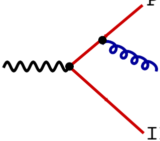
Antenna Subtraction in pQCD at NNLO



Prof. Nigel Glover
James Currie
IPPP Durham

RADCOR 2011
Mahabalipuram

University of Durham

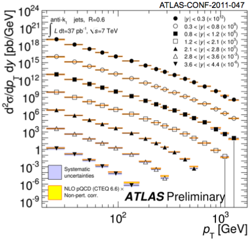
What To Expect From This Talk

- Motivation
  - Why is NNLO QCD desirable?
  - What can we use it for?
- Theoretical framework
  - Matrix element and phase space factorization
  - IR singularity cancellation
  - Implemented using Antenna Subtraction method
- Applications
  - QCD contribution to $pp \to$ jets @ NNLO
    - $q\bar{q} \to gggg$
    - $q\bar{q} \to r\bar{r}s\bar{s}$
- The wider project $\to$ "NNLOJET.f"

Motivation - Why QCD & Why NNLO?

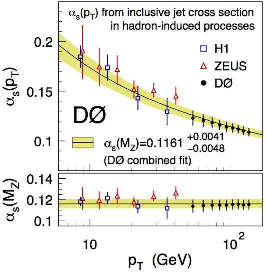
QCD:

- QCD dominates initial and final states
- Jets are good experimental objects
- PDF constraints, backgrounds
- α_s running from single experiment



NNLO:

- Experimental errors (JES) good
- Better description of hard scattering
- Massively improved scale dependence
- More realistic
  - jets/partial shower reconstruction
  - final state transverse momentum



Jet Cross Sections

The m jet cross section to

LO:

$$d\sigma_{LO} = \int_{d\Phi_m} d\sigma_B$$

NLO:

$$d\sigma_{NLO} = \int_{d\Phi_{m+1}} d\sigma^R_{NLO} + \int_{d\Phi_m} d\sigma^V_{NLO}$$

NNLO:

$$d\sigma_{NNLO} = \int_{d\Phi_{m+2}} d\sigma^{RR}_{NNLO} + \int_{d\Phi_{m+1}} d\sigma^{RV}_{NNLO} + \int_{d\Phi_m} d\sigma^{VV}_{NNLO}$$

Cross Section Pathologies

Renormalized cross sections contain infra-red (IR) singularities:

- Loop integrations $\sim$ Laurent expansion in $\epsilon = D - 4$
- Vanishing kinematic invariants $\sim$ $s_{ij} = (p_i + p_j)^2 \to 0$

Kinoshita-Lee-Nauenberg (KLN) theorem:

- Singularities cancel when summed over degenerate initial and final states

Problem:

- Need singularities in the same form (ϵ expansion)
  - Have to perform phase space integral analytically

Cross Section Pathologies

Renormalized cross sections contain infra-red (IR) singularities:

- Loop integrations $\sim$ Laurent expansion in $\epsilon = D - 4$
- Vanishing kinematic invariants $\sim$ $s_{ij} = (p_i + p_j)^2 \rightarrow 0$

Kinoshita-Lee-Nauenberg (KLN) theorem:

- Singularities cancel when summed over degenerate initial and final states

Problem:

- Need singularities in the same form (ϵ expansion)
  - Have to perform phase space integral analytically impossible!

Single Unresolved Tree Factorization

- Need to understand singular limits of the cross section
- Squared colour-ordered amplitudes obey universal factorization

1. Soft gluon emission, $p_j \to 0$

$$|\mathcal{M}^0_{m+1}(\cdots p_i, p_j, p_k, \cdots)|^2 \to S_{ijk}\ |\mathcal{M}^0_m(\cdots, \tilde{p}_I, \tilde{p}_K \cdots)|^2$$

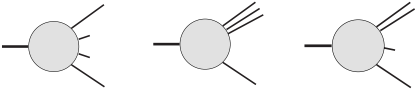
2. Collinear limit $p_j || p_k$

$$|\mathcal{M}^0_{m+1}(\cdots p_i, p_j, p_k, p_l, \cdots)|^2 \to \frac{P^0_{jk\to\tilde{K}}}{s_{jk}}\ |\mathcal{M}^0_m(\cdots, p_i, p_{\tilde{K}}, p_l, \cdots)|^2$$

Double Unresolved Tree Factorizaion

At NNLO we have new singular limits

- Double soft
- Triple collinear
- Soft and collinear



Factorization holds $\longrightarrow$ new universal functions

$$S_{abcd} \quad S_{d,abc} \quad P_{ijk\to\tilde{K}} \quad \tilde{P}_{ijk\to\tilde{K}}$$

Details of factorization depend on colour ordering of partons

[N.B. At NNLO also need factorization of one loop quantities $\longrightarrow$ $P^{(1)}_{ij\to\tilde{K}}$]

Phase Space Factorization

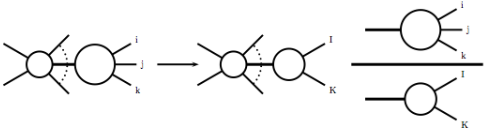
Also the phase space can be factorized with a phase space map, $\mathcal{O}$

$\mathcal{O}$ maps $\{p_X\} \subset \{p_n\}$ onto two composite momenta

$$\mathcal{O}(\{p_X\}) : \{p_n\} \mapsto \{\widetilde{p_m}\}$$

redfined momenta $\{\widetilde{p_m}\}$:

- remain on-shell
- conserve momentum



$$d\Phi_n(\{p_n\}) \quad = \quad d\Phi_m(\{\widetilde{p_m}\}) \cdot \underbrace{d\Phi_X(\{p_X\})}_{\text{"antenna PS"}}$$

Phase Space Factorization

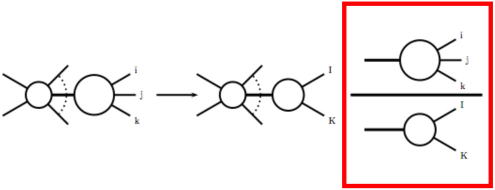
Also the phase space can be factorized with a phase space map, $\mathcal{O}$

$\mathcal{O}$ maps $\{p_X\} \subset \{p_n\}$ onto two composite momenta

$$\mathcal{O}(\{p_X\}) : \{p_n\} \mapsto \{\widetilde{p_m}\}$$

redfined momenta $\{\widetilde{p_m}\}$:

- remain on-shell
- conserve momentum



$$d\Phi_n(\{p_n\}) = d\Phi_m(\{\widetilde{p_m}\}) \cdot \underbrace{d\Phi_X(\{p_X\})}_{\text{“antenna PS”}}$$

Subtraction Method - counterterms

Construct a (local) counterterm, $\mathrm{d}\sigma^{RR,S}_{NNLO}$,

$$\int_{\mathrm{d}\Phi_{m+2}} \left[\mathrm{d}\sigma^{RR}_{NNLO} - \mathrm{d}\sigma^{RR,S}_{NNLO} \right]$$

- Mimics $\mathrm{d}\sigma^{RR}_{NNLO}$ in all singular limits
- New integrand is finite over the entire phase space
- Integrated numerically

Have to add $\mathrm{d}\sigma^{RR,S}_{NNLO}$ back in,

$$\int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{RR}_{NNLO} = \underbrace{\int_{\mathrm{d}\Phi_{m+2}} \left[\mathrm{d}\sigma^{RR}_{NNLO} - \mathrm{d}\sigma^{RR,S}_{NNLO} \right]}_{\text{finite}} + \underbrace{\int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{RR,S}_{NNLO}}_{\text{singular}}$$

Subtraction Method - singularity cancellation

Construct counterterm to reflect factorization

$$d\sigma_{NNLO}^{RR,S} \sim \underbrace{X(\{p_X\})}_{\text{antenna (singular)}} \cdot \underbrace{\overbrace{|\mathcal{M}(\{\widetilde{p_m}\})|^2}^{\text{reduced ME}} \cdot \underbrace{\mathcal{J}(\{\widetilde{p_m}\})}_{\text{jet function}}}_{\text{finite}}$$

To extract ϵ poles we only have to analytically integrate antenna function over antenna phase space

- Cancel against virtual ϵ poles
- Numerically integrate finite remainder

Finite Cross Section

Subtraction Method - singularity cancellation

Construct counterterm to reflect factorization

$$d\sigma_{NNLO}^{RR,S} \sim \underbrace{X(\{p_X\})}_{\substack{\text{antenna}\\ \text{(singular)}}} \cdot \underbrace{\overbrace{|\mathcal{M}(\{\widetilde{p_m}\})|^2}^{\text{reduced ME}} \cdot \underbrace{\mathcal{J}(\{\widetilde{p_m}\})}_{\text{jet function}}}_{\text{finite}}$$

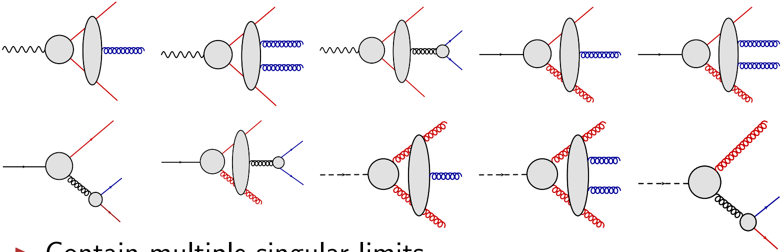
To extract ϵ poles we only have to analytically integrate antenna function over antenna phase space **not impossible!**

- Cancel against virtual ϵ poles
- Numerically integrate finite remainder

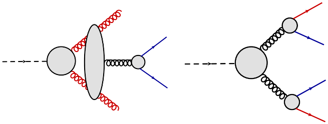
Finite Cross Section

Why Antenna Functions?

- Built from physical matrix elements



- Contain multiple singular limits
  - Regulate multiple singularities ☺
  - Spurious poles ☹
  - Azimuthal terms ☹



- Analytically integrable
  - D dimensions $\longrightarrow$ ϵ expansion ☺

Antenna Subtraction Toolbox

Many tools needed for implementation at NNLO:

- Final-final phase space mappings [Kosower '03]
- Tree + 1-loop antennae [Gehrmann-De Ridder, Gehrmann, Glover, '04, '05]
- Integrated final-final antennae [Gehrmann-De Ridder, Gehrmann, Glover, '05]

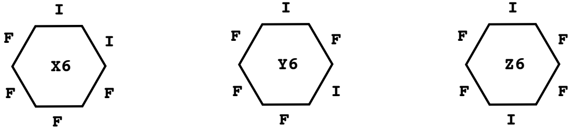
$\Rightarrow$ $e^+e^- \to$ 3 jets [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, '07, Weinzierl '08]

- Initial-final + initial-initial mappings [Daleo, Gehrmann, Maître, '07]
- Integrated IF antennae [Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, '10]
- Integrated II antennae [Boughezal, Gehrmann-De Ridder, Ritzmann, '10, Gehrmann, Monni '11] (20 master integrals remain)
- Double real gluon subtraction [Glover, Pires, '10]

$\Rightarrow$ $p\,p \to$ 2 jets

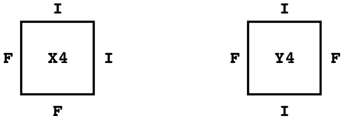
Matrix Element Structure

4 final + 2 initial state partons $\rightarrow$ 3 topologies $\mathcal{X}_6,\ \mathcal{Y}_6,\ \mathcal{Z}_6$



3 final + 2 initial state partons $\rightarrow$ 2 topologies $\mathcal{X}_5,\ \mathcal{Y}_5$

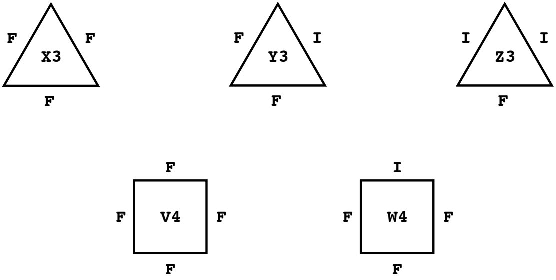
2 final + 2 initial state partons $\rightarrow$ 2 topologies $\mathcal{X}_4,\ \mathcal{Y}_4$



Constructing the Counterterm

$$d\sigma^{S}_{NNLO} = \sum_i (\text{Antenna})_i \otimes (\text{Reduced Matrix Element})_i$$

- Reduced matrix elements $\in \{\mathcal{X}_4, \mathcal{Y}_4\}\ ,\ \{\mathcal{X}_5, \mathcal{Y}_5\}$
- Antenna functions $\in \{\mathcal{X}_3, \mathcal{Y}_3, \mathcal{Z}_3\}\ ,\ \{\mathcal{V}_4, \mathcal{W}_4, \mathcal{X}_4, \mathcal{Y}_4\}$

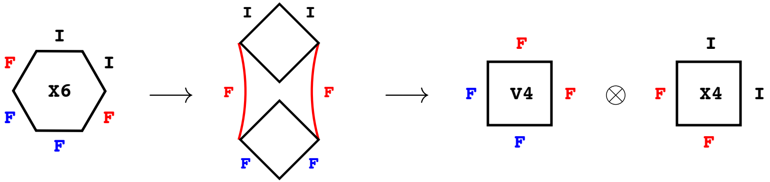


Constructing the Counterterm - Strategy

Define $\otimes$ operation diagrammatically

- Identify hard partons
- Stretch and pinch \ make insertions

e.g.



For each topology $\mathcal{T}$ remove singularities with counterterm $(\mathcal{A} \otimes \mathcal{R})$ such that,

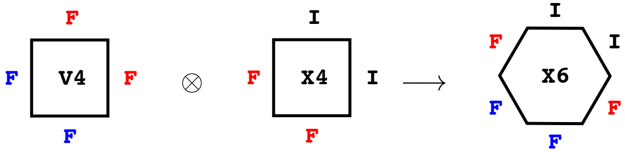
$$(\mathcal{A} \otimes \mathcal{R}) \ \in \ \mathcal{T}$$

Constructing the Counterterm - Strategy

Define $\otimes$ operation diagrammatically

- Identify hard partons
- Stretch and pinch \ make insertions

e.g.



For each topology $\mathcal{T}$ remove singularities with counterterm $(\mathcal{A} \otimes \mathcal{R})$ such that,

$$(\mathcal{A} \otimes \mathcal{R}) \in \mathcal{T}$$

General Kinematic Structure

$$
\begin{aligned}
d\sigma_{NNLO}^{S,\mathcal{X}_6} &= (\mathcal{X}_3 \otimes \mathcal{X}_5) + (\mathcal{Y}_3 \otimes \mathcal{X}_5) \\
&+ (\mathcal{V}_4 \otimes \mathcal{X}_4) - (\mathcal{X}_3 \otimes \mathcal{X}_3) \otimes \mathcal{X}_4 \\
&+ (\mathcal{W}_4 \otimes \mathcal{X}_4) - (\mathcal{X}_3 \otimes \mathcal{Y}_3) \otimes \mathcal{X}_4 - (\mathcal{Y}_3 \otimes \mathcal{Y}_3) \otimes \mathcal{X}_4 + \cdots
\end{aligned}
$$

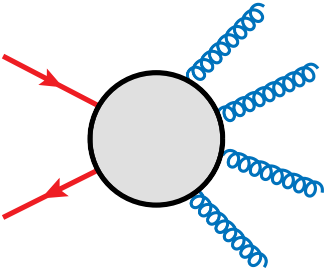
$$
\begin{aligned}
d\sigma_{NNLO}^{S,\mathcal{Y}_6} &= (\mathcal{X}_3 \otimes \mathcal{Y}_5) + (\mathcal{Y}_3 \otimes \mathcal{Y}_5) + (\mathcal{Z}_3 \otimes \mathcal{X}_5) \\
&+ (\mathcal{W}_4 \otimes \mathcal{Y}_4) - (\mathcal{X}_3 \otimes \mathcal{Y}_3) \otimes \mathcal{Y}_4 - (\mathcal{Y}_3 \otimes \mathcal{Y}_3) \otimes \mathcal{Y}_4 \\
&+ (\mathcal{Y}_4 \otimes \mathcal{X}_4) - (\mathcal{Z}_3 \otimes \mathcal{Z}_3) \otimes \mathcal{X}_4 + \cdots
\end{aligned}
$$

$$
\begin{aligned}
d\sigma_{NNLO}^{S,\mathcal{Z}_6} &= (\mathcal{Y}_3 \otimes \mathcal{Y}_5) \\
&+ (\mathcal{X}_4 \otimes \mathcal{X}_4) - (\mathcal{Y}_3 \otimes \mathcal{Z}_3) \otimes \mathcal{Z}_4 \\
&+ (\mathcal{Y}_4 \otimes \mathcal{Y}_4) - (\mathcal{Z}_3 \otimes \mathcal{Z}_3) \otimes \mathcal{Y}_4 + \cdots
\end{aligned}
$$

Application to $q\bar{q} \longrightarrow gggg$

$\mathrm{d}\sigma_{NNLO}^{RR,S}$ built from antennae:

- $F_4^0(g,g,g,g)$, $F_3^0(g,g,g)$
- $D_4^0(\hat{q},g,g,g)$, $D_3^0(\hat{q},g,g)$
- $\tilde{A}_4^0(\hat{q},g,g,\hat{\bar{q}})$, $A_3^0(\hat{q},g,\hat{\bar{q}})$

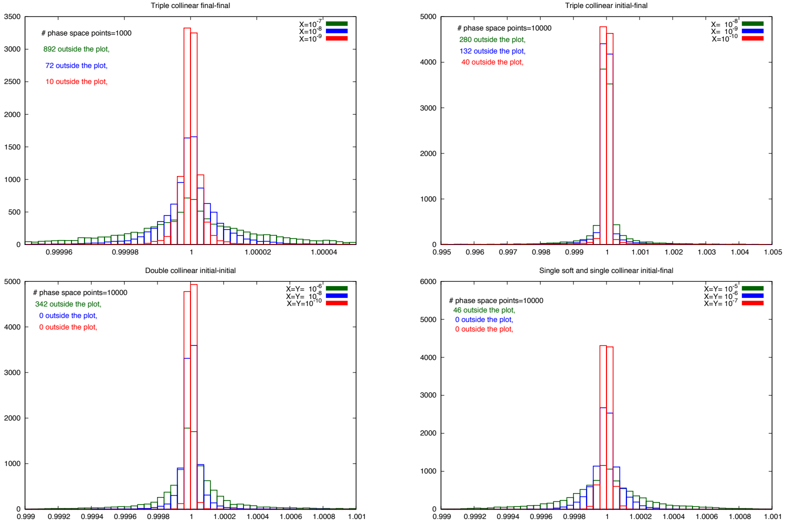


Imitates $\mathrm{d}\sigma_{NNLO}^{RR}$ in the IR limits:

- Triple collinear
- Double soft
- Soft-collinear
- Double collinear

} Final-Final, Initial-Final, Initial-Initial

Numerical Testing - Results for $q\bar{q} \longrightarrow gggg$



Application to $q\bar{q} \longrightarrow r\bar{r}s\bar{s}$

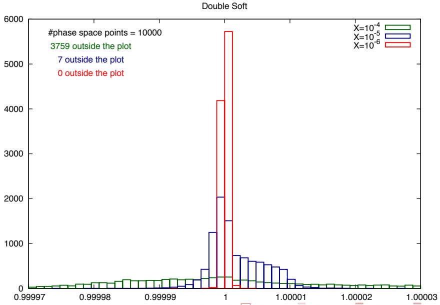
Full squared matrix element takes very simple form

$$|\mathbf{M}_6^0|^2 = N(N^2-1)\left\{|A_1^0|^2 + |A_2^0|^2 - \frac{1}{N^2}\left[|A_3^0|^2 + |A_4^0|^2 + |A_5^0|^2 - 3|A_6^0|^2\right]\right\}$$

- Singular limits of $|A_i^0|^2$ well understood
- Counterterm can be written down to all orders in N

$d\sigma_{NNLO}^{RR,S}$ built from antennae:

- $E_3^0(q, r, \bar{r})$
- $B_4^0(q, r, \bar{r}, \bar{q})$
- $H_4^0(q, \bar{q}, r, \bar{r})$



Looking to the Future

Short term:

- Tackle subleading colour terms systematically
- Mass produce double unresolved counterms
- Complete testing of auxiliary programs:
  - Jet function, phase space generator etc

Medium term:

- Monte Carlo integration of counterterms (convergence!)
- Real-Virtual counterterms + integration
- Assembly of final program

Long term:

- New processes γ + Jet, V + Jet, $\gamma\gamma$, VV ...

To Take Away

- NNLO QCD highly desirable for LHC physics
- Antenna formalism general
  - NLO, NNLO, tree, 1-loop
  - Coloured initial states
- Double real radiative correction (bottleneck) progress:
  - General structure understood
  - (semi)-Automation accelerating
- Final program under construction