Top mass effects in scalar and pseudo-scalar Higgs bosons production at NNLO for hadron colliders

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PHYSICS AT THE TERA SCALE Helmholtz Alliance

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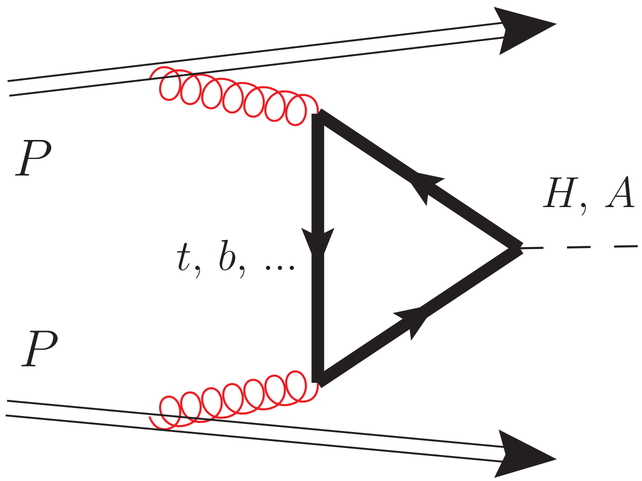
Higgs boson production at the LHC: $pp \to H(A) + X$

- Dominant channel (for intermediate m_H): $gg \to H$ via a top-quark loop



$\Longrightarrow$ **Very well studied process!**

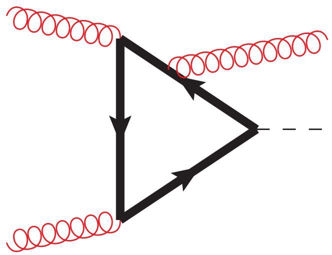
- **Scales of process**
  - $\sqrt{S_{part}} \sim 100 - 14000$ Gev
  - $m_H \sim 100 - 300$ Gev , $m_A \sim 100 - ?$ Gev
  - Top-quark mass $m_t \sim 173$ Gev

■ Leading order: Georgi, Glashow, Machacek, Nanopoulos '78 (full dependence on m_H/m_t)

QCD corrections: large!

- Next-to-leading order:

  Dawson; Djouadi, Spira, Zerwas '91(effective theory); Dawson and Kauffman '94(up to $\sim 1/m_t^2$); Spira et al '95 (exact)

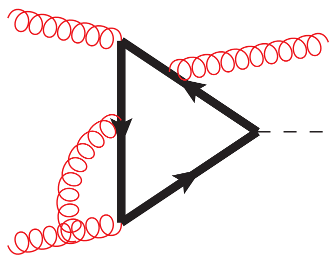


$\sim \mathcal{O}(70\%)$

- Next-to-next-to-leading order:

  Harlander, Kilgore '02(soft expansion); Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03

  Until recently, only available in the heavy top limit (Effective Field Theory)

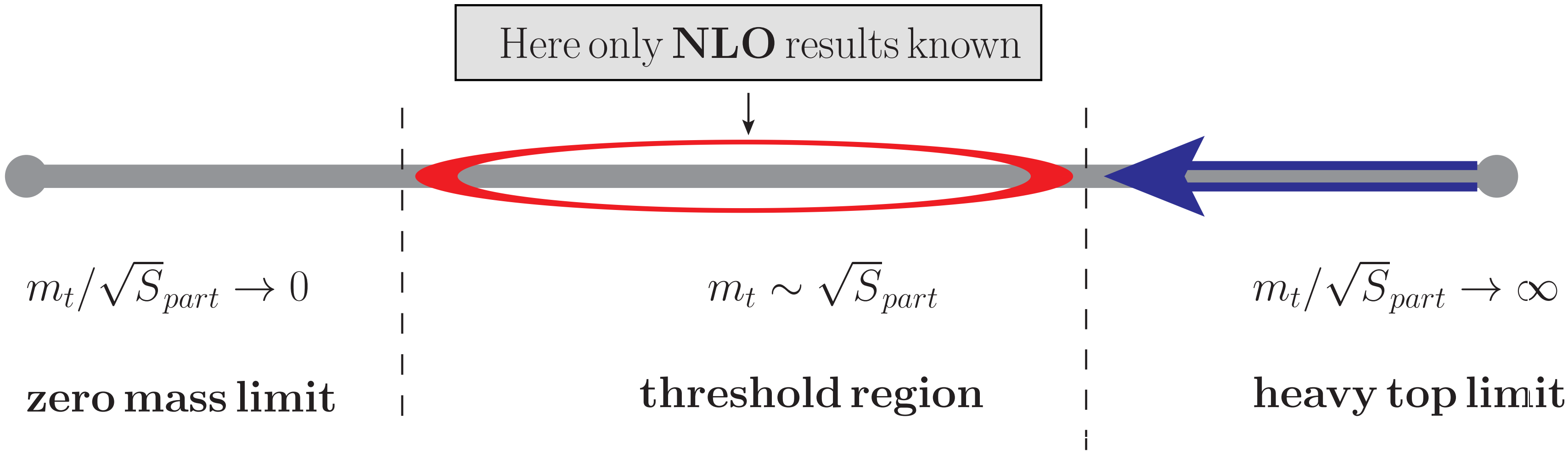


$\sim \mathcal{O}(10\%)$(scale uncert.: $\mathcal{O}(\text{few}\%)$)

Also available: $EW, QCD - EW, NNLO + NNLL, N^3LO$ threshold enhanced, π^2-resummation, $NNLO$ differential distributions...

Catani, de Florian, Grazzini, Nason; Ahrens, Becher, Neubert, Yang; Actis, Passarino, Sturm, Uccirati; Anastasiou, Boughezal, Petriello; de Florian, Grazzini; Moch,Vogt; ...

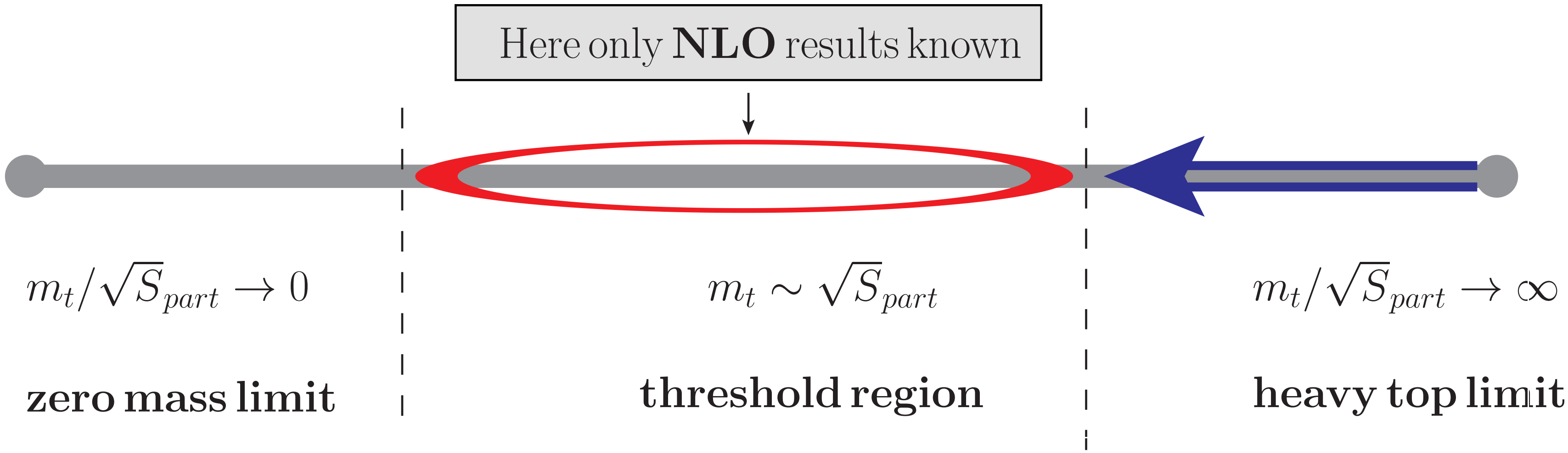
NNLO and top mass effects



H - scalar Higgs, A - pseudo-scalar Higgs

- *zero mass limit*
  H: Marzani et al'08, Harlander et al'09
  A: Caola, Marzani '11
- *effective field theory*, infinitely heavy top quark (no mass dependance)
  H: Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran et al. '03
  A: Harlander, Kilgore '02; Ravindran et al. '03; Anastasiou, Melnikov '03

NNLO and top mass effects



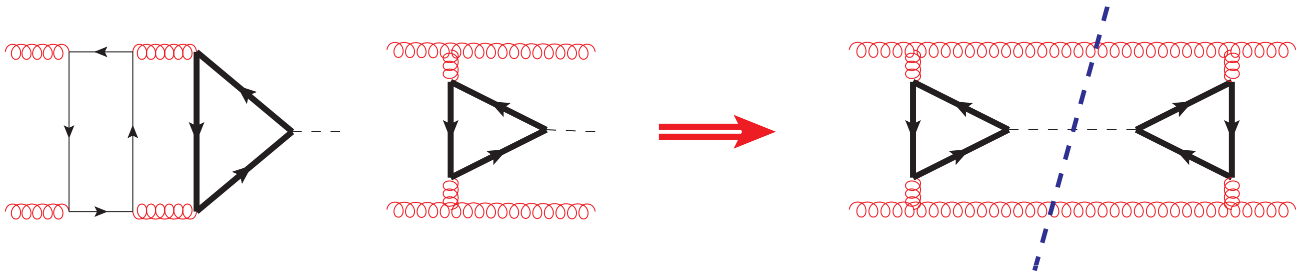
H - scalar Higgs, A - pseudo-scalar Higgs

- *expansion* in powers of $1/m_t^2$ + interpolations to *zero mass limit*
  H: Harlander, Ozeren '09 (soft expansion); Pak, MR, Steinhauser '10
  A: recent results by Pak, MR and Steinhauser JHEP **1109**, 088 (2011)

Main conclusion:
~ 120 GeV deviations from "$m_t \to \infty$" results are *small*,
~ 300 GeV in leading gg channel adds 9% for H, 22% for A at **NNLO** order $\to$

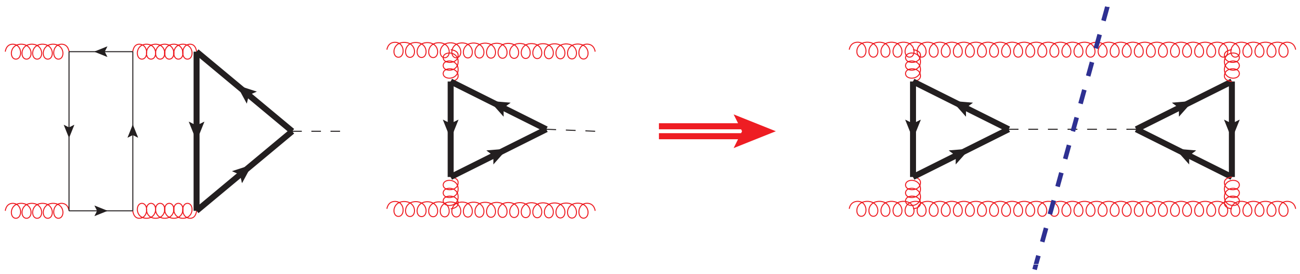
it results in the *total cross sections* to about 2% and 6%, respectively.

Optical theorem

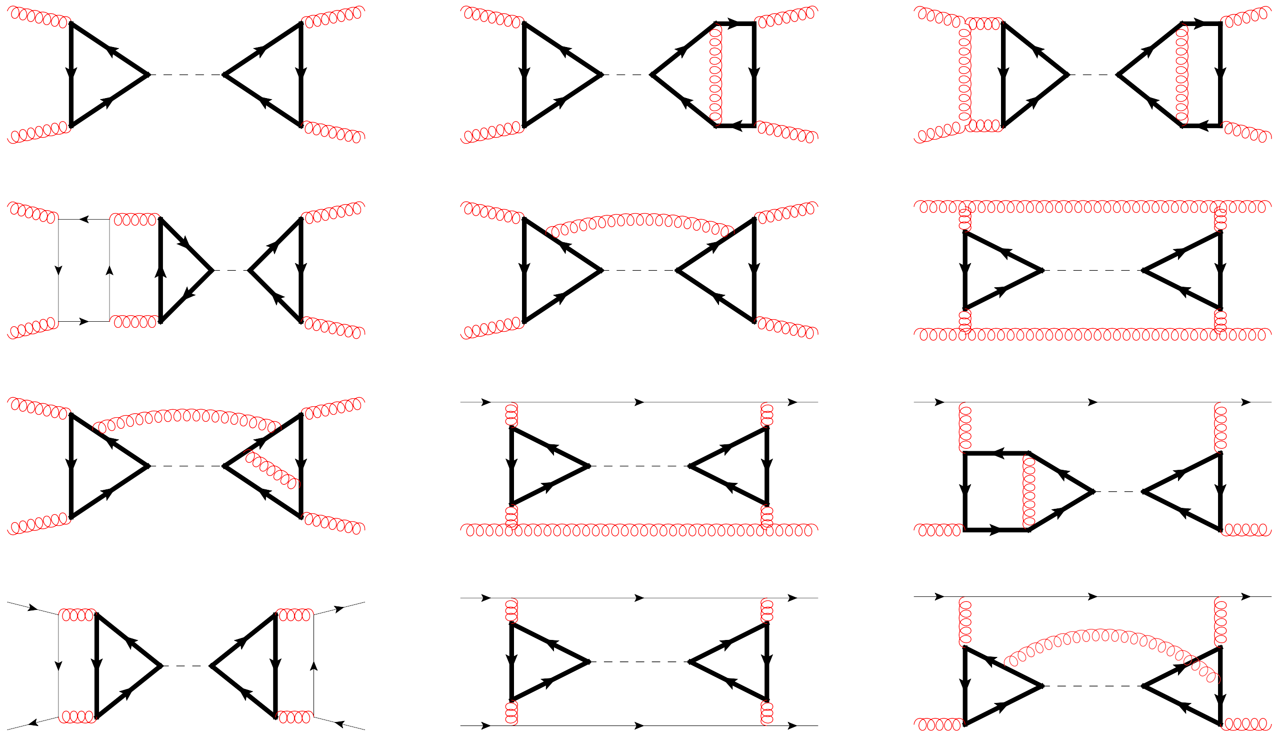


- *Imaginary parts* of forward scattering diags.

Optical theorem



- *Imaginary parts* of forward scattering diags.



- *Only cuts crossing* Higgs line should be considered.

Pseudo-scalar Higgs

Lagrangian:

$$\mathcal{L}_Y = -g_q^{Y,H} m_q^0 \frac{H^0}{v^0} \bar{q}^0 q^0 - g_q^{Y,A} m_q^0 \frac{A^0}{v^0} \bar{q}^0 i\gamma^5 q^0$$

$g_q^{Y,H}$ and $g_q^{Y,A}$ appears in many extensions of SM

e.g in MSSM: $g_t^{Y,H} \sim 1/\sin\beta$, $g_b^{Y,H} \sim 1/\cos\beta$, $g_t^{Y,A} \sim 1/\tan\beta$, $g_b^{Y,A} \sim \tan\beta$,

$\tan\beta$ - ratio of the Higgs field vacuum expectation values. Consider only small $\tan\beta$.

- Larin's prescription

$$\gamma_5 \to \frac{i}{24} \epsilon_{\mu\nu\rho\sigma} \gamma^{[\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma]}$$

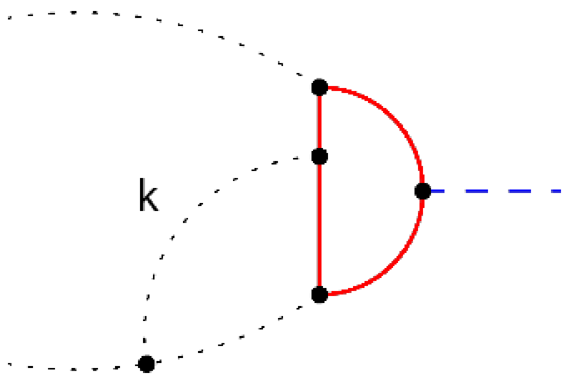
$$\epsilon^{\alpha\beta\gamma\delta}\epsilon_{\mu\nu\rho\sigma} = -g_\mu^{[\alpha} g_\nu^{\beta} g_\rho^{\gamma} g_\sigma^{\delta]} = - \begin{vmatrix} g_\mu^\alpha & g_\mu^\beta & g_\mu^\gamma & g_\mu^\delta \\ g_\nu^\alpha & g_\nu^\beta & g_\nu^\gamma & g_\nu^\delta \\ g_\rho^\alpha & g_\rho^\beta & g_\rho^\gamma & g_\rho^\delta \\ g_\sigma^\alpha & g_\sigma^\beta & g_\sigma^\gamma & g_\sigma^\delta \end{vmatrix}$$

+ finite renormalization of pseudo-scalar current through Z_5 !

! gratitude to K. Chetyrkin

Asymptotic expansion in m_H/m_t

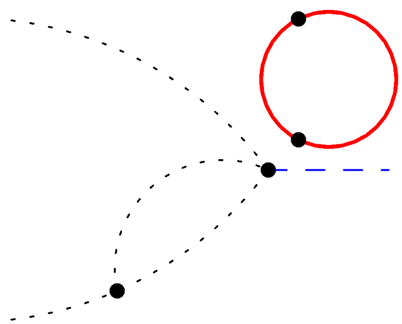
V.A. Smirnov ”Applied Asymptotic Expansions in Momenta and Masses“, Springer-Verlag 2003



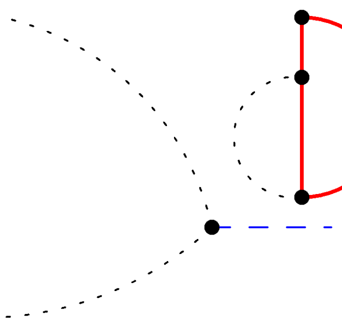
$\Longrightarrow$ loop momenta can be large or small,

use of *expansion by ”regions“:*

- $k \sim m_H, \sqrt{S_{part}} \Longrightarrow$



- $k \sim m_t \Longrightarrow$



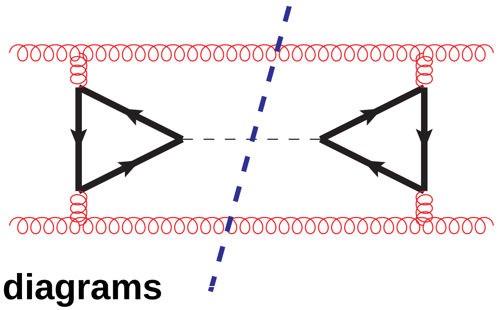
- **At NNLO, need to calculate** (1,2,3-loop tadpoles) $\otimes$ (2-loop 2$\rightarrow$1, 1-loop 2$\rightarrow$2, tree-level 2$\rightarrow$3 functions)

No need for higher order operators in EFT (no new Feynman rules, no ...)!!!

▲**To emphasize:** *same scale limitations* as for **EFT** !

Chain of calculations:

- Use of Optical Theorem:
  Imaginary Part of *4-loop* diagrams $\Rightarrow$



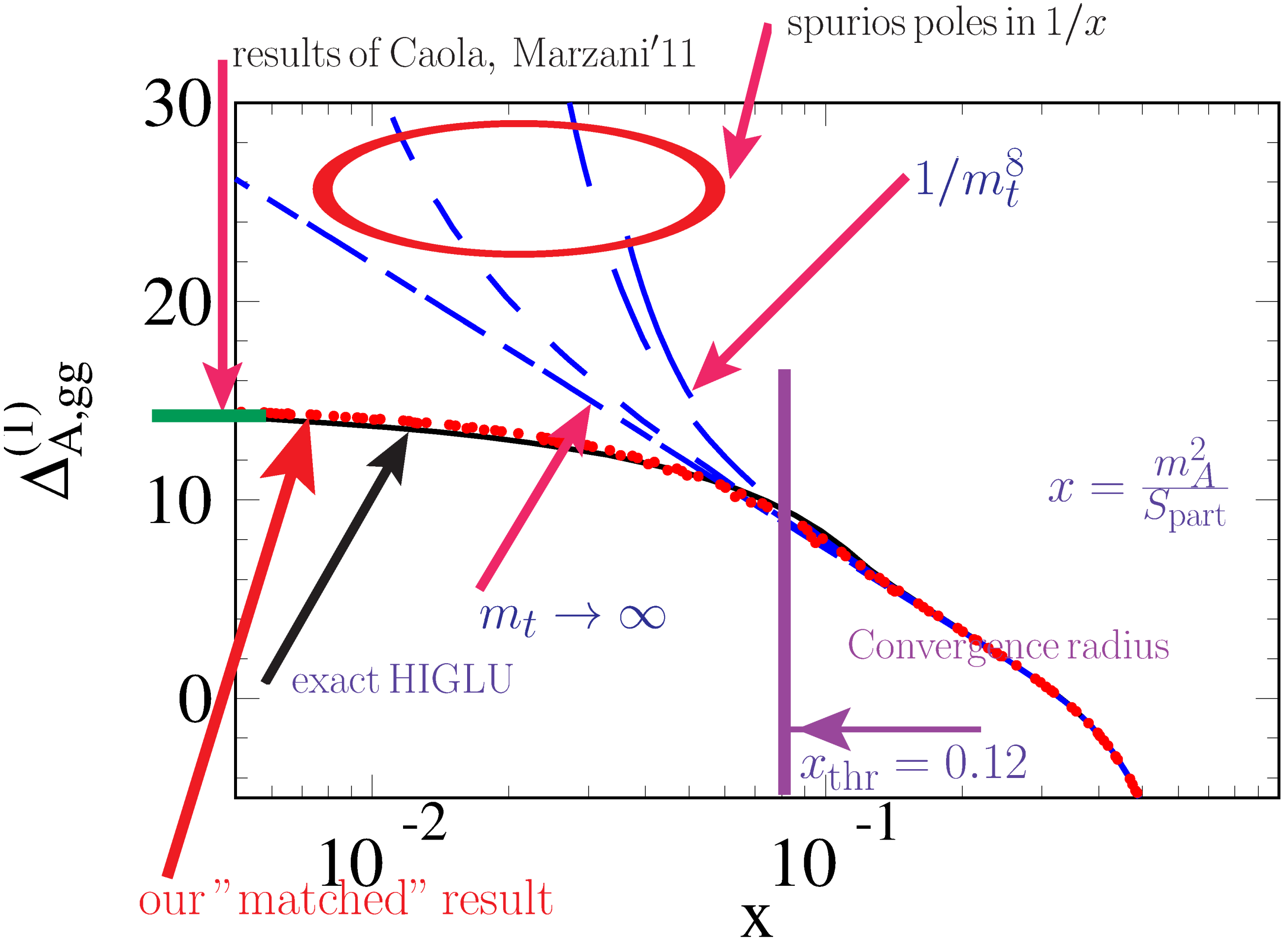
- Diagrams: QGRAF Nogueira '93, $\sim$ **20000 non-zero diagrams**
- Two independent calculations
  - Asymptotic expansion, mapping on pre-defined topologies *Q2E / EXP* and "in house" *Perl/C++* program
    1-,2-,3-loop massive vacuum bubbles $\times$ 1-, 2-loop forward scattering amplitudes
  - Calculations on FORM by *MATAD* and *by "in house" independent program*
- IBP Reduction by **Laporta algorithm** (own implement.) to masters, cut Higgs lines treated as normal propagators
- 2- and 3- particle cuts reintroduced again in masters,
  evaluated separately [Anastasiou, Melnikov '02], recalculated, extended by 1-2 orders in ε
- special treatment for virtual contribution!
- Convolution with Splitting Functions (plus-functions, HPLs) is done in Mellin space

$\sim$ **1 month @ TTP cluster, 100s Gb for intermediate calc., cross-checks, SU(N)**

Results: $\mathcal{O}\left(m_A^8/m_t^8\right)$ Full dependence on $x = m_A^2/S_{part}$.

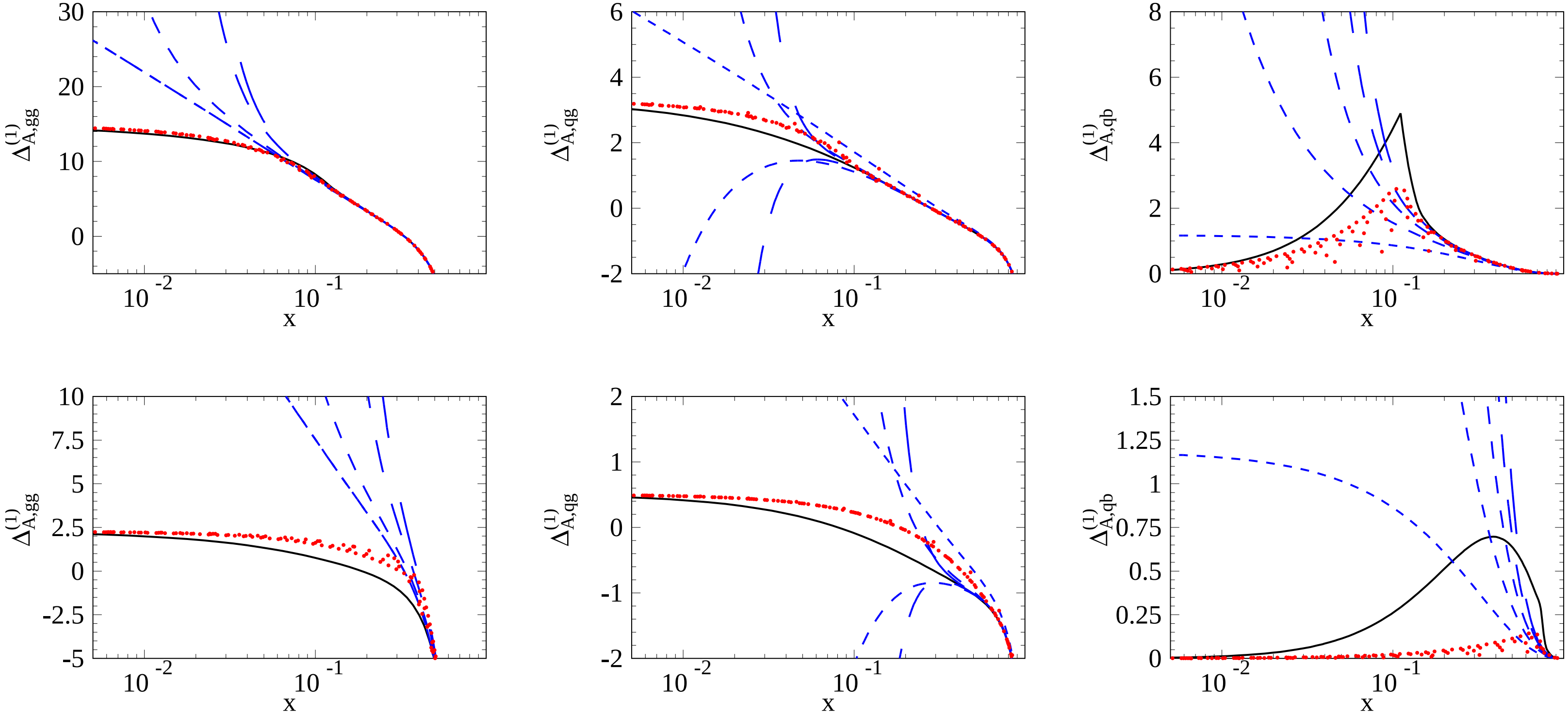
NLO results @ partonic level

$$\sigma_{part} = LO \times \left(\Delta^{(0)} + (\alpha_s/\pi)\Delta^{(1)} + (\alpha_s/\pi)^2\Delta^{(2)}\right)$$



- Caola, Marzani'11: $\Delta^{(1)}(x) \overset{x\to 0}{=} C + \mathcal{O}(x)$

NLO results @ partonic level

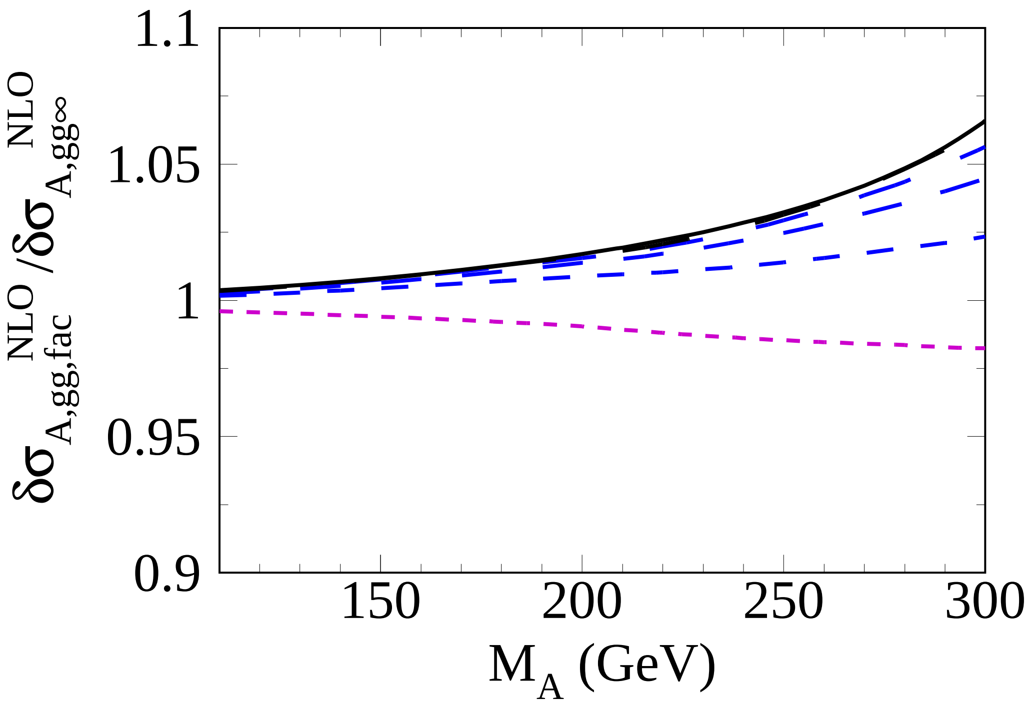


$m_A = 120$ GeV -top, $m_A = 300$ GeV - bottom.

- qg-channel hadr. contribution: $(-2)\%$ ($m_A = 110$ GeV) to (-7%) (m_A=300 GeV)
  deviation from *exact result* leads to 15% shift in qg NLO piece
- $q\bar{q}$ channel hadr. contribution $< 1\%$

NLO results @ hadr. level gg channel

$$\sigma_{pp' \to \Phi + X} = \sigma_{\Phi}^{\mathrm{LO}} + \delta\sigma_{\Phi}^{\mathrm{NLO}} + \delta\sigma_{\Phi}^{\mathrm{NNLO}}$$

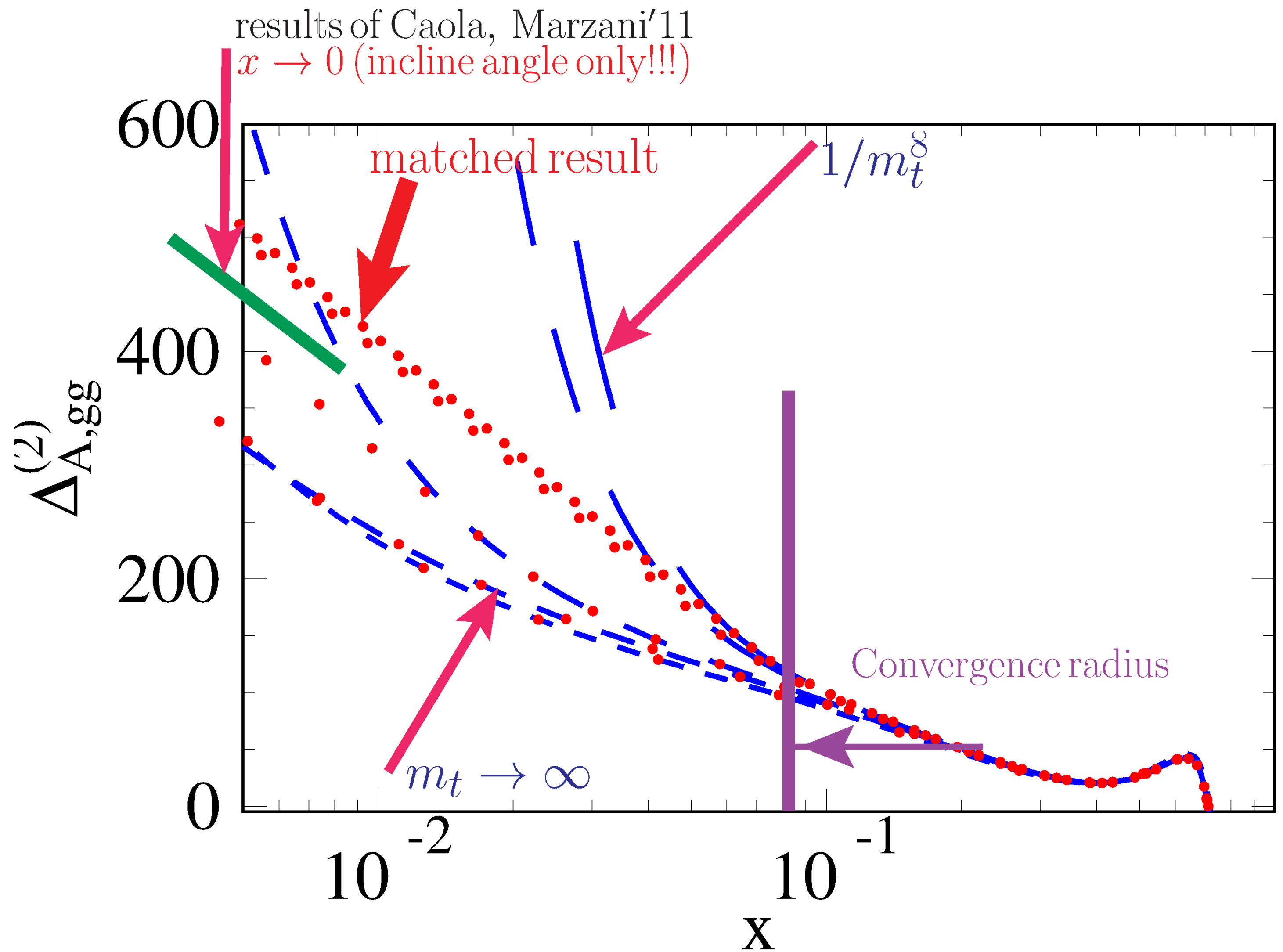


Ratio of the NLO parts to *the infinite-top mass approx.*
(made with **Effective Field Theory**) of

▲ **exact** result from **HIGLU** (black), ▲ our approximations $1/m_t^n$ (dashed)

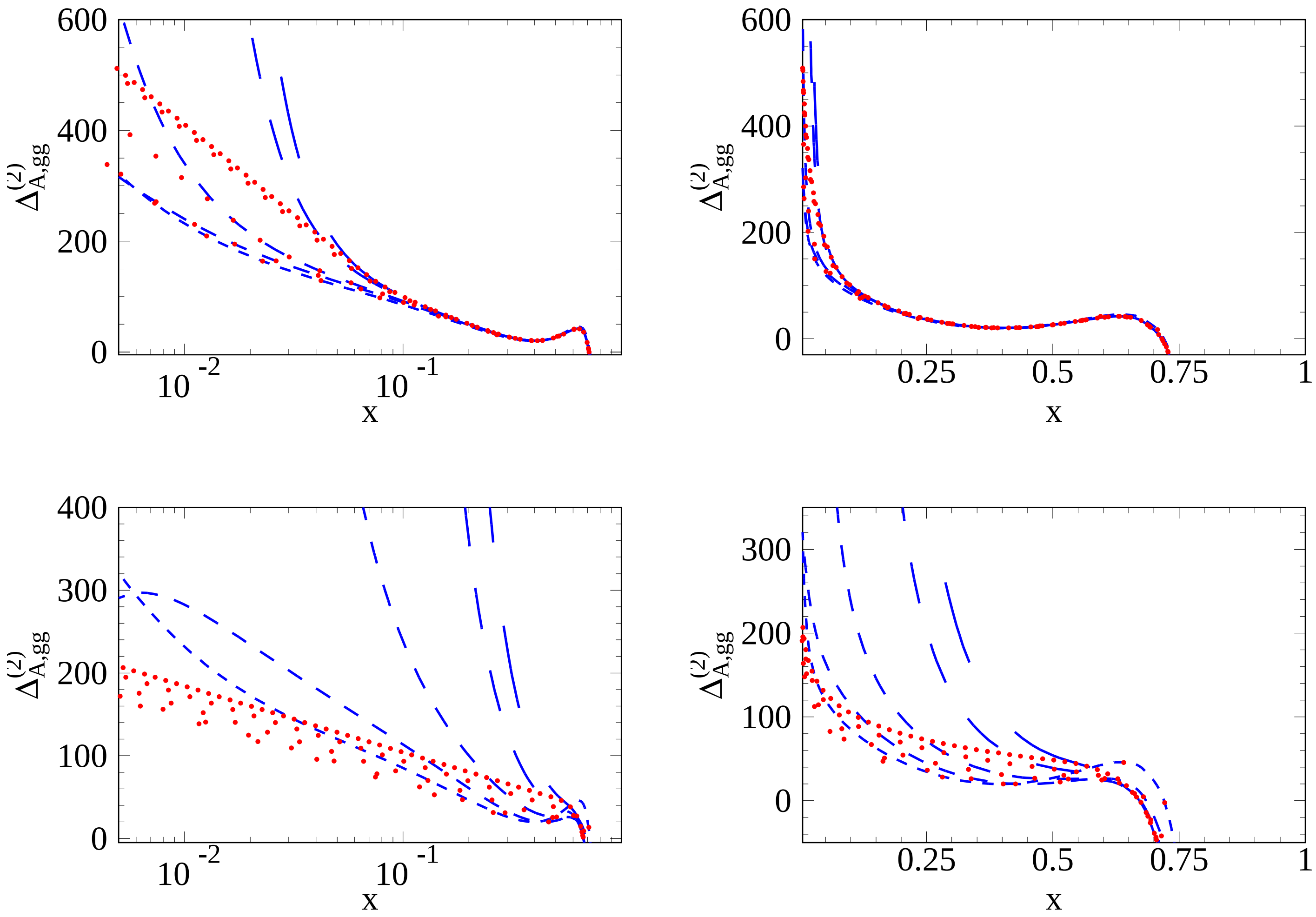
- m_A=120 GeV: deviations are *below* 1% level
- m_A=300 GeV: descrep. between **exact** and *the infinite-top mass approx.* $\sim 6\%$, perfect agreement of $\sim 1/m_t^8$ matched result with **exact** !!!

NNLO results @ part. level



- Caola, Marzani'11: $\Delta^{(2)}(x) \overset{x \to 0}{=} E \ln(x) + \mathcal{O}(1)$

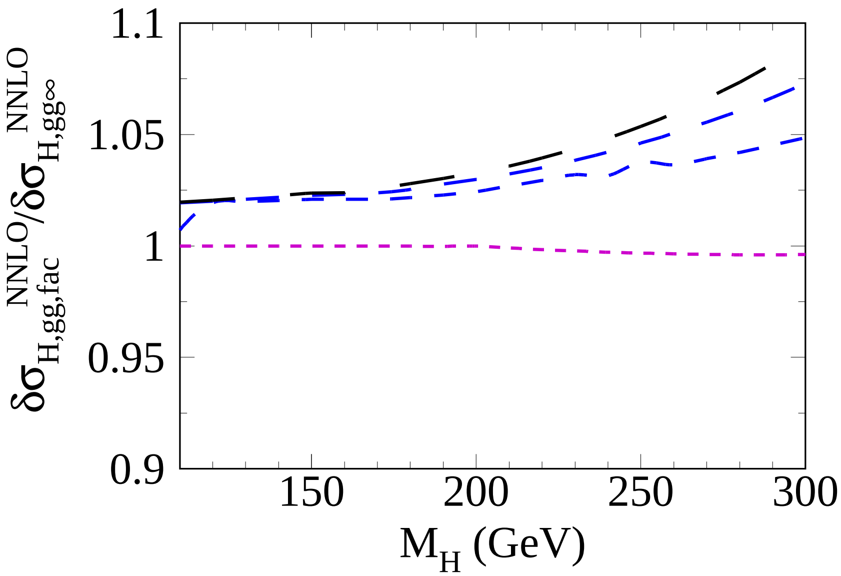
NNLO results @ partonic level

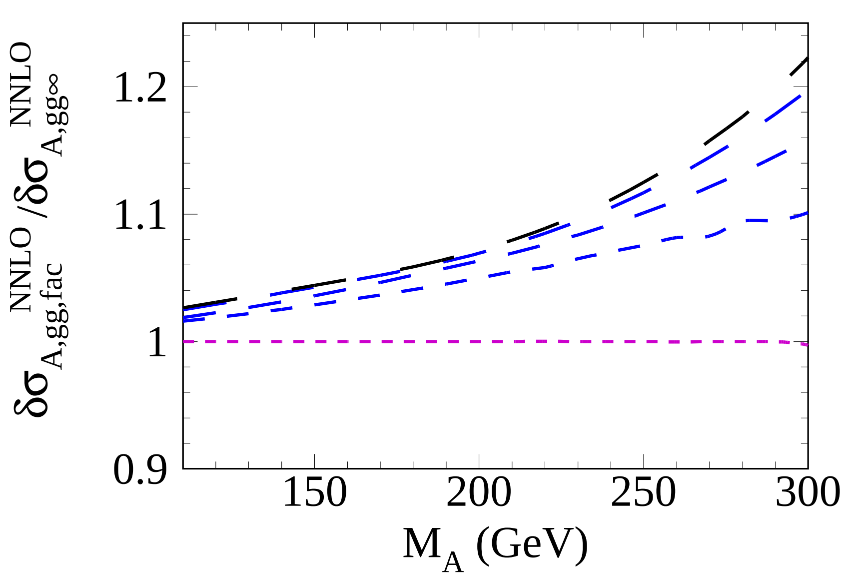


$m_A = 120$ GeV -top, $m_A = 300$ GeV - bottom.
left -logarithmic, right - linear x-scale.

NNLO results @ hadr. level

scalar H and pseudo-scalar A Higgs bosons:

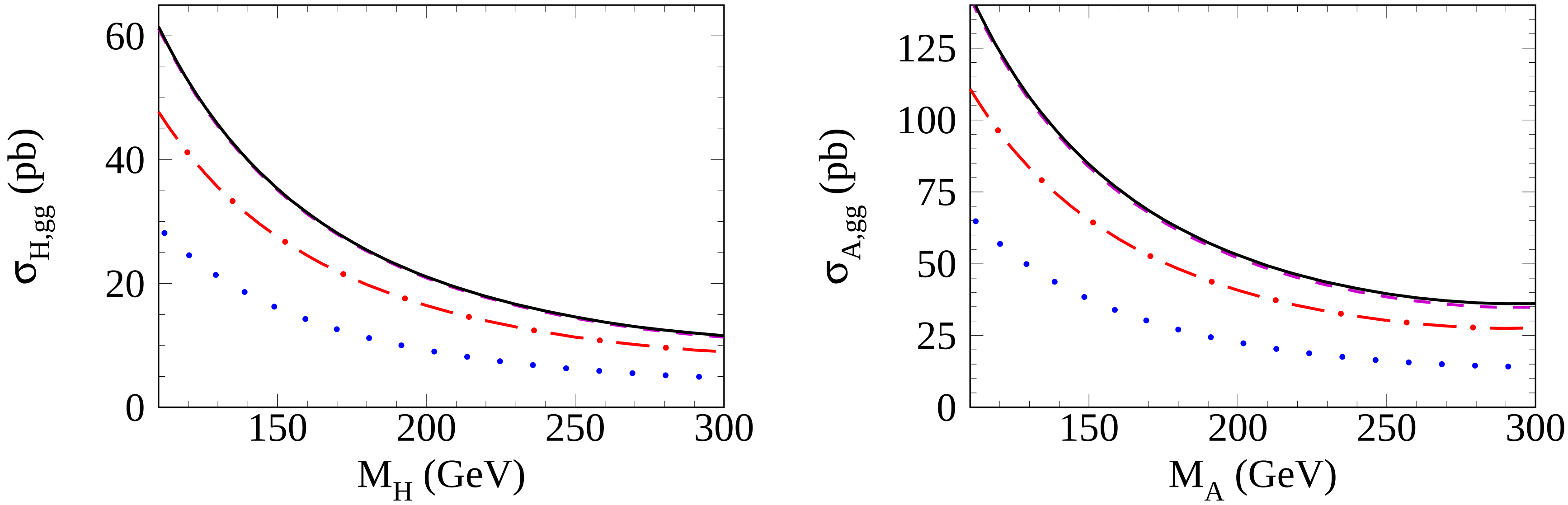




- corrections for pseudo-scalar case A to *the infinite-top mass approx.* from 2.5% to 22%
- for scalar H Higgs boson - at most 9%

Effects on total cross section at $\sqrt{s}$=14 TeV for gg channel

LO, NLO, NNLO



- small masses: deviation from *the infinite-top mass approx.* negligible
- at 300 GeV: scalar $\sim 2\%$, pseudo-scalar $\sim 6\%$

NNLO results: Channels involving quarks

- overall contribution from the qg, $q\bar{q}$, qq and qq' channels to the NNLO corrections $(-8)\%$ to $(-17)\%$, 0.1% to 0.2%, 0.08% and 0.3%
- $\Rightarrow$ One can use *the infinite-top mass approx.* for $q\bar{q}$, qq and qq' *even* if corrections are large!
- for the qg channel at $m_A = 300$ GeV corrections are $\sim 50\%$.
  Use of *the infinite-top mass approx.* may lead to 2% uncertainty in the full prediction.

Conclusions

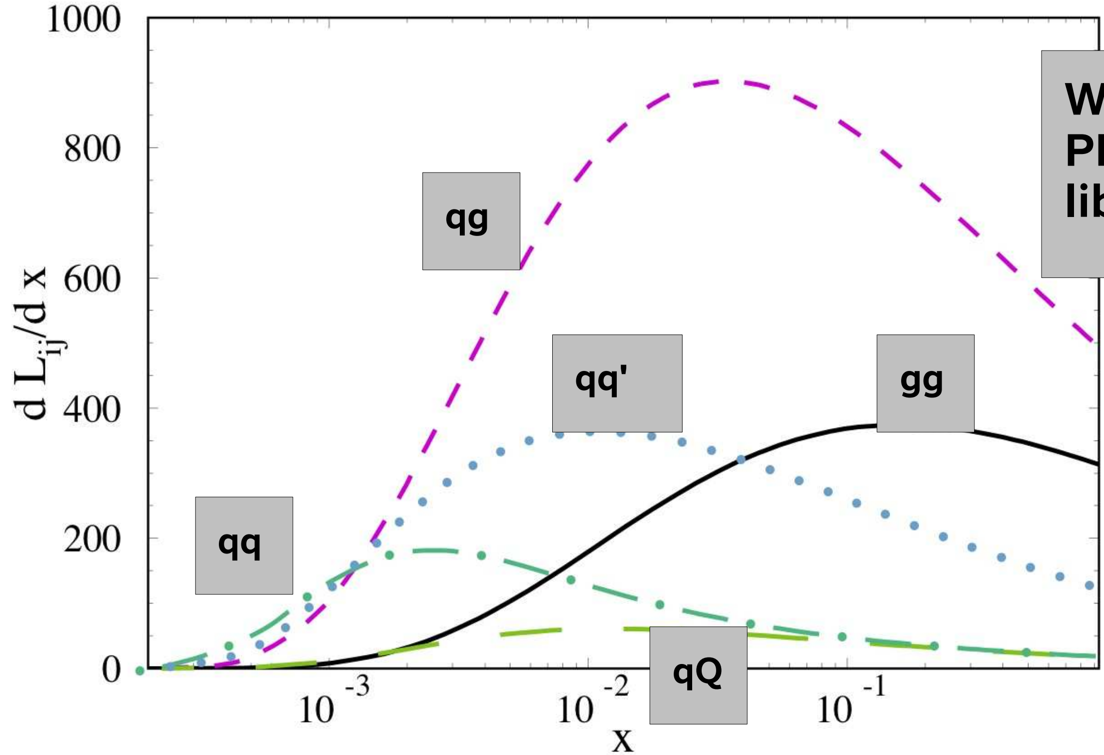
- Details in JHEP 1109 (2011) 088
- at NLO our approach reproduces *the exact results* with **very high precision**
- at NLO agreement between *the exact result* and *our approx.* ($\sim 1/M_t^8$ included) for the gg-channel $< 1\% \Rightarrow$
  confident that the same is *true* at NNLO
- our NNLO corrections deviate from *the infinite-top mass approx.* by $\sim 2\%$ for small masses.
  For higher masses larger, $m_\Phi = 300$ GeV: scalar 9%, pseudo-scalar 22%
- $\Rightarrow$ up to uncertainty $\sim 2\%$ for scalar and $\sim 6\%$ for pseudo-scalar Higgs bosons **save to use** *the infinite-top quark mass approx.*

  This is **NON-TRIVIAL** result, so far no explanation.

  For *better* precision one *should* consider top mass effects.
- For $m_\Phi = 300$ GeV accuracy of NNLO part of *the infinite-top quark mass approx.*:
  Tevatron $\rightarrow$ scalar 6%, pseudo-scalar 15%.
  LHC 7 TeV $\rightarrow$ 8% and 20%

Additional material

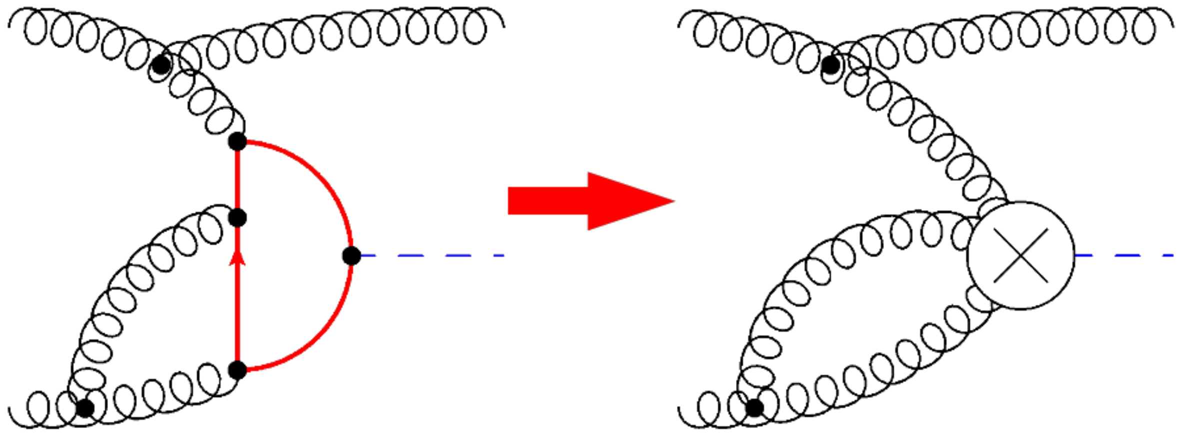
Luminosity functions: suppressed at x=0



We use MSTW2008 PDFs from LHAPDF library

$$\sigma_{pp\to H+X} = \sum_{ij=gg,\ldots} \int_{m_H^2/S_{part}}^{1} dx \left[\frac{d\,L_{ij}}{dx}\right](x)\,\sigma_{ij\to H+X}(x)$$

Effective Field Theory for heavy top limit



Effective Lagrangian :

$$L_{eff} \sim C \cdot H G_{\mu\nu} G^{\mu\nu}$$

[Shröder, Steinhauser; Chetyrkin, Kühn, Sturm; Spira et al]

- Assumptions $\frac{m_H}{m_t} \to 0$, $\frac{\sqrt{S_{part}}}{m_t} \to 0$
- EFT works reasonably good at NLO level (compared to known exact results).

NLO results @ hadr. level

$$\sigma_{pp' \to \Phi + X} = \sigma_{\Phi}^{\mathrm{LO}} + \delta\sigma_{\Phi}^{\mathrm{NLO}} + \delta\sigma_{\Phi}^{\mathrm{NNLO}}$$