Graviton plus vector boson production to NLO QCD

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in collaboration with

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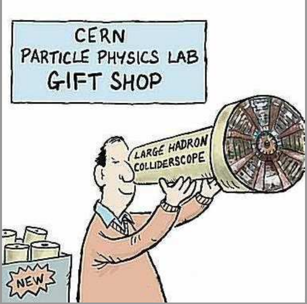
Outlines ....

- Introduction
- Collider signatures of KK Graviton
- Associated vector boson production to NLO at the LHC
- Results
- Summary

Introduction

The standard model (SM) of particle physics has been very successful in explaining the fundamental interactions of the elementary particles, and its predictions have been verified experimentally to a very good accuracy except for the discovery of the Higgs boson, the only elementary scalar particle in the SM.



There are many Beyond Standard Model (BSM) candidates, which are of great importances from different perspectives.

Some of them are :

1. **Large Extra Dimension (LED)**
2. Warped Extra Dimension
3. Universal Extra Dimension
4. Super Symmetry
5. Techni-Color
6. ....

Our main aim is to predict how LED model will affect the LHC results

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Our main aim is to predict how LED model will affect the LHC results *if it exists !!!!*

Main features of LED

[Arkani-Hamed, Dimopoulos, Dvali, PLB 429 (1998) 263]

- ✓ All SM particles are localised on a 3-brane and gravity can propagate in the full $4+d$ dimensional space-time
- ✓ The extra dimensions d are spatial and compactified on a d-dimensional torus of common radius $R/2\pi$
- ✓ These extra dimensions can be of macroscopic size and from some experimential bounds $d \geq 2$
- ✓ The fundamental Planck scale (M_S) in $4+d$ dimension is related to the Planck scale (M_P) in usual 4 space-time dimension as

$$\boxed{M_p^2 = C_d \; M_s^{2+d} \; R^d}$$

where, $C_d = 2 \; (4\pi)^{-\frac{d}{2}} / \Gamma(d/2)$

Main features of LED (contd....)

✓ Due to the compactification, graviton propagating in $4+d$ dimensional space-time manifests itself as a tower of massive graviton modes in 4-dimension

$$\boxed{m_{\vec{n}}^2 = \frac{4\pi^2 \vec{n}^2}{R^2}}$$

✓ The SM particles can feel these KK modes of the graviton

$$\boxed{\mathcal{L} = -\frac{\kappa}{2} \sum_{\vec{n}=0}^{\infty} T^{\mu\nu}(x) h_{\mu\nu}^{\vec{n}}(x)}$$

where $\kappa = \sqrt{16\pi}/M_P$, $T^{\mu\nu}$ is the energy-momentum tensor of localised SM fields and $h_{\mu\nu}^{\vec{n}}$ denotes the massive KK graviton

Address the large hierarchy between the Electro-weak and the Planck scale

Figure: Rebellious guy dreams of becoming a regular guy

Collider signatures of KK Graviton

Viable signatures of the ADD scenario at the LHC are possible by,

1. emission of real KK modes from the SM particles, leading to a missing energy signal

$$d\sigma = S_{\delta-1} \frac{\overline{M}_P^2}{M_D^{2+\delta}} \int dm \; m^{\delta-1} \; d\sigma_D^{(m)}$$

2. exchange of virtual KK modes between the SM particles, leading to an enhanced cross section

$$\mathcal{D}_{eff}(s) = \sum_{\vec{n}} \frac{1}{s - m_{\vec{n}}^2 + i\varepsilon}$$

[Giudice, Rattazzi, Wells, NPB 544 (1999) 3]

[Han, Lykken, Zhang, PRD 59 (1999) 105006]

If N denotes the number of KK modes with momentum within k and $k + dk$ along the extra dimension, then

$$\boxed{dN = k^{\delta-1}\ dk\ \Omega_{\delta-1}}$$

Placing the actual mass $(m = k/R)$ of a given KK mode and using the relation between M_p and M_s we get,

$$\boxed{dN = \frac{M_p^2}{M_s^{\delta+2}}\ m^{\delta-1}\ dm\ \Omega_{\delta-1}}$$

For calculating inclusive cross section,

$$\boxed{\frac{d^2\sigma}{dt\ dm} = \frac{M_p^2}{M_s^{\delta+2}}\ m^{\delta-1}\ \Omega_{\delta-1}\frac{d\sigma_m}{dt}}$$

The cross section for an individual KK mode is proportional to $\frac{1}{M_p^2}$,

$$\boxed{\frac{d^2\sigma}{dt\ dm} \sim \frac{1}{M_s^{\delta+2}}\ m^{\delta-1}\ \Omega_{\delta-1}}$$

Graviton plus Vector Boson Production

Associated vector boson production to NLO at the LHC

We have considered the graviton production in association with a vector boson at the LHC to NLO in QCD.

$$\boxed{P + P \longrightarrow V + G_{KK} + X}$$

where, $V = Z, W^{\pm}$.

[M. C. Kumar, P. Mathews, V. Ravindran, SS, J.Phys.G G38 (2011) 055001]

[M. C. Kumar, P. Mathews, V. Ravindran, SS, Nucl.Phys. B847 (2011) 54-92]

Motivation

- The production of vector boson $(Z, W^{\pm})$ with invisible graviton can give rise to a very large missing transverse momentum signal at the collider experiments
- This process has been studied at LO in context of lepton colliders

  [Cheung, Keung, PRD 60 (1999) 112003]

  [Giudice, Plehn, Strumia, NPB 706 (2005) 455]
- This is as well as studied in the context of hadron collider

  [Stefan Ask, EPJ C 60 (2009) 509]
- It has also been implemented in Pythia8
- Event selection and minimization of other SM contribution to the process $ZZ \to l\bar{l}\nu\bar{\nu}$, using MC@NLO and Pythia, is taken up in ATLAS detector simulation

  [G. Aad, et al., ATLAS Collaboration, CERN-OPEN-2008-020, arXiv: hep-ex/1004.5519]

LO diagrams

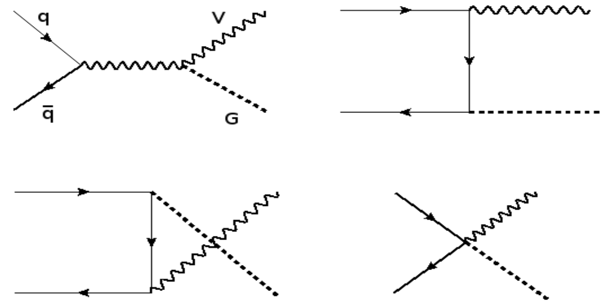

Figure: LO diagrams

The best way to find these diagrams is to consider the single born diagram $q\ \bar{q} \rightarrow V$ and attach the KK modes of graviton for being emmited in all possible way to it i.e, in fermion legs, boson leg and $q\ \bar{q}\ V$ vertex.

Real Gluon Emission Diagrams

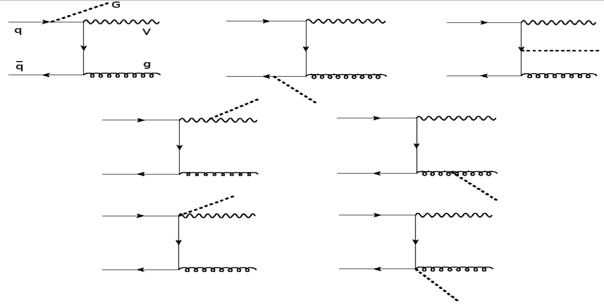

Figure: Real gluon emission diagrams

There are total 14 diagrams. Rest of the diagrams can readily be obtained if one invert the charge flow direction of the quark lines in this figure.

Virtual Gluon Emission Diagrams

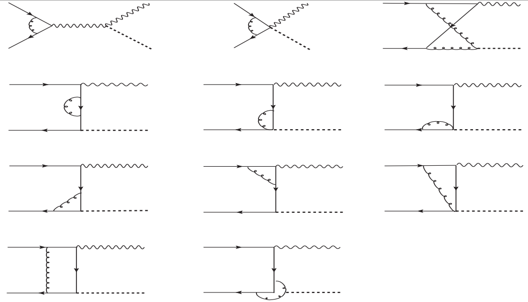

Figure: Virtual gluon emission diagrams

All together there are 27 diagrams out of which 8 diagrams contribute nothing in this process. Rest of the digrams can easily be obtained if the charge flow direction of the quark lines of the last eight diagrams of the Fig. are inverted.

Calculatioal Details

Through out our calculation we have used,

- massless quarks with $n_f = 5$
- $n = 4 + \epsilon$ for dimensional regularisation
- completely anti-commuting γ_5 prescription to deal with γ_5 in n-dimension
- algebric manipulation program FORM to evaluate analytical results
- the two cut-off phase space slicing method to tackle various singularities appearing in the NLO computation
- multi-dimensional integration package VEGAS in our numerical code to do 3-body phase space integration

Phase space slicing with two cut-off

[Harris, Owens, PRD 65 (2002) 094032]

Decomposition of three-body phase space used to calculate the two-to-three contribution to the partonic cross-section into two regions- soft(S) and hard(H):

$$\boxed{\sigma = \frac{1}{2\Phi} \int \overline{\sum} |M_3|^2 \, d\Gamma_3}$$

where, Φ is the usual flux factor, which depends on the partonic center-of-momentum energy squared s. $\overline{\sum}|M_3|^2$ is the two-to-three body squared matrix element avaraged over initial degrees of freedom and summed over final degrees of freedom and $d\Gamma_3$ is the three body phase space.

Decomposition

$$\boxed{\sigma = \sigma_S \ + \ \sigma_H}$$

$$\boxed{\sigma_{S(H)} = \frac{1}{2\Phi} \int\limits_{S(H)} \overline{\sum} |M_3|^2 \, d\Gamma_3}$$

The partitioning of phase space into S and H depends on a parameter δ_s, called soft cut-off. If, there is collinear singularity then,

$$\boxed{\frac{1}{2\Phi} \int\limits_{H} \overline{\sum} |M_3|^2 \, d\Gamma_3 = \frac{1}{2\Phi} \int\limits_{HC} \overline{\sum} |M_3|^2 \, d\Gamma_3 + \frac{1}{2\Phi} \int\limits_{H\overline{C}} \overline{\sum} |M_3|^2 \, d\Gamma_3}$$

This partitioning depends on a second cut-off δ_c, called collinear cut-off.

Phase Space Regions

The phase space region is divided in three parts depending on the cuts which are the following,

- soft region : $0 \leq E_5 \leq \frac{1}{2}\sqrt{s_{12}}\delta_s$
- hard region : $E_5 > \frac{1}{2}\sqrt{s_{12}}\delta_s$

In the HC region of phase space,

- any invariant (s_{ij} or t_{ij}) $< \delta_c \, s_{12}$.
- all gluons remain hard i.e., $E_5 > \delta_s \frac{1}{2}\sqrt{s_{12}}$.

When virtual contribution is added with these results in the same order, all the singularities cancel.
The rest of the region is free from any singularities.

Real Emission : Leading Pole Approximation

In the soft limit,

- Matrix element

$$M_3 \mid_{soft} \simeq -g_s \mu_r^{-\frac{\epsilon}{2}} \varepsilon_\sigma(p_5) T^a_{ij} \left(\frac{p_2^\sigma}{p_2.p_5} - \frac{p_1^\sigma}{p_1.p_5} \right) M_2$$

- Phase space $(p_5 \to 0)$

$$d\Gamma_3^{soft} = d\Gamma_2 \left(\frac{4\pi}{s_{12}} \right)^{-\frac{\epsilon}{2}} \frac{\Gamma(1+\epsilon/2)}{\Gamma(1+\epsilon)} \frac{1}{2(2\pi^2)} d\mathcal{S}$$

Further Steps

- Performing angular integrations over the Eikonal current
- Integrating E_5 integral over the soft limit

We are left with,

$$\boxed{d\hat{\sigma}_s = d\hat{\sigma}_0 \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1+\frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \left(\frac{4\pi\mu_R^2}{s} \right)^{-\frac{\epsilon}{2}} \right] C_F \left(\frac{8}{\epsilon^2} + \frac{8}{\epsilon}\, ln\ \delta_s + 4\, ln^2\ \delta_s \right)}$$

Real Emission : Leading Pole Approximation

In the collinear limit,

- Matrix element

$$\overline{\sum} \mid M_3(1,2 \to 3,4,5) \mid^2 |_{coll} \simeq \mid M_3(1,2' \to 3,4) \mid^2 g^2 \mu_r^{2\epsilon} P_{2'2}(z,\epsilon) \left(\frac{-2}{z t_{25}} \right)$$

- Phase space

$$d\Gamma_3 = d\Gamma_2 \frac{(4\pi)^{-\epsilon/2}}{16\pi^2 \Gamma(1+\epsilon/2)} dz dt_{25} (-(1-z)t_{25})^{\epsilon/2}$$

One Step Ahead

- Performing dt_{25} integral in the collinear region i.e,

$$0 < -t_{25} < \delta_c s_{12}$$

We are left with,

$$d\sigma_c = d\hat{\sigma}_0 \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1+\frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \left(\frac{4\pi\mu_R^2}{s} \right)^{-\frac{\epsilon}{2}} \right] \Big(\Big\{ G_{\bar{q}/p}(x_2,\mu_F) [\widetilde{G}_{q/p}(x_1,\mu_F) + G_{q/p}(x_1,\mu_F) \left(-\frac{1}{\epsilon} + ln \frac{s}{\mu_F^2} \right) A_{q\to q+g} + (q \leftrightarrow \bar{q}) \Big\} + (x_1 \leftrightarrow x_2) \Big) dx_1 dx_2$$

where, $A_{q\to q+g} = 2C_F \left(2ln\delta_s + \frac{3}{2} \right)$.

Virtual Gluon Exchange

- $M_{Virtual} \bigotimes M_{Born}$ gives the $\mathcal{O}(\alpha_{\int})$ contribution
- Integrating over loop momenta it looks like the following

$$\boxed{\mid M_{Virtual} \mid^2 = \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon} + C\right) \mid M_{Born} \mid^2}$$

After 2-body phase space integration, we finally get,

$$\boxed{d\hat{\sigma}_v = d\hat{\sigma}_0 \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1+\frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \left(\frac{4\pi\mu_R^2}{s}\right)^{-\frac{\epsilon}{2}}\right] C_F \left(-\frac{8}{\epsilon^2} + \frac{6}{\epsilon} \ + V_{finite}\right)}$$

At a Glance

$$d\hat{\sigma}_s = d\hat{\sigma}_0 \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1+\frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \left(\frac{4\pi\mu_R^2}{s} \right)^{-\frac{\epsilon}{2}} \right] C_F \left(\frac{8}{\epsilon^2} + \frac{8}{\epsilon}\ ln\ \delta_s + 4\ ln^2\ \delta_s \right)$$

$$d\sigma_c = d\hat{\sigma}_0 \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1+\frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \left(\frac{4\pi\mu_R^2}{s} \right)^{-\frac{\epsilon}{2}} \right] \Big(\Big\{ G_{\bar{q}/p}(x_2,\mu_F)[\widetilde{G}_{q/p}(x_1,\mu_F) + G_{q/p}(x_1,\mu_F) \left(-\frac{1}{\epsilon} + ln \frac{s}{\mu_F^2} \right) A_{q\to q+g} + (q \leftrightarrow \bar{q}) \Big\} + (x_1 \leftrightarrow x_2) \Big) dx_1 dx_2$$

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$$d\hat{\sigma}_v = d\hat{\sigma}_0 \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1+\frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \left(\frac{4\pi\mu_R^2}{s} \right)^{-\frac{\epsilon}{2}} \right] C_F \left(-\frac{8}{\epsilon^2} + \frac{6}{\epsilon} + V_{finite} \right)$$

Kinematical Cuts

The following cuts are used for our numerical studies,

1. $p_T^{Z,W} > p_T^{min}$
2. $p_T^{miss} > p_T^{min}$
3. $\mid y^{Z,W} \mid \leq 2.5$

The observable jet is defined as follows,

- $p_T^{jet} > 20$ GeV
- $\mid \eta^{jet} \mid \leq 2.5$

Results

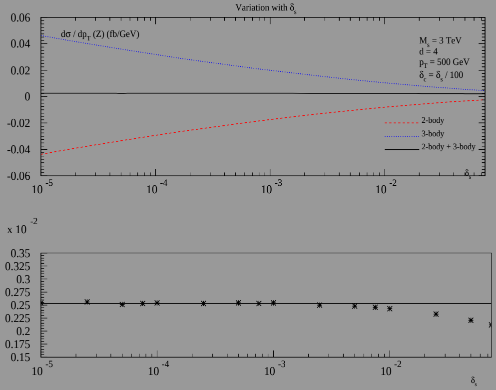

Figure: Variation of the transverse momentum distribution of Z boson with δ_s, keeping the ratio $\delta_s/\delta_c = 100$ fixed, for $M_s = 3$ TeV and $d = 4$.

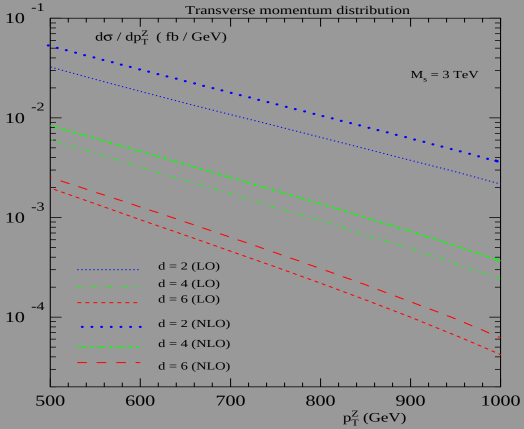

Figure: Transverse momentum distribution of the Z-boson for $M_s = 3$ TeV is shown for different values of the number of extra dimensions d.

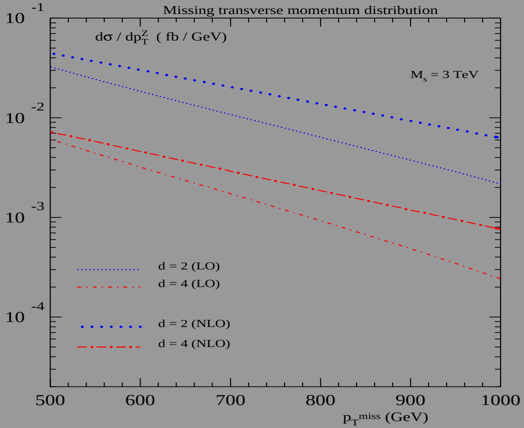

Figure: Missing transverse momentum distribution of the graviton produced in association with Z-boson at the LHC, for $M_s = 3$ TeV .

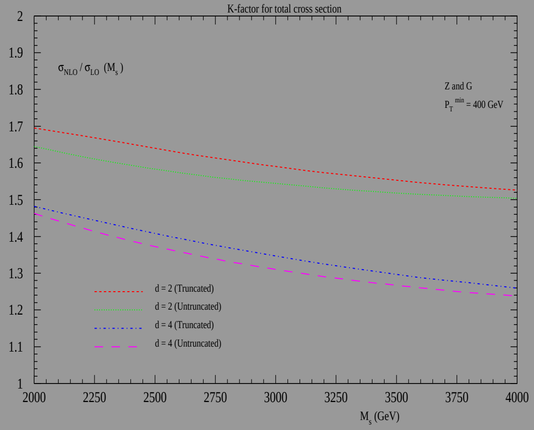

Figure: K-factors of the total cross section for the associated production of the Z-boson and the graviton at the LHC, given as a function of the scale M_s .

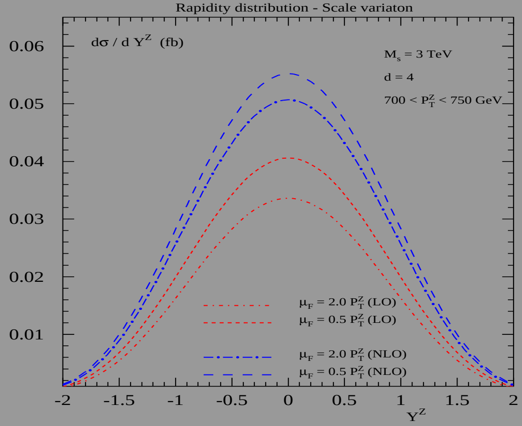

Figure: Scale uncertainties in the rapidity distribution of Z-boson for $M_s = 3$ TeV and $d = 4$.

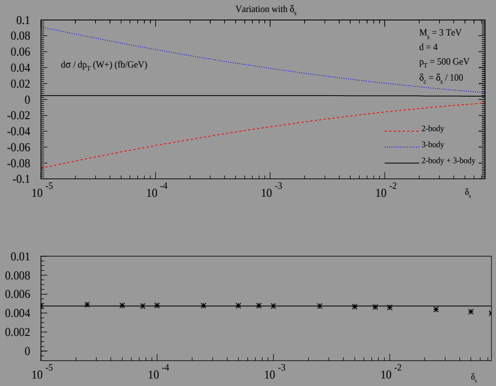

Figure: Variation of the transverse momentum distribution of W^+ boson with δ_s, keeping the ratio $\delta_s/\delta_c = 100$ fixed, for $M_s = 3$ TeV and $d = 4$.

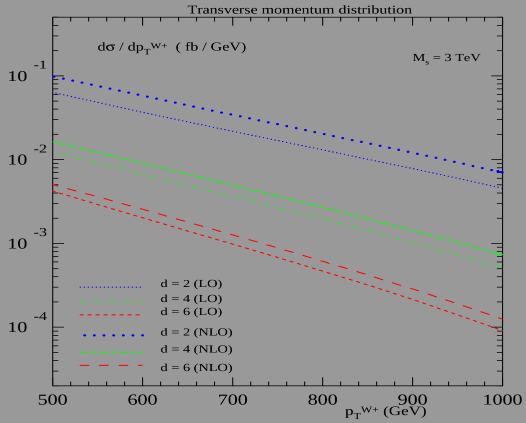

Figure: Transverse momentum distribution of the W^+-boson for $M_s = 3$ TeV is shown for different values of the number of extra dimensions d .

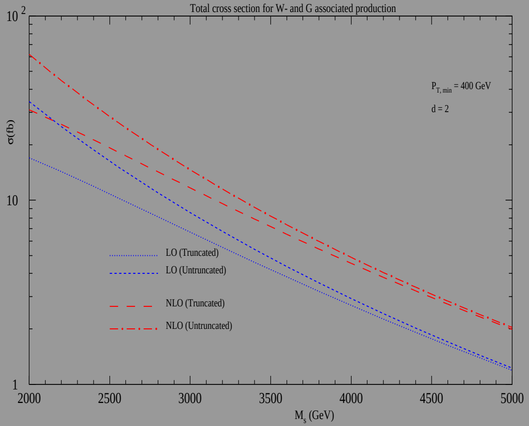

Figure: Total cross section for the associated production of W^- boson and the graviton at the LHC, given as a function of M_s for $d = 2$.

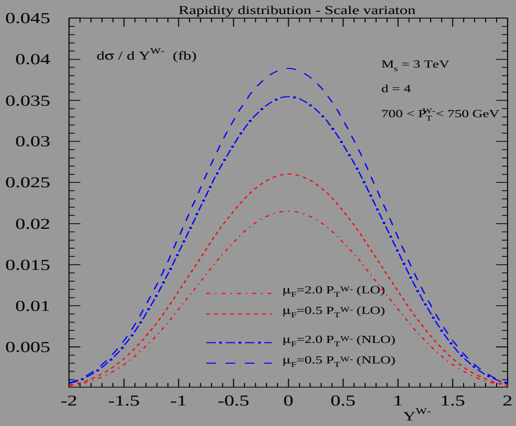

Figure: The scale uncertainties in the rapidity distribution of W^- boson for $M_s = 3$ TeV and $d = 4$.

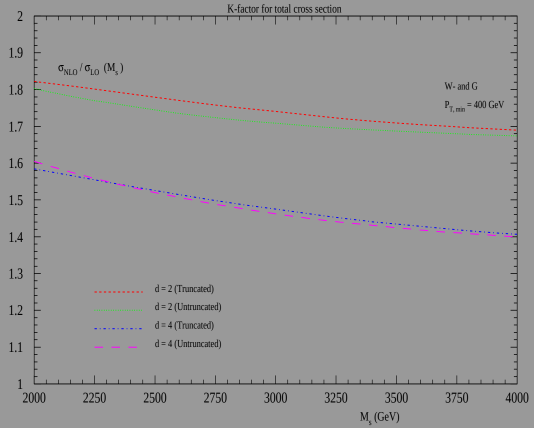

Figure: K-factors of the total cross section for the associated production of W^- boson and the graviton at the LHC, given as a function of the scale M_s .

Summary

- Full NLO correction is done for the process $P + P \longrightarrow Z/W^{\pm} + G$
- Effects of LED for these processes are shown explicitly
- Scale uncertainty is minimised in the NLO improved results
- Ultra-violet sensitivities of the results are checked

THANK YOU

KK Reduction

In the $\mathcal{M}_4 \times \mathcal{K}$, x^μ denotes the co-ordinates in $\mathcal{M}_4$ and y^i denotes the co-ordinates in $\mathcal{K}$, where,
$\mu = 0, 1, 2, 3$ and $i = 1, 2, \ldots, \delta$.

For simplicity, let us assume that $\mathcal{K}$ is S^1. In other wowds, we are assuming one extra spatial dimension which is circularly compactified.

$$y \longrightarrow y + 2\pi R$$

where, R is the radius of the compactified circle.

In this space-time geometry $(5D)$, we want to examine the complex Klein-Gordon action.

In this case, $\Phi(x^\mu, y)$ must be periodic under the condition $y \longrightarrow y + 2\pi R$, i.e.,

$$\Phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \phi_n(x^\mu) \exp\left(\frac{iny}{R}\right)$$

and, the $5D$ Klein-Gordon action is,

$$\mathcal{S}_5 = \int d^4x \int_0^{2\pi R} dy \left[\frac{1}{2}(\partial_M \Phi)^\star (\partial^M \Phi) - \frac{1}{2} m_0^2 \Phi^\star \Phi\right]$$

where, m_0 is the mass of the field in $5D$.

We now use the orthogonal property of the exponential wave-functions with different frequencies, i.e,

$$\int_0^{2\pi R} dy \; e^{i(n-m)y/R} \;=\; 2\pi R \; \delta_{mn}$$

After doing the y-integration, what is left is the following,

$$\mathcal{S}_4 = \int d^4x \sum_n \left[\frac{1}{2}(\partial_\mu \phi_n)^\star (\partial^\mu \phi_n) - \frac{1}{2}\left(m_0^2 + \frac{n^2}{R^2}\right)\phi_n^\star \phi_n\right]$$

This is nothing but the corresponding action in $4D$.

So, the mass term in $4D$ is,

$$m^2 = m_0^2 + \frac{n^2}{R^2}$$

where, $p_5 = n/R$ is the quantized momentum in the compactified fifth dimension.

More generally, for δ extra spatial dimensions on circle with radius R_i , $(i = 1, 2, \ldots, \delta)$

$$m^2 = m_0^2 + \sum_{i=1}^{\delta} \frac{n_i^2}{R_i^2}$$

So, there is infinite tower of Kaluza-Klein (KK) states associated with our known fields.