RADCOR2011

The GRACE project

QCD   SUSY   Multi-loop

KEK   Junpei FUJIMOTO
on behalf of GRACE group

Plan

1. Introduction
2. QCD application
3. SUSY application
4. Multi-loop application
5. Summary

1. Introduction

GRACE is the generator of event generators

- Feynman rules based on SM, MSSM.
- Any orders of Feynman diagrams can be generated automatically
- 1-loop diagrams of SM and MSSM can be evaluated.
- 1-loop integrals up to 4-point functions are equipped.
- 1-loop integrals up to 6-point functions are evaluated with reduction method.
- 2-loop integrals up to 4-point functions are evaluated numerically.

GRACE Members

KEK : Y. Kurihara, T. Kaneko, T. Ishikawa, F. Yuasa,
Y. Shimizu, N. Hamaguchi, J. F.

M.G.U. : M. Kuroda,

Kogakuin U. : K. Kato, N. Nakazawa, K. Tobimatsu

Chiba U. C. : M. Jimbo,

Seikei U. : T. Kon, T. Inoue, T. Jujo,
T. Koike, H. Kataoka

T. M. C. : Y. Yasui

2. QCD application GR@PPA for LHC

Y. Kurihara, S. Odaka, S. Tsuno, K. Kato, T. Ishikawa, Y. Yasui

- pp collision processes generated with GRACE
- LO(v2.8)
- some NLO (v3.0 coming soon)
- M.E. & Parton Shower with matching scheme
- Interface to PYTHIA, HERWIG through LHA format

GR@PPA 2.8

released on Nov. 17, 2010

http://atlas.kek.jp/physics/nlo-wg/grappa.html

Les Houches 2011
S. Odaka and Y. Kurihara

- Tree-level event generation with an initial-state jet matching for single ($W^{\pm}$, Z) and double ($W^{+}W^{-}$, $ZW^{\pm}$, ZZ) weak-boson production processes at hadron collisions
  - Smooth p_T spectrum of the weak-boson system from p_T = 0 up to the phase-space limit
  - Finite at $p_T = 0$, and matches the 1-jet matrix elements at high p_T
  - Suitable for efficiency/acceptance evaluations
- Other features:
  - $W^{\pm}/Z$ decays are included in the matrix elements.
    - Exact phase-space and spin effects at the tree level
  - Finite decay widths of $W^{\pm}$ and Z

Z production at Tevatron

GR@PPA 2.8 + PYTHIA 6.4

PYTHIA for simulating soft PS ($1.0 < Q < 4.6$ GeV), primordial k_T ($\langle k_T \rangle = 2.0$ GeV/c), hadronization, and decays

Histograms: simulation

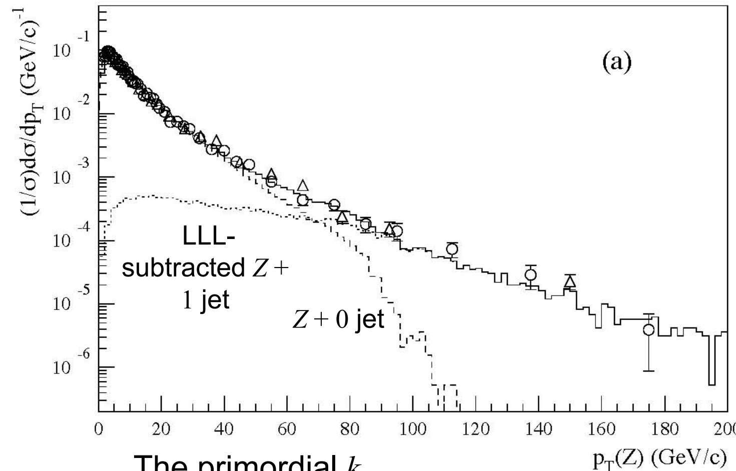
Circles: CDF, Phys. Rev. Lett. 84, 845 (2000)

Triangles: D0, Phys. Rev. D 61, 032004 (2000)

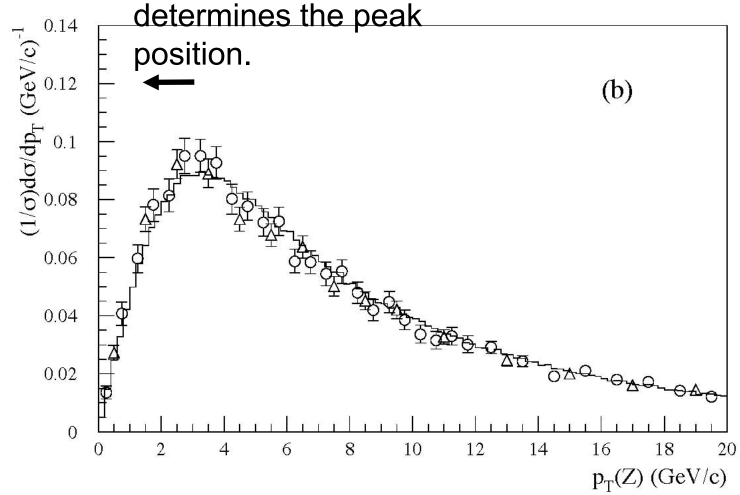
Nearly perfect through the entire measurement range

No tunable parameter in GR@PPA

The $\langle k_T \rangle$ value in PYTHIA for LHC has to be tuned using the measurement results.



The primordial k_T determines the peak position.



W^+W^- production at LHC

Plot: GR@PPA 2.8 + PYTHIA 6.4

Histograms

solid: MC@NLO (IL1 = IL2 = 1) + HERWIG

dashed: PYTHIA 6.4 (new PS)

Reasonable agreement with MC@NLO

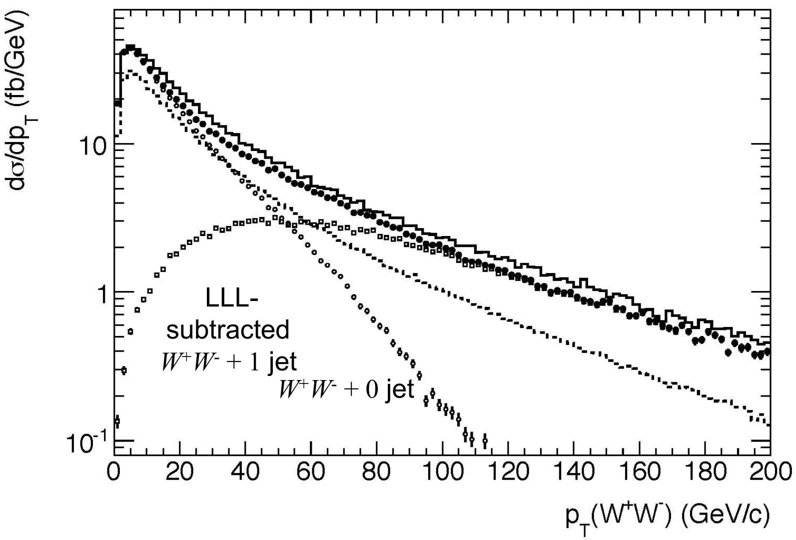
Virtual corrections are yet to be included in GR@PPA.

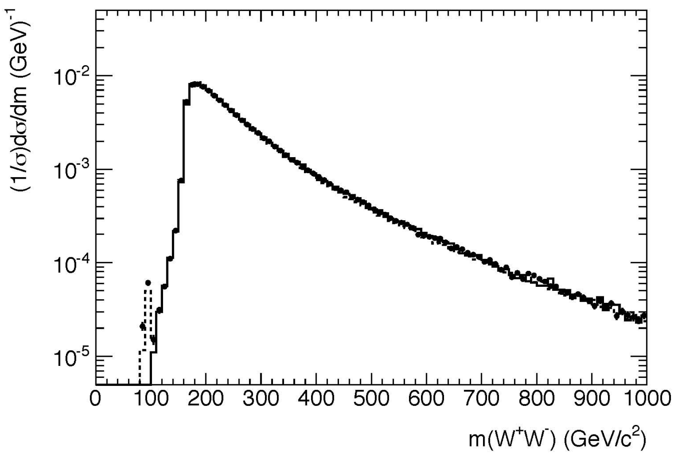
Difference in the applied PS

Significant difference from PYTHIA

Hard radiations are based on an approximation in PYTHIA.

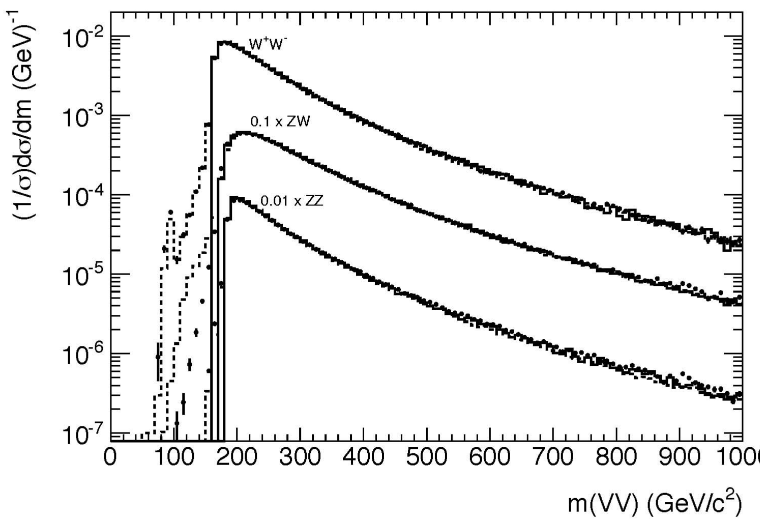
No significant difference between the three simulations in the W^+W^- invariant mass spectrum, except for a small Z peak





Difference to MC@NLO

m_{VV} at LHC



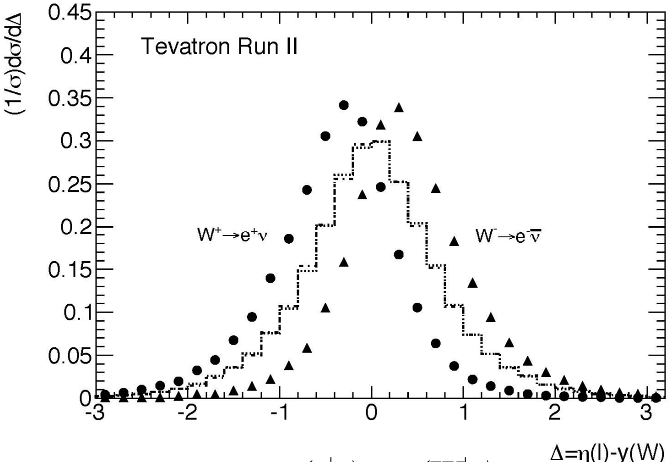
Plots: GR@PPA 2.8 + PYTHIA 6.4

Histograms

solid: MC@NLO (IL1 = IL2 = 7) + HERWIG

dashed: PYTHIA 6.4 (new PS)

$ZW^{\pm}$ production at Tevatron



$\eta(e^{\pm}) - y(W^{\pm})$

Plots: GR@PPA 2.8 + PYTHIA 6.4

Histograms: MC@NLO (IL1 = IL2 = 7) + HERWIG

No decay width nor spin effect in this mode of MC@NLO

IL1 = IL2 = 1 can be applied to the W^+W^- production only.

Total sum: p_T distribution of the $\gamma\gamma$ system

after the isolation cut

Reasonable agreement with ResBos

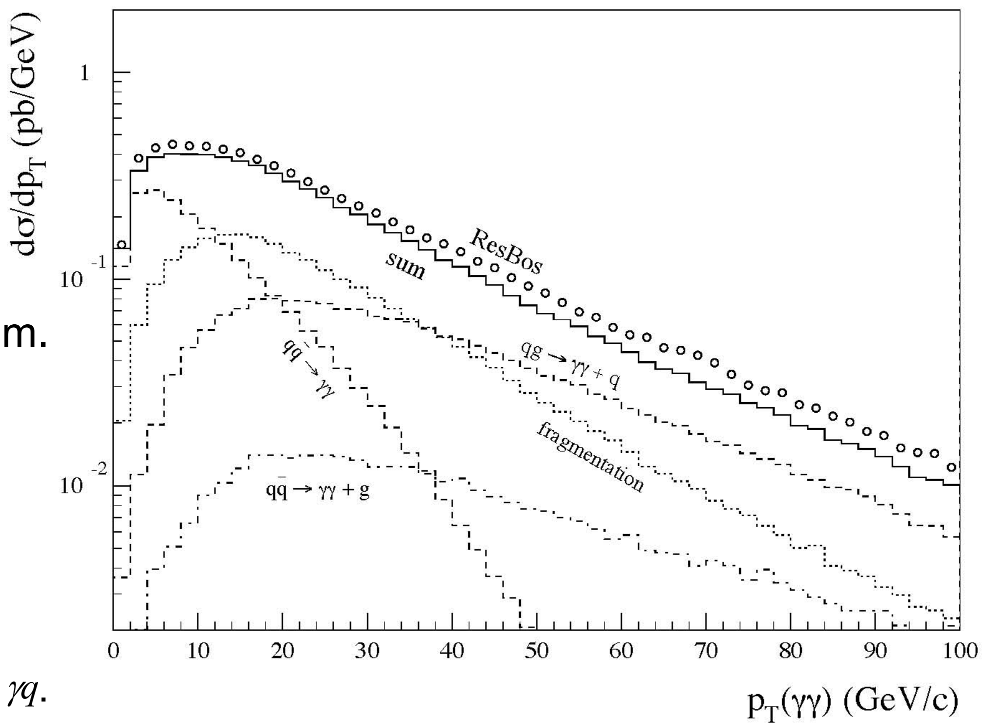
ResBos: resummed NLO calculation.
Here, $gg \to \gamma\gamma$ is not included.

ResBos: 15.5 pb
GR@PPA + PYTHIA: 13.7 pb

$q\bar{q} \to \gamma\gamma$ is less than 1/3 of the sum.

- $q\bar{q} \to \gamma\gamma$: 4.1 pb (30%)
- fragmentation: 5.1 pb (37%)
- $qg \to \gamma\gamma + q$: 3.7 pb (27%)
- $q\bar{q} \to \gamma\gamma + g$: 0.8 pb (11%)

This separation is not physical, but is a result when we separate soft/hard radiations at $\mu_F = p_T$ of $q\bar{q} \to \gamma\gamma$ or $qg \to \gamma q$.



In any case, the contribution from $qg \to \gamma\gamma + q$ with large ΔR(γ-jet) cannot be ignored.
There must be a large deficit in the estimation with $q\bar{q} \to \gamma\gamma$ and the fragmentation only.
Besides, $qg \to \gamma\gamma + q$ has an event topology quite different from them.

3. SUSY application GRACE/SUSY-loop

T.Inoue, T. Koike, D. Jujo, Y. Kataoka, T. Kon, M.Jimbo,
T. Ishikawa, Y. Kurihara, K. Kato, M. Kuroda

∗ tree level : Ref. **Comput.Phys.Commun.153:106-134,2003**
download : http://minami-home.kek.jp/

∗ 1-loop : Ref. **Phys.Rev.D75:113002,2007**

1. Feynman diagrams
2. Physical amplitudes
3. Phase space Integration
4. Event generation
5. Various Self-checks

Input file

$\tilde{t}_1 \to b e^+ \tilde{\nu}_e$

```
Model="mssmnlg_j2_q.mdl";
 Process;
  elwk= {2,2}; qcd={0,2};
  Initial={st1};
  Final  ={b positron
snu-e};
kinem="1301";
Pend;
```

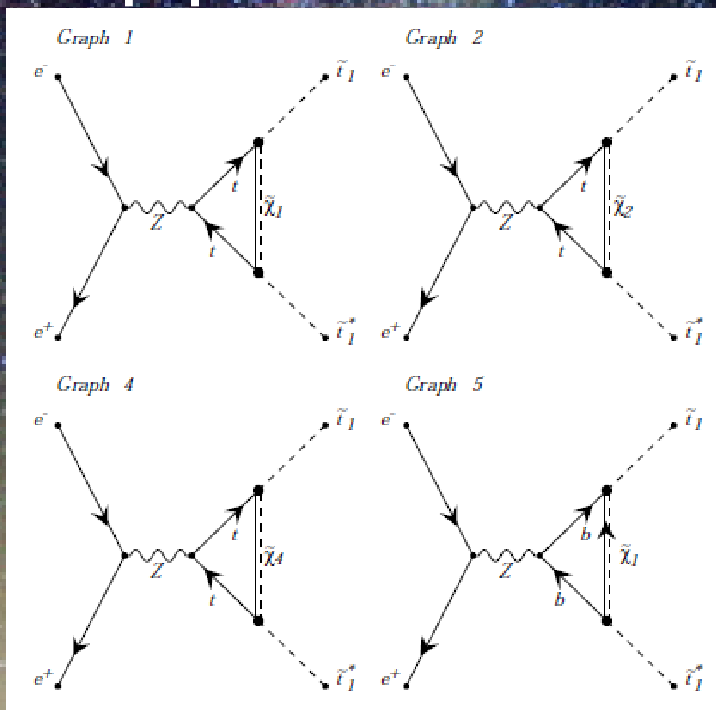
$e^+ e^- \to \tilde{t}_1 \bar{\tilde{t}}_1$

```
Model="mssmnlg_j2_q.mdl";
Process
elwk={4,2};
  Initial={electron,
positron};
  Final  ={st1, anti-st1};
kinem="2201";
Pend;
```

JPS meeting 2011 fall
T. Inoue

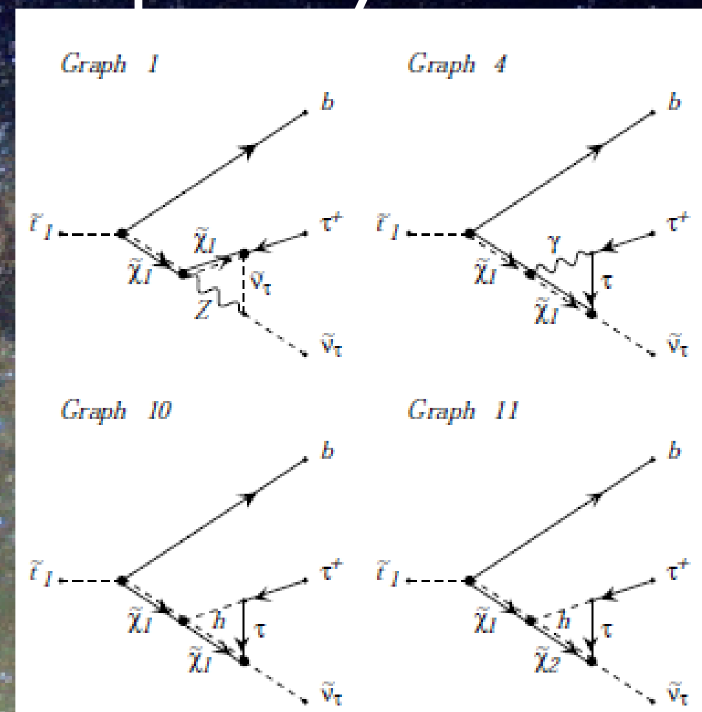
stop physics in ILC

stop production



Tree 4 diagrams
QCD1-loop 252 diagrams
ELW1-loop 1251 diagrams

stop decay



Tree 2 diagrams
QCD1-loop 12 diagrams
ELW1-loop 490 diagrams

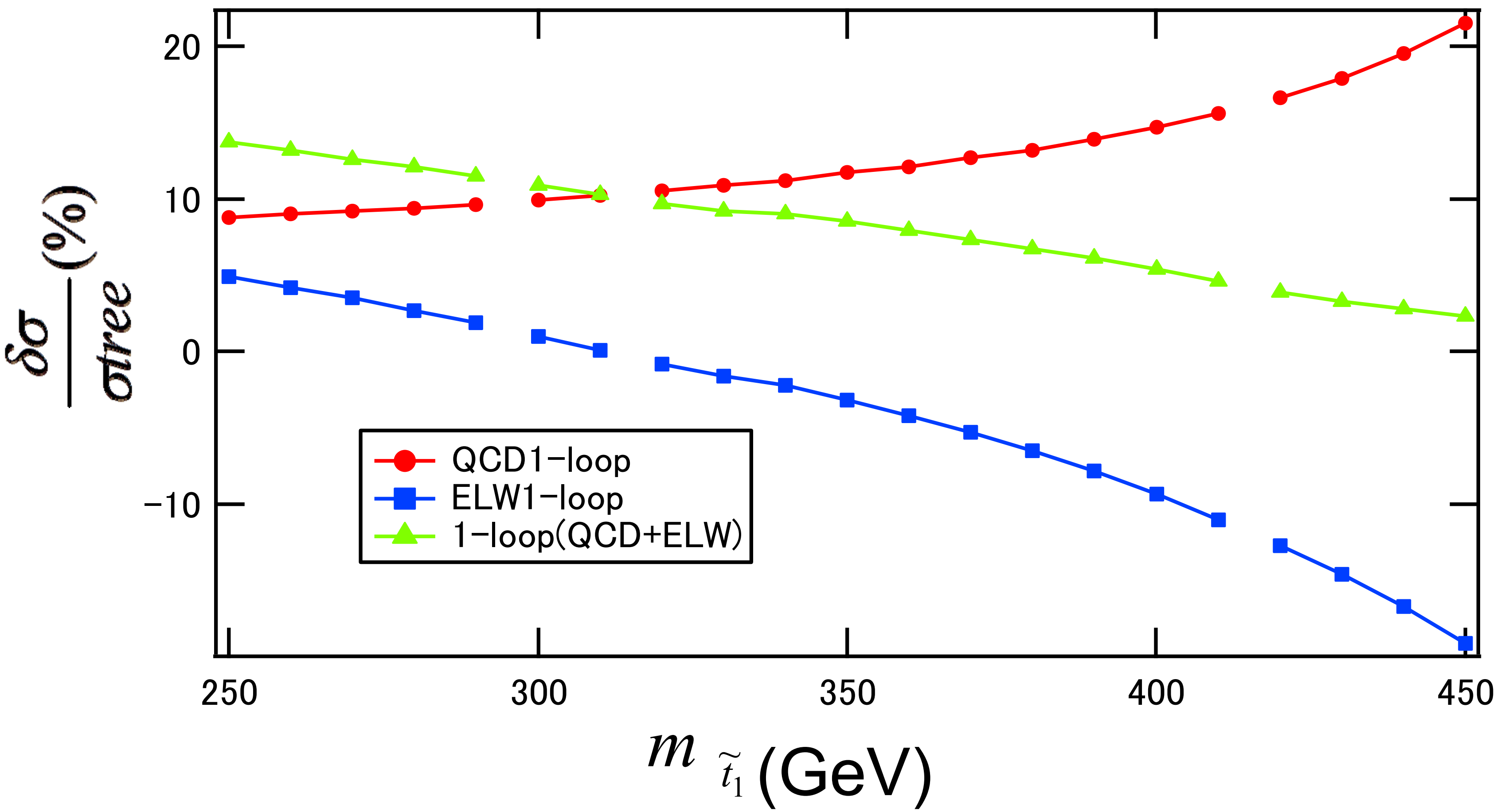
MSSM parameters

setA	Input		
tanβ	7	$m_{\tilde{b}_1}$	330GeV
μ	−500GeV	$\theta_{\tilde{b}}$	0.6π
M_2	300GeV	m_A	300GeV
$m_{\tilde{\ell}_1^+}$	170GeV	$m_{\tilde{g}}$	1042GeV
$m_{\tilde{\ell}_2^+}$	175GeV	$m_{\tilde{\chi}_1^0}$	146GeV
$\theta_{e,\mu}$	0.01π	$m_{\tilde{\chi}_1^+}$	294GeV
θ_τ	0.2π		
$m_{\tilde{\nu}_{e,\mu}}$	151GeV		
$m_{\tilde{\nu}_\tau}$	152GeV		
$m_{\tilde{t}_1}$	200~450GeV		
$m_{\tilde{t}_2}$	600GeV		
$\theta_{\tilde{t}}$	0.8π		

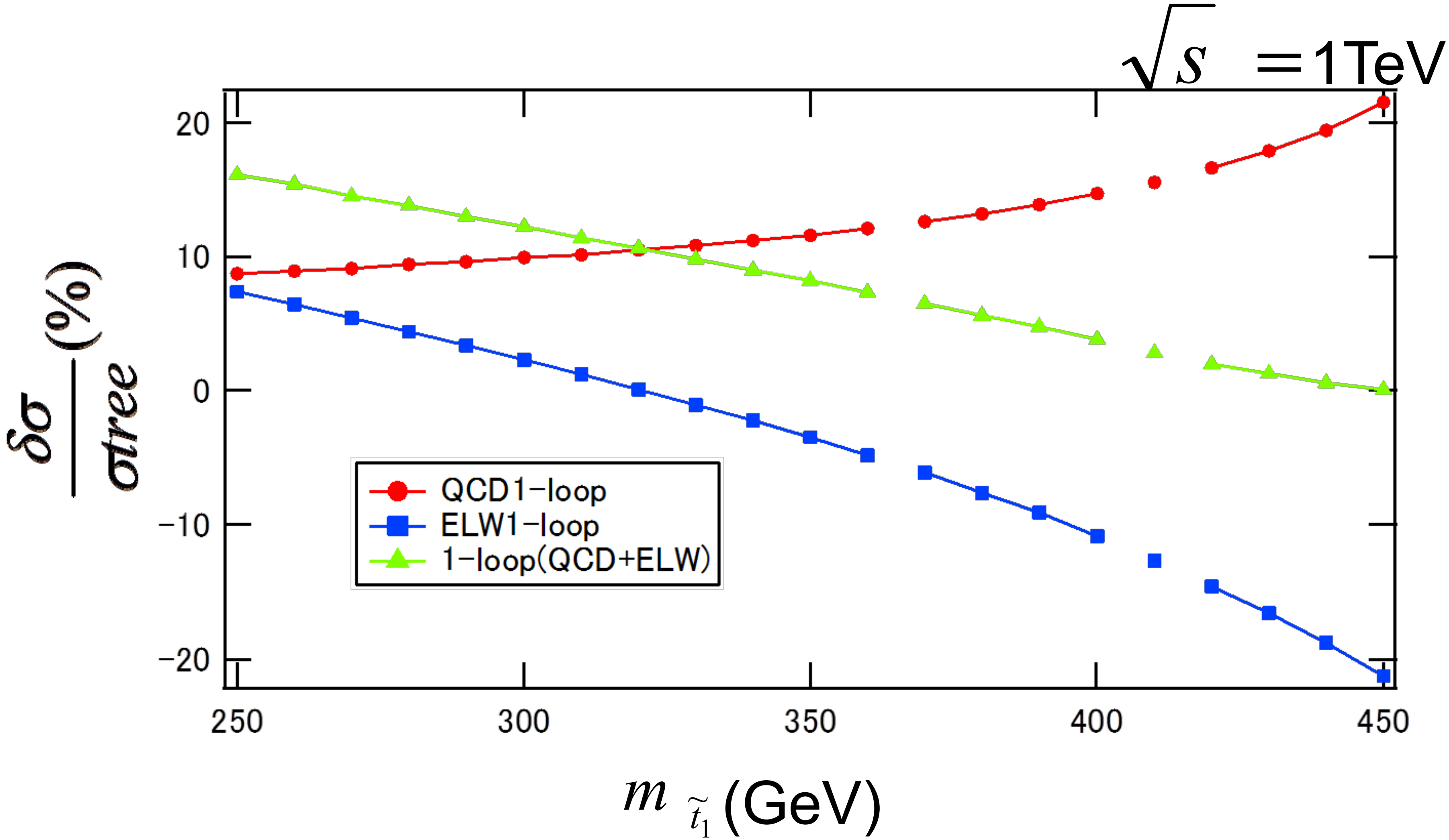
setB	input		
tanβ	10	$m_{\tilde{b}_1}$	330GeV
μ	−750GeV	$\theta_{\tilde{b}}$	0.6π
M_2	400GeV	m_A	525GeV
$m_{\tilde{\ell}_1^+}$	325GeV	$m_{\tilde{g}}$	1389GeV
$m_{\tilde{\ell}_2^+}$	370GeV	$m_{\tilde{\chi}_1^0}$	194GeV
$\theta_{e,\mu}$	0.05π	$m_{\tilde{\chi}_1^+}$	396GeV
θ_τ	0.2π		
$m_{\tilde{\nu}_{e,\mu}}$	316GeV		
$m_{\tilde{\nu}_\tau}$	328GeV		
$m_{\tilde{t}_1}$	200~450GeV		
$m_{\tilde{t}_2}$	480GeV		
$\theta_{\tilde{t}}$	0.8π		

Correction factor $e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1$ (setA)

$\sqrt{s} = 1\text{TeV}$



Correction factor $e^+e^- \to \tilde{t}_1\overline{\tilde{t}_1}$ (setB)



R.C. to stop decays

$200 \le m_{\tilde{t}_1} \le 290$ (GeV) (setA)

Decay Processes	$\tilde{t}_1 \to b e^+ \tilde{\nu}_e$	$\tilde{t}_1 \to b \tau^+ \tilde{\nu}_\tau$	$\tilde{t}_1 \to b \tilde{e}_1^+ \nu_e$	$\tilde{t}_1 \to b \tilde{\tau}_1^+ \nu_\tau$	$\tilde{t}_1 \to b \tilde{\tau}_2^+ \nu_\tau$
QCD corr.	15%~16%	15%~16%	13%~16%	11%~15%	15%~20%
ELWK corr.	9%~14%	7%~14%	17%~20%	15%~18%	11%~27%
ELWK+ QCD corr (Max.)	30%	30%	36%	31%	47%

4. Multi-loop application

F.Yuasa, E. de Doncker, N.Hamaguchi, T.Ishikawa, Y. Shimizu, Y. Kurihara, K.Kato, T. Koike, T. Kaneko, T. Ueda, J.F.

Direct Computation Method (DCM)

E. De Doncker, Y.Shimizu J.F. F. Yuasa Comput. Phys. Comm. 159('04)145.

1st step

Let ε be finite as $\varepsilon_l = \frac{\varepsilon_0}{(A_c)^l}$, $A_c > 1$

with l=0,1,2,….

ε_0 and acceleration constant A_c are positive numbers and given by hand.

$$I = \lim_{\varepsilon \to 0} \int_0^1 dx \int_0^{1-x} dy \frac{1}{D(x,y) - i\varepsilon}$$

Denominator becomes 0 in the integration domain for some choices of the parameters.

2nd step

Evaluate the integral I numerically and get the sequence of $I(\varepsilon_l)$.

$$\Re(I(\varepsilon_l)) = \int_0^1 dx \int_0^{1-x} dy \frac{D(x,y)}{D(x,y)^2 + \varepsilon_l^2},$$

$$\Im(I(\varepsilon_l)) = \int_0^1 dx \int_0^{1-x} dy \frac{\varepsilon_l}{D(x,y)^2 + \varepsilon_l^2}$$

3rd step

Extrapolate the sequence $I(\varepsilon_l)$ to the limit (ε -> 0) and determine I.

$$\Re(I) = \lim_{\varepsilon \to 0}\{\Re(I(\varepsilon))\},$$

$$\Im(I) = \lim_{\varepsilon \to 0}\{\Im(I(\varepsilon))\}$$

Numerical Integration

- DQAGE

R.Piessens E. De Doncker, C.W.Uberhuber, D.K.Kahaner;
" Quadpack – a subroutine package for automatic integration", Springer-Verlag, 1983

$$I = \int_a^b f(x)dx \approx \sum_{i=1}^{n} \omega_i f(x_i)$$

an adaptive quadrature routine where sampling points are chosen by Gauss-Kronrod quadrature rule

- Double Exponential formulae

H.Takahashi and M.Mori;
"Double Exponential Formulas for Numerical Integration",
Bull.R.I.M.S.,Kyoto Univ.,9,pp.721-741(1974).

$$I = \int_{-1}^{1} f(x)dx = \int_{-\infty}^{\infty} f(g(t))g'(t)dt \approx h \sum_{j=-N}^{N} \omega_j f(x_j)$$

$$x = g(t) \qquad g(t) = \tanh\left(\frac{\pi}{2}\sinh(t)\right) \qquad g'(t) = \frac{\frac{\pi}{2}\cosh(t)}{\cosh^2\left(\frac{\pi}{2}\sinh(t)\right)}$$

$$x_j = g(hj) \qquad \omega_j = g'(hj)$$

➢DE is not adaptive, i.e., no subdivisions are performed; whereas DQAGE subdivides towards the singularity.

➢DQAGE and DE (for 1dim) can be used iteratively for multi-dimensional integration.

Extrapolation

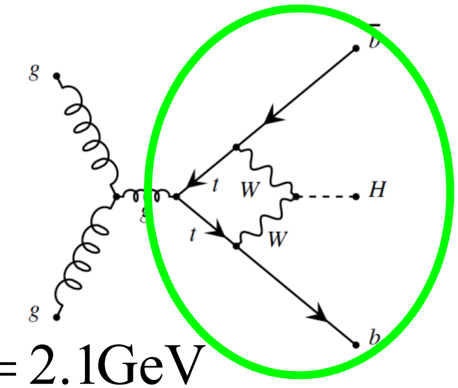
- Extrapolation is used to obtain the limit of the sequence $I(\varepsilon_l)$ and Wynn's epsilon algorithm is used to accelerate convergence of the sequence.
  - P.Wynn Mathematical Tables and Other Aids to Computation, Vol. 10, No. 54 (Apr., 1956), pp.97.
  - SIAM J. Numer. Anal. **3** (1966) 91.
- This is valid under quite general conditions and does not require any specific information to be supplied about the sequence.

1^{st} example: 1-loop box integral with complex masses reported in PoS(ACAT08)122

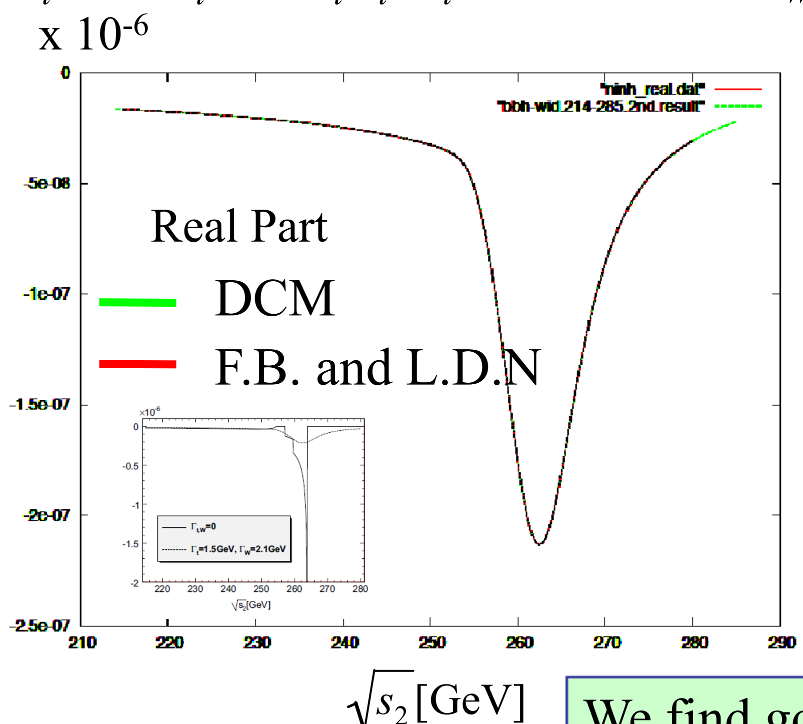
F.Boudjema and LE Duc Ninh Phys.Rev.D78:093005,2008

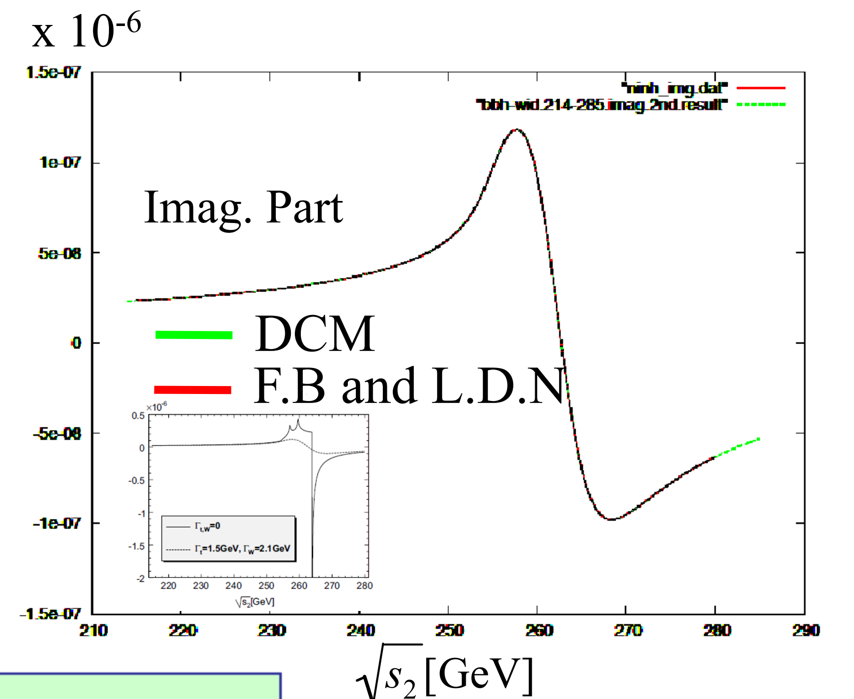
A box diagram contributing to $gg \to bb\,H$ that can develop a Landau singularity for $M_H \geq 2M_W$ and $\sqrt{s} \geq 2m_t$.

$m_t = 174$ GeV, $M_W = 80.3766$ GeV, $\sqrt{s} = 353$ GeV and $M_H = 165$ GeV.



$$m_t^2 \to m_t^2 - i m_t \Gamma_t, \Gamma_t = 1.5\text{GeV} \quad M_W^2 \to M_W^2 - i M_W \Gamma_w, \Gamma_W = 2.1\text{GeV}$$





We find good agreement.

2nd example : selfenergy, vertex and box integral

The program SYS

S. Laporta, Int. J. Mod. Phys. A15 (2000) 5087

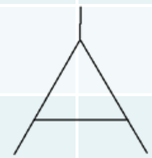
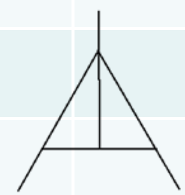
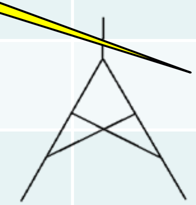
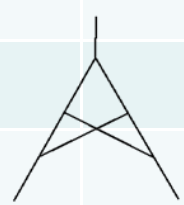
Parameters:

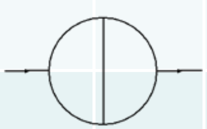
- $m_1 = \ldots = m_N = 1$
- $s = t = 1$ and $u = 2$

DE-DCM: Double Exp. Formulae is used

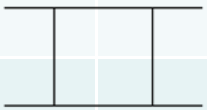
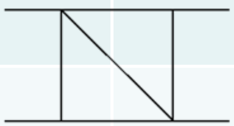
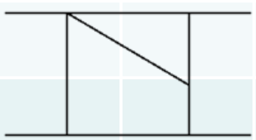
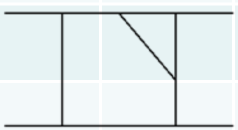
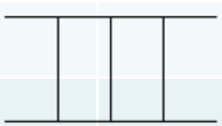
34.3 sec

1.0 x 10^{-3} sec

L	N		finite term
1	3	SYS	0.671253105748
		DQ-DCM	0.67125310574800
		DE-DCM	0.671253105748005
2	5	SYS	0.937139527315
		DQ-DCM	0.937139
		DE-DCM	0.93713952731 4984
2	6	SYS	0.2711563494022
		DQ-DCM	0.2711563491
		DE-DCM	0.2711559
2	6	SYS	0.173896742268
		DQ-DCM	0.173432
		DE-DCM	0.17390

L	N		finite term
2	5	SYS	0.9236318265199
		DQ-DCM	0.92363182651986 4
		DE-DCM	0.9236

2^{nd} example : selfenergy, vertex and box integral contd.

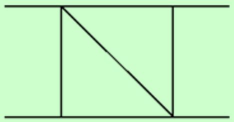
L	N		finite term
1	4	SYS	0.3455029252972
		DQ-DCM	0.34550292529718
		DE-DCM	0.345502925289537
2	5	SYS	0.9509235623171
		DQ-DCM	0.95092
		DE-DCM	0.95092
2	6	SYS	0.2762092225359
		DQ-DCM	(skipped)
		DE-DCM	0.276209223589
2	7	SYS	0.1723367907503
		DQ-DCM	(skipped)
		DE-DCM	0.1723367907501
2	7	SYS	0.1036407209893
		DQ-DCM	0.10364072096
		DE-DCM	0.1036407209892

2 x 10^{-3}sec

95.3sec

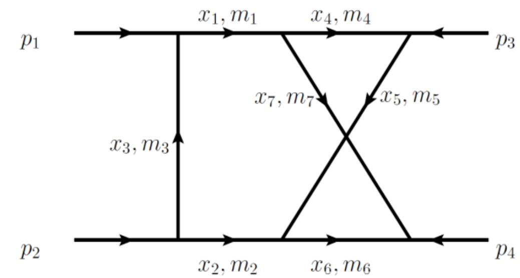
Note 1:
We find good agreement.

Note 2:
We get the non-planar diagram as 0.08535139 (47.6 sec) while no result is available by the program SYS.



Note 3:
When function D does not vanish in the integration domain, we do not need the extrapolation and CPU time required is short.

Two-loop non-planar box



$$I = -\int_0^1 dx_1\, dx_2\, dx_3\, dx_4\, dx_5\, dx_6\, dx_7\, \delta(1 - \sum_{\ell=1}^{7} x_\ell) \frac{\mathcal{C}}{(\mathcal{D} - i\epsilon\mathcal{C})^3}$$

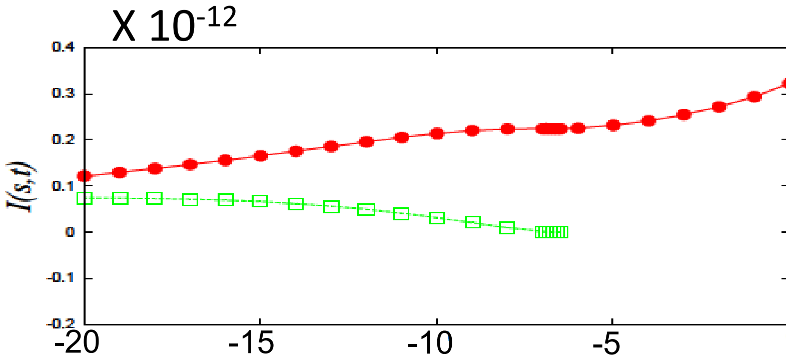
$$\begin{aligned}
\mathcal{D} &= -\mathcal{C} \sum x_\ell m_\ell^2 \\
&+ \{s(x_1x_2x_4 + x_1x_2x_5 + x_1x_2x_6 + x_1x_2x_7 + x_1x_5x_6 + x_2x_4x_7 - x_3x_4x_6) \\
&+ t(x_3(-x_4x_6 + x_5x_7)) \\
&+ p_1^2(x_3(x_1x_4 + x_1x_5 + x_1x_6 + x_1x_7 + x_4x_6 + x_4x_7)) \\
&+ p_2^2(x_3(x_2x_4 + x_2x_5 + x_2x_6 + x_2x_7 + x_4x_6 + x_5x_6)) \\
&+ p_3^2(x_1x_4x_5 + x_1x_5x_7 + x_2x_4x_5 + x_2x_4x_6 + x_3x_4x_5 + x_3x_4x_6 + x_4x_5x_6 + x_4x_5x_7) \\
&+ p_4^2(x_1x_4x_6 + x_1x_6x_7 + x_2x_5x_7 + x_2x_6x_7 + x_3x_4x_6 + x_3x_6x_7 + x_4x_6x_7 + x_5x_6x_7)\}
\end{aligned}$$

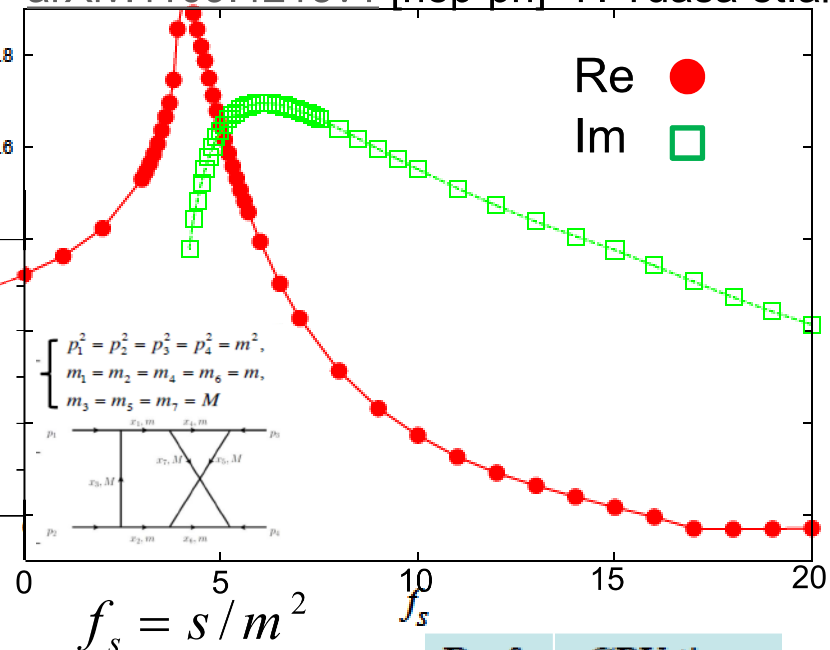
$$\mathcal{C} = (x_1 + x_2 + x_3 + x_4 + x_5)(x_1 + x_2 + x_3 + x_6 + x_7) - (x_1 + x_2 + x_3)^2$$

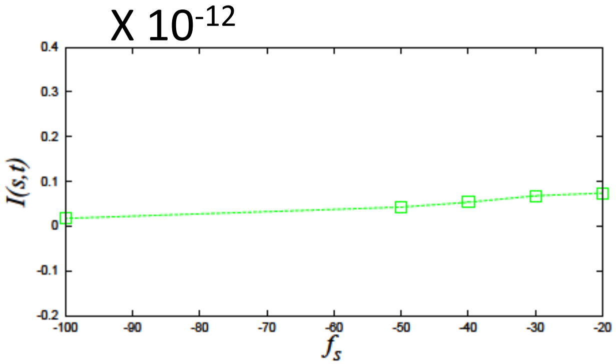
arXiv:1109.4213v1 [hep-ph] F. Yuasa et.al.

Numerical results of Two-loop non-planar box with masses

m=50 GeV, M = 90 GeV, t = -100^2 GeV^2

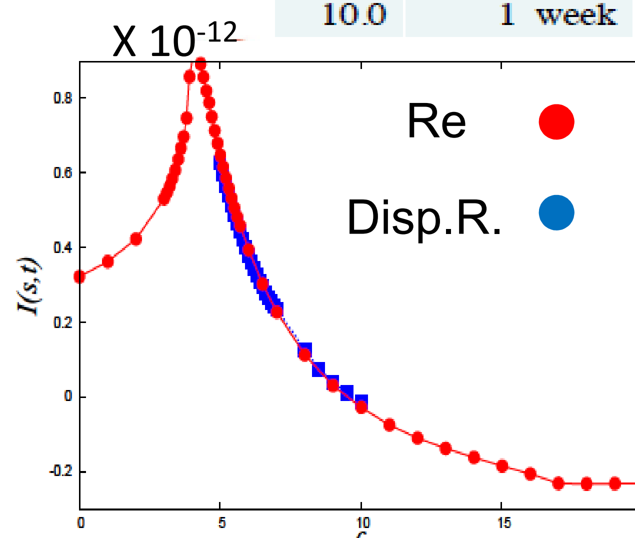






Intel(R) Xeon(R) X5460 @ 3.16GHz

Re. fs	CPU time
6.0	16 hours
7.0	2 days
10.0	1 week



Dispersion Relation

$$\Re(I(s)) = \frac{1}{\pi}\left(\mathrm{P}\int_{-\infty}^{s_0'} \frac{\Im(I(s'))}{s-s'}ds' + \mathrm{P}\int_{s_0}^{\infty} \frac{\Im(I(s'))}{s-s'}ds'\right),$$

where s_0 and s_0' are the threshold in s-channel and that in u-channel,

4. Summary

- gr@ppa2.8 is ready for $W^+W^-, ZW^{\pm}, ZZ, \gamma\gamma$
- gr@ppa3.0 with NLO is coming soon
- ELWK R.C. to Stop production and decays for ILC are indispensable.
- DCM(Direct Computation Method) works well for loop integrals.