Generalized Parton Distributions of the Photon

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- Deeply Virtual Compton Scattering
- Generalized parton Distributions
- GPDs of the photon
- GPDs in position space
- Conclusions

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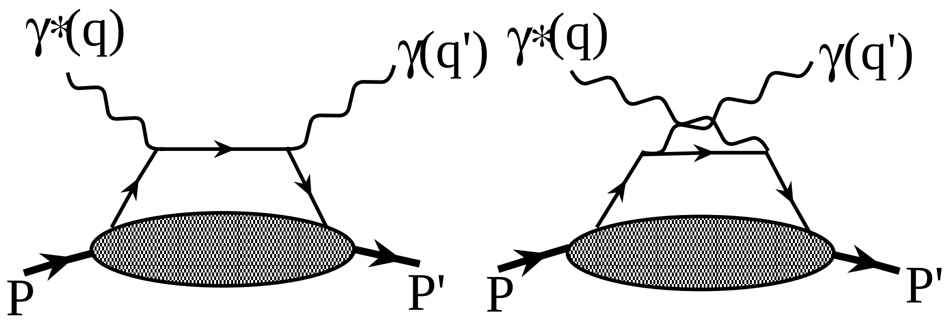
In collaboration with Sreeraj Nair

Deeply Virtual Compton Scattering (DVCS)



- $\gamma^* p \to \gamma p$ (exclusive process)
- Momentum of final proton P' different from that of the initial proton
- $Q^2 = -q^2$ large, squared momentum transfer $t = (P' - P)^2$ fixed
- Final photon $\to$ real ($q'^2 = 0$)

- Factorization applies : DVCS amplitude $\to$ short distance (perturbative part) $*$ 'soft part'

(generalized parton distributions)

DVCS Kinematics and Reference Frame

Consider the process $\gamma^*(q) + p(P) \to \gamma(q') + p(P')$

Component notation $V^{\pm} = V^0 \pm V^z$ and $V^2 = V^+V^- - (V^{\perp})^2$
Momentum transfer $\Delta = P - P'$, $t = \Delta^2$

Momenta of initial and final proton :

$$P = \left(P^+ \, , \, \vec{0}_\perp \, , \, \frac{M^2}{P^+} \right) \, , P' = \left((1-\zeta)P^+ \, , \, -\vec{\Delta}_\perp \, , \, \frac{M^2 + \vec{\Delta}_\perp^2}{(1-\zeta)P^+} \right)$$

$$t \; = \; 2P \cdot \Delta \; = \; -\frac{\zeta^2 M^2 + \vec{\Delta}_\perp^2}{1-\zeta}$$

$\zeta = \frac{Q^2}{2P \cdot q}$: skewness variable;
For DVCS, $-q^2 = Q^2$ is large compared to the masses and $\mid t \mid$

Choose a frame where the incident space-like photon carries $q^+ = 0$

DVCS (contd.)

DVCS amplitude

$$M^{IJ}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) = \epsilon^I_\mu \, \epsilon^{*J}_\nu \, M^{\mu\nu}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) = -e_q^2 \, \frac{1}{2\bar{P}^+} \int_{\zeta-1}^{1} \mathrm{d}x$$
$$\times \quad \Big\{ t^{IJ}(x,\zeta) \, \bar{U}(P') \Big[H(x,\zeta,t) \, \gamma^+ + E(x,\zeta,t) \, \frac{i}{2M} \, \sigma^{+\alpha}(-\Delta_\alpha) \Big] U(P) \Big\} ,$$

where $\bar{P} = \frac{1}{2}(P' + P)$,
x is the fraction of the proton momentum carried by the active quark
For circularly polarized initial and final photons

$$t^{\uparrow\uparrow}(x,\zeta) = \quad t^{\downarrow\downarrow}(x,\zeta) = \frac{1}{x - i\epsilon} + \frac{1}{x - \zeta + i\epsilon}$$

Contributions from longitudinally polarized photons are suppressed

$$F_{\lambda,\lambda'} = \int \frac{dy^-}{8\pi} \, e^{ixP^+y^-/2} \, \langle P', \lambda' | \bar{\psi}(0) \, \gamma^+ \, \psi(y) \, | P, \lambda \rangle \Big|_{y^+=0, y_\perp=0}$$
$$= \quad \frac{1}{2\bar{P}^+} \, \bar{U}(P', \lambda') \Big[H(x,\zeta,t) \, \gamma^+ + E(x,\zeta,t) \, \frac{i}{2M} \, \sigma^{+\alpha}(-\Delta_\alpha) \Big] U(P, \lambda),$$

DVCS contd.

DVCS amplitude contains

$$\int \frac{dy^-}{8\pi} \, e^{ixP^+y^-/2} \, \langle P', \lambda' | \bar{\psi}(0) \, \gamma^+ \, \psi(y) \, | P, \lambda \rangle \Big|_{y^+=0, y_\perp=0}$$

$$= \frac{1}{2\bar{P}^+} \, \bar{U}(P', \lambda') \left[H(x, \zeta, t) \, \gamma^+ + E(x, \zeta, t) \, \frac{i}{2M} \, \sigma^{+\alpha}(-\Delta_\alpha) \right] U(P, \lambda),$$

$H(x, \zeta, t)$ and $E(x, \zeta, t)$ are chiral even GPDs; as well as $\tilde{H}(x, \zeta, t)$ and $\tilde{E}(x, \zeta, t)$

$$\int \frac{dy^-}{8\pi} \, e^{ixP^+y^-/2} \, \langle P', \lambda' | \bar{\psi}(0) \, \gamma^+ \gamma_5 \, \psi(y) \, | P, \lambda \rangle \Big|_{y^+=0, y_\perp=0}$$

$$= \frac{1}{2\bar{P}^+} \, \bar{U}(P', \lambda') \left[\tilde{H}(x, \zeta, t) \, \gamma^+ \gamma_5 + \tilde{E}(x, \zeta, t) \, \frac{\gamma_5 \Delta^+}{2M} \right] U(P, \lambda),$$

Momentum transfer $\Delta = P - P'$, $t = \Delta^2$, $\zeta = \frac{\Delta^+}{P^+}$

x is the fraction of the proton momentum carried by the active quark

Generalized Parton Distributions : Properties

- GPDs $H(x,\zeta,t)$, $E(x,\zeta,t)$, $\tilde{H}(x,\zeta,t)$ and $\tilde{E}(x,\zeta,t)$ contribute at leading order in $1/Q$
- In the forward limit, $H(x,0,0) = q(x)$ (unpol. quark distribution) and $\tilde{H}(x,0,0) = \Delta q(x,0,0)$ (polarized quark distribution)
- No simple relation for E and $\tilde{E}$ in the forward limit
- Moments of GPDs give nucleon form factors

$$\int_{-1}^{1} dx H(x,\zeta,t) = F_1(t); \qquad \int_{-1}^{1} dx E(x,\zeta,t) = F_2(t)$$
$$\int_{-1}^{1} dx \tilde{H}(x,\zeta,t) = g_A(t); \qquad \int_{-1}^{1} dx \tilde{E}(x,\zeta,t) = g_P(t)$$

- Second moment [X. Ji (1997)]

$$\int_{-1}^{1} dx x [H_q + E_q] = A_q(t) + B_q(t)$$

$J_{q,g} = \frac{1}{2}[A_{q,g}(0) + B_{q,g}(0)]; \qquad J_q + J_g = \frac{1}{2}$

- $J_{q,g} \rightarrow$ Fraction of the nucleon spin carried by quarks (gluons)

Why GPDs/DVCS are Interesting ??

- Initial proton momentum differs from final proton momentum : GPDs no longer represent squared amplitudes (like ordinary pdfs) and thus do not have a probability interpretation.

- Richer in content about proton structure than pdfs

- GPDs give a unified picture of both exclusive and inclusive processes

- Second moment of the GPDs gives the fraction of the nucleon spin carried by the quarks.

- Polarization of the target may change due to the scattering : rich spin structure of the GPDs $\longrightarrow$ still unknown orbital angular momentum of the partons (quarks, gluons).

- Momentum transfer has a transverse component; leads to information about the transverse structure of the target

M. Burkardt, 2000

- 3D spatial structure of the nucleon in terms of the GPDs

X. Ji, 2003

- Being measured in experiments worldwide : JLab, DESY (HERA and HERMES), CERN (COMPASS).

Generalized parton distributions of the photon

- Consider DVCS process on a photon $\gamma^*(Q)\gamma \to \gamma\gamma$
- At leading order in α and zeroth order in α_s it was shown that the amplitude factorizes
- Upto leading log terms it can be written in terms of the GPDs of the photon
- Mometum transfered between the initial state and the final state photon is purely in the longitudinal direction

S. Friot, B. Pire, L. Szymanowski, Phys. Lett. **B 645** 153 (2007)

GPDs of the photon are defined as

$$F^q = \int \frac{dy^-}{8\pi} e^{\frac{-iP^+y^-}{2}} \langle \gamma(P') \mid \bar{\psi}(0)\gamma^+\psi(y^-) \mid \gamma(P)\rangle;$$

$$\tilde{F}^q = \int \frac{dy^-}{8\pi} e^{\frac{-iP^+y^-}{2}} \langle \gamma(P') \mid \bar{\psi}(0)\gamma^+\gamma^5\psi(y^-) \mid \gamma(P)\rangle.$$

Chosen light-front gauge $A^+ = 0$

$\tilde{F}^q$ can be calculated from terms of the form $\epsilon^2_\lambda \epsilon^{1*}_\lambda - \epsilon^1_\lambda \epsilon^{2*}_\lambda$ in the amplitude

Generalized parton distributions of the photon

We first consider the momentum transfer to be purely in the transverse direction; $t = (P - P')^2 = -(\Delta^\perp)^2$ where P and P' are the momenta of the initial and final (real) photon respectively

Skewness ζ is zero

GPDs of the photon can be expressed in terms of the photon light-front wave functions

$$\begin{aligned} F^q &= \int d^2q^\perp dx_1 \delta(x - x_1) \psi_2^*(x_1, q^\perp - (1 - x_1)\Delta^\perp) \psi_2(x_1, q^\perp) \\ &\quad - \int d^2q^\perp dx_1 \delta(1 + x - x_1) \psi_2^*(x_1, q^\perp + x_1 \Delta^\perp) \psi_2(x_1, q^\perp) \end{aligned}$$

Two-particle light-front wave function corresponds to $q\bar{q}$

First term corresponds to active quark and the second term antiquark

GPDs of the photon

Two particle light-front wave functions can be calculated in perturbation theory

We get

$$
\begin{aligned}
F^q \quad = \quad & \sum_q \frac{\alpha e_q^2}{4\pi^2}\Big[((1-x)^2+x^2)(I_1+I_2+LI_3)+2m^2 I_3\Big]\theta(x)\theta(1-x) \\
& -\sum_q \frac{\alpha e_q^2}{4\pi^2}\Big[((1+x)^2+x^2)(I_1'+I_2'+L'I_3')+2m^2 I_3'\Big]\theta(-x)\theta(1+x)
\end{aligned}
$$

Sum indicates sum over different quark flavors

$L = -2m^2 + 2m^2 x(1-x) - (\Delta^\perp)^2 (1-x)^2$,

$L' = -2m^2 - 2m^2 x(1+x) - (\Delta^\perp)^2 (1+x)^2$

The integrals can be written as

$$
I_1 = \int \frac{d^2 q^\perp}{D} = \pi Log\Big[\frac{\Lambda^2}{\mu^2 - m^2 x(1-x) + m^2}\Big] = I_2
$$

$$
I_3 = \int \frac{d^2 q^\perp}{DD'} = \int_0^1 d\alpha \frac{\pi}{P(x,\alpha,(\Delta^\perp)^2)}
$$

GPDs of the photon

where $D = (q^{\perp})^2 - m^2 x(1-x) + m^2$ and
$D' = (q^{\perp})^2 + (\Delta^{\perp})^2 (1-x)^2 - 2q^{\perp} \cdot \Delta^{\perp}(1-x) - m^2 x(1-x) + m^2$, and
$P(x, \alpha, (\Delta^{\perp})^2) = -m^2 x(1-x) + m^2 + \alpha(1-\alpha)(1-x)^2 (\Delta^{\perp})^2$

Zeroth order in α_s results are scale dependent; this scale dependence in our approach comes from the upper limit of the transverse momentum integration $\Lambda = Q$

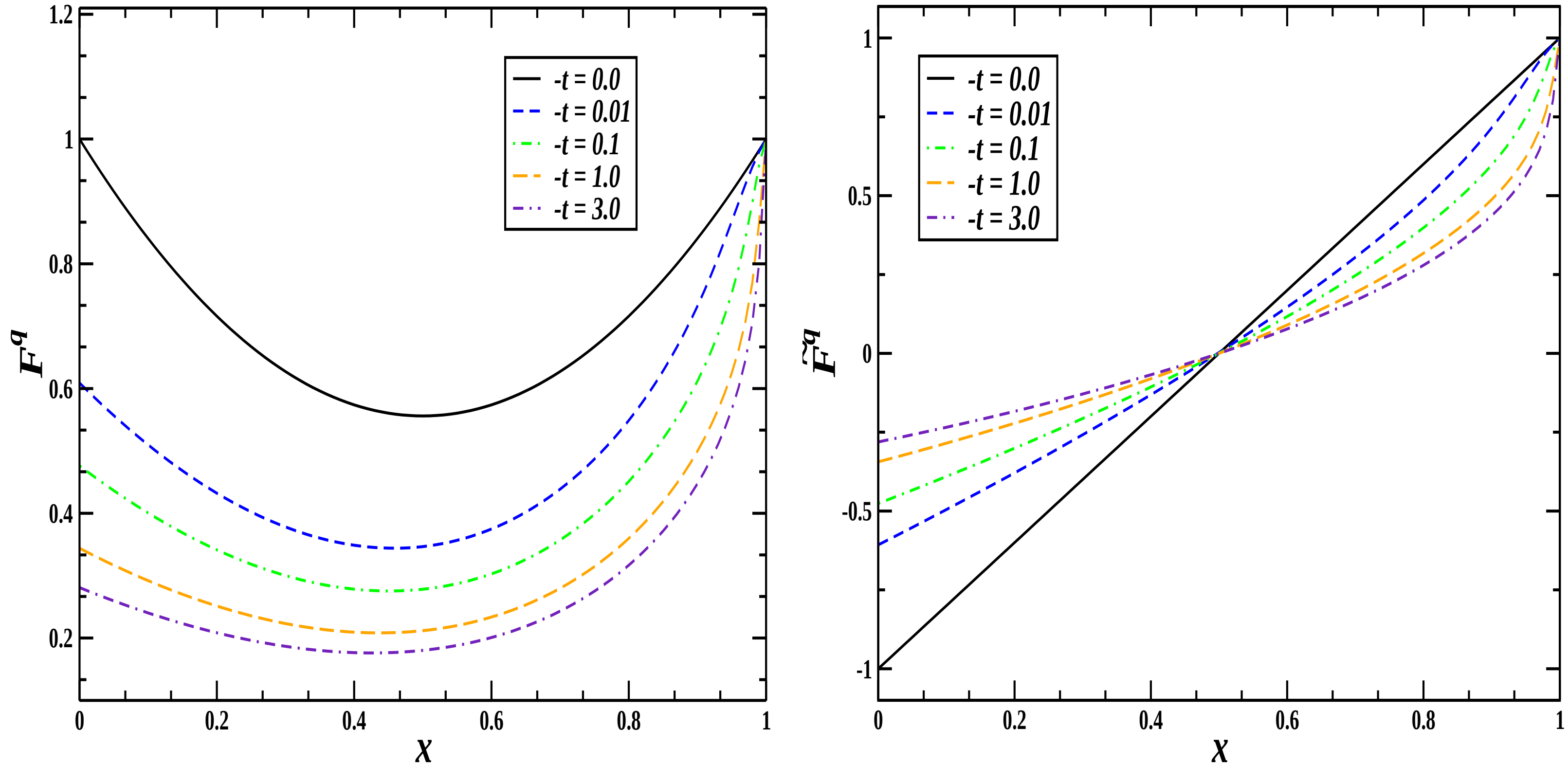
For the antiquark contributions

$$I'_1 = \int \frac{d^2 q^{\perp}}{H} = \pi Log\left[\frac{\Lambda^2}{\mu^2 + m^2 x(1+x) + m^2}\right] = I'_2$$

$$I'_3 = \int \frac{d^2 q^{\perp}}{H H'} = \int_0^1 d\alpha \frac{\pi}{Q(x, \alpha, (\Delta^{\perp})^2)}$$

$H = (q^{\perp})^2 + m^2 x(1+x) + m^2$ and $H' = (q^{\perp})^2 + (\Delta^{\perp})^2 (1+x)^2 + 2q^{\perp} \cdot \Delta^{\perp}(1+x) +$ $m^2 x(1+x) + m^2$, and $Q(x, \alpha, (\Delta^{\perp})^2) = m^2 x(1+x) + m^2 + \alpha(1-\alpha)(1+x)^2 (\Delta^{\perp})^2$

GPDs of the photon



- Only the quark contribution is shown
- We took $Q = \Lambda = 20 GeV$
- As $x \to 1$, most of the momentum is carried by the quark in the photon and the GPDs become independent of t

AM, S. Nair (2011)

GPDs of the photon in impact parameter space

Fourier transform with respect to the transverse momentum transfer $\Delta_\perp$ gives GPDs in impact parameter space

Burkardt (2000)

$$
\begin{aligned}
q(x,b) &= \frac{1}{(2\pi)^2}\int d^2\Delta^\perp e^{-i\Delta^\perp\cdot b^\perp} F^q(x,t) \\
&= \frac{1}{2\pi}\int \Delta d\Delta J_0(\Delta b) F^q(x,t),
\end{aligned}
$$

$$
\begin{aligned}
\tilde{q}(x,b) &= \frac{1}{(2\pi)^2}\int d^2\Delta^\perp e^{-i\Delta^\perp\cdot b^\perp} \tilde{F}^q(x,t) \\
&= \frac{1}{2\pi}\int \Delta d\Delta J_0(\Delta b) \tilde{F}^q(x,t),
\end{aligned}
$$

where $\Delta = |\Delta^\perp|$ and $b = |b^\perp|$

GPDs of the photon in impact parameter space

Introduced in analogy with the impact parameter dependent parton distribution of the proton

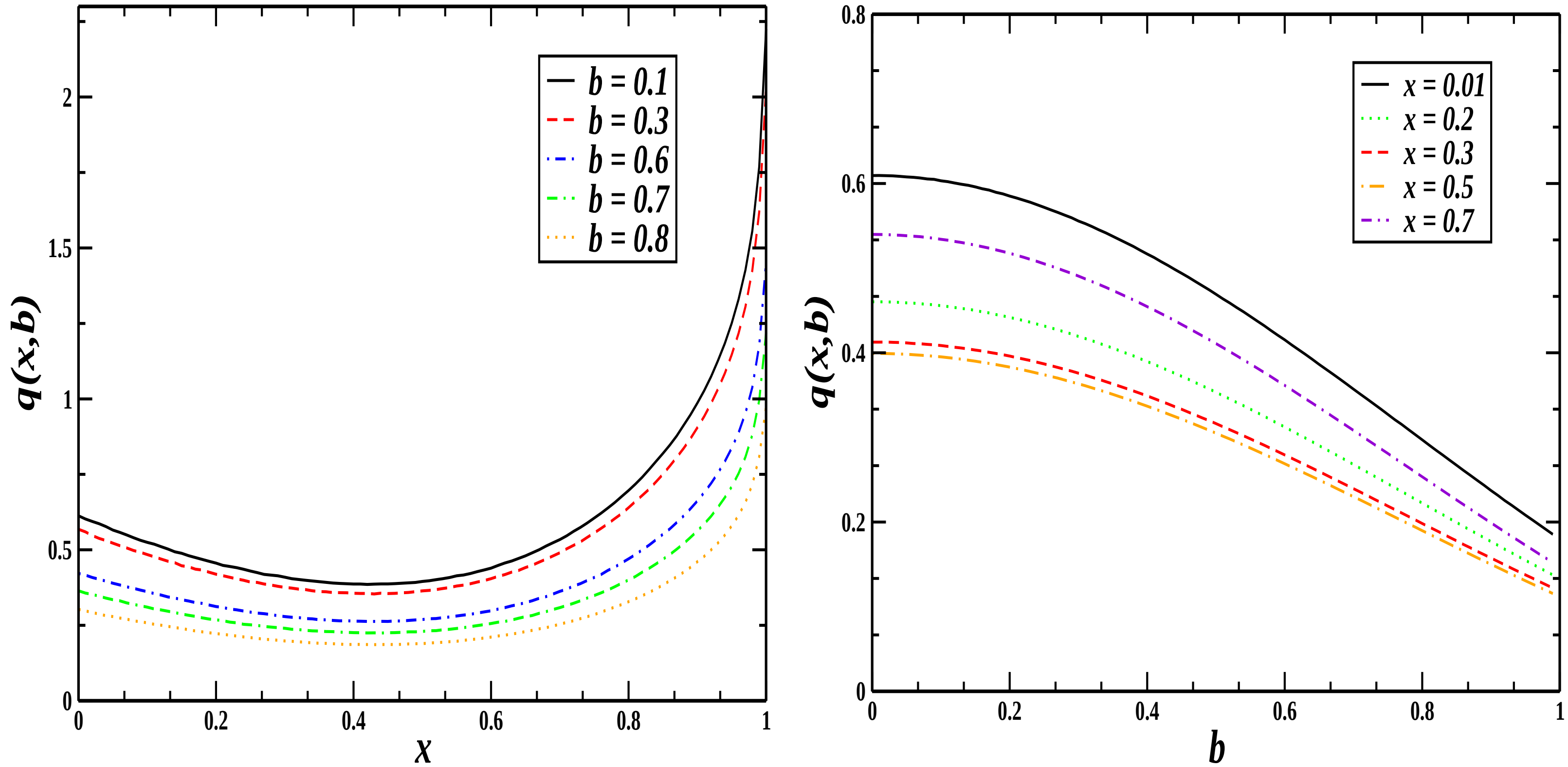
Have probabilistic interpretation : $q(x, b)$ is the probability of finding a quark of momentum fraction x and at transverse distance b from the center of the photon : parton distributions of the photon in the transverse plane

Impact parameter distribution for a polarized photon is given by $\tilde{q}(x, b^{\perp})$

New insight to the transverse 'shape' of the photon

Used a cutoff on $\Delta^{\perp}$ integration satisfying $t << Q^2$; DVCS kinematics

GPDs of the photon in impact parameter space

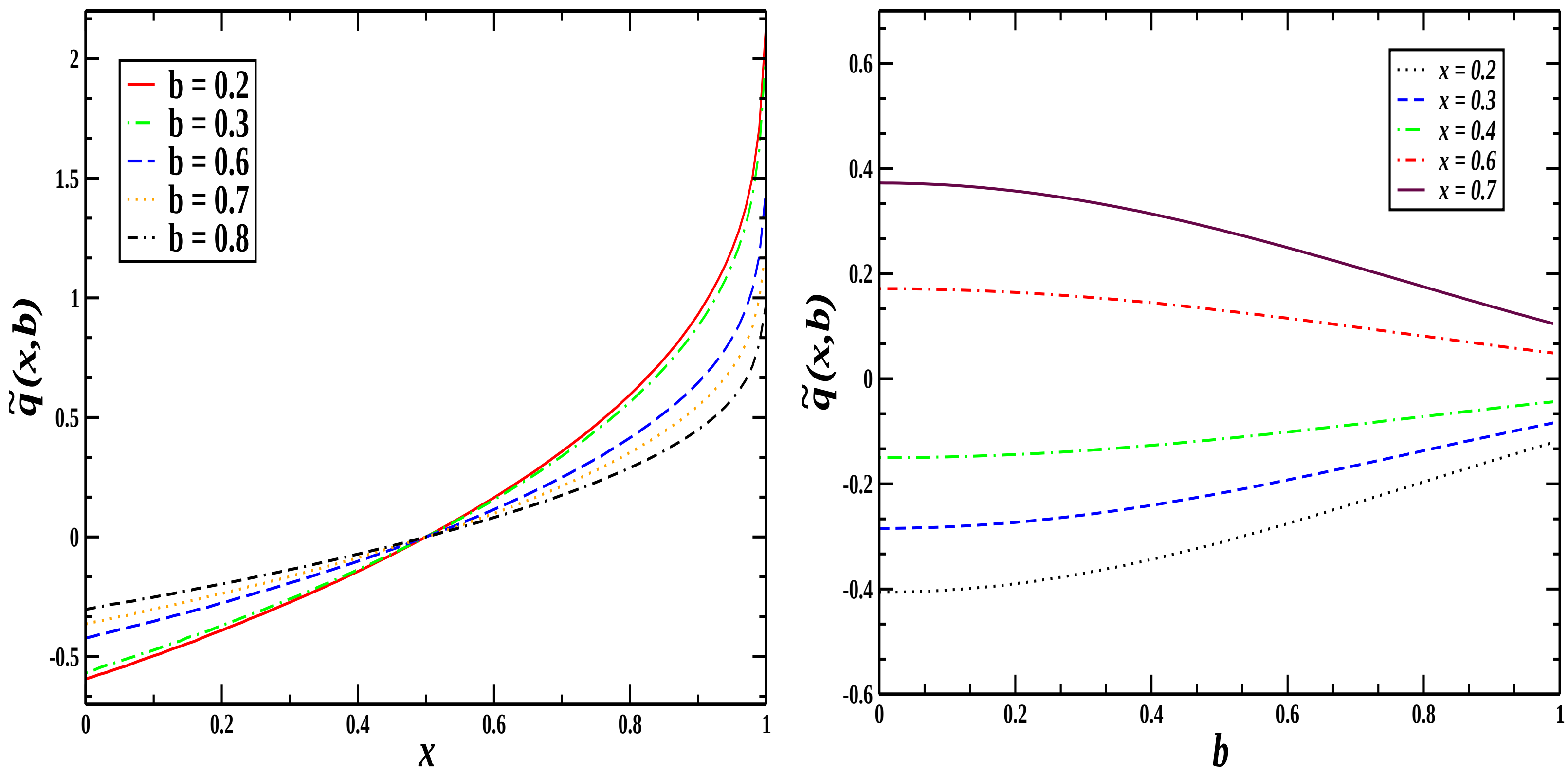


We have taken Λ = 20 GeV and Δ_{max}= 3 GeV where Δ_{max} is the upper limit in the Δ integration. b is in GeV^{-1} and $q(x,b)$ is in GeV^2

In the ideal definition the Fourier transform over Δ should be from 0 to ∞. In this case the $\Delta^\perp$ independent terms in F^q and $\tilde{F}^q$ would give $\delta^2(b^\perp)$ in the impact parameter space

This means in the case of no transverse momentum transfer, the photon behaves like a point particle in transverse position space. The distribution in transverse space is a unique feature accessible only when there is non-zero momentum transfer in the transverse direction

GPDs of the photon in impact parameter space

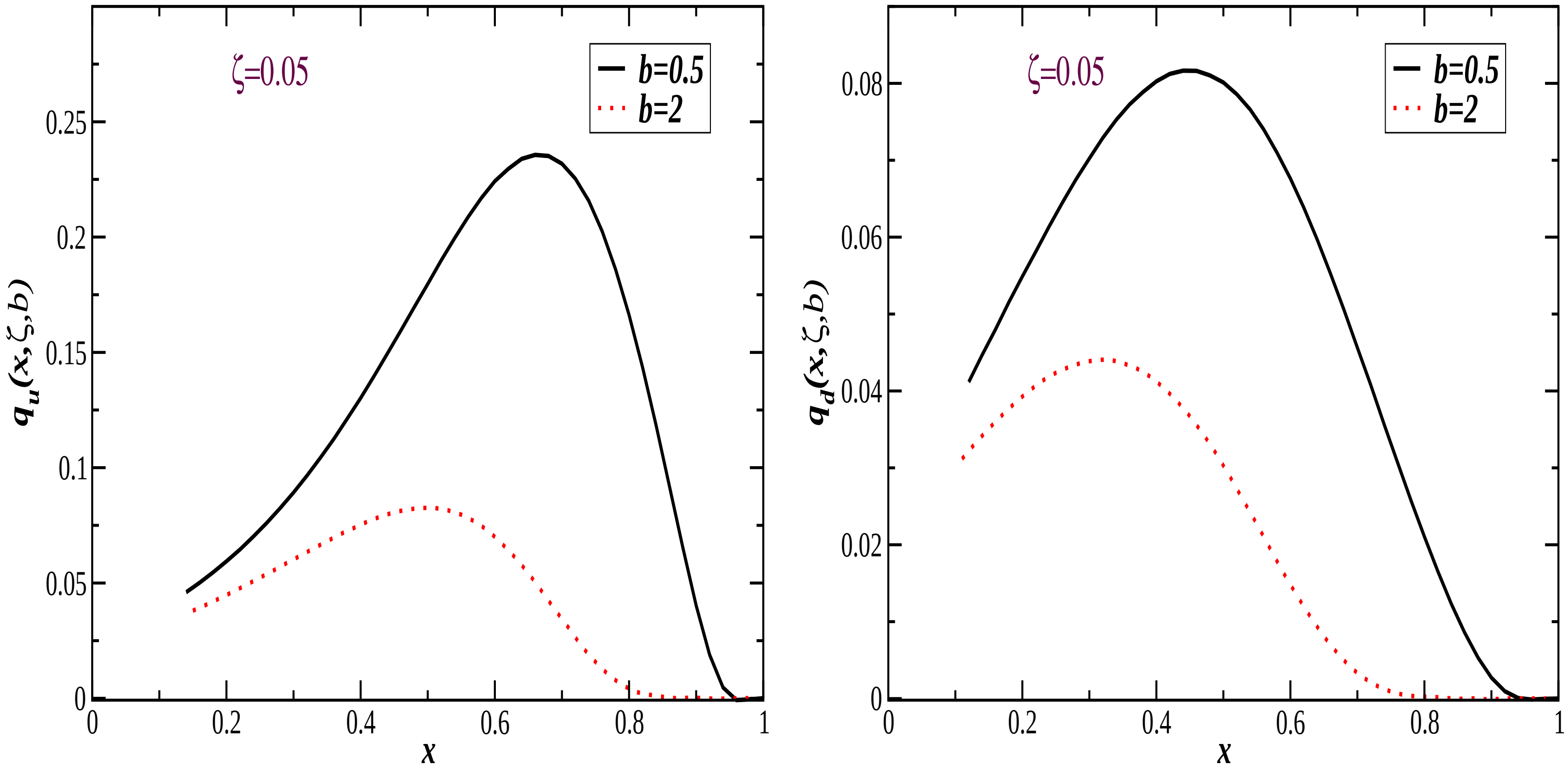


The behavior in impact parameter space is qualitatively different phenomenological models of proton GPDs

Parton distribution is more dispersed when the q and $\bar{q}$ share almost equal momenta

AM, S. Nair (2011)

GPD model of the proton in impact parameter space



- Used parametrization of

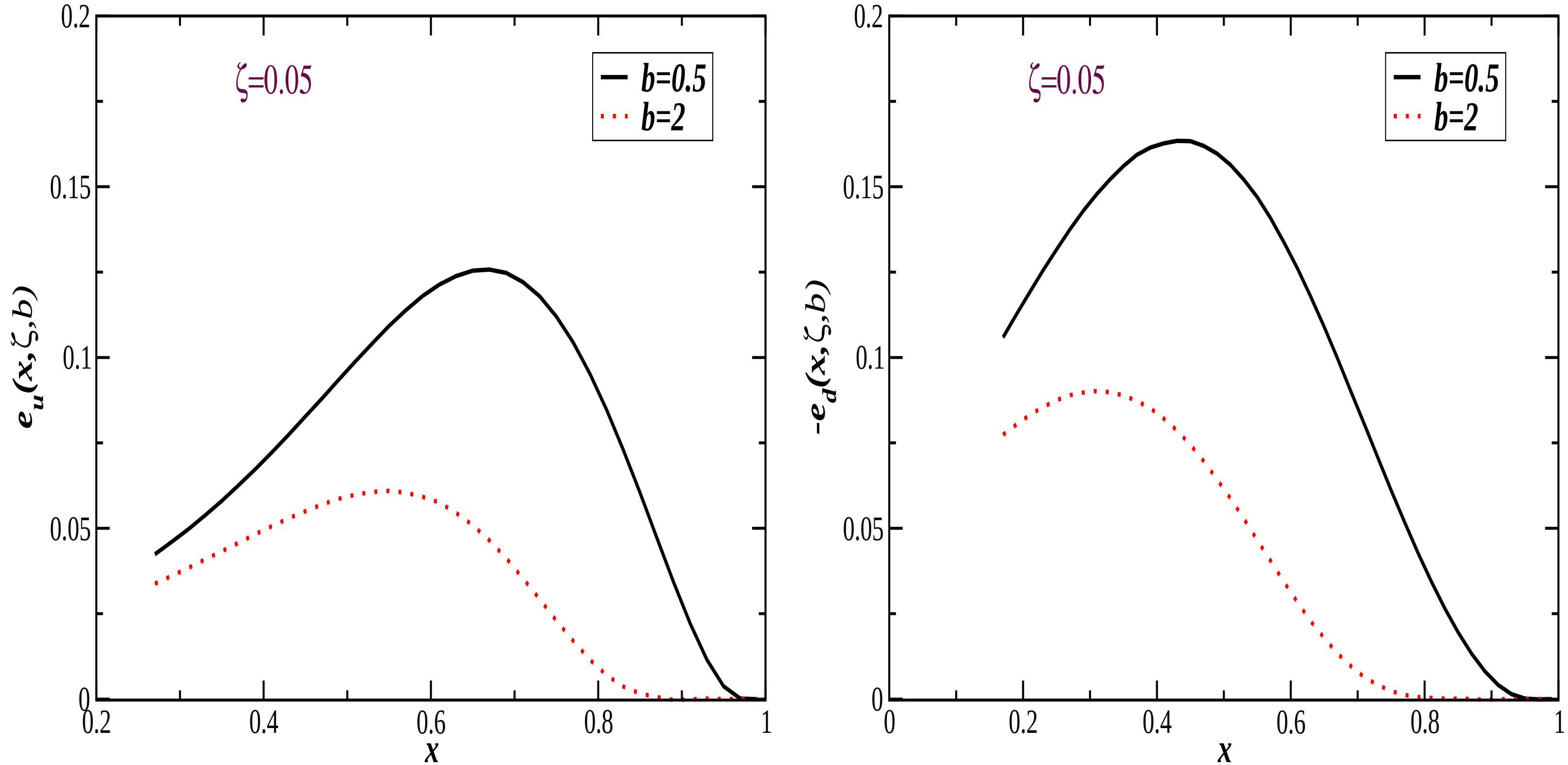
Ahmad, Honkanen, Liuti, Taneja (2009)

- Parametrization obtained by simultaneously fitting the experimental data on nucleon form factor and DIS structure functions; generalized form for non-zero skewness

- Spectator model with Regge-type term at input scale

Manohar, Mukherjee, Chakrabarti, PRD (2011)

GPD model of the proton in impact parameter space



- Used parametrization of

Ahmad, Honkanen, Liuti, Taneja (2009)

- We have used parametrizations (u and d quark) at scale 0.09 GeV^2

Manohar, Mukherjee, Chakrabarti, PRD (2011)

Non-zero Skewness ζ

More general case : momentum transfer between the initial and final photon has both transverse and longitudinal components

$$
\begin{aligned}
q(x,\zeta,b) &= \frac{1}{(2\pi)^2}\int d^2\Delta^\perp e^{-i\Delta^\perp\cdot b^\perp} F^q(x,\zeta,t) \\
&= \frac{1}{2\pi}\int \Delta d\Delta J_0(\Delta b) F^q(x,\zeta,t),
\end{aligned}
$$

$$
\begin{aligned}
\tilde{q}(x,\zeta,b) &= \frac{1}{(2\pi)^2}\int d^2\Delta^\perp e^{-i\Delta^\perp\cdot b^\perp} \tilde{F}^q(x,\zeta,t) \\
&= \frac{1}{2\pi}\int \Delta d\Delta J_0(\Delta b) \tilde{F}^q(x,\zeta,t),
\end{aligned}
$$

We took the frame

$$
\begin{aligned}
P &= \Big(P^+ \,,\, 0^\perp \,,\, 0 \Big) \,, \\
P' &= \left((1-\zeta)P^+ \,,\, -\Delta^\perp \,,\, \frac{\Delta^{\perp 2}}{(1-\zeta)P^+} \right) \,,
\end{aligned}
$$

Non-zero Skewness ζ

Repeat the calculation of F^q and $\tilde{F}^q$ when both ζ and $\Delta^\perp$ are non-zero

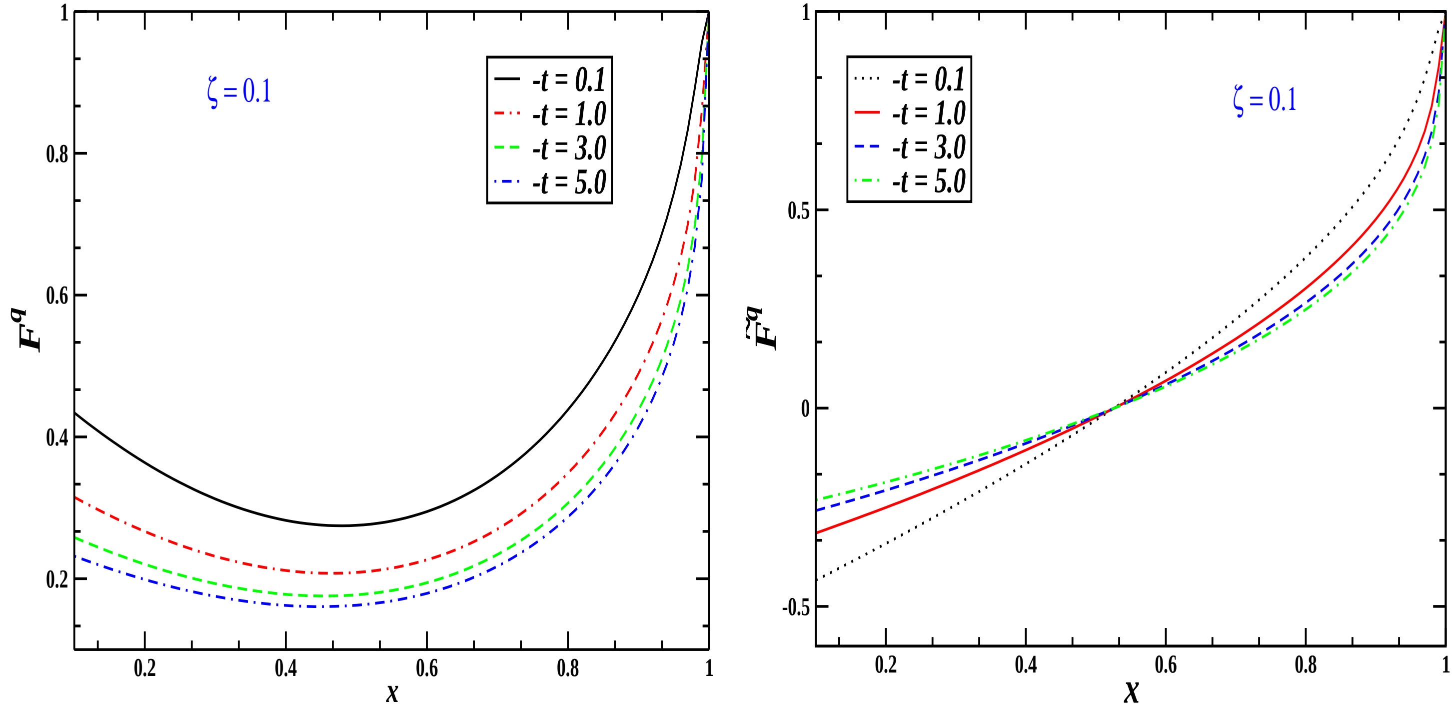
We limit ourselves to the kinematical region $1 > x > \zeta$ and $-1 < x < \zeta - 1$ where only the two-particle LFWFs contribute

When the skewness ζ is non-zero, GPDs in impact parameter space do not have a probabilistic interpretation

They are still interesting as they now probe the partons when the initial photon is displaced from the final photon in the transverse impact parameter space. This relative shift does not vanish when the GPDs are integrated over x in the amplitude

Diehl(2002)

Photon GPDs for non-zero ζ

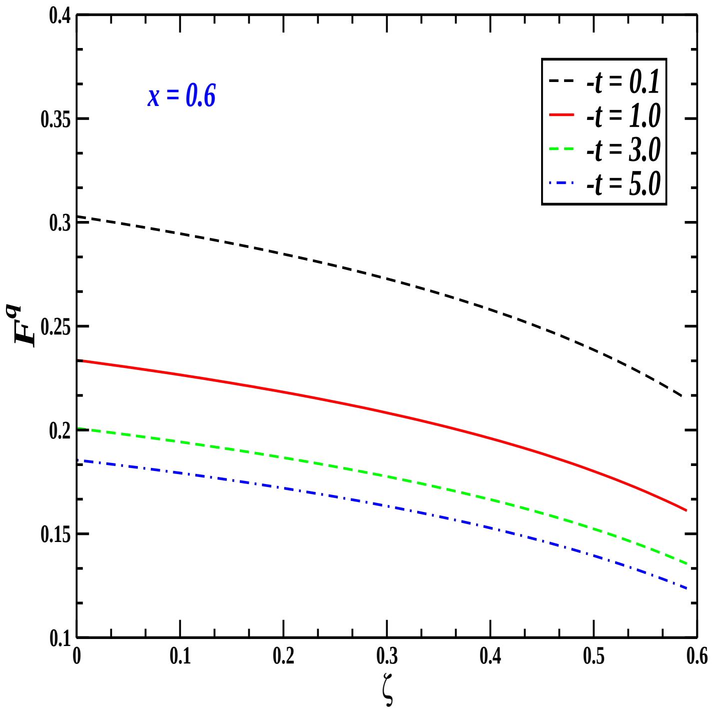


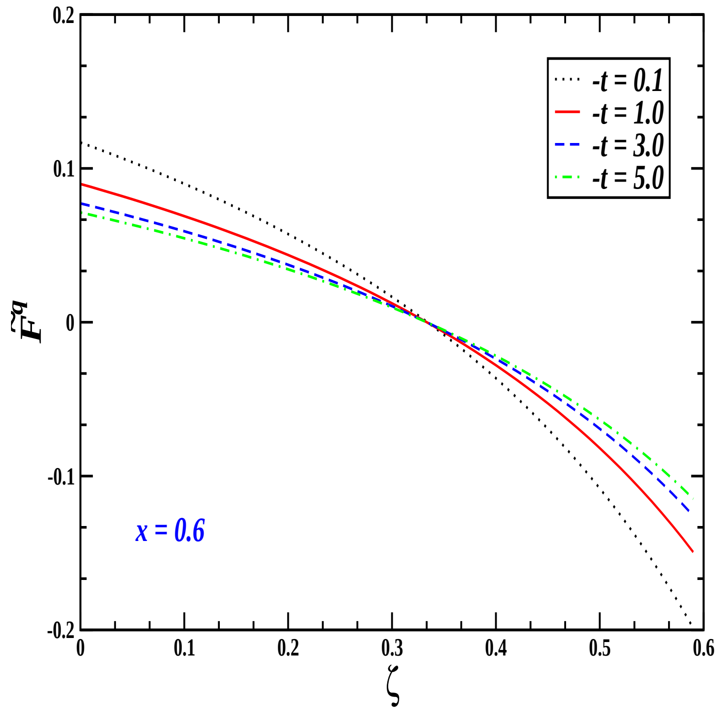
At leading order in α and zeroth order in α_s the results are logarithmically dependent on the scale

We took a fixed scale $\Lambda = 20\mathrm{GeV}$

AM, S. Nair (In Preparation)

Photon GPDs for non-zero ζ





We took a fixed scale $\Lambda = 20\mathrm{GeV}$

Helicity non-flip contributions; $x > \zeta$ region

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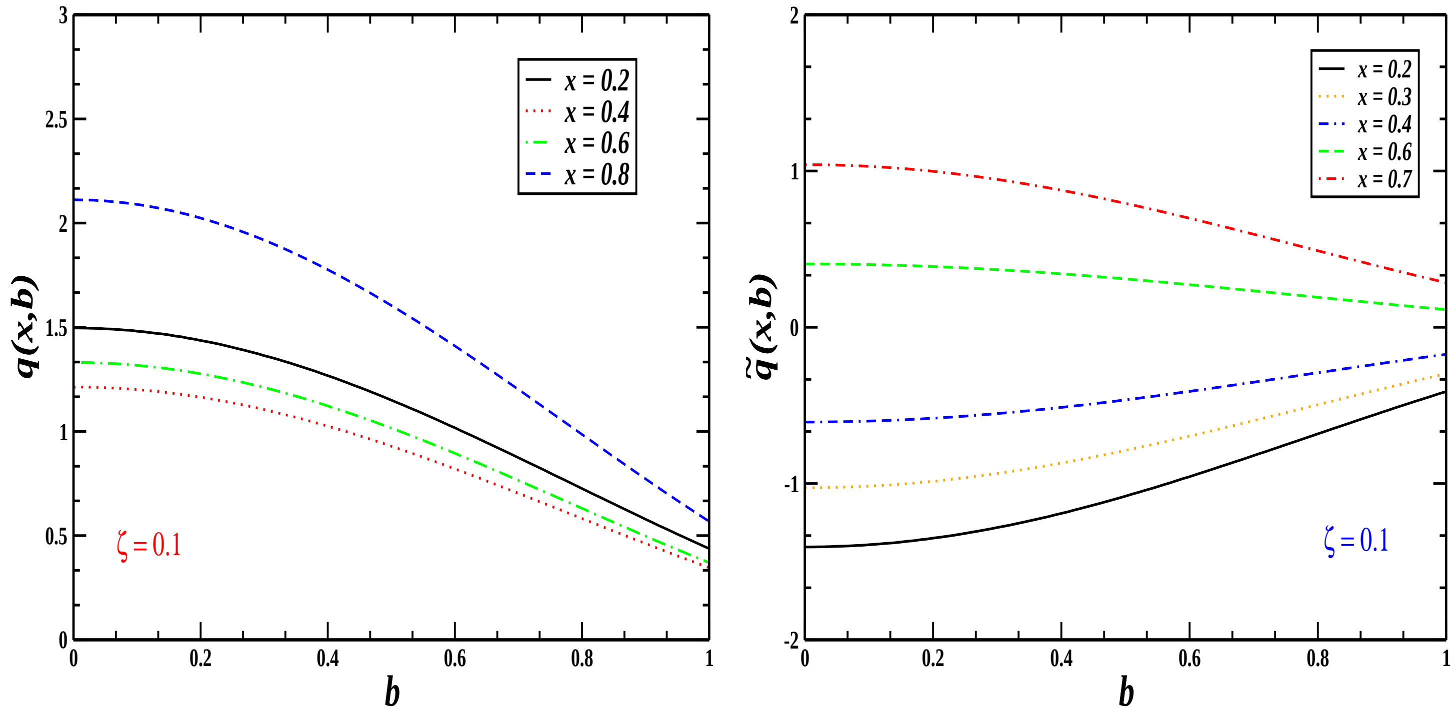
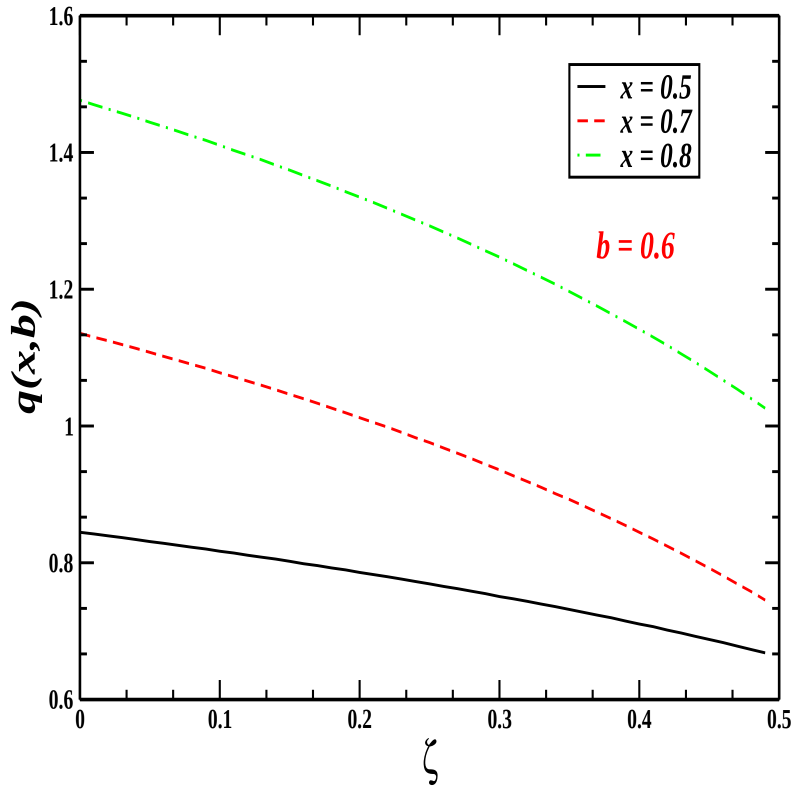
Photon GPDs for non-zero ζ

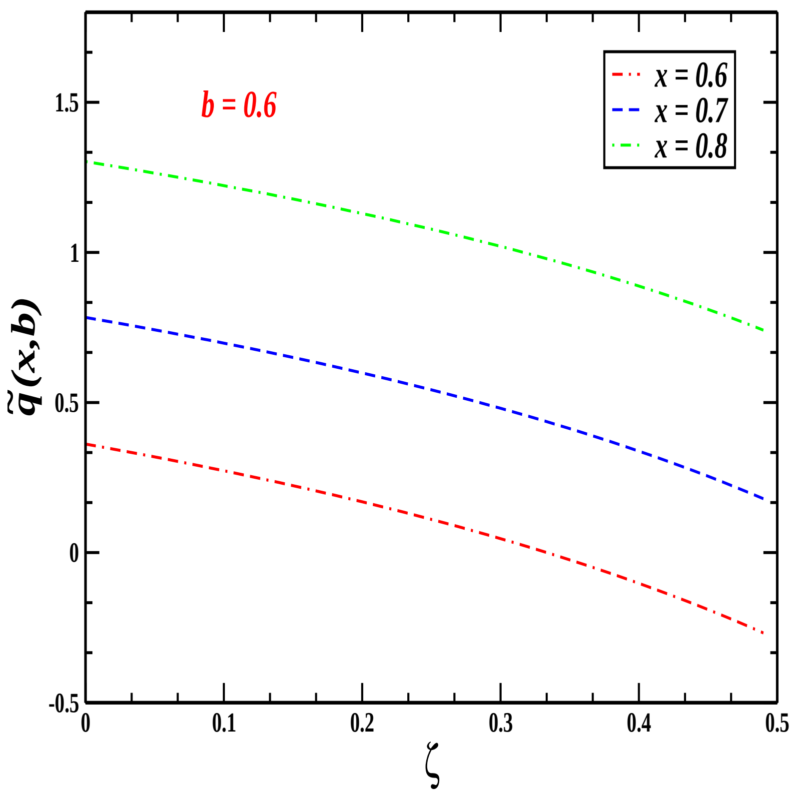


We have taken Λ = 20 GeV and Δ_{max}= 3GeV where Δ_{max} is the upper limit in the Δ integration. b is in GeV^{-1}

AM, S. Nair (In Preparation)

Photon GPDs for non-zero ζ





we have taken Λ = 20 GeV and Δ_{max}= 3GeV where Δ_{max} is the upper limit in the Δ integration. b is in GeV^{-1}

AM, S. Nair (In Preparation)

GPDs in Longitudinal Position Space

Connection with Wigner distribution : Belitsky, Ji, Yuan (2003)

DVCS amplitude for the proton in longitudinal position space : analogy with diffraction pattern in optics

Brodsky, Chakrabarti, Harindranath, Mukherjee, Vary (2006)

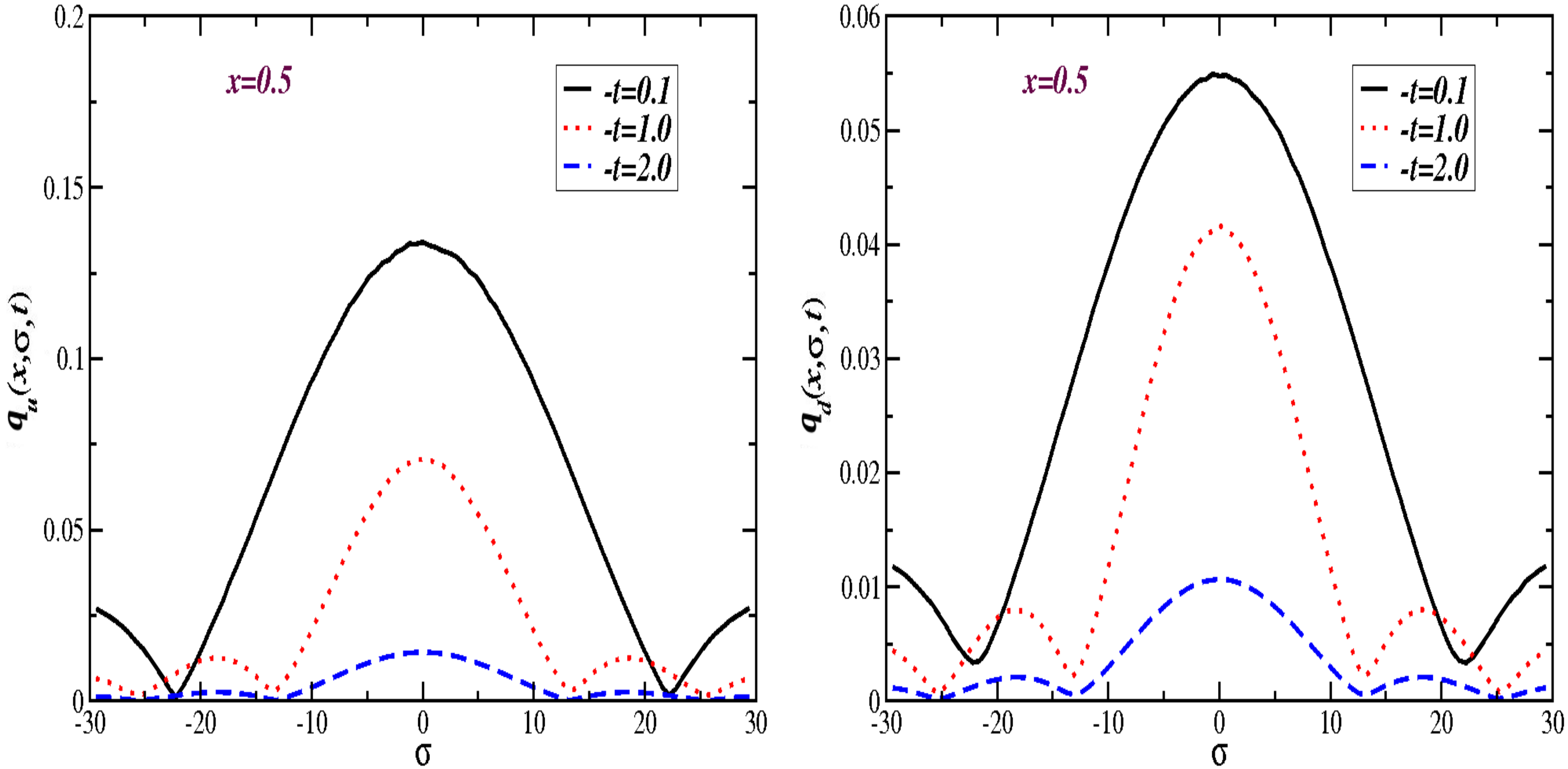
We define a boost invariant impact parameter conjugate to the longitudinal momentum transfer as $\sigma = \frac{1}{2} b^- P^+$

$$\begin{aligned} \mathcal{F}(x, \sigma, t) &= \frac{1}{2\pi} \int_0^{\zeta_f} d\zeta e^{i \frac{1}{2} P^+ \zeta b^-} H(x, \zeta, t) \\ &= \frac{1}{2\pi} \int_0^{\zeta_f} d\zeta e^{i \sigma \zeta} F(x, \zeta, t). \end{aligned}$$

Upper limit is the maximum ζ value allowed for fixed $-t$

To get the complete picture both $x > \zeta$ and $x < \zeta$ contributions will have to be considered

Proton GPDs in longitudinal position space



Diffraction pattern somewhat similar to that in optics is observed with well defined maxima and minima

Pattern with similar general behaviour is observed in other models

The diffraction pattern remains when both contributions $x > \zeta$ and $x < \zeta$ are taken into account in the DVCS amplitude

Manohar, Mukherjee, Chakrabarti, PRD (2011)

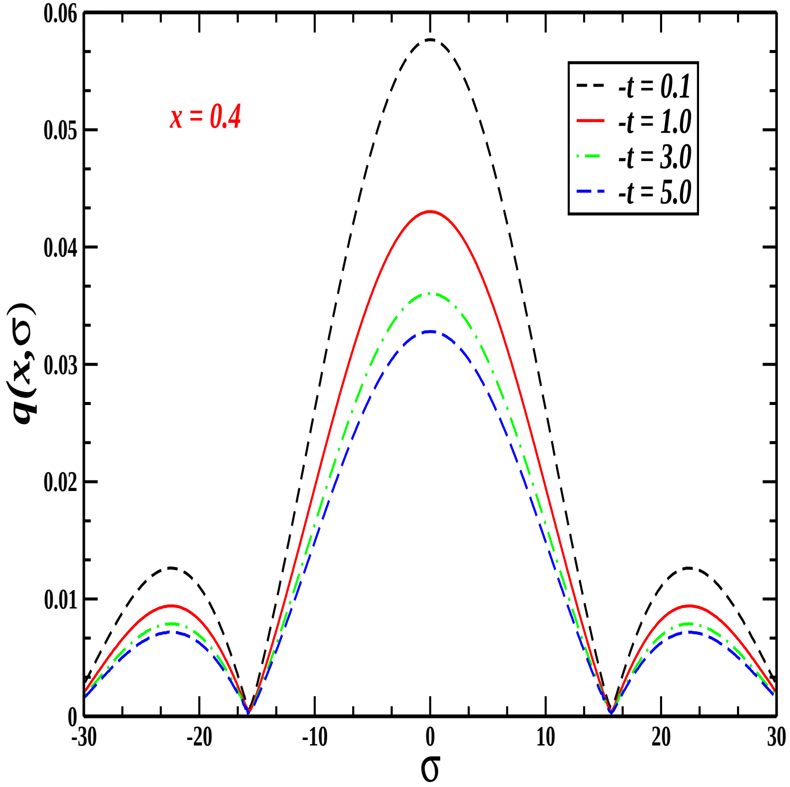
Optics Analog

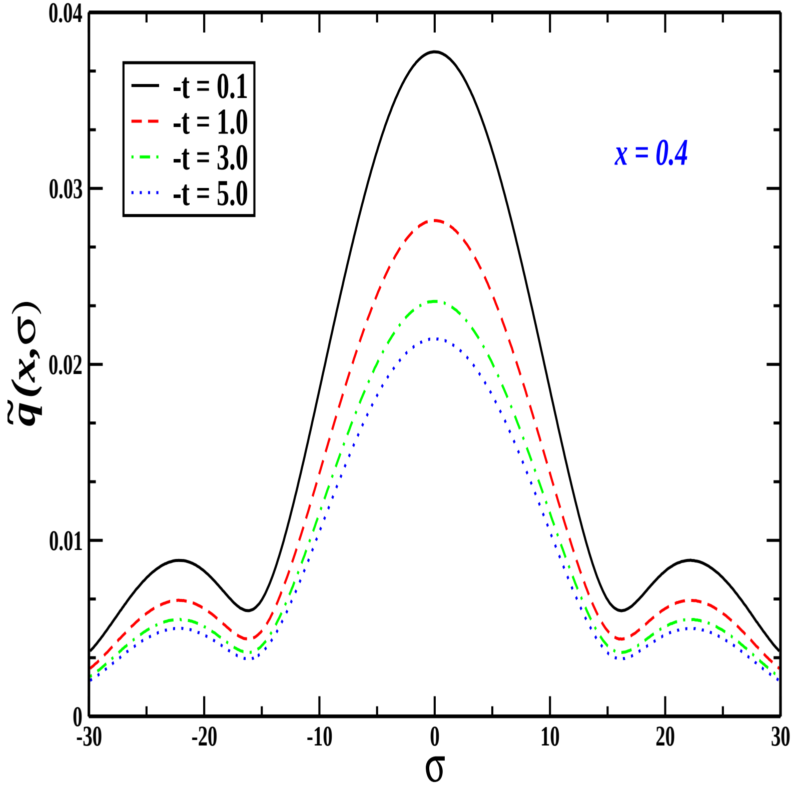
(i) Finite range of ζ integration act as a slit of finite width and provides a necessary condition for the occurrence of diffraction pattern in the Fourier transform
(ii) In analogy with optical diffraction, where the positions of the first minima are inversely proportional to the slit width, here we expect their positions to be inversely proportional to ζ_{max}. Since ζ_{max} increases with -t, the position of the first minimum moves to a smaller value of σ
(iii) For fixed $-t$, higher minima appear at positions which are integral multiples of the lowest minimum : in analogy with diffraction in optics
(iv) However the appearance of the pattern also depends on the x, ζ interplay in the GPD of the proton

- Scattering photons in DVCS provides the complete Lorentz-invariant light front coordinate space structure of a hadron

Brodsky,Chakrabarti,Harindranath,AM,Vary;2006,2007.

Photon GPDs in longitudinal position space





No prominent diffraction pattern in $\tilde{F}$
Position of the minima does not depend on t
For a complete analysis also $x < \zeta$ contribution needs to be taken into account

AM, S. Nair (In Preparation)

Discussions

•Presented a first calculation of the generalized parton distributions of the photon, both polarized and unpolarized, when the momentum transfer in the transverse direction is non-zero; at zeroth order in α_s and leading order in α

•We calculated at leading logarithmic order and also kept the mass terms at the vertex. The GPDs are logarithmically dependent on the scale

• Considered both quark and the antiquark contributions : GPDs probe the two-particle $q\bar{q}$ structure of the photon

• Taking a Fourier transform (FT) with respect to $\Delta_\perp$ we obtain impact parameter dependent parton distribution of the photon : distinctive features compared to the proton

• Taking the Fourier transform with respect to the skewness we express the GPDs in longitudinal coordinate space : diffraction pattern similar to optics

•It is to be noted that a complete understanding of the photon GPDs beyond leading logs would require also the non-pointlike hadronic contributions which will be model dependent. However, the GPDs of the photon calculated here may act as interesting tools to understand the partonic substructure of the photon. Accessing them in experiment is a challenge