Radiative Corrections 2011
Mamallapuram, India

Production of KK-gravitons in association with a boson via gluon fusion in the LHC

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September 30, 2011

Plan

- **Introduction**
  A little about the extra-dimensional models – ADD / RS

- **Gluon Fusion: KK-Graviton + Boson**
  Consider only the color singlet SM bosons

  1. **KK-Graviton + Higgs**
     The process, Results
  2. **KK-Graviton + Photon**
     The process
  3. **KK-Graviton + Z Boson**
     Some results

Introduction

- **What are these?** The number of spacial dimensions is assumed to be $3 + d$. These extra dimensions are generally assumed to be compactified. Gravity can freely propagate through the extra-dimensions but, depending on the model, the SM fields can either propagate in the bulk or live on a boundary of the bulk.
- **Motivation:** Can solve the hierarchy problem – provide an effective gravity scale.
- **Phenomenologically interesting:** Possibility of experimental access to extra dimensions.
- **Different Models:** Different models with different number and size (scale) of the extra-dimensions.
- We shall consider two such models.
  1. **ADD:** Number of extra-dimensions, $d \geq 2$ and the scale of the extra-dimensions can be as large as a μm.
  2. **RS:** Number of extra-dimensions, $d = 1$ and the scale of the extra-dimension is slightly larger than the Planck length.

Introduction

Arkani-Hamed, Dimopoulos and Dvali

- Can assume a $4+d$ dimensional bulk with the extra-dimensions compactified on a d-dimensional torus of radius $R/2\pi$ for simplicity.
  Han, Lykken, Zhang, PRD, 1999
- In the 3+1 dimensional picture gravity is treated as an effective theory valid below some scale, $M_{\rm S}$. This scale of the extra-dimensional theory, which acts like the ultraviolet cutoff scale of the effective theory, is related to the Planck constant ($M_{\rm PL}$) as,

$$M_{\rm S}^{d+2} = \frac{(4\pi)^{d/2}\Gamma(d/2)}{2R^d} M_{\rm PL}^2 .$$

- The masses of the different KK-modes of the graviton are given by,

$$M_{\vec{n}}^2 \equiv M_{\{n_1, n_2, \ldots, n_d\}}^2 = \frac{4\pi^2}{R^2} \sum_{i=1}^{d} \left(n_i^2\right) ,$$

  where each $n_i = \{0, 1, 2, \ldots\}$.
- Mass difference between two successive modes decreases with increasing R.

Introduction

- KK-gravitons interact with the SM fields with the gravitational coupling,

$$\mathcal{L} \supset -\frac{\kappa}{2} \sum_{\vec{n}} T^{\mu\nu} G^{\vec{n}}_{\mu\nu}, \quad \kappa = \sqrt{16\pi G_{\rm N}}$$

- The cross-section for production of a single on-shell KK-graviton mode, $\sigma\left(M^2_{\vec{n}}\right)$ is suppressed by $1/M^2_{\rm PL}$.
- For large R, one can make a continuum approximation,

$$\rho\left(M^2_{\vec{n}}\right) dM^2_{\vec{n}} = \frac{R^d M^{(d-2)}_{\vec{n}}}{(4\pi)^{d/2}\Gamma(d/2)} dM^2_{\vec{n}} .$$

- The cross-section for the on-shell KK-graviton production,

$$\sigma = \int \sigma\left(M^2_{\vec{n}}\right) \rho\left(M^2_{\vec{n}}\right)\, dM^2_{\vec{n}} .$$

  Due to the density of the KK-graviton modes, there is an enhancement factor of $M^2_{\rm PL}/M^{d+2}_{\rm S}$. Therefore, the cross-section, σ is ultimately suppressed only by a factor of $1/M^{d+2}_{\rm S}$.
- If the mass scale is of the order of a few TeVs, then it may be possible to see the signatures of this theory in the LHC.

Randall and Sundrum

- Only one extra-dimension.
- The five dimensional bulk is warped with the following space-time metric,

$$ds^2 = \left(e^{-2kR_c|\phi|}\right) \eta_{\mu\nu} dx^\mu dx^\nu + R_c^2 d\phi^2$$

– the extra dimension ϕ is assumed to be compactified on a $S^1/\mathbf{Z}_2$ orbifold.

- At the fixed points of the orbifold, there are two 3-branes – the IR where the SM fields are localized and the UV brane.
- Due to the warped nature of the bulk space-time the graviton mass spectrum in this model is quite different from that of the ADD model.
- The mass of n^{th} KK mode of the graviton can be written as,

$$M_n = x_n kW \equiv x_n m_0,$$

where x_n denotes the n^{th} zero of the Bessel function $J_1(x)$ ($x_n \approx 3.83, 7.02, 10.17 \cdots$) and W is the warp factor,

$$W = e^{-\pi kR_c}.$$

Introduction

- In the IR brane, all the massive KK modes of graviton couple with the SM fields with an effective gravitational coupling $\kappa_{\rm IR}$, given as,

$$\kappa_{\rm IR} = \frac{\kappa}{W} \equiv \sqrt{2}\frac{c_0}{m_0}.$$

- The massless zeroth mode couples with SM fields with the gravitational coupling κ and hence its effects can safely be ignored.
- The quantities, $c_0 = k/\bar{M}_{\rm PL} = \sqrt{8\pi}k/M_{\rm PL}$ and $m_0 = kW$ are the two free parameters of the model.
- The RS model requires the scale k to be smaller than the 5 dimensional Planck scale. However taking k too small will not solve the hierarchy problem. A common choice is to take $0.01 \lesssim c_0 \lesssim 0.1$.
- The other parameter, m_0 sets the mass scales of different KK modes of the graviton. We shall consider only the first excited mode of the graviton with mass $M_1 = x_1 m_0 \approx 3.83\, m_0$.

Introduction

Experimental Limits

- **ADD Graviton**

  Recent experimental limits on the mass scale, M_S are of TeV order for $d \geq 2$.

d	2	3	4	5	6
M_S (TeV)	1.80	2.23	1.84	1.63	1.46

  CMS, JHEP **1105**, 085 (2011)

- **RS Graviton**
  - Tevatron sets the limit on M_1 as 1058 GeV (612 GeV) for $c_0 = 0.1$ (0.01).
  - For values of c_0 ranging from 0.01 to 0.1, at 95% C.L., graviton masses below 368 to 952 GeV are excluded by CMS. (CMS Public Wiki)

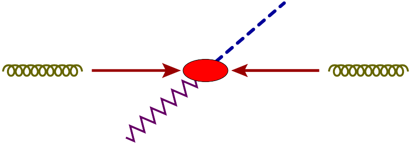
Introduction

- One can look into associated production of graviton with some other boson for a possible signal.
- Several studies on associated production of graviton with bosons

1. **Gluon jet** (ADD model)
   S Karg *et al*, Phys. Rev. D **81**, 094036 (2010) [arXiv:0911.5095 [hep-ph]]
2. **Photon** (ADD model)
   X Gao *et al*, Phys. Rev. D **81**, 036008 (2010) [arXiv:0912.0199 [hep-ph]]
3. **Massive EW vector bosons** (ADD model)
   M C Kumar *et al*, J. Phys. G **38**, 055001 (2011) [arXiv:1004.5519 [hep-ph]]
   A. Shivaji, V. Ravindran and P. Agrawal, in preperation.
4. **Higgs** (ADD model & RS model)
   A Shivaji, SM, P Agrawal, arXiv:1108.4561 [hep-ph].

Gluon Fusion

KK-Graviton + Higgs



KK-Graviton + Higgs

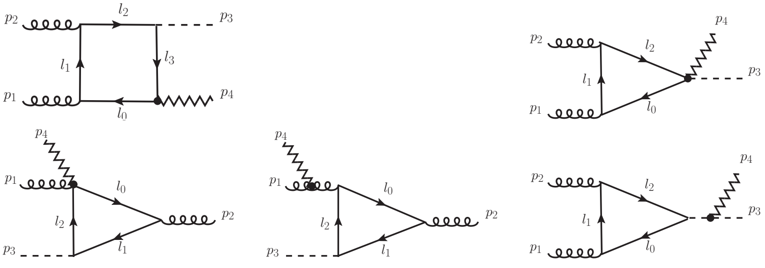
The Dominant Channel

- In the LHC gluon PDF dominates over all quark distribution functions.
- The coupling of the light quarks to the Higgs boson is very small.
- No contribution from the diagrams that involve a Higgs-quark-quark-graviton ($hqqG_{\rm KK}$) vertex; this vertex is proportional to the metric, $\eta_{\mu\nu}$, which when contracted with the graviton polarization tensor gives zero.
- Other qq initiated processes mediate via fusions of heavy electroweak vector bosons whose heavy masses and the small electroweak couplings make the contribution negligible.

Therefore, at the LHC, the gluon fusion mechanism, *i.e.* $gg \to hG_{\rm KK}$ is the dominant channel for the production of a Higgs boson in association with a graviton.

KK-Graviton + Higgs

- No tree level contribution to the $gg \to hG_{\mathrm{KK}}$ process. The first non vanishing contribution comes from the diagrams with a quark loop ($\mathcal{O}(g_s^2 \kappa y_q)$).
- However, since a Yukawa coupling (y_q) is present at the Higgs-quark-quark (hqq) vertex, all the light-quark loop contributions are negligible.
- Only the top-quark loop contribution is significant.
- Diagrams containing three gluon vertices are absent due to color conservation.
- Six box diagrams and twelve triangle diagrams. Half of them are independent.



KK-Graviton + Higgs

Calculation

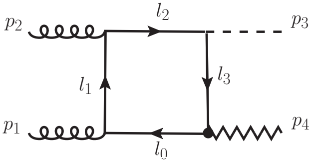
1. Traces associated with the top-quark loop computed using FORM.

2. At this stage, the amplitude contains tensor integrals:
   4-rank tensor-box integral ($D^{\mu\nu\rho\sigma}$), 2-rank tensor-triangle integral ($C^{\mu\nu}$),

$$D^{\mu\nu\rho\sigma} = \int \frac{d^n l_0}{(2\pi)^n} \frac{l_0^\mu l_0^\nu l_0^\rho l_0^\sigma}{D_0 D_1 D_2 D_3},$$

$$C^{\mu\nu} = \int \frac{d^n l_0}{(2\pi)^n} \frac{l_0^\mu l_0^\nu}{D_0 D_1 D_2},$$

$$D_i = l_i^2 - m_t^2 + i\varepsilon, \quad n = 4 - 2\epsilon.$$



3. The tensor integrals are reduced into the standard scalar integrals – A_0, B_0, C_0 and D_0 using the reduction scheme developed by Oldenborgh and Vermaseren. Ultimately scalars are computed using FF (LoopTools).

4. Helicity basis for the polarization vectors used to calculate the amplitude.

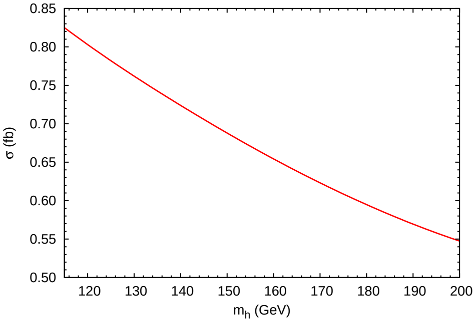
KK-Graviton + Higgs

Amplitude has to be checked for UV finiteness and gauge invariance.

1. **UV Finiteness:** Individual box and triangle diagrams can be UV divergent, but the total amplitude should be UV finite.
   - Tested the UV finiteness of the total amplitude by varying the renormalization scale (μ_{R}) over ten orders of magnitude.
   - The amplitude is indeed independent of the actual value of μ_{R}.
   - The triangle and box amplitudes are separately UV finite (each triangle diagram is UV finite by itself).
2. **Gauge Invariance:**
   - Checked the gauge invariance of the amplitude with respect to both the gluons. This has been done by replacing the polarization vector of either of the gluons by its momentum ($\varepsilon^{\mu}(p_i) \to p_i^{\mu}$).
   - Some of the triangle diagrams are separately gauge invariant with respect to both the gluons.
   - Gauge invariance checked with respect to the graviton polarization tensor also.

KK-Graviton + Higgs

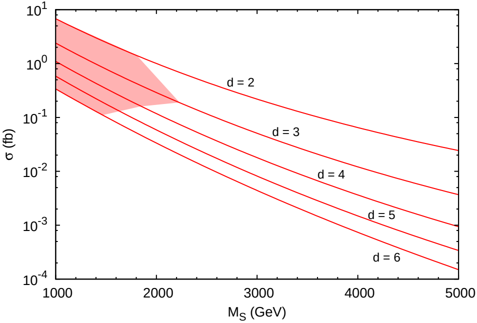
Dependence on m_h



- The number of extra dimensions, $d = 2$, and $M_{\mathrm{S}} = 2$ TeV.
- Cuts on the transverse momentum and rapidity of the Higgs: $p_{\mathrm{T}}^{h} > 20$ GeV, $|\eta^{h}| < 2.5$.
- PDF used: NLO CTEQ6 ($\overline{\mathrm{MS}}$ scheme). PDF dependence: 5 – 15%
- Scale dependence: $\Delta\sigma \sim 15 - 20\%$ for variation of factorization scale by a factor of 2.

KK-Graviton + Higgs

Dependence on M_S

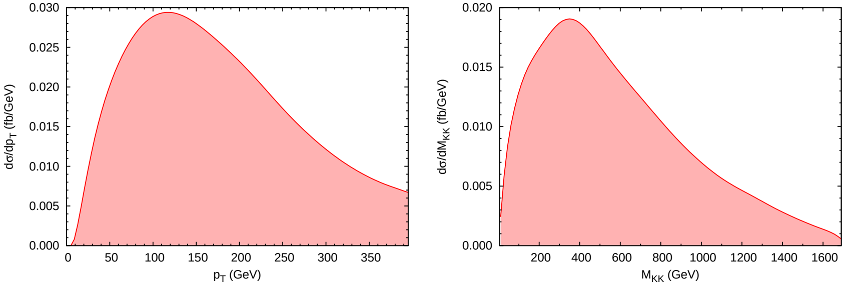


$m_h = 120$ GeV

- Large cancellation between the box-diagrams contribution and the triangle-diagrams contribution. This reduces the amplitude by two-to-three orders of magnitude.
- For the ADD model, one extra cut: $M(hG_{\rm KK}) \leq M_{\rm S}$. For $d = 2$ and $M_{\rm S} = 2$ TeV, the difference in the truncated and untruncated cross-sections is about 15% for $m_h = 120$ GeV. This difference increases to about 60% for $d = 4$ and keeps decreasing with the increase in the cut-off scale $M_{\rm S}$.

KK-Graviton + Higgs

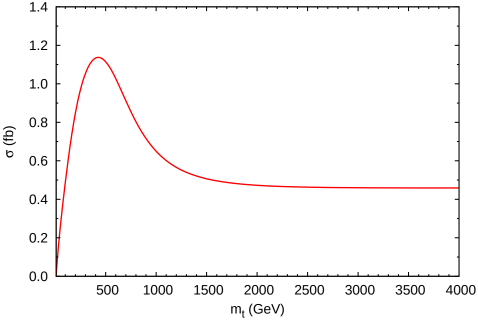
$p_T^{h/G}$ and M_{KK} Distributions

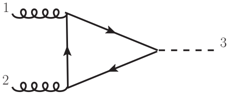


- The transverse momentum distribution of the Higgs boson has a peak at about 120-130 GeV.
- The density of graviton states increases with the increase in the mass of the graviton – the cross-section gets contribution from mostly large values of the G_{KK} mass. However, after about $M_{KK} \approx 400$ GeV, the phase space suppression takes over and the cross-section starts to decrease.

KK-Graviton + Higgs

Violation of Decoupling Theorem





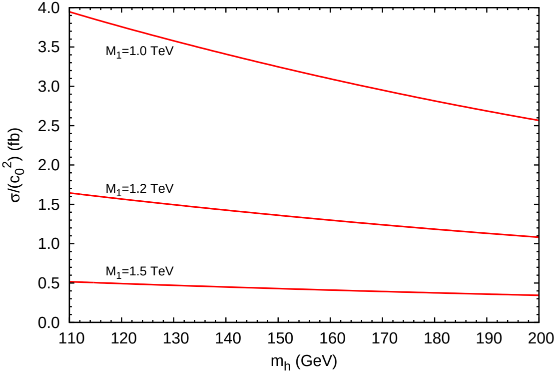
- In the SM the *ggh* coupling via top loop:

$$C_{ggh}^{\mathrm{SM}} = -\frac{m_t}{v} \times \frac{1}{2} \times \frac{\alpha_s}{4\pi m_t} \mathcal{F}\left(\frac{4m_t^2}{m_h^2}\right),$$

$$F(\tau) = -2\tau\left(1 + (1-\tau)\arcsin^2(1/\sqrt{\tau})\right) \to -\frac{4}{3}\,(\text{for } \tau \gg 1)$$

KK-Graviton + Higgs

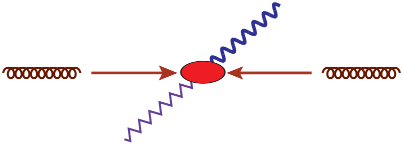
Cross-section in the RS Model



- For $0.01 \lesssim c_0 \lesssim 0.1$, the cross-section is more than an order of magnitude smaller than that in the ADD model. For example for $c_0 = 0.075$, $M_1 = 1$ TeV and $m_h = 120$ GeV the cross-section is only about 0.02 fb.

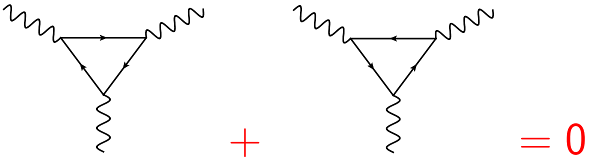
Gluon Fusion

KK-Graviton + Photon



KK-Graviton + Photon

Furry's Theorem



- Photons: negative C-parity,

$$\mathcal{C}A_\mu(x)\mathcal{C}^\dagger = -A_\mu(x)$$

- The photon 3-point Green's function,

$$\begin{aligned}\Gamma_{\mu_1\mu_2\mu_3} &\propto \left\langle 0 \left| \mathrm{T}\left[A_{\mu_1}(x_1)A_{\mu_2}(x_2)A_{\mu_3}(x_3)\exp\left[i\int\mathcal{L}_{int}^{\mathrm{QED}}\right]\right]\right| 0\right\rangle, \\ &\propto \left\langle 0 \left| \mathrm{T}\left[\mathcal{C}^\dagger\mathcal{C}A_{\mu_1}\mathcal{C}^\dagger\mathcal{C}A_{\mu_2}\mathcal{C}^\dagger\mathcal{C}A_{\mu_3}\mathcal{C}^\dagger\mathcal{C}\exp\left[i\int\mathcal{L}_{int}^{\mathrm{QED}}\right]\mathcal{C}^\dagger\mathcal{C}\right]\right| 0\right\rangle \\ &= (-1)^3\Gamma_{\mu_1\mu_2\mu_3}\end{aligned}$$

- This is true for n number of photons if $n =$ odd.
- As long as $\mathcal{L}_{int}$ is invariant under $\mathcal{C}$ this is true at all orders of perturbation theory.

KK-Graviton + Photon

Furry's Theorem with 2 Gluons

- Gluons carry color charges and hence are not eigenstates of $\mathcal{C}$. One cannot expect for Furry's theorem to work for gluons and indeed three gluon vertex exists even at the tree level. However, the theorem still works if we replace any two photons by two gluons.
- Since QCD interactions are invariant under charge conjugation and we know how the quarks transform, one can derive a transformation rule for gluons [Tyutin, Lokhvitskii, Russian Physics Journal, **25**, 346 (1982)],

$$\mathcal{C}\, G^a_\mu(x)\mathcal{C}^\dagger = -\,[\Lambda]^{ab}\, G^b_\mu(x), \quad [\tau^a]^{\mathrm{T}} = [\Lambda]^{ab}\,\tau^b.$$
$$\Rightarrow \quad f^{abc}\,[\Lambda]^{aa'}\,[\Lambda]^{bb'}\,[\Lambda]^{cc'} = -f^{a'b'c'},$$

The matrix Λ is a diagonal matrix with the non-zero elements being ± 1, i.e., $\Lambda = \Lambda^{\mathrm{T}}$ and $\Lambda^2 = 1$.
- The Green's function with n-number of external photons and two external gluons,

$$\Gamma_{\{\mu_i\}\nu_1\nu_2} \propto \delta^{ab}\left\langle 0\left|\mathrm{T}\left[A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)G^a_{\nu_1}(y_1)G^b_{\nu_2}(y_2)\exp\left[i\int\mathcal{L}_{int}\right]\right]\right|0\right\rangle.$$

Just like before we can insert $\mathcal{C}^\dagger\mathcal{C}$'s to get

$$\Gamma_{\{\mu_i\}\nu_1\nu_2} = (-1)^n(\Lambda)^2\Gamma_{\{\mu_i\}\nu_1\nu_2} = (-1)^n\Gamma_{\{\mu_i\}\nu_1\nu_2},$$

KK-Graviton + Photon

Charge Parity of Gravitons

- Since gravitons are assumed to carry no known quantum number and are bosonic in nature, graviton states are eigenstates of $\mathcal{C}$.
- As gravity couples only to the energy-momentum tensor, one would expect that it should not differentiate between an electron and a positron. Hence, it is natural to assume that the gravitational interaction with electron is invariant under charge conjugation.
- The graviton-electron interaction is given by

$$\begin{aligned}\mathcal{L}_e &= h^{\vec{n}}\eta_{\mu\nu}\bar{\psi}_e i\gamma^\mu\partial^\nu\psi_e + \frac{1}{2}\bar{\psi}_e i\gamma^\mu\left(\partial_\mu h^{\vec{n}}\right)\psi_e - m_e h^{\vec{n}}\bar{\psi}_e\psi_e \\ &\quad - h^{\vec{n}}_{\mu\nu}\bar{\psi}_e i\gamma^\mu\partial^\nu\psi_e - \frac{1}{2}\bar{\psi}_e i\gamma^\mu\left(\partial^\nu h^{\vec{n}}_{\mu\nu}\right)\psi_e\end{aligned}$$

- The graviton field transforms as

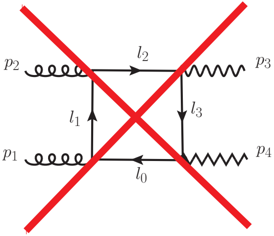
$$\mathcal{C}h^{\vec{n}}_{\{\mu\nu\}}(x)\mathcal{C}^\dagger = h^{\vec{n}}_{\{\mu\nu\}}(x),$$

i.e., it has +ve C-parity.

KK-Graviton + Photon

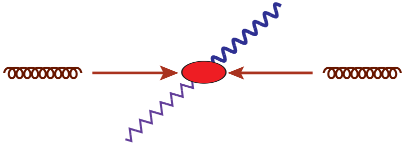
No Gluon Fusion Process

- Since gravitons have positive C-parity, Furry's theorem still works with any number of external gravitons.
- The Green's function with odd number of photons and any number of gravitons vanishes if we include only change conjugation invariant interactions. Similarly two gluons can not fuse into odd number of photons and any number of gravitons.
- This result is true at any order of perturbation theory if we don't include weak corrections.



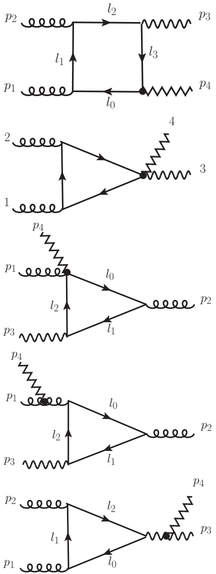
Gluon Fusion

KK-Graviton + Z Boson



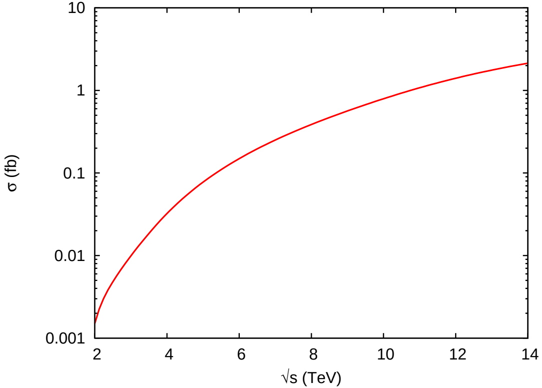
KK-Graviton + Z Boson

No Vector Contribution



ADD Model

Thanks to Ambresh for the plot.



- No contribution from the vector part. Only the axial part contributes.
  A. Shivaji, V. Ravindran and P. Agrawal, in preperation.

Summary

- Studied KK-graviton + Higgs production at the LHC for both ADD and RS models. Due to large cancellation between the box and triangle contributions cross-section is of the order of a fb for ADD for $d = 2$. For RS the cross-section is even smaller.
- Due to -ve C-parity of the photon, $gg \to G_{\rm KK}\gamma$ vanishes.
- In case of $gg \to G_{\rm KK} Z$ only the axial part contributes.

Thank You