Herwiri2: CEEX Electroweak Corrections in a Hadronic MC

S.A. Yost

The Citadel – The Military College of South Carolina

with Miroslav Hejna and Valerie Halyo (Princeton) and B.L.F. Ward (Baylor)

Vector Boson Production

- Vector Boson Production is an important process at the LHC, and electro-weak radiative corrections will be needed to obtain measurements on the percent level.
- While studying these corrections using the state-of-the-art electro-weak corrections of Horace, we (Nadia Adam, Valerie Halyo, and S.Y.) found that the EW contribution to $pp \to Z/\gamma^* \to f\overline{f}$ is typically several percent, requiring precise control if measurements become possible at the percent level, as anticipated.
- For precision calculations, multiple approaches and calculations provide valuable cross-checks. Also, it is desirable to have the QCD and QED corrections fully integrated in the same event generator, *e.g.* NLO EWK + NLO QCD.

QCD⊗QED Exponentiation

- In my 2005 RADCOR talk, I described plans by B.F.L. Ward and myself to develop such a generator, and presented an overall structure based on QCD⊗QED nonabelian exponentiation.
- Motivation: the successful application of YFS exponentiation to to BHLUMI, BHWIDE, $\mathcal{KK}$ MC KORALZ, KORALW, and related programs to achieve high precision in LEP processes.
- These programs benefit from a very efficient representation of N-photon phase space with complete control over the soft and collinear singularities for arbitrary numbers of photons.
- Real and Virtual IR singularities cancel exactly to infinite order.
- The non-abelian extension should have the same advantages for N-gluon amplitudes The IR singularity cancelation is more complicated, but is still gauranteed at all orders.

The HERWIRI Project

- The class of programs based on this idea has since been named HERWIRI, for High Energy Radiation With Infra-Red Improvements, a name acknowledging the fact that our present efforts build upon one of the leading shower generators, HERWIG.
- The structure is not tied to a particular shower, and our ultimate goal is a complete shower-generator based entirely on QCD$\otimes$QED nonabelian exponentiation with exact $\mathcal{O}(\alpha_s^2, \alpha_s\alpha, \alpha^2)$ residuals.
- The first program to be publicly released in this series was HERWIRI1, which applied the proposed exponentiation to the shower's splitting kernels.
- Work on incorporating $\mathcal{O}(\alpha)$ (or better) EWK corrections in the same exponentiation paradigm began in parallel, and is close to the point of producing results. This program will be called HERWIRI2. It is independent of HERWIRI1, although the two can be combined.
- Future work would use this framework to develop a self-contained program combining YFS exponentiated photon emission with exponentiated gluon emission.

EWK Corrections in a Hadronic Context

Some other projects that have combined EWK and Hadronic Physics in MC generators:

- PHOTOS (E. Barberio, B. van Eijk, P. Golonka, Z. Wąs) is an add-on generator which adds multi-photon emission to charged final state particles, using YFS-inspired exponentiation.
- ZINHAC and WINHAC describe single W or Z production at hadron colliders with $\mathcal{O}(\alpha)$ YFS exponentiation (EEX) (S. Jadach, W. Płaczek, A. Siódmok)
- HORACE combines exact $\mathcal{O}(\alpha)$ EWK corrections with a QED DGLAP shower. (C.M. Carloni Calame, G. Montagna, O. Nicrosini, M. Treccani, A. Vicini.)
- MRST 2004 (A.D. Martin, R.G. Roberts, W.J. Stirling, R.S. Thorne) included PDFs with QED corrections.

HERWIRI2

- The success of YFS exponentiation in the precision event generator $\mathcal{KK}$ MC (S. Jadach, B.F.L. Ward, and Z. Wąs) for $e^+e^- \to Z\gamma^* \to f\overline{f}$ provides a natural starting point for incorporating EWK corrections to the parton-level process.
- HERWIRI2 is a hybrid of $\mathcal{KK}$ MC with a with hadronic event generator, HERWIG in its present incarnation.
- The two programs function largely independently:
  - HERWIG generates the parton momenta and shower.
  - $\mathcal{KK}$ MC provides a more precise calculation of the hard process and generates multiple ISR and FSR photon emission.
- $\mathcal{KK}$ MC was designed to be upgradable to processes beyond just the e^+e^- scattering of interest at LEP: thus, the ability to select incoming quarks already exists.

$\mathcal{KK}$ MC

- $\mathcal{KK}$ MC is a precision event generator for $e^+e^- \to f\overline{f} + n\gamma$, $f = \mu, \tau, d, u, s, c, b$ for CMS energies from $2m_\tau$ to 1 TeV. The precision tag for LEP2 was $0.2 - 1$%.
- ISR and FSR γ emission is calculated up to $\mathcal{O}(\alpha^2)$, including interference.
- The MC structure is based on YFS exponentiation, including residuals calculated perturbatively to the relevant orders in $\alpha^k L^l$. ($L = \ln(s/m_e^2)$). CEEX mode: $\alpha, \alpha L, \alpha^2 L^2, \alpha^2 L$.
- Exact collinear bremsstrahlung for up to three γ's.
- $\mathcal{O}(\alpha)$ EWK corrections and more are included via DIZET 6.21.
- Beamstrahlung can be modeled over a wide range via a built-in or user-defined distribution.
- Final state hadronization is supported via JETSET.
- τ decay is simulated using TAUOLA.

Coherent Exclusive Exponentiation

- CEEX was introduced for pragmatic reasons, the traditional exponentiation (EEX) of spin-summed cross sections suffered from a proliferation of interference terms in processes with multiple diagrams, limiting its utility in reaching the desired 0.2% precision tag for LEP2.
- CEEX works at the level of spinor helicity amplitudes, greatly facilitating the calculation of effects such as ISR-FSR interference, which are included in $\mathcal{KK}$ MC, and therefore HERWIRI2.
- CEEX is maximally exclusive: all real photons radiated are kept in the event record, no matter how soft or collinear. There is no need to "integrate out" a region of soft phase space because the exponentiated amplitudes are well-behaved at $k = 0$. (HERWIRI1 implements this for soft gluons.)

CEEX Formalism

The CEEX cross section for $q\bar{q} \to f\bar{f}$ has the form

$$\sigma = \frac{1}{\text{flux}} \sum_{n=0}^{\infty} \int d\text{PS}\; \rho^{(n)}_{\text{CEEX}}(\vec{p}, \vec{k})$$

where

$$\rho^{(n)}_{\text{CEEX}} = \frac{1}{n!} e^{Y(\vec{p}, E_{\min})} \sum_{\text{hel.}} \left| \mathcal{M} \begin{pmatrix} \vec{p} & \vec{k} \\ \vec{\lambda} & \vec{\mu} \end{pmatrix} \right|^2$$

The YFS form factor is

$$Y(\vec{p}, E_{\min}) = Q_i^2 Y(p_1, p_2, E_{\min}) + Q_f^2 Y(p_3, p_4, E_{\min}) + Q_i Q_f Y(p_1, p_3, E_{\min})$$

$$+ Q_i Q_f Y(p_2, p_4, E_{\min}) - Q_i Q_f Y(p_1, p_4, E_{\min}) - Q_i Q_f Y(p_2, p_3, E_{\min})$$

$$Y(p_i, p_j, E_{\min}) = 2\alpha \widetilde{B}(p_i, p_j, E_{\min}) + 2\alpha \text{Re}\, B(p_i, p_j, E_{\min})$$

$$\widetilde{B} = \int_{k^0 > E_{\min}} \frac{d^3k}{k^0} \left(\frac{p_i}{p_i \cdot k} - \frac{p_j}{p_j \cdot k} \right)^2, \qquad B = \frac{i}{(2\pi)^3} \int \frac{d^3k}{k^2} \left(\frac{2p_i + k}{2p_i \cdot k + k^2} - \frac{2p_j - k}{2p_j \cdot k - k^2} \right).$$

CEEX Formalism

The n-photon helicity-spinor amplitude can be expanded in terms of order α^r having the form

$$\mathcal{M}_n^{(r)} = \sum_{\mathcal{P}} \prod_{i=1}^{n} \mathcal{S}_i^{(\mathcal{P}_j)} \left[\beta_0^{(r)} \begin{pmatrix} \vec{p} & ; X_{\mathcal{P}} \\ \vec{\lambda} & \end{pmatrix} + \sum_{j=1}^{n} \frac{\beta_1^{(r)} \begin{pmatrix} \vec{p} & k & ; X_{\mathcal{P}} \\ \vec{\lambda} & \mu & \end{pmatrix}}{\mathcal{S}_j^{(\mathcal{P}_j)}} \right.$$

$$\left. + \cdots + \sum_{1<j_1<\cdots<j_n} \frac{\beta_n^{(r)} \begin{pmatrix} \vec{p} & \vec{k} & ; X_{\mathcal{P}} \\ \vec{\lambda} & \vec{\mu} & \end{pmatrix}}{\mathcal{S}_{j_1}^{(\mathcal{P}_{j_1})} \cdots \mathcal{S}_{j_n}^{(\mathcal{P}_{j_n})}} \right]$$

with residual spinor amplitudes $\beta_i^{(r)}$ and complex soft photon factors $\mathcal{S}_j$ with the property

$$\left| \mathcal{S}_j^{(\mathcal{P}_j)} \right| = -2\pi\alpha Q^2 \left(\frac{p_a}{p_a \cdot k_j} - \frac{p_b}{p_b \cdot k_j} \right)^2$$

where Q, p_a, p_b belong to the initial or final fermions depending on the partition $\mathcal{P}_j$.

ElectroWeak Corrections

$\mathcal{KK}$ MC incorporates the DIZET library (version 6.2) from the semi-analytical program ZFITTER by A. Akhudov, A. Arbuzov, M. Awramik, D. Bardin, M. Bilenky, P. Christova, M. Czakon, A. Frietas, M. Gruenewald, L. Kalinovskaya, A. Olchevsky, S. Riemann, T. Riemann.

- The γ and Z propagators are multiplied by vacuum polarization factors:

$$H_\gamma = \frac{1}{2 - \Pi_\gamma}, \qquad H_Z = 4\sin^2(2\theta_{\rm W})\frac{\rho_{\rm EW} G_\mu M_Z^2}{8\pi\alpha\sqrt{2}}.$$

- Vertex corrections are incorporated into the coupling of Z to f via form factors in the vector coupling:

$$g_V^{(Z,f)} = \frac{T_3^{(f)}}{\sin(2\theta_W)} - Q_f F_{\rm v}^{(f)}(s)\tan\theta_W.$$

- Box diagrams contain these plus a new angle-dependent form-factor in the doubly-vector component:

$$g_V^{(Z,i)} g_V^{(Z,f)} = \frac{T_3^{(i)}T_3^{(f)} - 2T_3^{(i)}Q_f F_{\rm v}^{(f)}(s) - 2Q_i T_3^{(f)} F_{\rm v}^{(i)}(s) + 4Q_i Q_f F_{\rm box}^{(i,f)}(s,t)}{\sin^2(2\theta_W)}.$$

The correction factors are calculated at the beginning of a run and stored in tables.

Combining $\mathcal{KK}$ MC with a Shower

- The goal is to build a shower generator based on QED$\otimes$QCD exponentiation. As a first step in combining exponentiated QED with a shower, we can take an external shower, and use a parton-level hard process calculated using the CEEX formalism, which can be calculated with known per-mil level precision in $\mathcal{KK}$ MC.
- The Drell-Yan cross section with multiple-photon emission can be expressed as an integral over the parton-level process $q_i(p_1)\overline{q}_i(p_2) \rightarrow f(p_3)\overline{f}(p_4) + n\gamma(k)$, integrated over phase space and summed over photons.
- The parton momenta p_1, p_2 are generated using parton distribution functions giving a process at CMS energy q and momentum fractions x_1, x_2 such that $q^2 = x_1 x_2 s$:

$$\sigma_{\rm DY} = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} \sum_i f_i(q, x_1) f_{\overline{i}}(q, x_2) \sigma_i(q^2) \delta(q^2 - x_1 x_2 s),$$

where the final state phase space includes p_3, p_4 and k_i, $i = 1, \cdots, n$ and multiple gluon radiation + hadronization is included through a shower.

Combining $\mathcal{KK}$ MC with a Shower

- The parton-level cross section $\sigma_i(q^2)$ can be calculated by $\mathcal{KK}$ MC, which integrates over a final space phase space with two fermions and an arbitrary number of photons:

$$\sigma_i(q^2) = \sum_{n=0}^{\infty} \int d\mathrm{PS}_{2+n} \sigma_i(\vec{p}, \vec{k})$$

- The QCD and QED scales have a heirarchy such that gluon emission occurs first, so the shower is permitted to run first, generating the scale q for the hard process, which is then calculated with electro-weak corrections and multiple photon emission.

- We are using HERWIG as the shower generator, which creates the hard process first, at Born level, in subroutine HWEPRO (HWHDYP), and then passes it to the cascade generator HWBGEN.

- HERWIRI2 finds the Z/γ^* and the partons interacting with it in the event record. The initial partons define p_1, p_2, which are transformed to the CM frame and projected on-shell to create a starting point for $\mathcal{KK}$ MC.

Combining $\mathcal{KK}$ MC with a Shower

- $\mathcal{KK}$ MC generates the final fermion momenta p_3, p_4 and photons k_i (both ISR and FSR.) The generated particles are transformed back to the lab frame and placed in the event record.
- The appropriate weight is constructed for the combined event and can be used with rejection to generate unweighted events, if desired.
- This HERWIRI2 weight is a product of the HERWIG and $\mathcal{KK}$ MC weights with a common factor removed and appropriate additional factors required because the scale and incoming fermion vary in $\mathcal{KK}$ MC.
- In addition to the basic DY process, HERWIG generates "Compton" events $g + q \to q + Z/\gamma^*$. About 10% of the events have this form. This is factorized into gluon emission times a hard EW process at a shifted value of q^2. These have a different profile in the event record, but can be processed by $\mathcal{KK}$ MC as well.
- There is also a third class of events with the emission of an additional hard gluon. About 1% of the events have this form, and also have a significant shift of the Z energy from its generation scale.

MC Weights

With a change of variables, the DY cross section in HERWIG can be expressed as

$$\begin{aligned}
\sigma_{\rm DY} &= \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\sum_i f_i(q,x_1)f_{\bar{i}}(q,x_2)\sigma_i(q^2)\delta(q^2 - x_1x_2s) \\
&= \int_{q_{\rm min}}^{q_{\rm max}} dqP(q)\int_{q^2/s}^{1}\frac{dx_1}{x_1}\sum_i P_i \quad W_{\rm HW}^{(i)}(q^2,x_1) \\
&= \langle W_{\rm HW}\rangle
\end{aligned}$$

where $P(q)$ is a normalized, integrable, crude probability distribution for q, P_i is the crude probability of generating parton i, and $W_{\rm HW}$ is the HERWIG event weight. This weight depends only on the hard Born cross section and is not altered by the shower.

$$P(q) = \frac{1}{2}[P_\gamma(q) + P_Z(q)], \qquad P_\gamma(q) = \frac{N_\gamma}{q^4}, \qquad P_Z(q) = \frac{N_2 q}{(q^2 - M_Z^2) + \Gamma_Z^2 M_Z^2}$$

The HERWIG Event Weight

- The HERWIG event weight is

$$W_{\rm HW} = \sum_i W_{\rm HW}^{(i)}, \qquad W_{\rm HW}^{(i)} = \frac{1}{P(q)} f_i(q,x_1) f_{\bar{i}}(q,x_2) \ln\left(\frac{s}{q^2}\right) \sigma_{\rm HW}^{(i)}(q^2)$$

and the corresponding probability for selecting parton i is

$$P_i = W_{\rm HW}^{(i)} / W_{\rm HW}$$

- We have chosen to introduce EW corrections in a "minimally invasive" way, incorporating them in a form factor

$$F_{EW}^{(i)}(q^2) = \frac{\sigma_i(q^2)}{\sigma_{\rm Born}^{(i)}(q^2)}$$

- $\mathcal{KK}$ MC will calculate the EW form factor, and multiply it by the Herwig Born cross section.

$$\sigma_{\rm HW+EW} = \langle W_{\rm Tot} \rangle \,, \qquad W_{\rm Tot} = F_{EW}^{(i)}(q^2) W_{\rm HW} = W_{\rm HW} \frac{\sigma_{\rm KK}^{(i)}(q^2)}{\sigma_{\rm Born}^{(i)}(q^2)}.$$

$\mathcal{KK}$ MC Generator Structure

- The $\mathcal{KK}$ MC cross section is calculated using a “primary distribution”

$$\frac{d\sigma^{(i)}_{\rm Pri}(s,v)}{dv} = \sigma^{(i)}_{\rm Born}(s(1-v))\frac{1}{2}\left(1+\frac{1}{\sqrt{1-v}}\right)\overline{\gamma}_i v^{\overline{\gamma}_i-1} v_{\rm min}^{\gamma_i-\overline{\gamma}_i}$$

  with

$$\gamma_i = \frac{2\alpha}{\pi}Q_i^2\left[\ln\left(\frac{s}{m_i^2}\right)-1\right], \qquad \overline{\gamma}_i = \frac{2\alpha}{\pi}Q_i^2\ln\left(\frac{s}{m_i^2}\right)$$

  to generate the factor v giving the fraction of s remaining after ISR photon emission, $s_X = s(1-v)$.

- The $\mathcal{KK}$ MC cross section is

$$\sigma(q^2) = \int d\sigma_{\rm Pri}\frac{d\sigma_{\rm Cru}}{d\sigma_{\rm Pri}}\frac{d\sigma_{\rm Mod}}{d\sigma_{\rm Cru}} = \sigma_{\rm Pri}\left\langle W_{\rm Cru}W_{\rm Mod}\right\rangle .$$

  $W_{\rm Cru}$ is calculated during ISR generation and $W_{\rm Mod}$ is generated after s_X is available.

Combined Generator HERWIRI2

- We want to use HERWIG and $\mathcal{KK}$ MC together to calculate

$$\sigma_{\mathrm{Tot}} = \left\langle W_{\mathrm{HW}} \frac{\sigma_i(q^2)}{\sigma^{(i)\star}_{\mathrm{Born}}(q^2)} \right\rangle = \left\langle W_{\mathrm{HW}} \sigma^{(i)}_{\mathrm{Pri}}(q^2) \frac{W^{(i)}_{\mathrm{Cru}} W^{(i)}_{\mathrm{Mod}}}{\sigma^{(i)\star}_{\mathrm{Born}}(q^2)} \right\rangle ,$$

- This average *could* be calculated using a joint probability distribution for q and v, $D(q,v) = P(q) d\sigma_{\mathrm{Pri}}/dv$, with $P(q)$ from HERWIG.
- An adaptive MC (S. Jadach's FOAM) could calculate the normalization of the distribution at the beginning of the run, in a similar manner to how $\mathcal{KK}$ MC presently integrates the one-dimensional primary distribution. In fact, to account for beamsstrahlung, $\mathcal{KK}$ MC permits such a distribution, in up to three variables, to be introduced by the user.
- As a first step, we have tried to run HERWIRI2 using $\mathcal{KK}$ MC's one-dimensional primary distribution. This requires fixing an overall scale q_0 to initialize $\mathcal{KK}$ MC (*e.g.*, $q_0 = M_Z$).

Combined Generator HERWIRI2

- The built-in primary distribution for electrons at scale q_0 can be used for the low-level generation of v. The transformation from this distribution to a distribution at HERWIG's generated scale q for quark i is then obtained by a change of variables.

$$\sigma_{\rm Tot} = \sigma^{(e)}_{\rm Pri} \left\langle W_{\rm HW} \left(\frac{d\sigma^{(i)}_{\rm Pri}(q^2, v)}{d\sigma^{(e)}_{\rm Pri}(q_0^2, v)} \right) \left(\frac{W^{(i)}_{\rm Crud} W^{(i)}_{\rm Mod}}{\sigma^{(i)\star}_{\rm Born}(q^2)} \right) \right\rangle$$

with

$$\frac{d\sigma^{(i)}_{\rm Pri}(q^2, v)}{d\sigma^{(e)}_{\rm Pri}(q_0^2, v)} = W^{(i)}_\gamma \frac{\sigma^{(i)}_{\rm Born}(q^2(1-v))}{\sigma^{(e)}_{\rm Born}(q_0^2(1-v))} \; ,$$

where

$$W_\gamma = \frac{\overline{\gamma}_i}{\overline{\gamma}_e} \left(\frac{v}{v_{\rm min}} \right)^{\overline{\gamma}_i - \overline{\gamma}_e} v_{\rm min}^{\gamma_i - \gamma_e} .$$

The γ factors are calculated using q^2/m_i^2 for parton i and q_0^2/m_e^2 for the electron.

Combined Weight

- Shuffling the numerators and denominators about gives the expression used in HERWIRI2:

$$\sigma_{\rm Tot} = \langle W_{\rm HW} W_{\rm Mod} W_{\rm Karl} W_{\rm FF} W_\gamma \rangle$$

  with two new weights

$$W_{\rm Karl} = \frac{\sigma^{(e)}_{\rm Pri} W^{(i)}_{\rm Crud}}{\sigma^{(e)}_{\rm Born}(q_0^2(1-v))}, \qquad W_{\rm FF} = \frac{\sigma^{(i)}_{\rm Born}(q^2(1-v))}{\sigma^{(i)\star}_{\rm Born}(q^2)}.$$

- The weights can be calculated by insuring that each subroutine is initialized for either an electron or parton i, as appropriate.

- Since $\sigma_{\rm Pri}$ is calculated before generation begins, $\mathcal{KK}$ MC can be pre-initialized with its standard values. The primary distribution is, in fact, hard-wired, and cannot be changed. The weight $W_{\rm Crud}$ for a quark requires a little rewriting.

- $W_{\rm Mod}$ requires only passing the correct variables, since the CEEX module that calculates it already anticipated being called with different fermions.

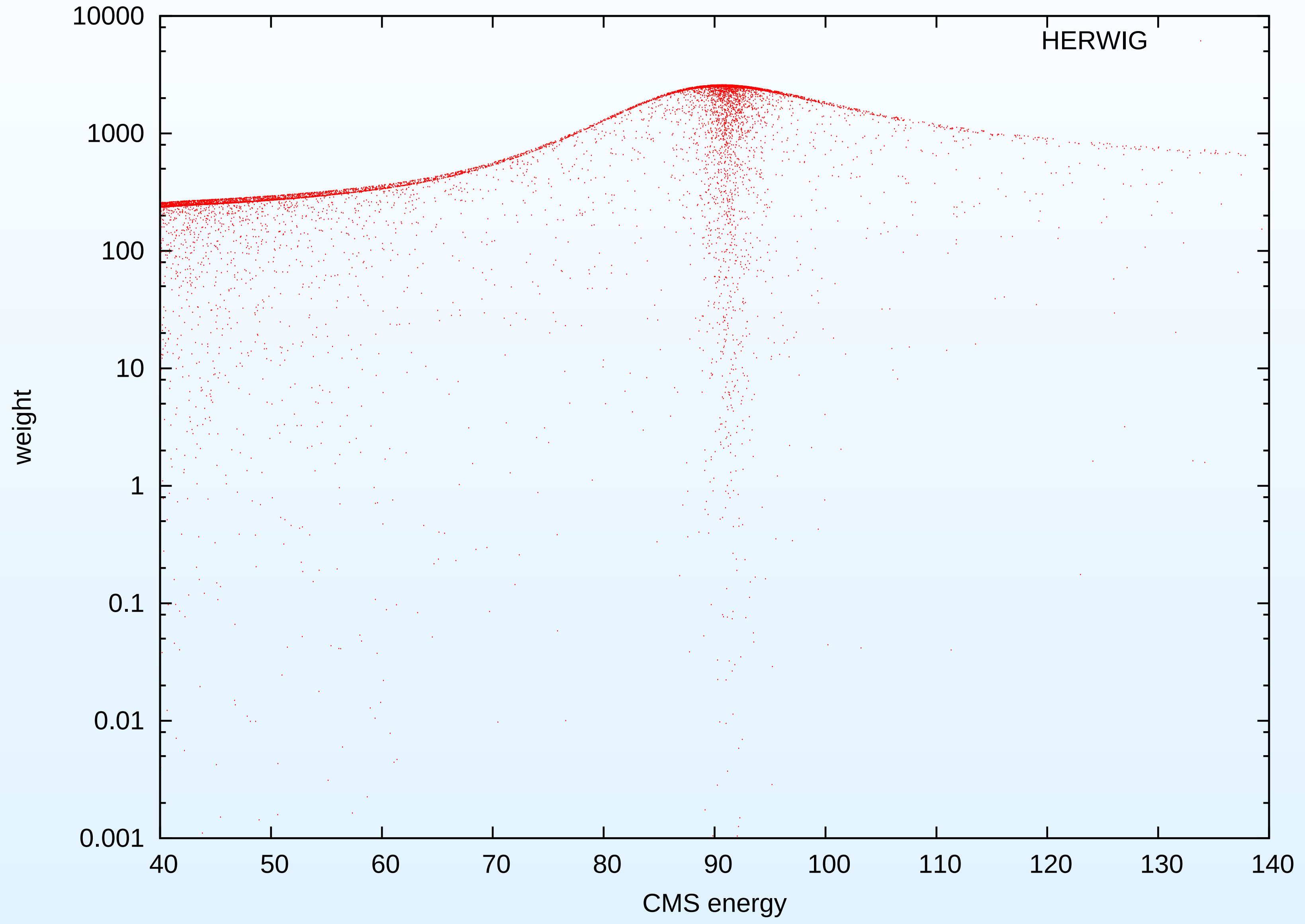
HERWIRI2 Without ISR

- HERWIRI2 is still in testing! All results here are very preliminary.
- Turning ISR is useful as a first test because the weights simplify: $W_{\mathrm{Karl}} = W_\gamma = 1$. Since $v = 0$, there are no scale shifts in the weights, leading to a much narrower weight distribution.
- A very short test run (10,000 events) with 5 TeV proton beams, ISR off, and CMS energies between 40 and 140 GeV gives cross sections

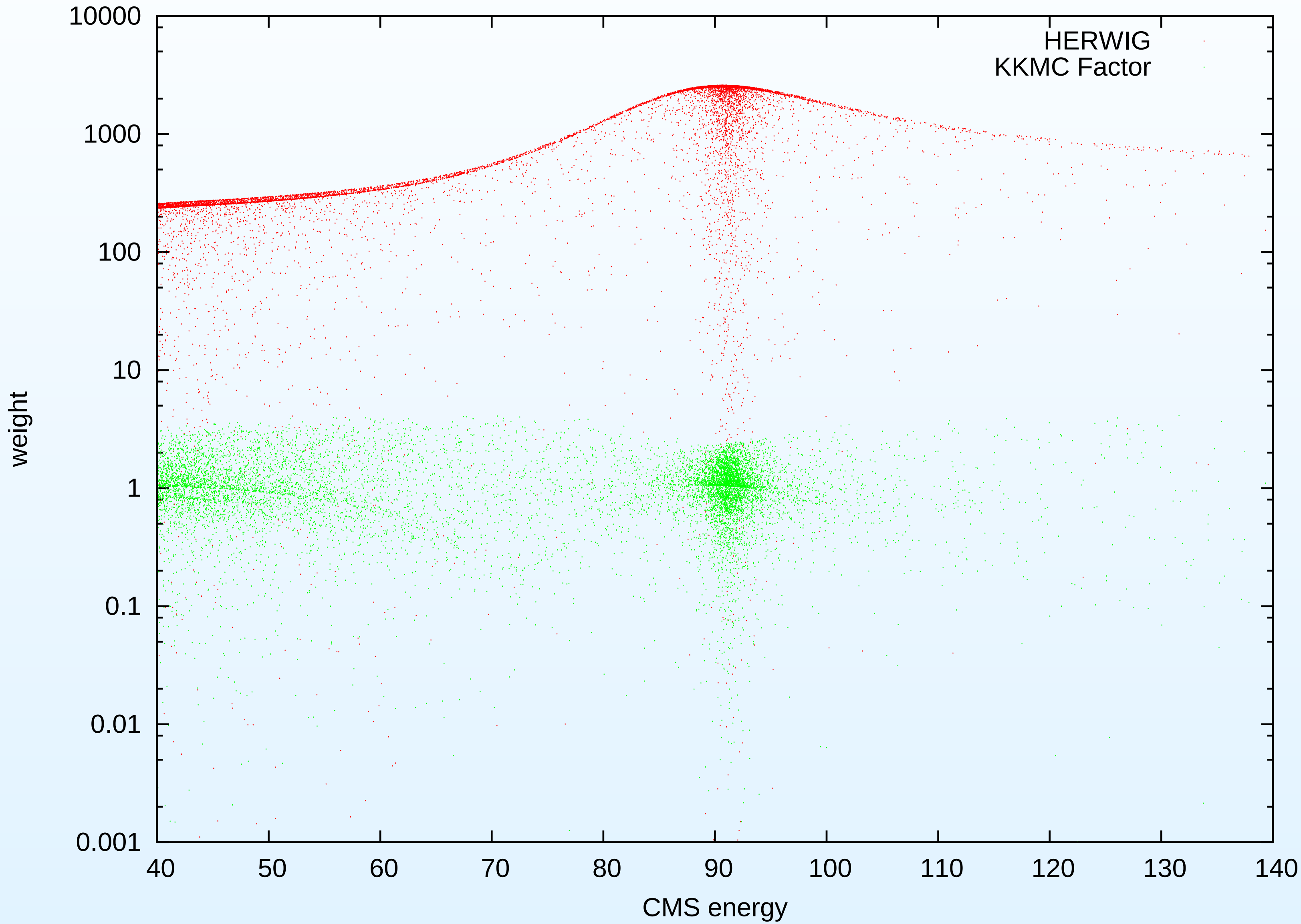
HERWIG CS	1095 ± 10 pb
HERWIRI2 CS	1176 ± 13 pb

- Thus we find an increase of about 7% from EW corrections, not far from the effect of approximately 4% we (N. Adam, V. Halyo, S.Y.) found using HORACE for the same cross section.
- An average of 0.6 photons per event is generated, with an average total energy of 1.6 GeV.

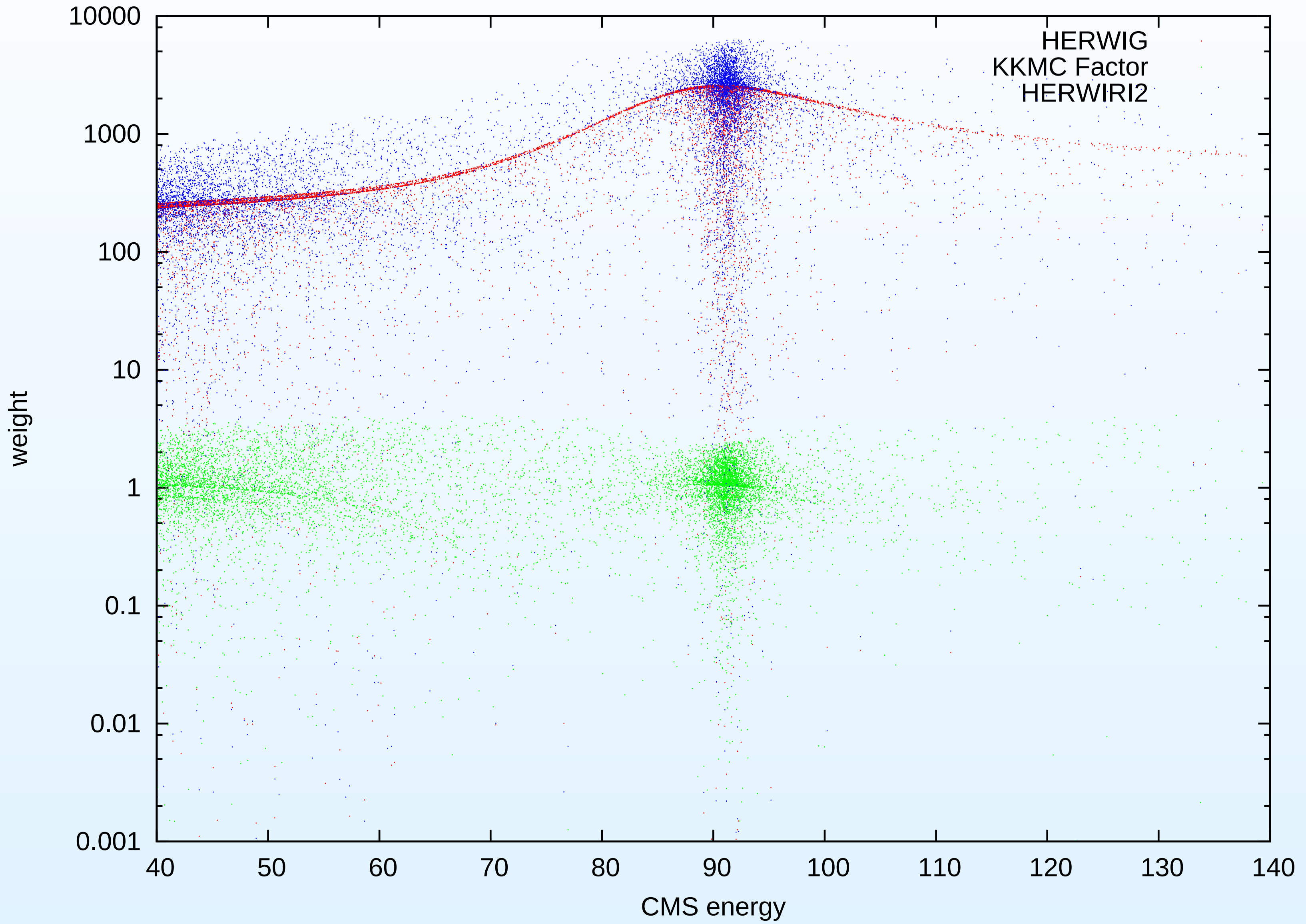
Weight Distributions, No ISR



Weight Distributions, No ISR



Weight Distributions, No ISR



HERWIRI2 with ISR: Beta Testing

- The version of HERWIRI2 with ISR is still deeply in testing mode, and likely to be subject to change, especially once the two-dimensional primary distribution is introduced. For the same parameters and run length, it gives a cross section of 1212 ± 109 pb, which is 11% above HERWIG, certainly compatible with other results given the large statistical errors.
- The large statistical errors are an issue: The fixed initialization scale creates a wide weight distribution. It appears this will just be a stop-gap until a better primary distribution is introduced.
- W_γ is the main problem, with an average of 3.3 ± 1.2 in this run, and weights ranging up to 7402. It may be possible to improve this with a better initialization scheme taking into account the multiple quark types.
- The weight in this case must be considered a work in progress.

Summary

- $\mathcal{KK}$ MC HERWIRI2 is the first implementation of CEEX in a hadronic context.
- The weights still need to be fine-tuned in the presence of ISR, but it is expected that the remaining construction can be completed in the near future.
- It will be interesting to see the effects of γ ISR, which is not present otherwise (except in some MRST PDFs).
- Comparisons with other hadronic/EW MC's will be in order once HERWIRI2 passes all its internal tests.
- HERWIRI2 is a step toward our goal of an event generator based on nonabelian QED$\otimes$QCD exponentiation and exact $\mathcal{O}(\alpha_s^2, \alpha_s\alpha, \alpha^2)$ residuals.