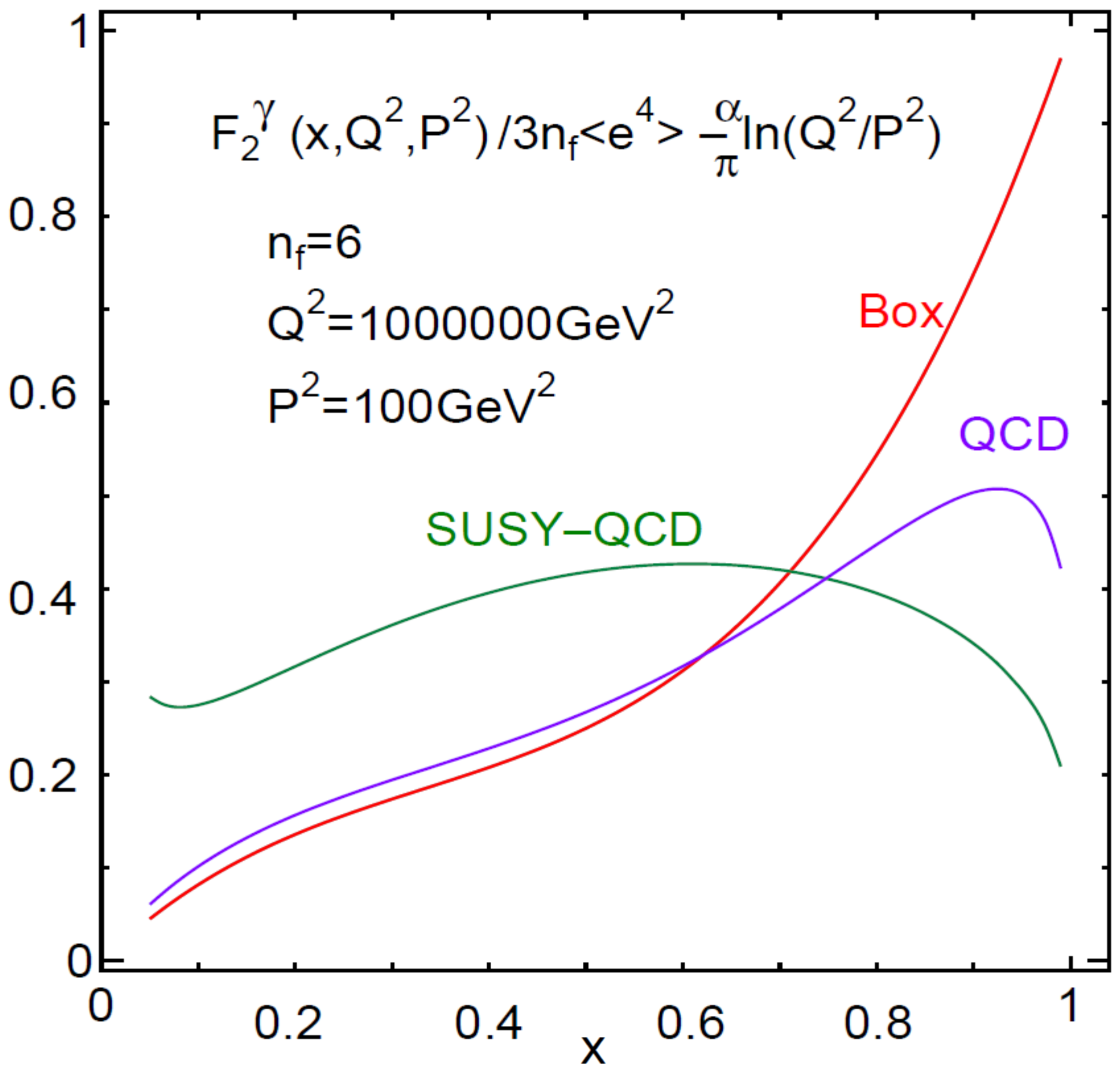
singlet

$$\frac{d}{d\ln Q^2}\phi = P_{\phi\phi} \otimes \phi + P_{\phi V} \otimes V + K_{\phi}^{(S)}$$
$$\frac{d}{d\ln Q^2}V = P_{V\phi} \otimes \phi + P_{VV} \otimes V \qquad K_{\phi}^{(S)} = K_{q}^{(S)} + K_{s}^{(S)}$$

Non-singlet
$$\frac{d}{d\ln Q^2}\phi_{NS} = P_{\phi\phi} \otimes \phi_{NS} + K_{\phi}^{(NS)}$$
$$K_{\phi}^{(NS)} = K_{q}^{(NS)} + K_{s}^{(NS)}$$

⇒ Mixing easily solved !

Massless SUSY QCD and Non-SUSY QCD



➢heavy mass effects can be incorporated by the boundary conditions:

$$\lambda(n, Q^2 = m_\lambda^2) = 0, \; s_{LS}(n, Q^2 = m_{sq}^2) = 0, \; q_H(n, Q^2 = m_H^2) = 0,$$
$$s_H(n, Q^2 = m_{sq}^2) = 0, \; s_{LNS}(n, Q^2 = m_{sq}^2) = 0$$

m_λ gluino mass m_{sq} squark mass m_H heavy quark mass

$$q^\gamma(n,t) \equiv \int_0^1 dx x^{n-1} q^\gamma(x, Q^2, P^2)$$

$$q^\gamma(n, t=0) = \Big(0, \hat{\lambda}(n), 0, \hat{s}_{LS}(n), \hat{q}_H(n), \hat{s}_H(n), 0, \hat{s}_{LNS}(n)\Big)$$

$$q^\gamma(n,t) = \frac{\alpha}{8\pi\beta_0}\frac{4\pi}{\alpha_s(t)} K_n^{(0)} \sum_i P_i^n \frac{1}{1+d_i^n}\left\{1 - \left[\frac{\alpha_s(t)}{\alpha_s(0)}\right]^{1+d_i^n}\right\}$$
$$+ q^\gamma(n,0) \sum_i P_i^n \left[\frac{\alpha_s(t)}{\alpha_s(0)}\right]^{d_i^n}$$

Numerical analysis

By solving the coupled boundary conditions

$$q_j^{\gamma}(t = t_{m_j}) = 0, \quad t_{m_j} = \frac{2}{\beta_0} \ln \frac{\alpha_s(P^2)}{\alpha_s(m_j^2)}$$

to determine the initial conditions:

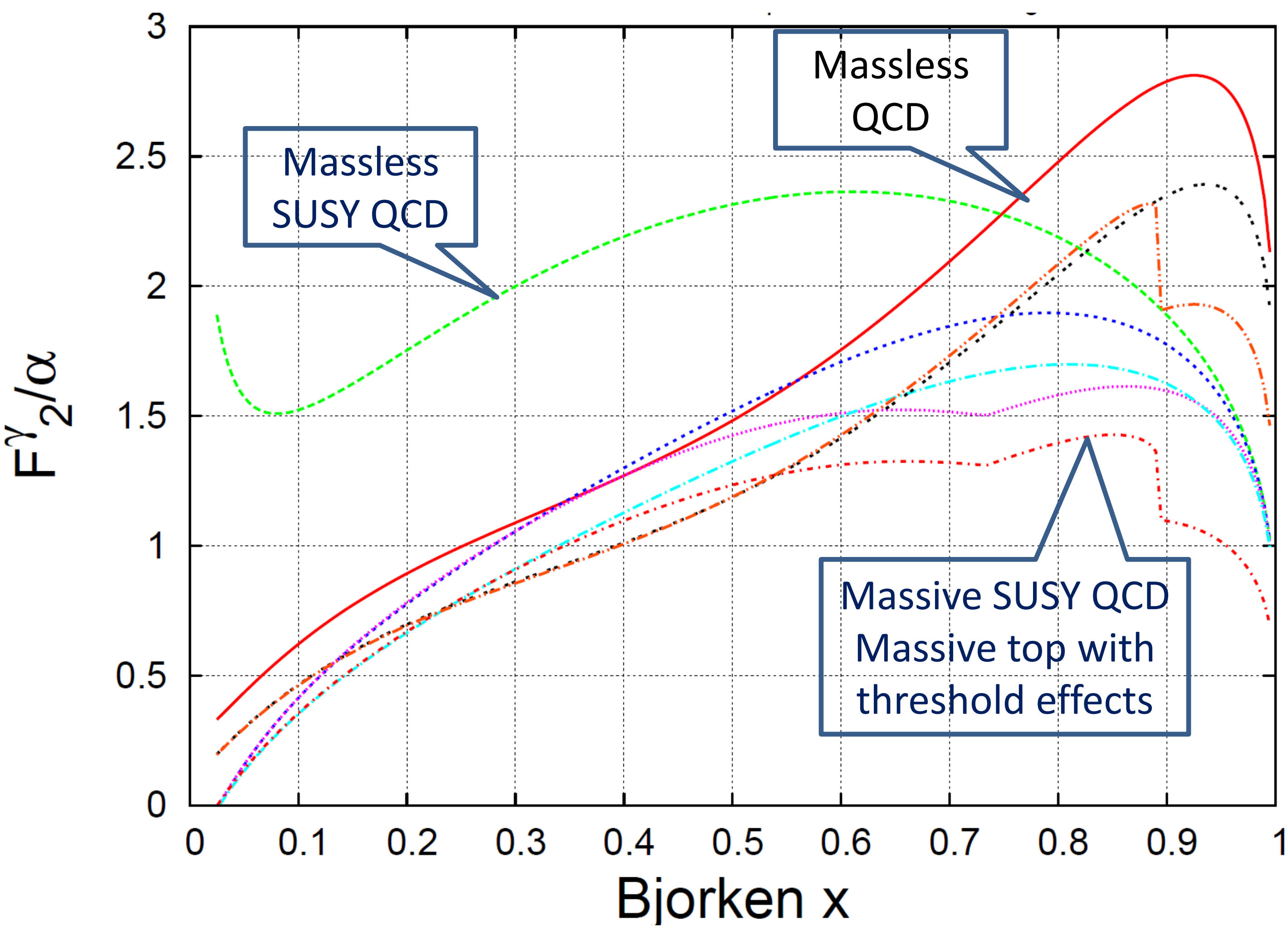
$$q^{\gamma}(n, t = 0) = \Big(0, \hat{\lambda}(n), 0, \hat{s}_{LS}(n), \hat{q}_H(n), \hat{s}_H(n), 0, \hat{s}_{LNS}(n)\Big)$$

Parameter set

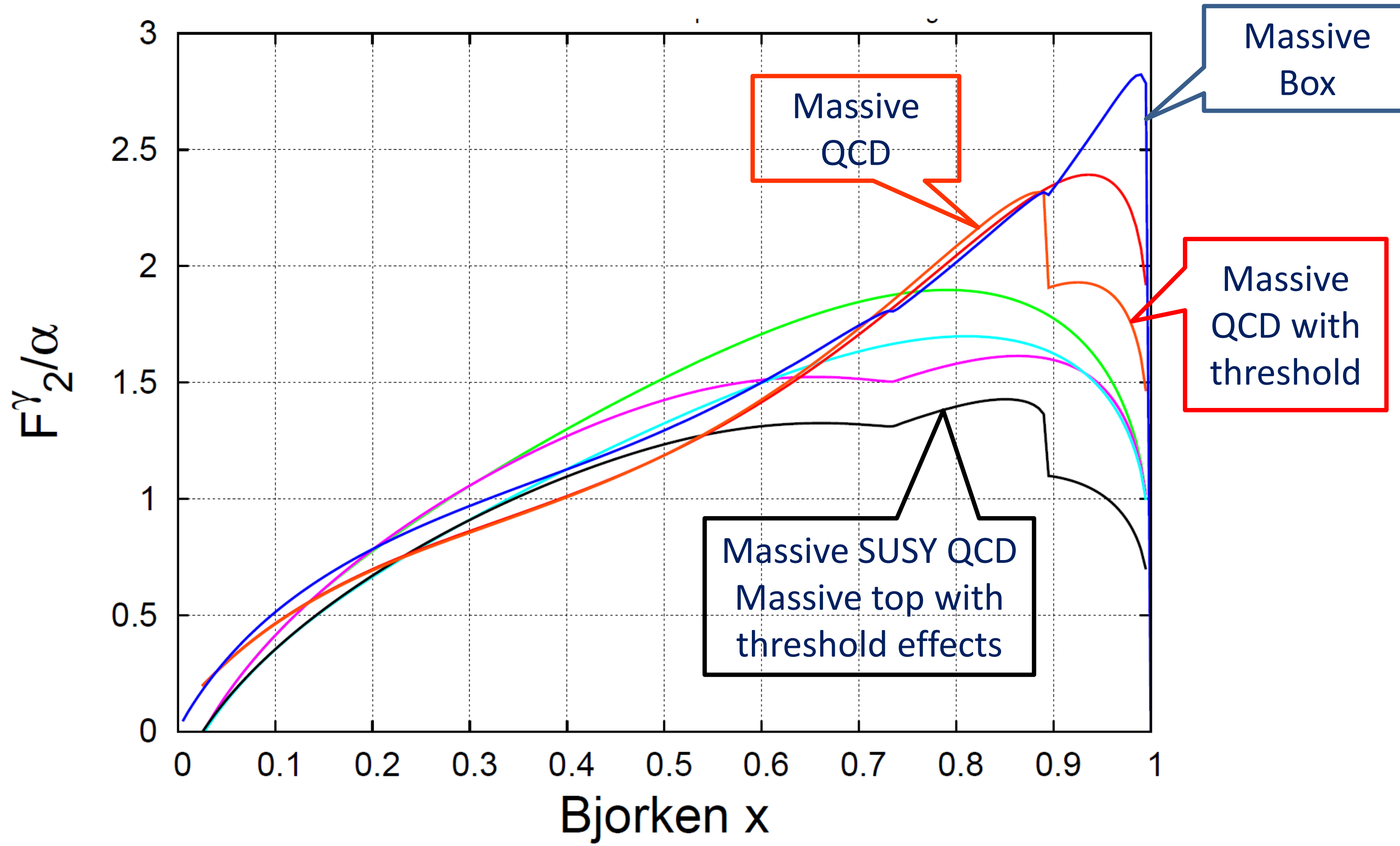
Inverse Mellin transform

$$m_{squark} = 300[GeV]$$
$$m_{\lambda} = 700[GeV]$$
$$Q^2 = 1000000[(GeV)^2]$$
$$P^2 = 100[(GeV)^2]$$

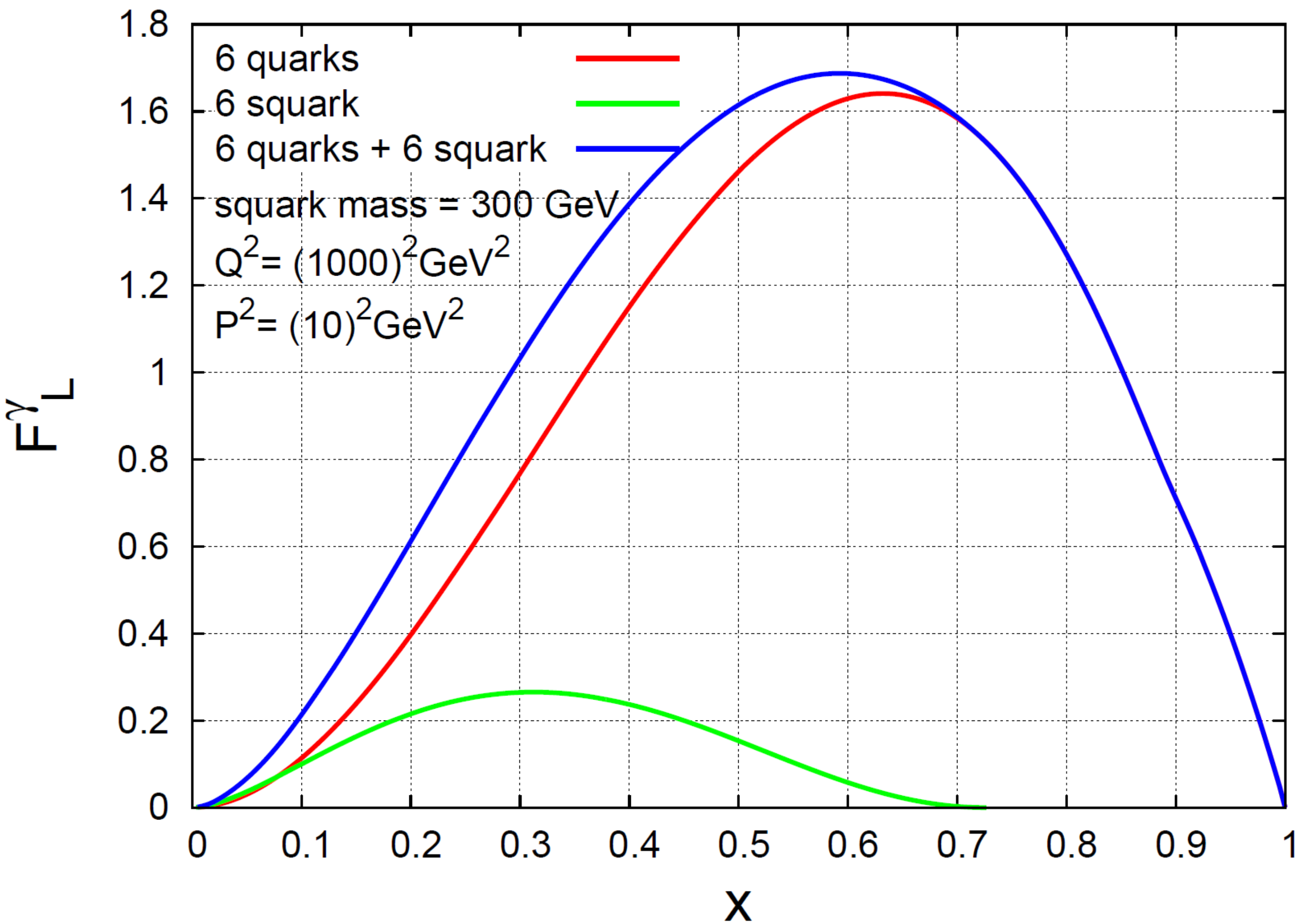
Massive SUSY QCD and Non-SUSY QCD



Massive SUSY QCD and Non-SUSY QCD



F^{γ}_L in QCD and SUSY QCD



5. Conclusion and Outlook

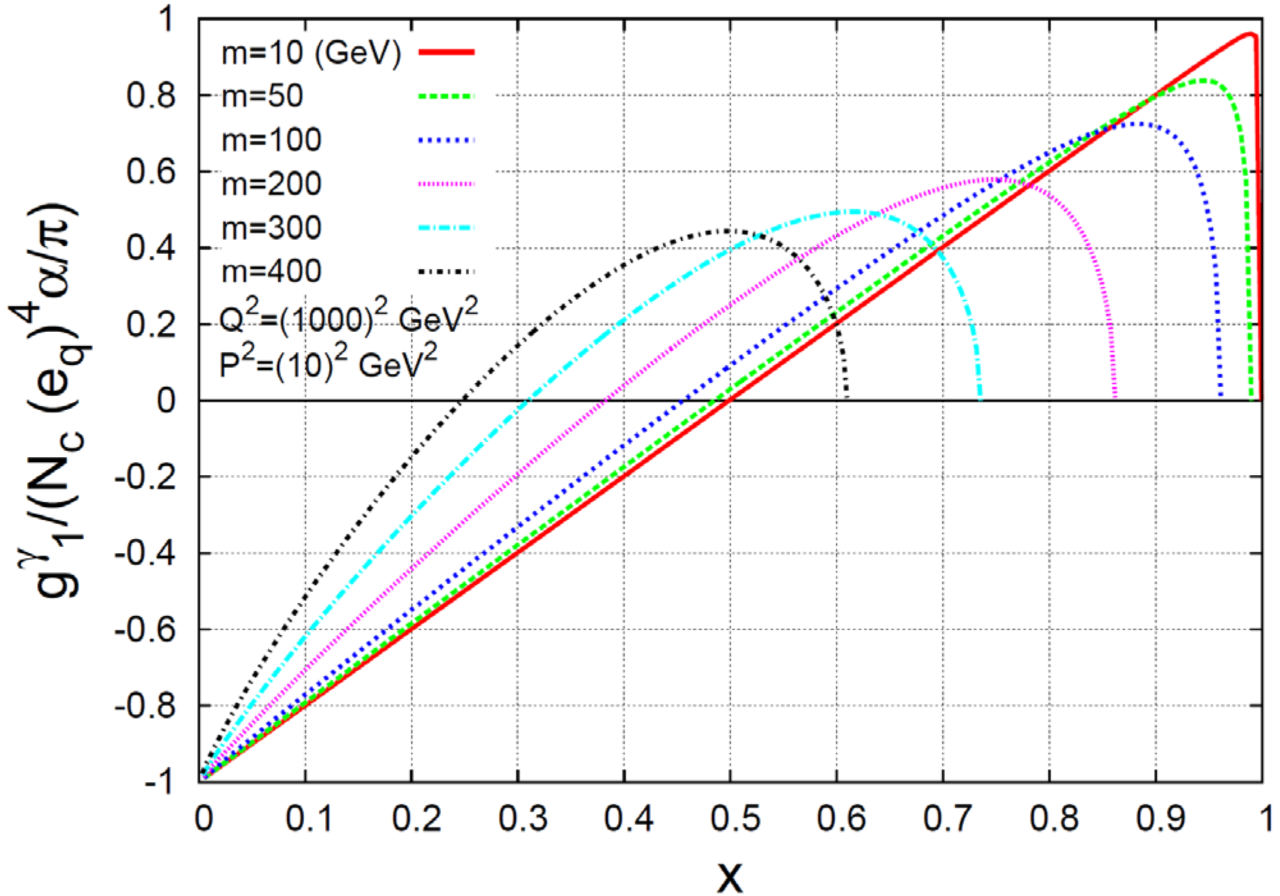
- We have calculated the eight virtual photon structure functions in squark parton case and have numerically studied their contributions
- We have found three positivity constraints in the squark case reduce to two equalities and one inequality in contrast to the quark case
- Inequality $|g_1| \leq F_1$ holds for the real photon
- We confirmed the vanishing g_1 sum rule in real photon target as in the quark case

- SUSY QCD effects added by DGLAP equation with modified boundary conditions to include squark mass effects
- At small x, there is no significant differences between massless and massive cases
- At large x, significant heavy mass effects exist
- Possible squark signature at small x in W^{τ}_{TT}
- The kinematic parameters chosen for Q^2, P^2 and m^2 are just illustrative values and are not necessarily realistic values in the ILC region.

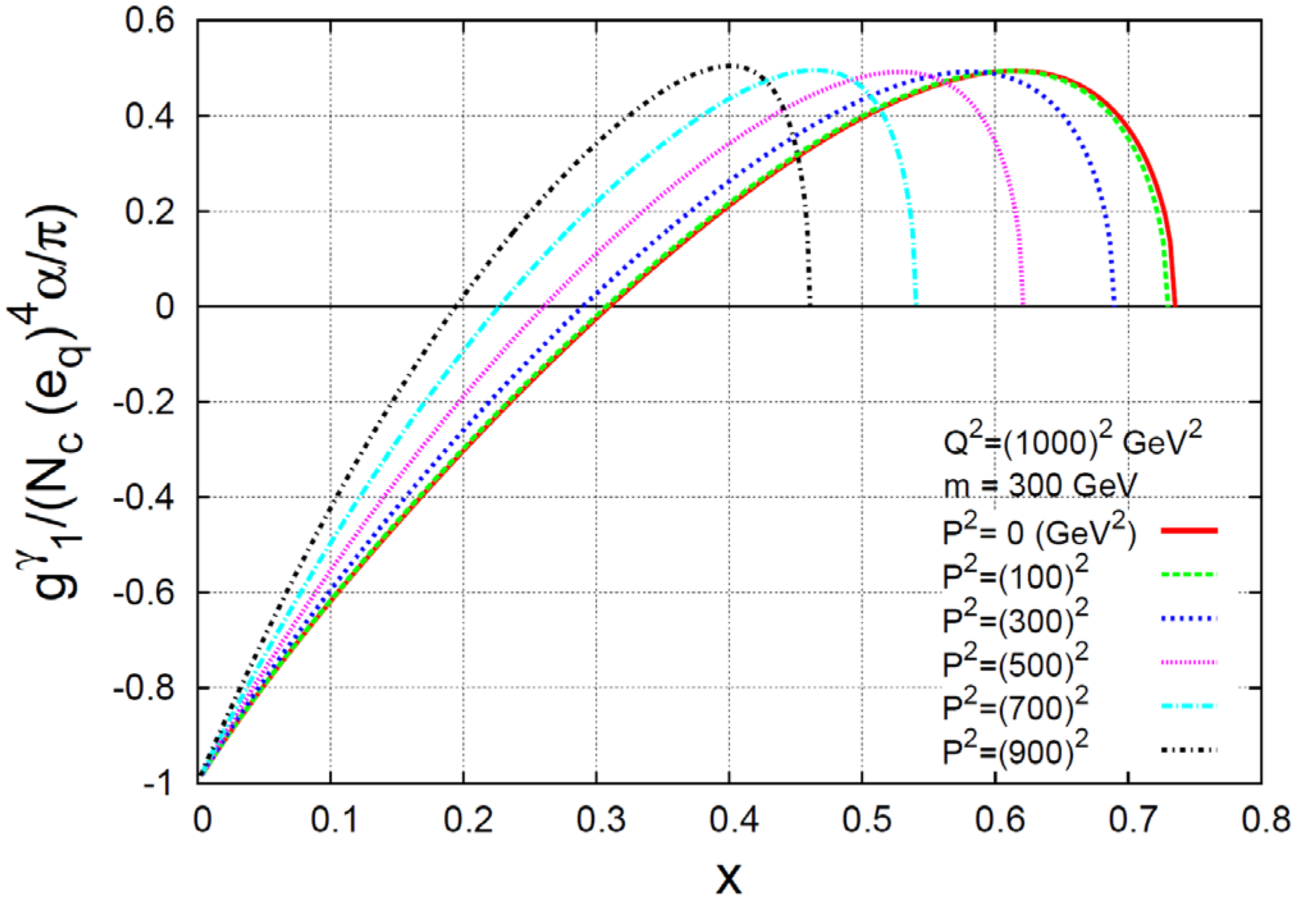
- The parameters can be freely scaled up or scaled down since we have general formulas for the structure functions, which only depend on the ratios such as m^2/Q^2, P^2/Q^2. The heavy squark mass, m, could be set larger than 1 TeV as the recently reported results from the ATLAS/CMS group at LHC.

Back up slides

g_1 with various mass



g_1 with various P^2



Analytical Results (4 W's in real photon)

$$W_{TT} = N_c \frac{\alpha}{\pi} e_q^4 \left[L\ \tau x \left\{ \frac{1}{2}\tau x + (2x-1) \right\} + \beta \left\{ \tau x(1-x) + 2x^2 - 2x + 1 \right\} \right]$$

$$W_{LT} = N_c \frac{\alpha}{\pi} e_q^4 \left[L \left\{ \tau x^2 + 2x(1-x) \right\} - 6\beta x(1-x) \right]$$

$$W_{TT}^a = N_c \frac{\alpha}{\pi} e_q^4 \left[L\ \tau x + \beta(2x-1) \right]$$

$$W_{TT}^\tau = N_c \frac{\alpha}{\pi} e_q^4 \left[L \left\{ 2\tau \left(1 + \frac{1}{4}\tau \right) x^2 \right\} + \beta \left\{ 2x^2 + \tau x(1-x) \right\} \right]$$

with $L = \ln \frac{1+\beta}{1-\beta} \quad \tau = \frac{4m^2}{Q^2} \quad \beta = \sqrt{1 - \frac{\tau x}{(1-x)}}$,

The others (W_{TL} , W_{LL}, W^{τ}_{TL} , $W^{\tau a}_{TL}$) vanish in real limit (P→0)

Analytical Results (8 W's in virtual photon) –(1)

For simplicity we use the following definitions:

$$L = \ln \frac{1+\beta\widetilde{\beta}}{1-\beta\widetilde{\beta}} \qquad \widetilde{\beta} = \sqrt{1 - \frac{P^2 Q^2}{(p\cdot q)^2}} = \sqrt{1 - \frac{4x^2P^2}{Q^2}}$$

and $\beta = \sqrt{1 - \frac{4m^2}{(p+q)^2}} = \sqrt{1 + \frac{4m^2 x}{xP^2 + (x-1)Q^2}}$

$$W_{TT} = N_c \frac{\alpha}{\pi} e_q^4 \Big[L \frac{1}{4\widetilde{\beta}^5} \frac{1}{x} (1-\beta^2\widetilde{\beta}^2)(4x(1-x) - 1 + \widetilde{\beta}^2) \Big\{ 2x \left(\frac{m^2}{Q^2} - \frac{P^2}{Q^2}(x^2+x-1) \right)$$
$$-\frac{1}{2}(1-\widetilde{\beta}^2) \left[(1-x)(1-\beta^2) + x\beta^2 \frac{P^2}{Q^2} \right] + (2x-1) \Big\} + \frac{\beta}{\widetilde{\beta}^4} \Big\{ 2x \Big(\frac{P^2}{Q^2}(2x(x^2-4x+2)-1)$$
$$-\frac{2m^2}{Q^2}(x-1) \Big) + \frac{1}{2}(1-\widetilde{\beta}^2) \left[\frac{m^2}{Q^2}(8x^2-8x-2) + \frac{P^2}{Q^2}(12x^2-8x+1) \right]$$
$$+\frac{1}{4}(1-\widetilde{\beta}^2)^2 \frac{1}{x} \left[(1-x)(1-\beta^2) + x\beta^2 \frac{P^2}{Q^2} \right] + (2x^2-2x+1) \Big\} \Big]$$

$$W_{TL} = N_c \frac{\alpha}{\pi} e_q^4 2x(1-2x)^2 \frac{P^2}{Q^2} \Big[-\frac{1}{2\widetilde{\beta}^5} L \Big\{ 2x \left(\frac{P^2}{Q^2}(2x^2-2x+1) - \frac{2m^2}{Q^2} \right)$$
$$+(1-\widetilde{\beta}^2) \left[(1-x)(1-\beta^2) + x\beta^2 \frac{P^2}{Q^2} \right] + 2(x-1) \Big\} + 3\beta \frac{1}{\widetilde{\beta}^4} \left(\frac{P^2}{Q^2} x + (x-1) \right) \Big]$$

Analytical Results (8 W's in virtual photon) –(2)

$$W_{LT} = N_c \frac{\alpha}{\pi} e_q^4 \left(1 - \frac{2P^2 x}{Q^2}\right)^2 \left[-\frac{1}{\widetilde{\beta}^5} L \left\{2x^2 \left(\frac{P^2}{Q^2}(2x^2 - 2x + 1) - \frac{2m^2}{Q^2}\right)\right.\right.$$

$$\left.\left. +(1 - \widetilde{\beta}^2) \left[x(1-x)(1-\beta^2) + x^2\beta^2 \frac{P^2}{Q^2}\right] - 2x(1-x)\right\} + \frac{6\beta x}{\widetilde{\beta}^4} \left(\frac{P^2}{Q^2} x + (x-1)\right)\right]$$

$$W_{LL} = N_c \frac{\alpha}{\pi} e_q^4 \frac{P^2}{Q^2} x \left[-\frac{1}{\widetilde{\beta}^5} L (2x-1)(2x-1+\widetilde{\beta}^2) \left(2\frac{P^2}{Q^2} x(2x+3) + 6x - 7\right)\right.$$

$$+ \frac{1}{\widetilde{\beta}^4} \frac{8\beta x}{1 - \beta^2\widetilde{\beta}^2} \frac{\frac{P^2}{Q^2} x + (x-1)}{(2x-1)^2 - \widetilde{\beta}^2} \left\{-2\left[4(x-1)\left(\frac{8m^2x^2}{Q^2} - (1-\widetilde{\beta}^2)x(2x-3)\right) + (1-\widetilde{\beta}^2)\right]\right.$$

$$+2(1-\widetilde{\beta}^2)x \left[\frac{8m^2}{Q^2}(4x^2 - 4x - 1) + \frac{P^2}{Q^2}(20x^2 - 20x + 1)\right]$$

$$\left.\left. +4(1-\widetilde{\beta}^2)^2 \left[(1-x)(1-\beta^2) + x\beta^2 \frac{P^2}{Q^2}\right] + 2x(1-2x)^2\right\}\right]$$

$$W_{TT}^a = N_c \frac{\alpha}{\pi} e_q^4 \left[\frac{1}{\widetilde{\beta}^3} L \left\{-(1-\widetilde{\beta}^2)\left[1 - \beta^2(1-x) + x\beta^2 \frac{P^2}{Q^2}\right] + \frac{4m^2 x}{Q^2} + 1 - \widetilde{\beta}^2\right\}\right.$$

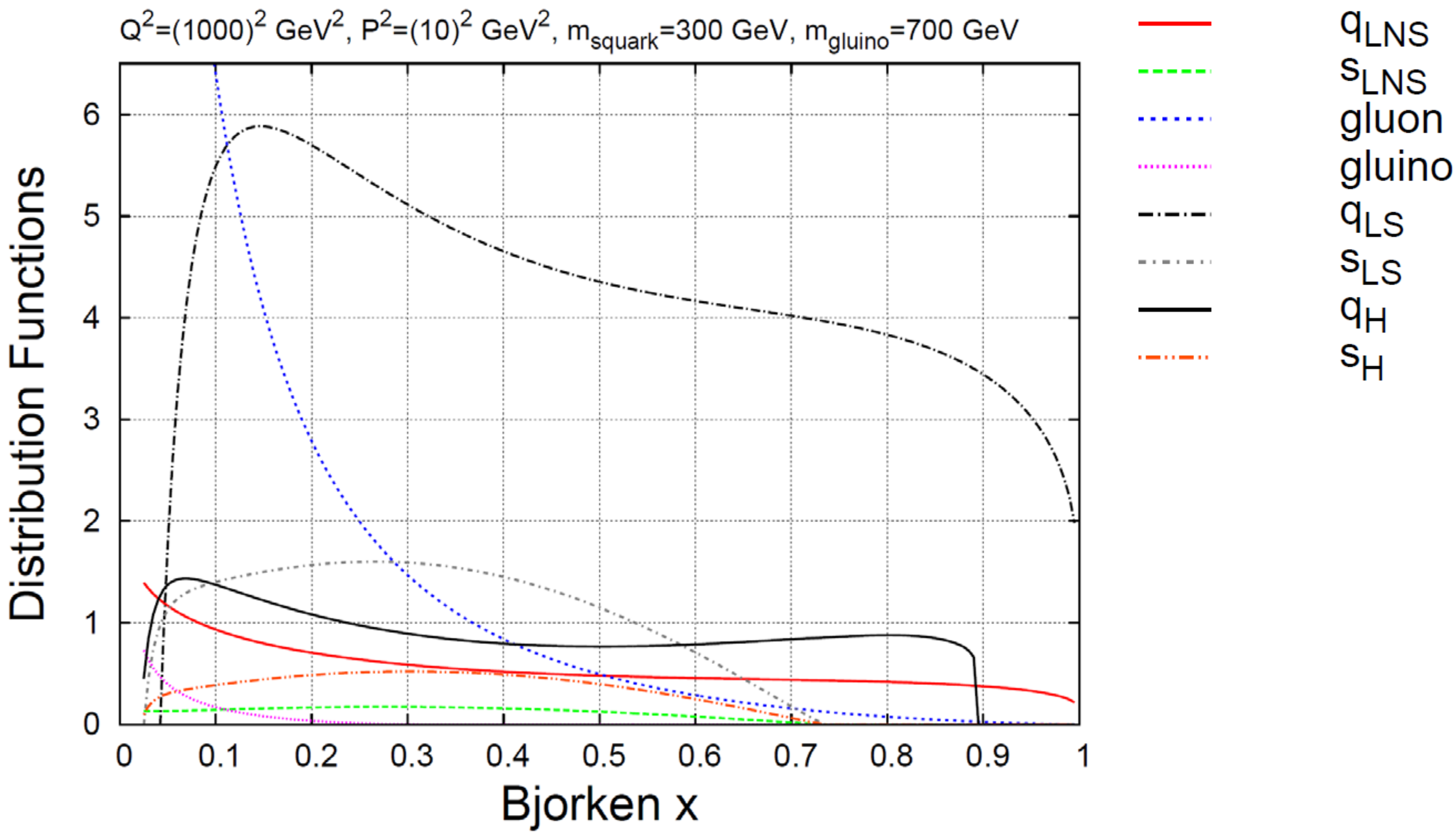
$$\left. + \frac{\beta}{\widetilde{\beta}^2}(2x-1)\left(1 - \frac{2P^2}{Q^2} x\right)\right]$$

Analytical Results (8 W's in virtual photon) –(3)

$$W_{TT}^{\tau} = N_c \frac{\alpha}{\pi} e_q^4 \left[\frac{1}{\widetilde{\beta}^5} L \left\{ -2x(1-\widetilde{\beta}^2) \left[1 - \beta^2(1-x) + x\beta^2 \frac{P^2}{Q^2} \right] + 2(1-\beta^2)x(1-x) \right.\right.$$
$$\left. + \frac{1}{2}(1-\widetilde{\beta}^2)(3+\beta^2) + 8x^2 - 6x(1-\widetilde{\beta}^2) \right\} \left\{ -\frac{P^2}{Q^2} \left[(1-\beta^2)x(1-x) \right.\right.$$
$$\left.\left. + \frac{1}{4}\beta^2(1-\widetilde{\beta}^2) + x^2 \right] + \frac{m^2}{Q^2} + \frac{P^2}{Q^2}x \right\} + \frac{\beta}{\widetilde{\beta}^4} \left\{ \frac{1}{4}\frac{P^2}{Q^2}(1-\widetilde{\beta}^2)^2 + \frac{1}{2}\frac{P^2}{Q^2}(1-\widetilde{\beta}^2) \right.$$
$$\times (20x^2 - 12x + 1) + (1-\widetilde{\beta}^2)(x^2 - 6x + 2) + 2x^2$$
$$\left.\left. - \frac{1}{4x}\widetilde{\beta}^2(1-\beta^2)[4x(1-x) - (1-\widetilde{\beta}^2)] \left(\frac{P^2}{Q^2}x + x - 1 \right) \right\} \right]$$

$$W_{TL}^{\tau} = N_c \frac{\alpha}{\pi} e_q^4 \sqrt{1-\widetilde{\beta}^2} \left[\frac{1}{2\widetilde{\beta}^5} L \left(4x \left[\frac{P^2}{Q^2}(2x(x-1)(x-3) - 1) - \frac{2m^2}{Q^2}(x-1) \right] + (1-\widetilde{\beta}^2) \right.\right.$$
$$\times \left(\frac{m^2}{Q^2}(8x^2 - 8x - 2) + \frac{P^2}{Q^2}(8x^2 - 8x + 1) \right) + 2\frac{P^2}{Q^2}x(1-\widetilde{\beta}^2) \left[(1-x)(1-\beta^2) + x\beta^2 \frac{P^2}{Q^2} \right]$$
$$\left. \left. + (1-2x)^2 \right) - \frac{2\beta}{\widetilde{\beta}^4} \left(x - 1 + x\frac{P^2}{Q^2} \right) \left(3x - 1 - x(8x-3)\frac{P^2}{Q^2} \right) \right]$$

$$W_{TL}^{\tau a} = N_c \frac{\alpha}{\pi} e_q^4 \sqrt{1-\widetilde{\beta}^2}$$
$$\times \left[-\frac{1}{2\widetilde{\beta}^3} L \left\{ (1-\widetilde{\beta}^2) \left[1 - \beta^2(1-x) + x\beta^2 \frac{P^2}{Q^2} \right] - 2x \left(1 + \frac{2m^2 + P^2}{Q^2} \right) + 1 \right\} \right.$$
$$\left. - \frac{\beta}{\widetilde{\beta}^2} \left\{ (x-1) + x\frac{P^2}{Q^2} \right\} \right]$$


Q^2=(1000)^2 GeV^2, P^2=(10)^2 GeV^2, m_squark=300 GeV, m_gluino=700 GeV
Distribution Functions
Bjorken x
0
1
2
3
4
5
6
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
q_LNS
s_LNS
gluon
gluino
q_LS
s_LS
q_H
s_H