



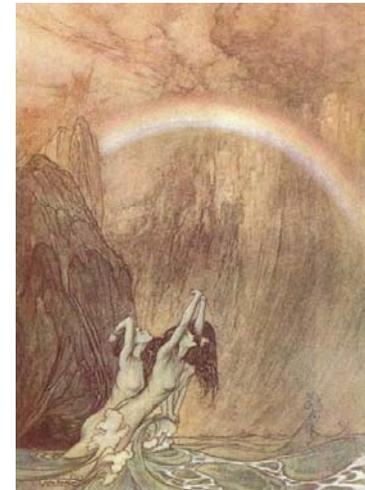
(not the session-chair)



# SEARCH OF THE RINGS

## Approaches to Cherenkov Ring Finding and Reconstruction in HEP

Guy Wilkinson, Oxford University  
RICH 2007, Trieste, October 2007



## First some excuses, admissions and warnings

Final title and nature of talk does not exactly address remit I was given ('Review of the analysis techniques currently in use, including a comparison of measured/expected PID performance in different experiments') – but I hope its an adequate approximation.

I name check plenty of experiments, some old – review doesn't lend itself to 'state-of-the-art' perspective – but use LHCb for a fair few of the examples.

Whenever possible I have ducked the nitty-gritty, eg. giving exact expressions for likelihoods + optimisation tricks. If you want to know more, then wait for the next talks. This is just the hors d'oeuvre.

I have received plenty of help (especially from Chris Jones, Raluca Muresan and Nico Di Bari), but any errors are of my own making.

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Challenges of Cherenkov Ring Pattern Recognition

Reconstructing the Cherenkov Angle

Track-based likelihood approaches – local and global

Hough Transforms – spatial and angular

Other possibilities

Summary

# Challenges in RICH Pattern Recognition

LHCb: RICH 1  
(revolved !)

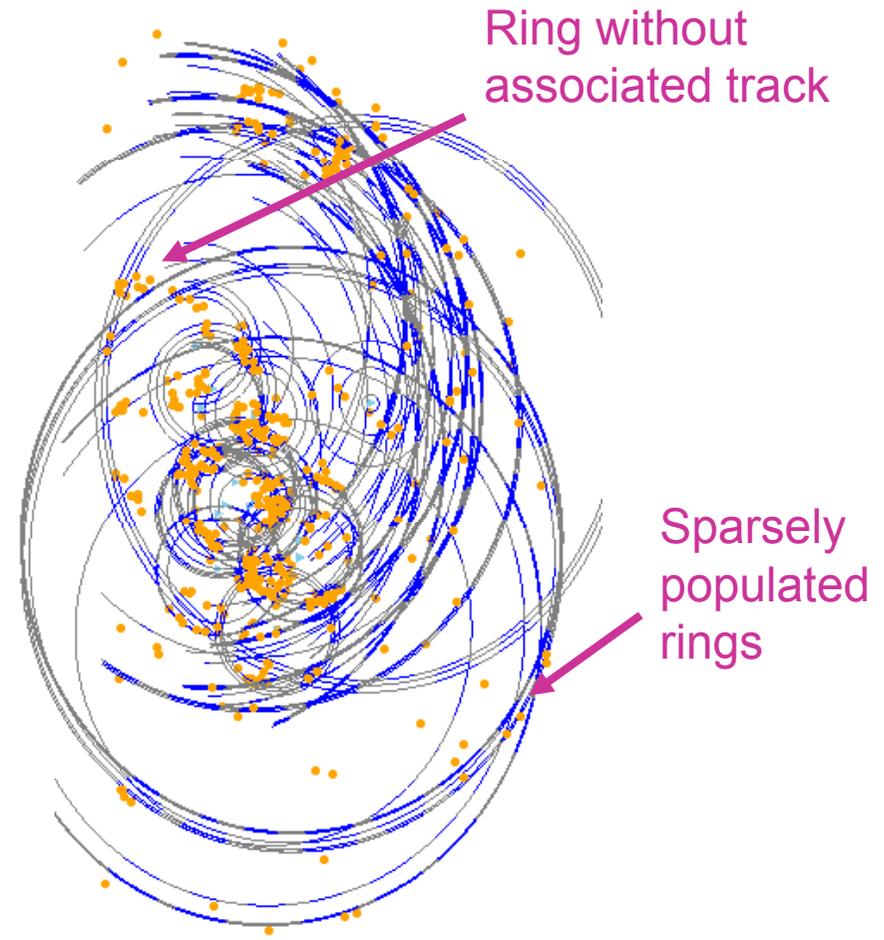
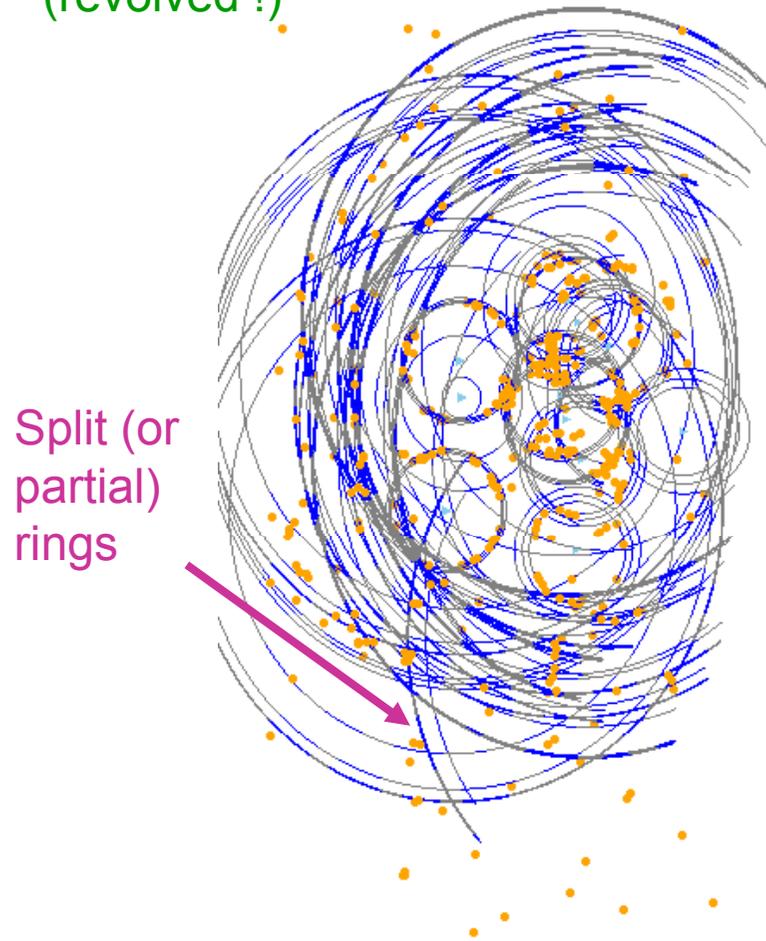


Complicated environment !  
Lesson 1: main source of  
background is other rings.



# Challenges in RICH Pattern Recognition

LHCb: RICH 1  
(revolved !)



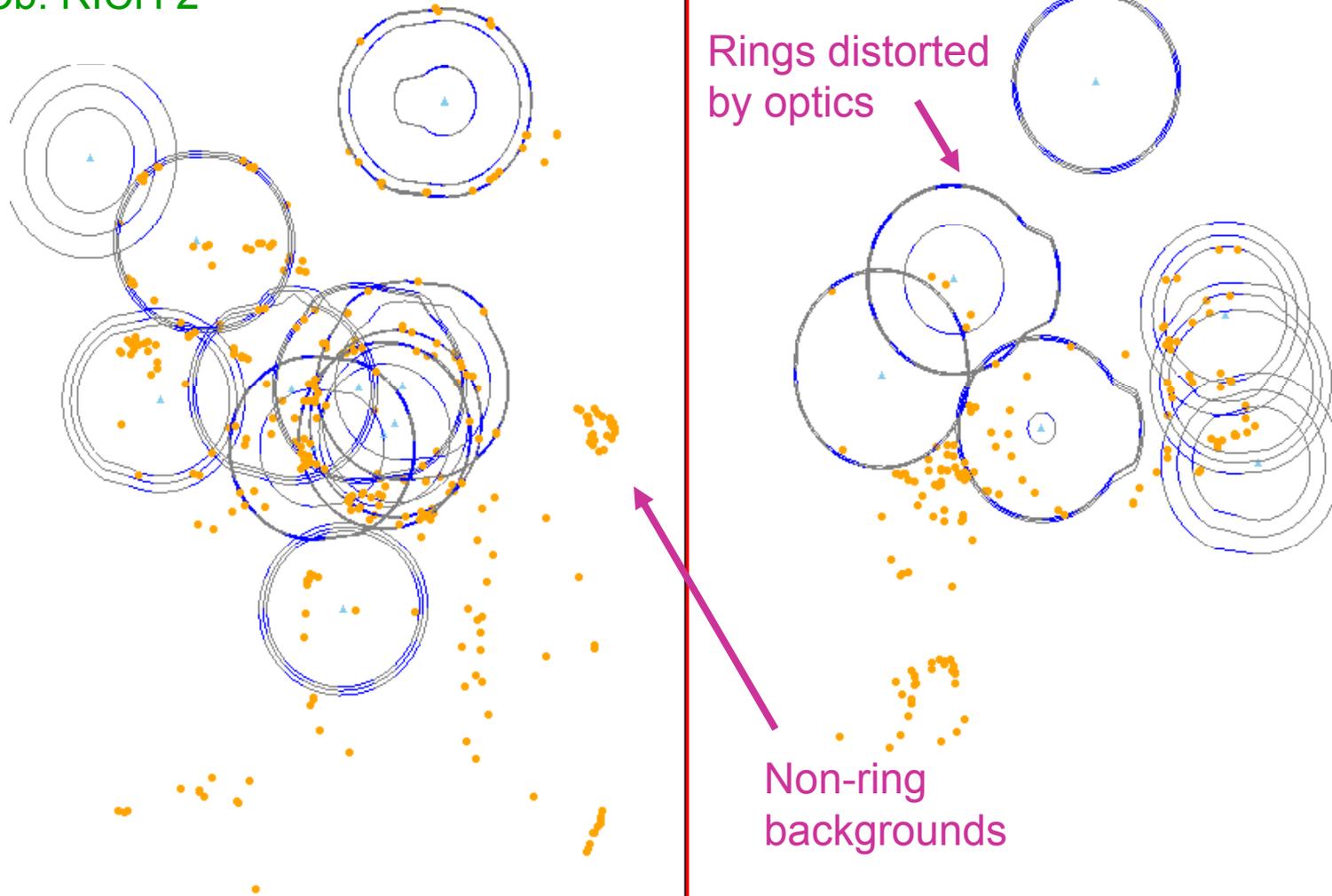
# Challenges in RICH Pattern Recognition

LHCb: RICH 2



# Challenges in RICH Pattern Recognition

LHCb: RICH 2



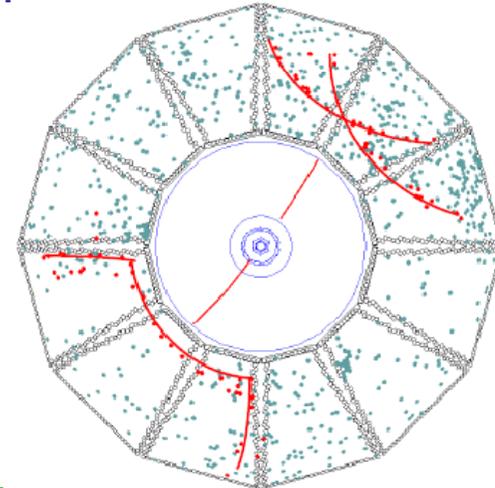
# Working in $\theta_c$ space

Can perform RICH reconstruction in coordinates of photodetector plane, or in 'Cherenkov angle space'. Latter preferable since:

- In many RICHes Cherenkov rings will be partial, distorted (ie. ellipses) or disjoint – or all three.

An extreme example: BaBar DIRC →

Not a trivial pattern to find !



- When transformed back into Cherenkov space, however, hits will accumulate at the same  $\theta_c$ , albeit smeared by resolution. Very helpful for subsequent pattern recognition & PID, however performed.
- But this transformation requires that we know where the observed photon originated – we must assume a parent track!

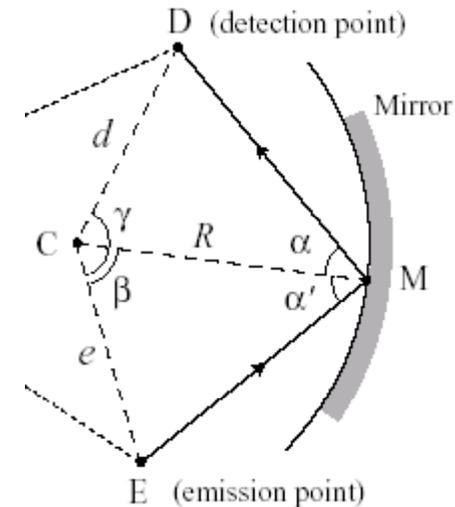
(However, from knowledge of behaviour of optics, it may still be possible, to correct back to approximate circles, without resorting to track information.)

# Determining $\theta_C$

Knowing detection point, mirror optics and assuming emission point, one may calculate reflection point, M

(Forty, Schneider, LHCb-98-040)

$$4e^2d^2 \sin^4 \beta - 4e^2d_yR \sin^3 \beta + (d_y^2R^2 + (e + d_x)^2R^2 - 4e^2d^2) \sin^2 \beta + 2ed_y(e - d_x)R \sin \beta + (e^2 - R^2)d_y^2 = 0 .$$



From reflection point, and knowing track direction, can calculate Cherenkov angles,  $\theta_C$  and  $\Phi_C$

Corrections needed (usually through iterative procedure) to correct for refractions in eg. photodetector windows.

Exercise equally possible for proximity focused device.

In performing calculation one must assume a certain hit-track association, which may well be incorrect – this is a problem for pattern recognition to solve.

# Likelihood Algorithms

Likelihood approach is most common method of pattern-recognition + PID (note - it performs both steps!) for experiments where tracking info is available.

eg. LHCb, BaBar, CLEO-c, Hermes, HERA-B, DELPHI, SLD...

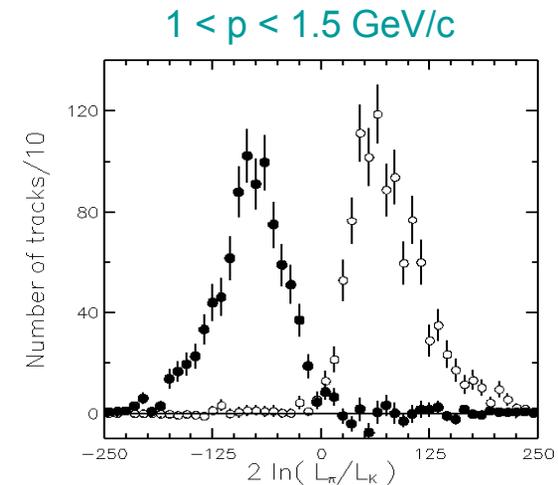
For a given set of photons which are candidates to be associated with the track, formulate a likelihood for each particle hypothesis (e,  $\mu$ ,  $\pi$ , K, p). Eg. for CLEO-c:

$$L_h = \prod_{\gamma} \left\{ P_{background} + \sum_{\text{optical paths, } j} P_{optical, j} \cdot P_{signal}(\theta_{\gamma}^j | \theta_{exp.}^{h, j} \sigma_{\theta}^j) \right\}$$

background distribution

there may be several paths by which photon has reached detector

expect a certain number of photons, at a certain angle, with a certain resolution

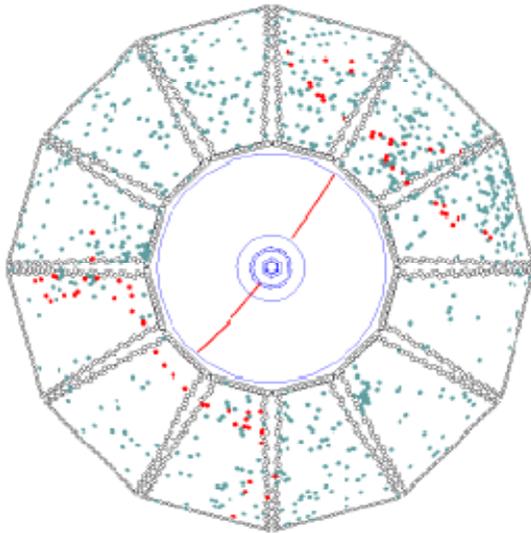


Ratio of likelihoods, or difference of log-likelihoods then gives a statistically meaningful quantity that can be cut on to distinguish between hypotheses.

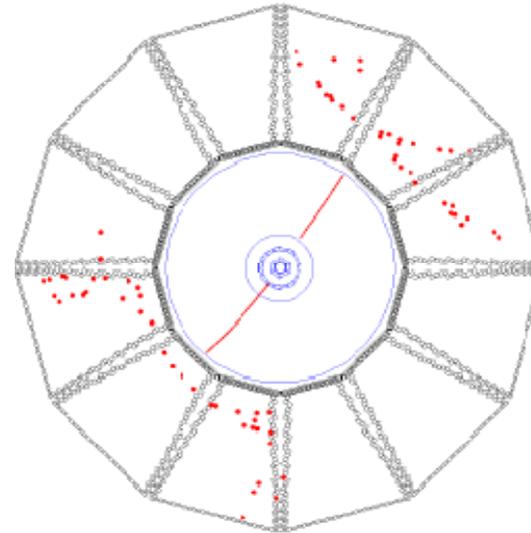
# Including more information in likelihood

It may be that the detector provides additional information that can be included in likelihood. Good example is the BaBar DIRC where PMT times [ $\sigma(\Delta t)=1.7$  ns] are compared with expected photon arrival time, after propagation in quartz bar.

Power of timing information clear from simple cut applied to dilepton event:



$\pm 300$  ns window:  
500-1300 background hits/event

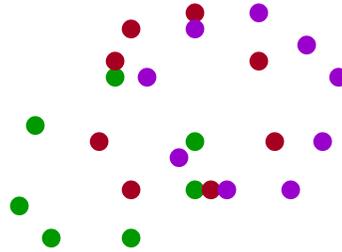


$\pm 8$  ns window:  
1-2 background hits/sector/event

# Global likelihoods

Very often it is advantageous to calculate a single (log) likelihood for all event, being the (sum) product of the likelihoods for all of the tracks in all radiators.

- In high-multiplicity environments, the background to each signal ring is... other signal rings!



Only way to get an unbiased estimate for each track is to consider entire event simultaneously.

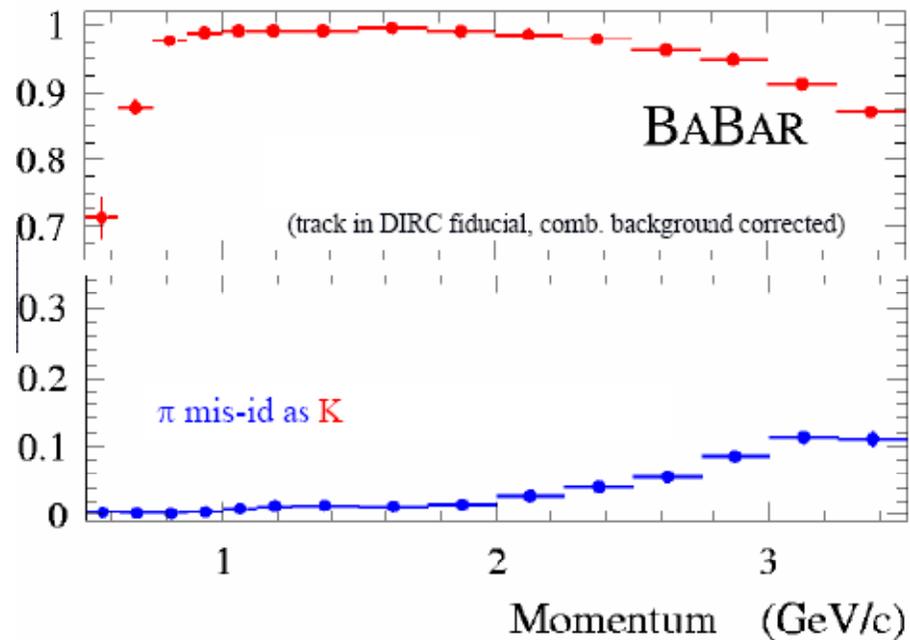
- In experiments with  $>1$  radiator or  $>1$  counters (eg. LHCb 3 radiators in 2 counters, SLD liquid and gas, HERMES aerogel and gas...) this is a convenient way to make best use of all information.

Negative likelihood must be minimised – flip each track hypothesis in turn until convergence is attained.

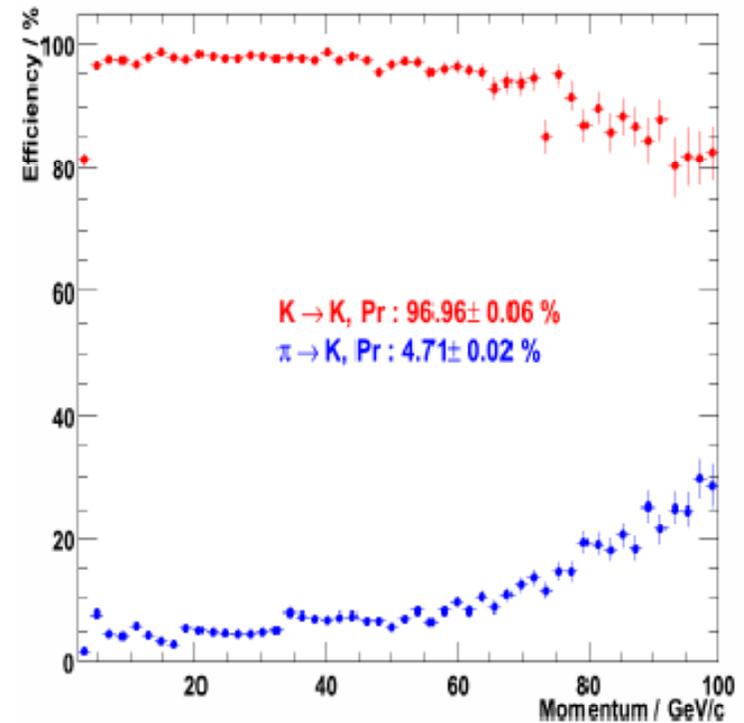
# Performance of likelihood algorithms

Kaon identification efficiency, and  $\pi$  misid efficiency:

BaBar:  $L_K > L_\pi$

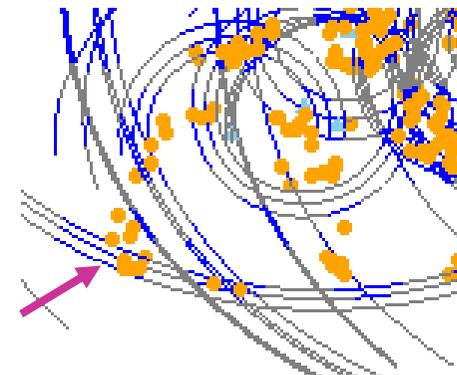


LHCb: K (or p) preferred hypothesis



## Drawbacks/dangers in global likelihood approach

- Global likelihood approach assumes that all hits in photodetectors can be described by the assumed PDF. There may be background hits that aren't.  
Particular danger from rings from secondaries which have no reconstructed track associated.

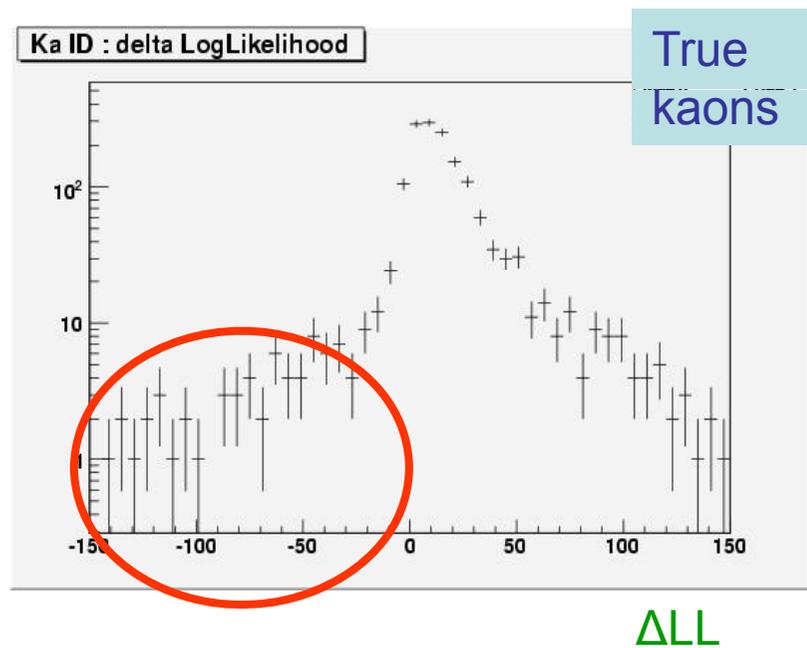


- Furthermore, considering all tracks may make procedure slow
- The LL fit may well do the job of pattern recognition well, but it may not provide the optimal PID information.
  - 1) Correlated errors, eg. from tracking, are likely not to have been accounted for, therefore not giving the optimal discrimination for certain rings. Therefore a refit may be useful.
  - 2) In analysis may not want to cut on PID information, but instead use the RICH results as one of several variables in event by event fit. Here, may want a Gaussian variable.

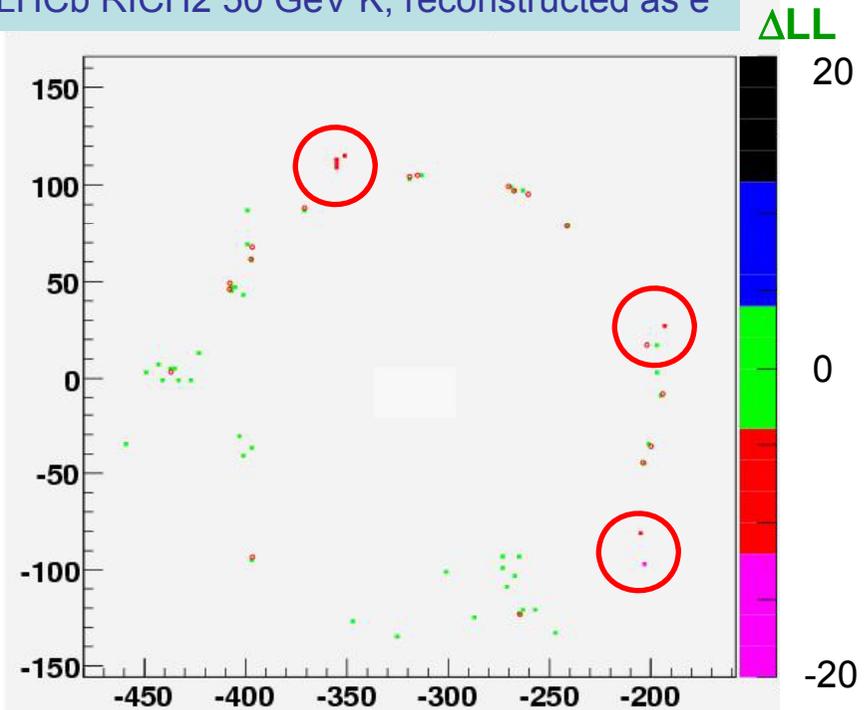
# Tails in likelihood distributions

Background from rings which have no reconstructed track cause headaches for global LL approaches, as such contributions are difficult (impossible?) to account for in LL → tails in performance distributions

LHCb example:



LHCb RICH2 50 GeV K, reconstructed as e

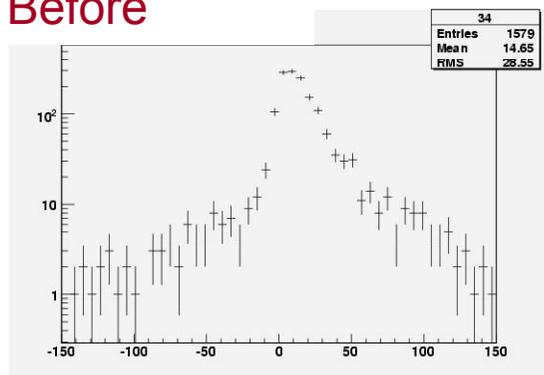


Circles indicate background hits which bias result

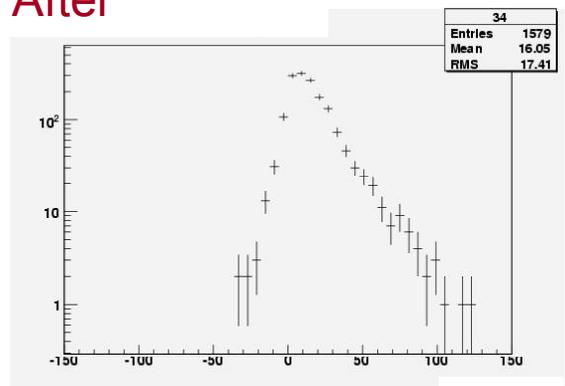
# Truncating Contribution of Outliers

Effect of outliers can be suppressed by limiting contribution they make to LL. For example in LHCb, truncate probability of pixels which contribute a probability  $< 10^{-3}$  to  $\Delta LL$

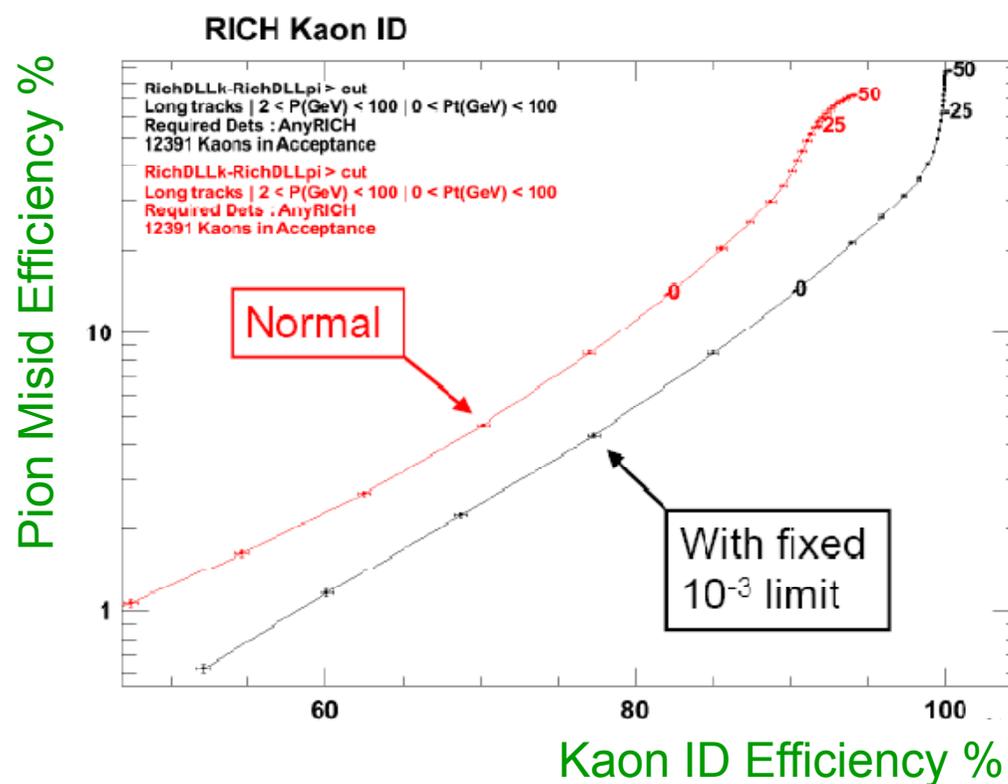
Before



After



$\Delta LL$



# Ring Refit

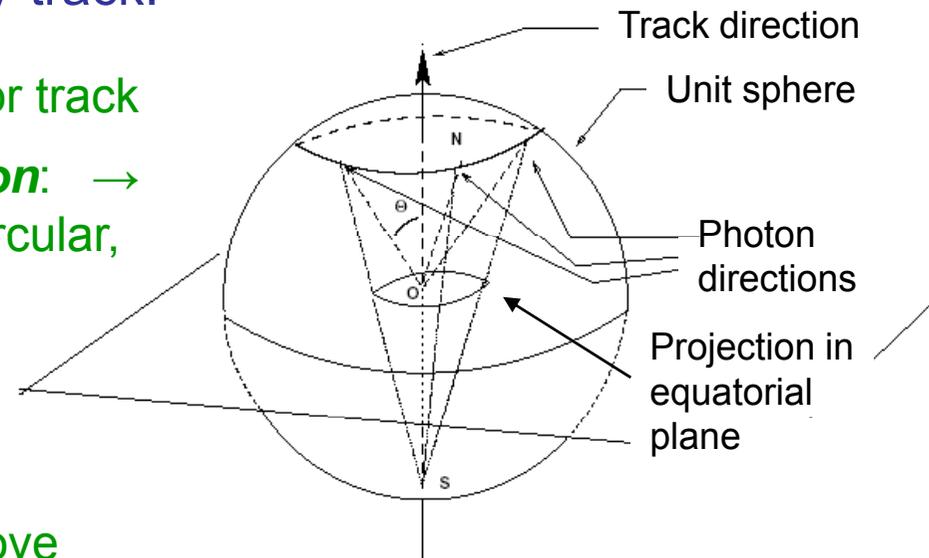
In likelihood approach it is difficult to account for correlated errors, for instance that on associated track direction, which affects all hits on ring

ie.  $\sigma_{\text{ring}} = \sqrt{\sigma_{\text{pe}}^2 / N_{\text{pe}} + \sigma_{\text{track}}^2}$  not  $\sigma_{\text{ring}} = \sigma_{\text{pe}} / \sqrt{N_{\text{pe}}}$

Can limit performance, particularly at high  $p$  where resolution important

One solution - refit rings, track-by-track:

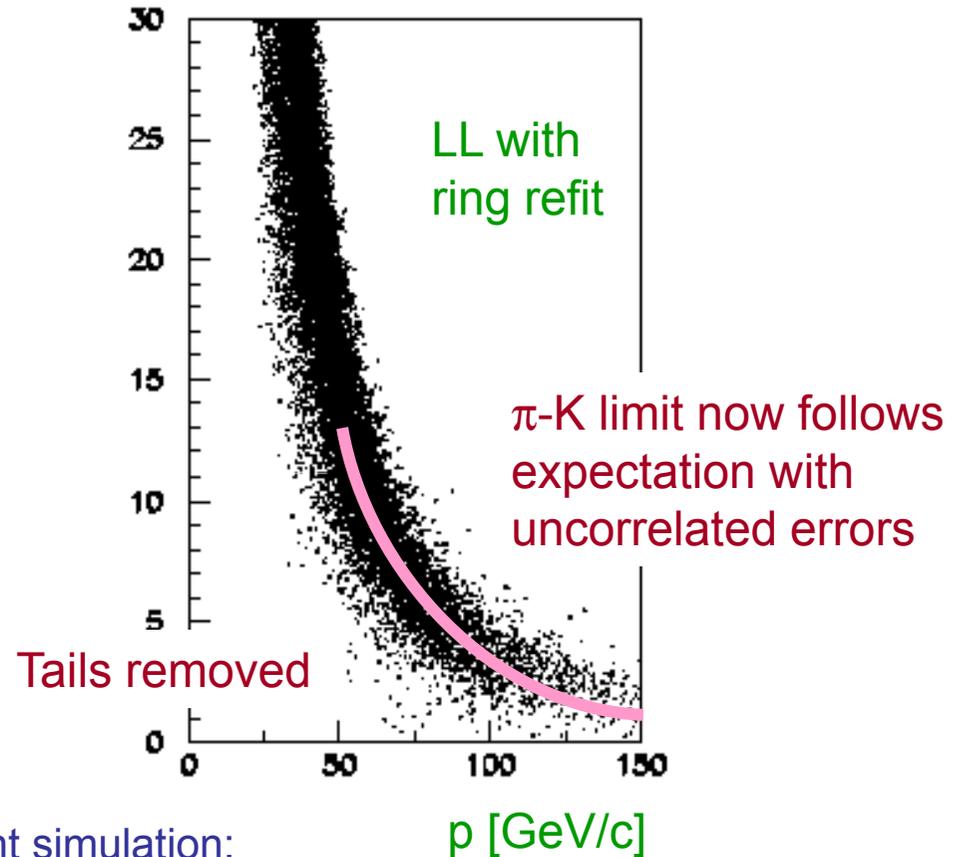
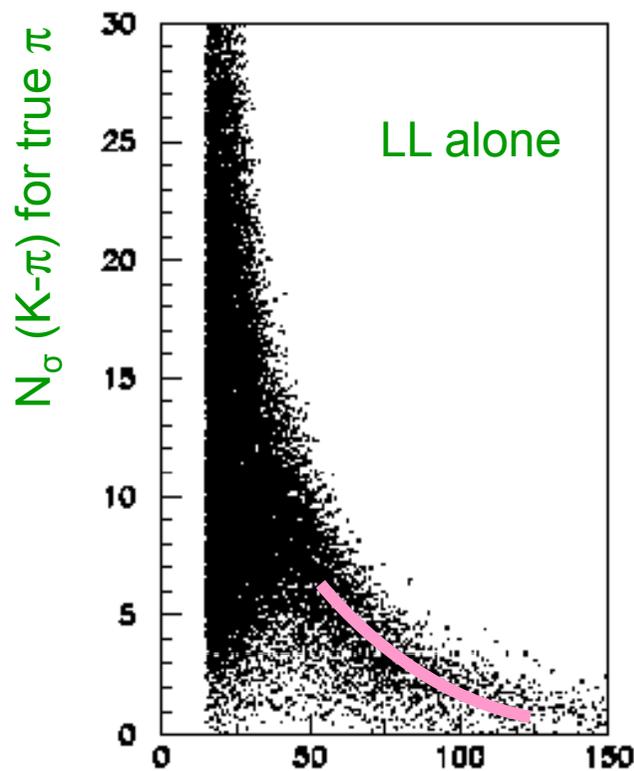
- Take set of hits favoured by LL for track
- Make **stereographical projection**: → Projected rings are essentially circular, even if track direction imperfect
- Fit radius (and centre) of circle without any more need of track information, hence get  $\theta_c$ . Iterate fit to discard outliers and to improve original track direction estimate.



[ Benayoun et al., NIM A426 (1999) 283;  
Benayoun and Jones, LHCb-2004-057. ]

# Benefits of Ring Refit

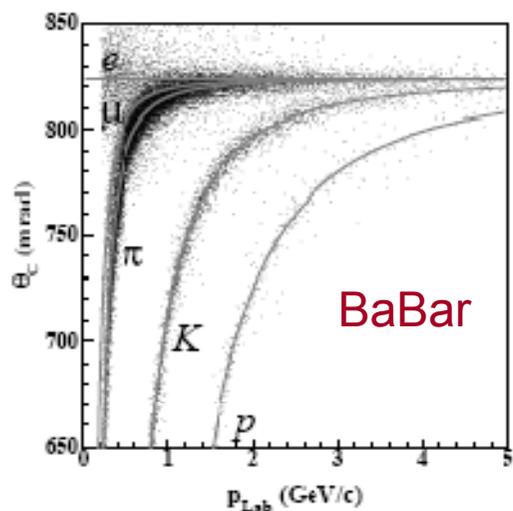
Ring-refit exercise performed in LHCb RICH 2 both eliminates outliers & tails (à la LL cut) and extends PID performance at high momentum



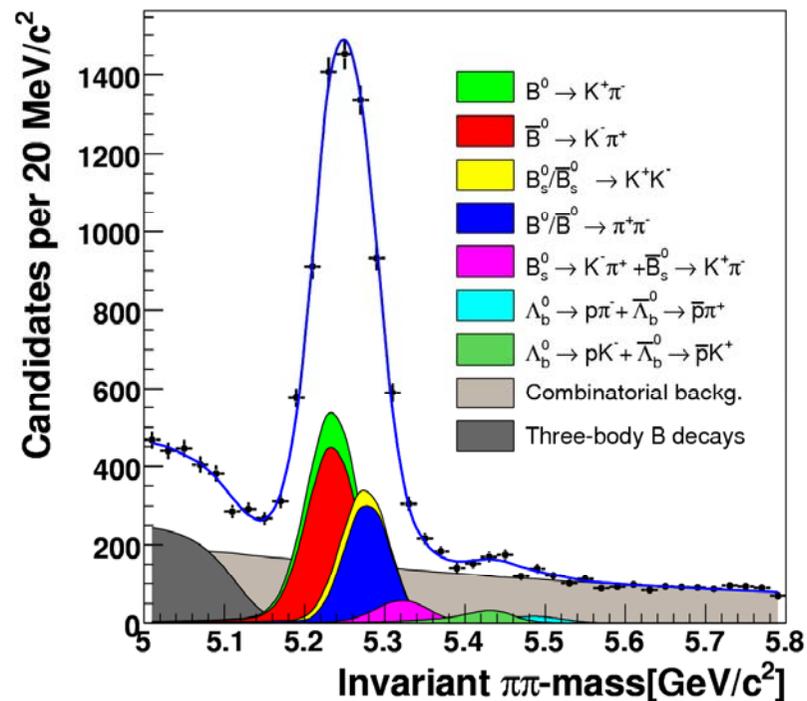
(Caution: plots not made with most recent simulation; absolute performance is now different.)

# Using PID information in event-by-event fits

In seeking to fit relative contributions to  $B \rightarrow hh$  spectrum (as BaBar have done for  $B^0$ , and CDF also, incld  $B_s$  and  $\Lambda_b$ , using  $dE/dx$ ) it is desirable to use event-by-event fit exploiting invariant mass, knowledge of lineshapes and PID information.

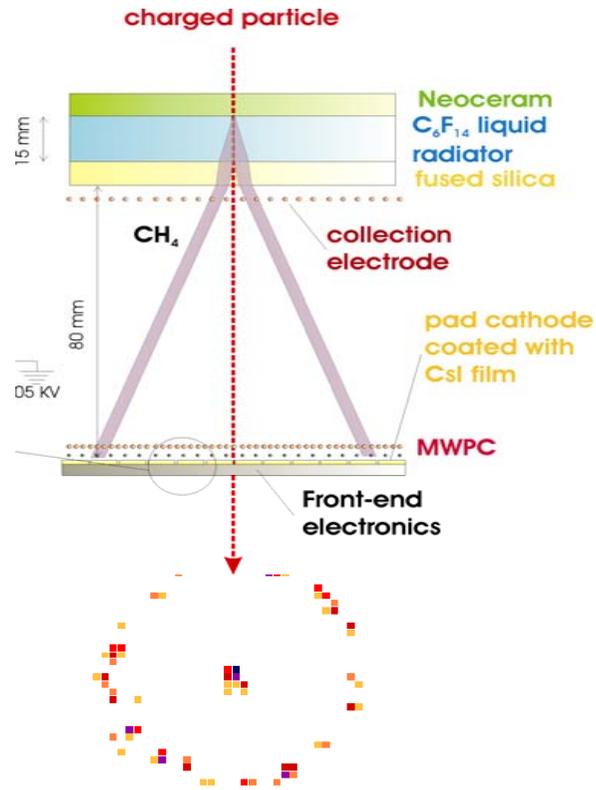


CDF fit using  $dE/dx$ ,  $1 \text{ fb}^{-1}$



BaBar chooses to use the measured Cherenkov angle for ring. Has no meaning in global LL, but is rather the result of an individual track-by-track fit.

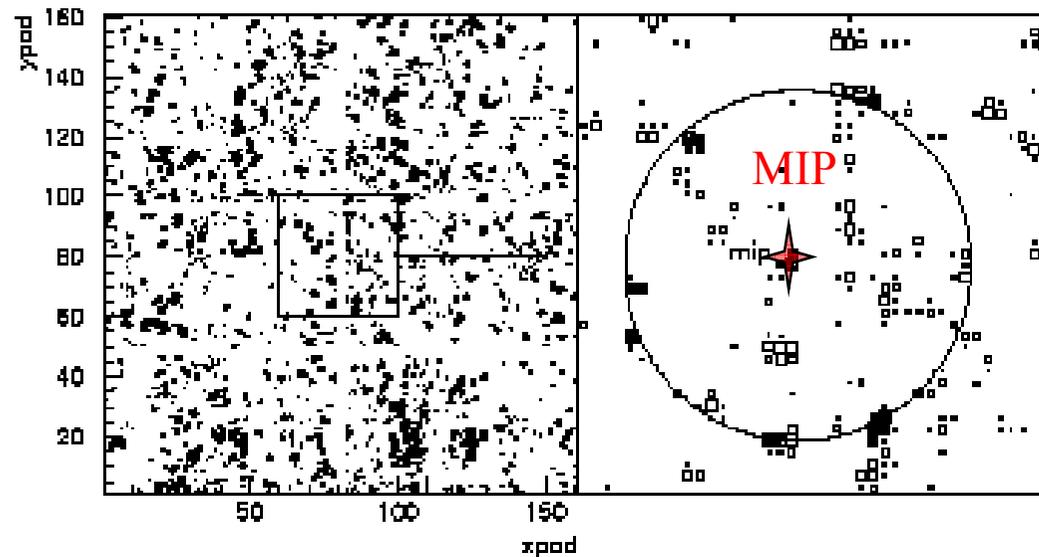
# Ring-finding in the ALICE HPMID



$$n_{pe} \approx 20 @ \beta = 1$$

Resolution per p.e. = 10-15 mrad,  
depending on track inclination

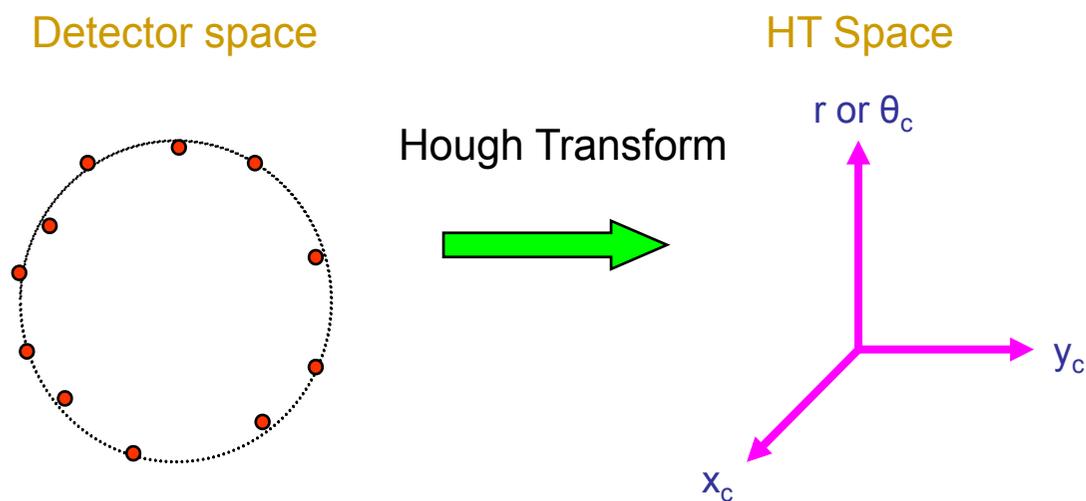
Pb-Pb collisions,  $dN/dy=6000$ :  
50 particles/m<sup>2</sup>  
(pad occupancy 13%)



Challenge of ring finding and PID in  
high multiplicity environment !

# Pattern Recognition with Hough Transforms

Hough Transform: common technique in both tracking & ring finding.  
Attractive features - unaffected by topological gaps in curves, split images, and is rather robust against noise.



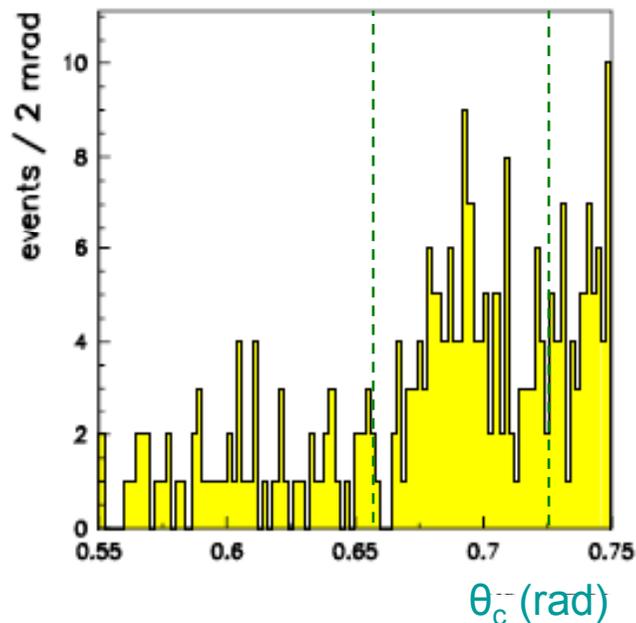
Each point gives surface in HT space. Intersection of surfaces gives ring parameters. Find by peak hunting in suitably binned histogram.

Usual practice: look for centre OR radius, ie. reduce to 2-d or 1-d problem.

# ALICE Hough Transform

First extrapolate tracks to HMPID and look for matching MIP signals. Either track extrapolation or MIP signal may be used as 'impact point'

Then transform nearby hits into  $\theta_c$  space and histogram – this is the Hough Transform. The PID then amounts to peak finding in histogram.



But high background from nearby and overlapping rings means that peak is not a priori obvious to find.

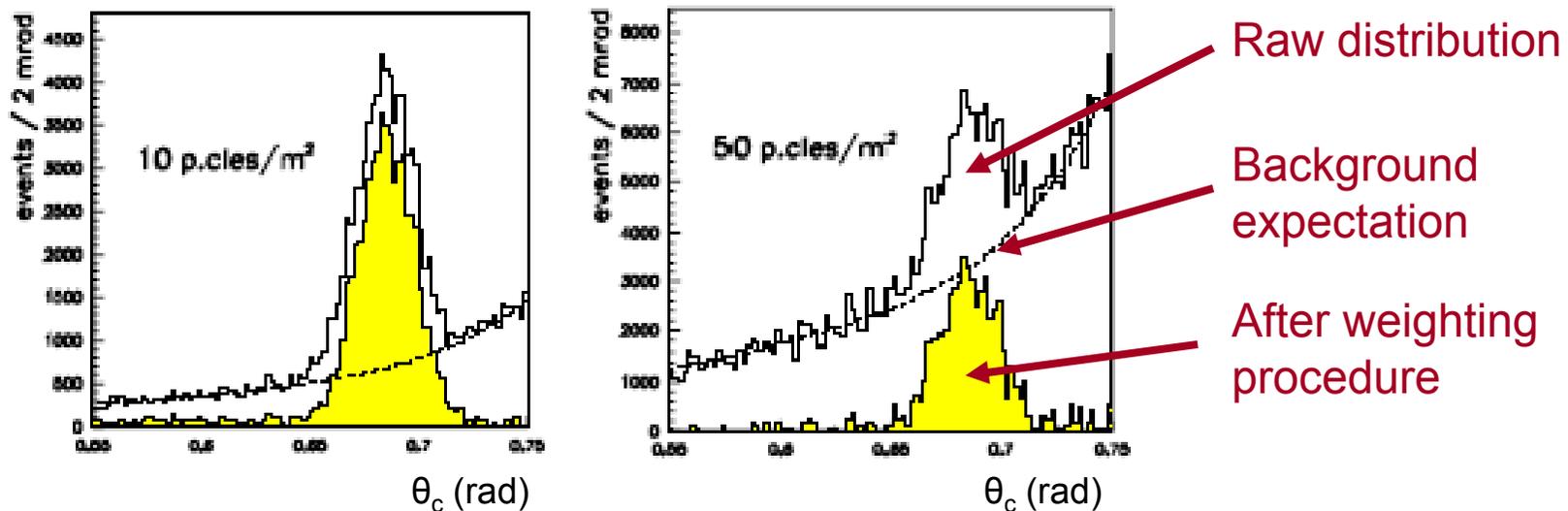
← Hits in Cherenkov angle space around single track (simulated event)

Simple average of entries will not give true Cherenkov angle !

# ALICE Hough Transform

Knowing the distribution of rings to be essentially uniform, can calculate expected background distribution in 'Hough Transfer Space' (HTS) (a.k.a. Cherenkov angle space). When incrementing bins in HTS, use weight, rather than integer, to reflect background subtracted significance

MC (over many events) at 2 different particle densities

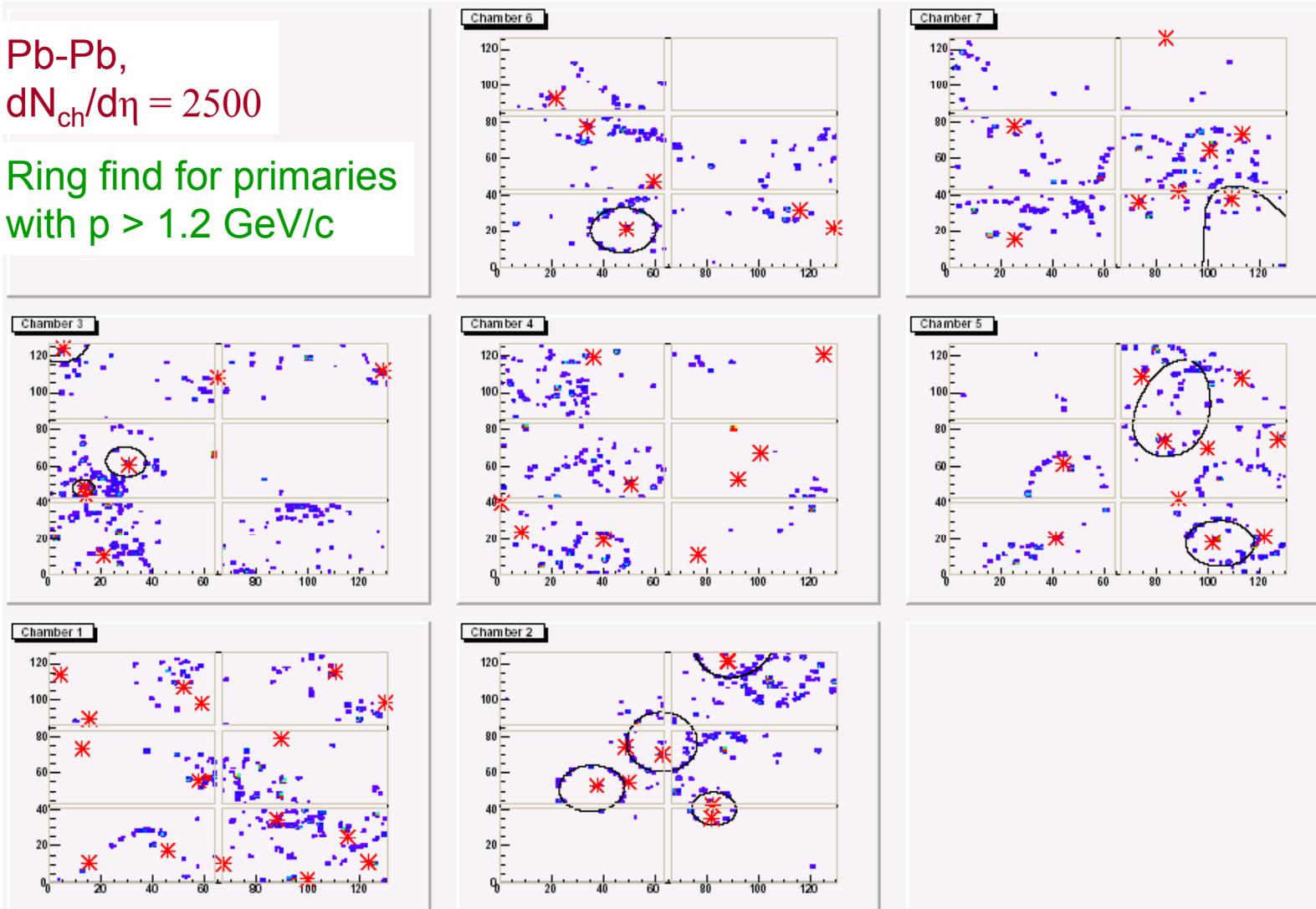


Make weighted average of entries in sliding window of width 3-4 times expected single p.e. resolution. Iterative procedure allows knowledge of track direction to be improved.

# HMPID Reconstructed Event

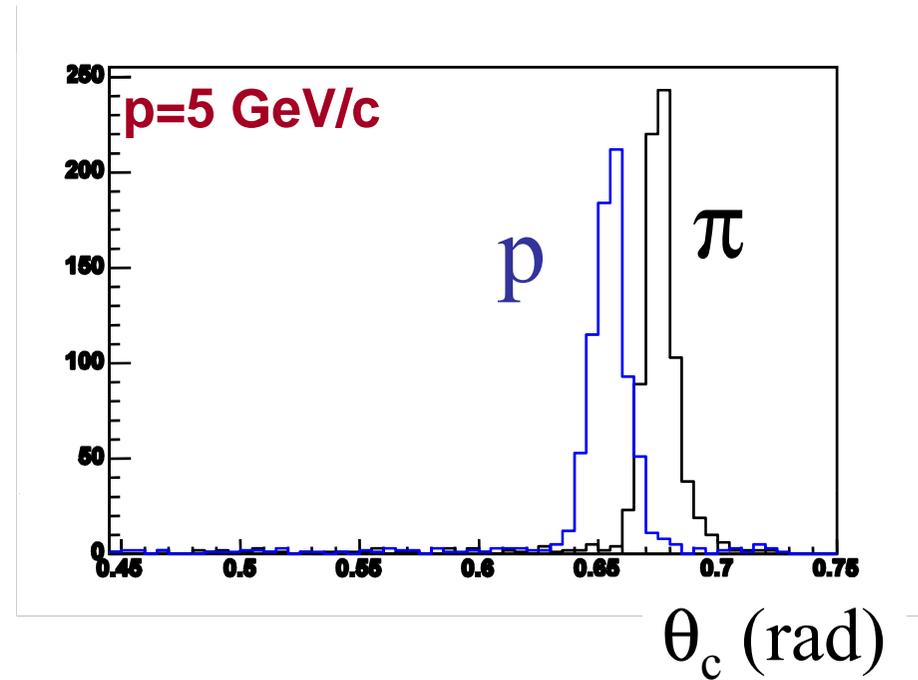
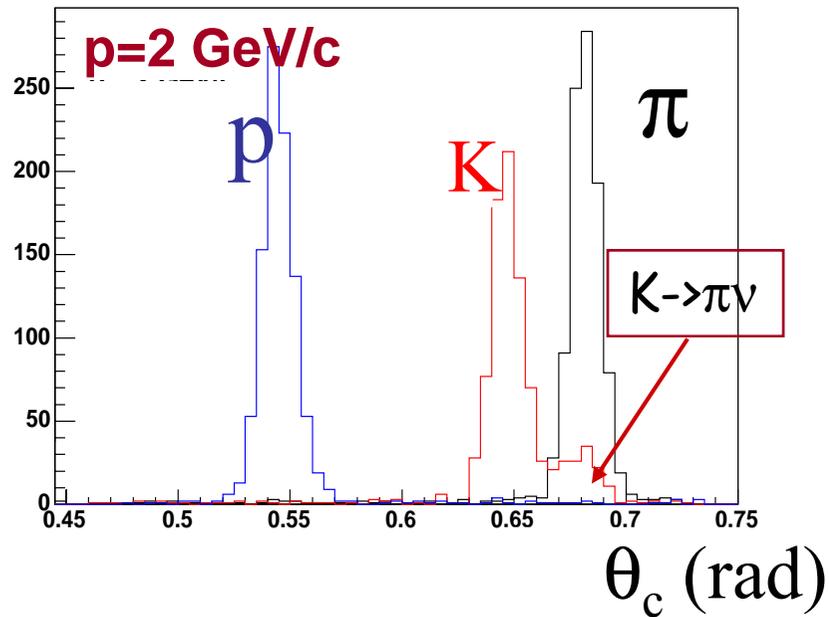
Pb-Pb,  
 $dN_{ch}/d\eta = 2500$

Ring find for primaries  
with  $p > 1.2 \text{ GeV}/c$



# HMPID PID Performance

Single  $\pi$ , K, p superimposed to Pb-Pb collisions,  $dN/dy=6000$ :

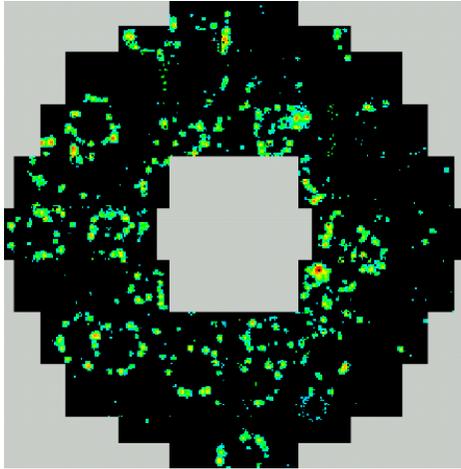


$\theta_c$  resolution  $\sim 6$  mrad,  
Particle Separation @  $3\sigma$  :

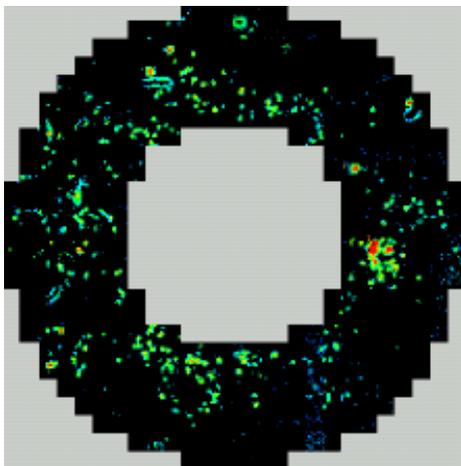
$\left\{ \begin{array}{l} \pi/K \text{ up to } 3 \text{ GeV}/c \\ p/K \text{ up to } 5 \text{ GeV}/c \end{array} \right.$

## Hough Transforms in Heavy Ion Physics: another example

CERES RICH 1 : 11 hits per  $\beta = 1$   
track with single p.e. res = 1 mrad



CERES RICH 2 : 12 hits per  $\beta = 1$   
track with single p.e. res = 0.8 mrad



A frequent goal in HI experiments is to use RICHes to search for low mass  $e^+e^-$  pairs.

So task is to hunt for pairs of saturated rings.

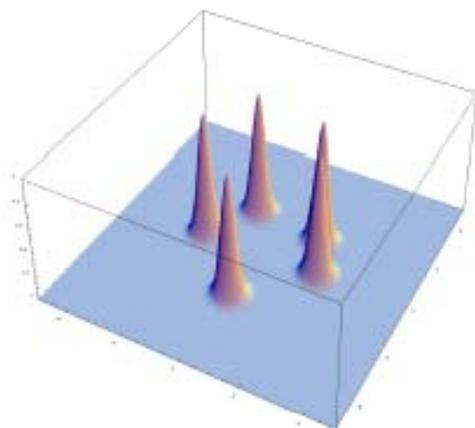
Backgrounds: close-lying rings produced by photon conversions and Dalitz decays.

← Good example: CERES at CERN SPS

Common approach: Hough Transform, in which  $\theta_c$  is assumed as maximal, and this time one looks for ring centres. (So this is a *track independent* method!)

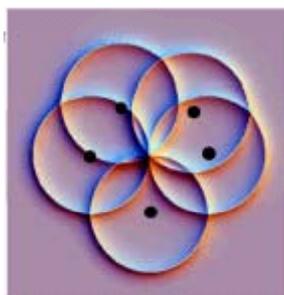
# Saturated Ring Finding at CERES

Schematic of approach:

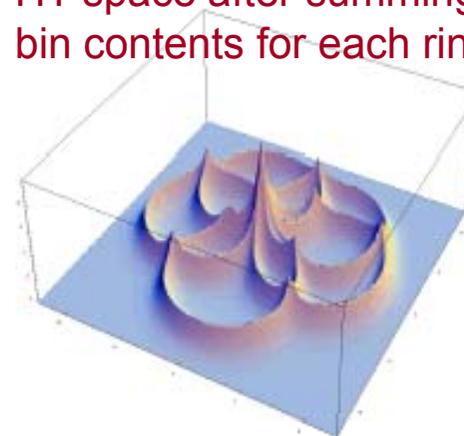


Photon hits (ie. centre-of-gravity of pad clusters in photodetector)

Circles drawn around each hit



HT space after summing bin contents for each ring.



Peaks define candidate ring centres

Refinements:

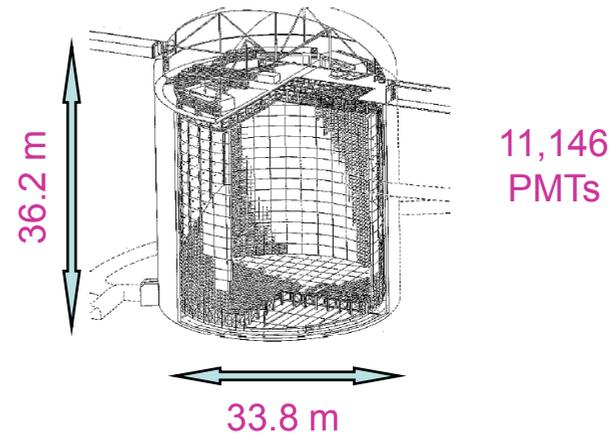
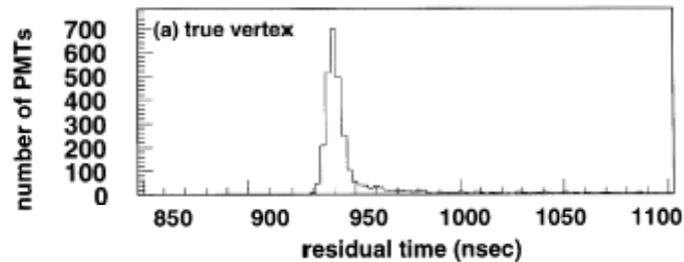
- Smooth bins in HT space by averaging over cells.
- Mask candidate rings & do 2<sup>nd</sup> pass HT to look for overlapping rings.
- Improve knowledge of ring centre perform 'robust fit' – reweighted least-squares method which minimises bias from outliers [NIM A 371 (1996) 243.]

Electrons found with efficiency of 68% and ring centre with  $\sigma$  of 0.8 mrad

# Hough Transforms in Neutrino Physics: SuperK

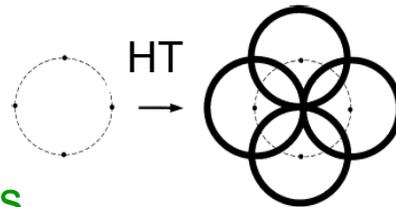
No tracking info available in SuperK: standalone ring-finding essential

Firstly find event vertex position based on spread of hit PMT times

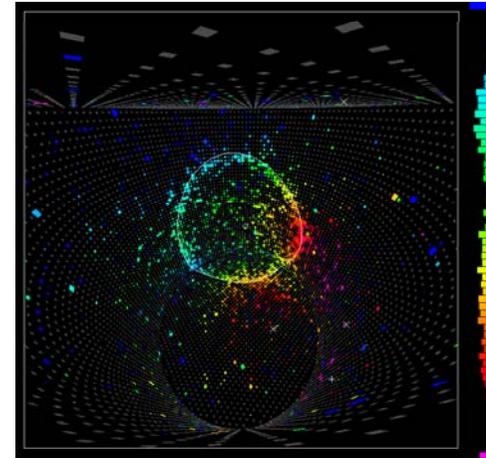


Find vertex to resolution of  $\sim 30$  cm  
Initial direction indicator also available.

Then perform HT: draw saturated ( $42^\circ$ ) rings around hit tubes to look for ring centres and hence directions.

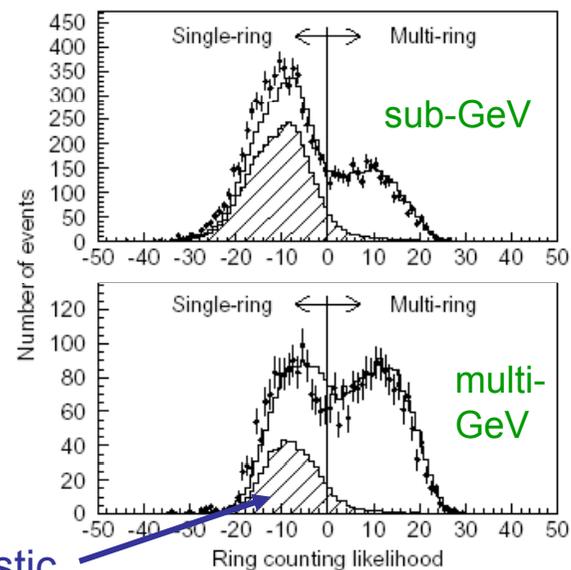
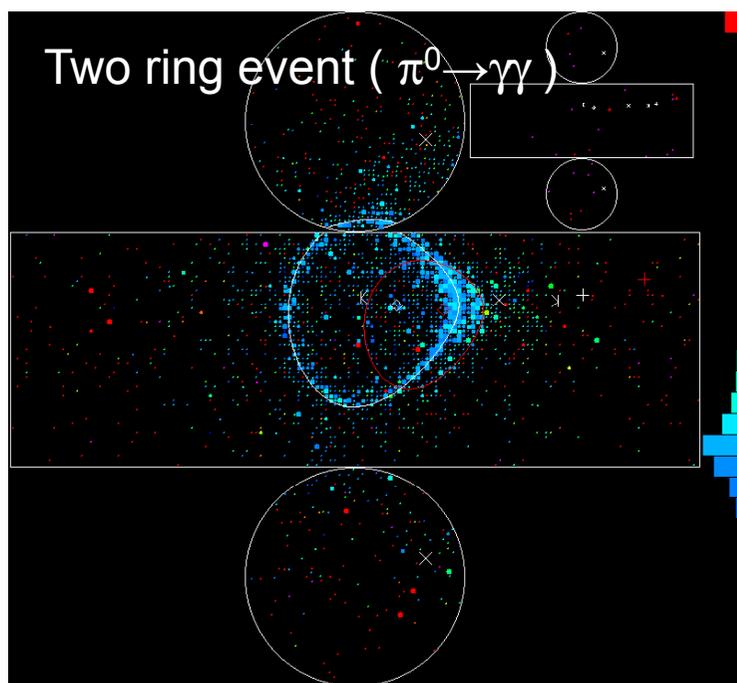


Iterate, to look for multiple ring candidates

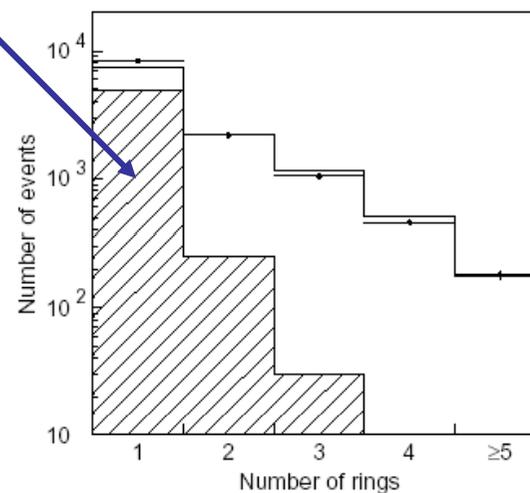


# Resolving Multiple Rings

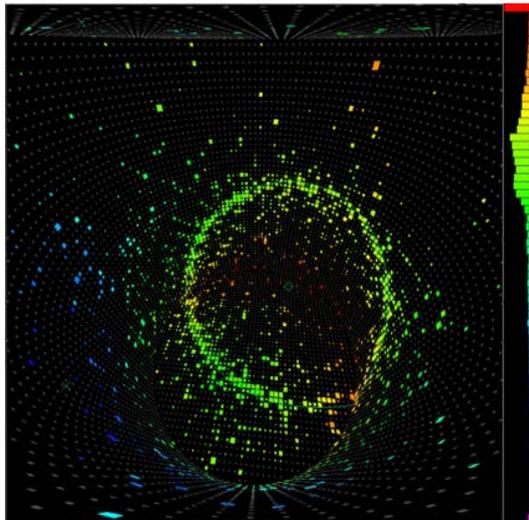
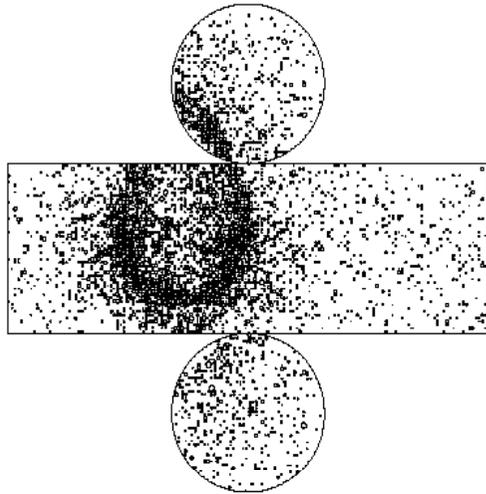
Events can contain several rings.  
So likelihood built up based on  
expected ring distribution to decide  
whether there are 1, 2 or more.



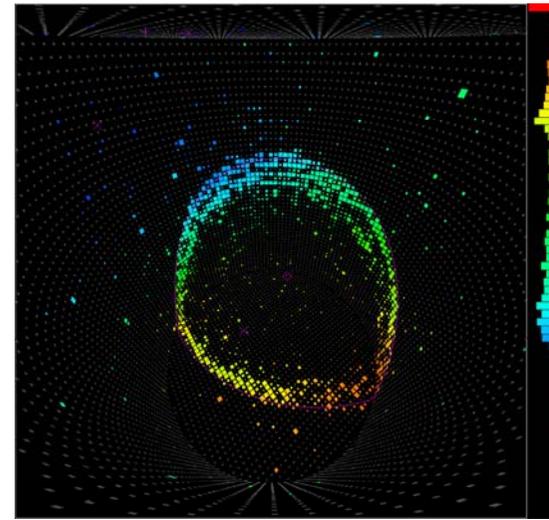
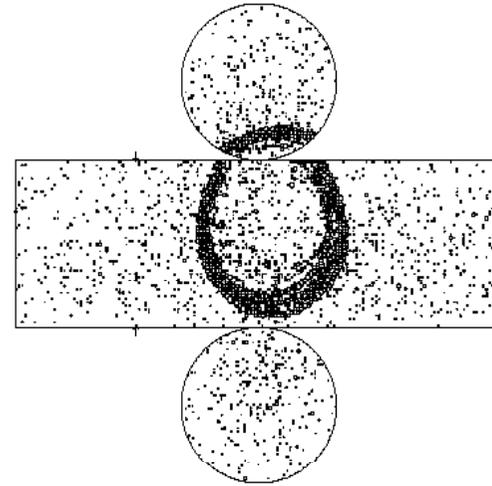
CC quasi-elastic



2 electron candidates:



2 muon candidates:



# Particle Identification

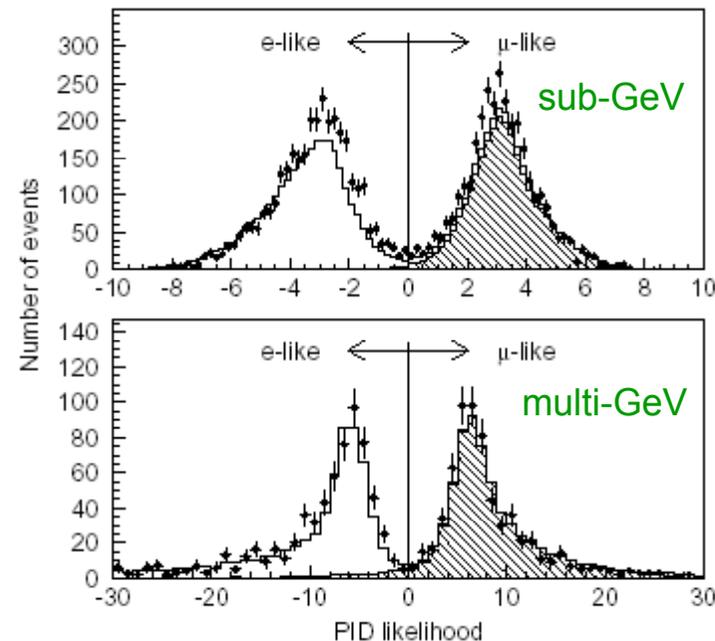
Likelihood built up based on how well rings agree with fuzzy electron or sharp muon hypothesis, plus opening angle information.

Performance on single ring events:

	Id eff	Misid eff
CC $\nu_\mu$	95.8 %	0.7%
CC $\nu_e$	93.2 %	0.8%

Finally, rings re-fit making use of expected light pattern by PID to improve p and angular resolution.

Momentum resolution  $\sim 3\%$   
 Angular resolution  $\sim 2-3^\circ$



# Other Approaches to Ring-Finding

Other approaches have been explored for RICH reconstruction. Of particular interest are those which proceed independently of track information, eg:

- Fuzzy Clustering Methods
- Deformable Templates
- Metropolis-Hasting Markov Chains (→)
- etc .....

None have shown sufficient competitiveness with standard approaches to warrant becoming default algorithm for reconstruction, but some show promise as a complementary tool in certain applications.

# Ring Finding with a Markov Chain

Sample parameter space of ring position and size by use of a Metropolis-Hastings Markov Chain Monte Carlo (MCMC)

What is a MCMC? A Bayesian grounded approach which samples from a proposal distribution in a correlated manner

→ advantage, possibility of sampling parameter space efficiently

If  $\mathbf{R}$  is a collection of rings, and  $\mathbf{H}$  the collection of hits

$$p(\mathbf{R} | \mathbf{H}) = p(\mathbf{H} | \mathbf{R}) p(\mathbf{R})$$

what we are interested in –  
an unknown function telling  
us the probability of  $\mathbf{R}$  having  
given rise to  $\mathbf{H}$

probability of  
getting  $\mathbf{H}$  given  $\mathbf{R}$

probability of having  $\mathbf{R}$

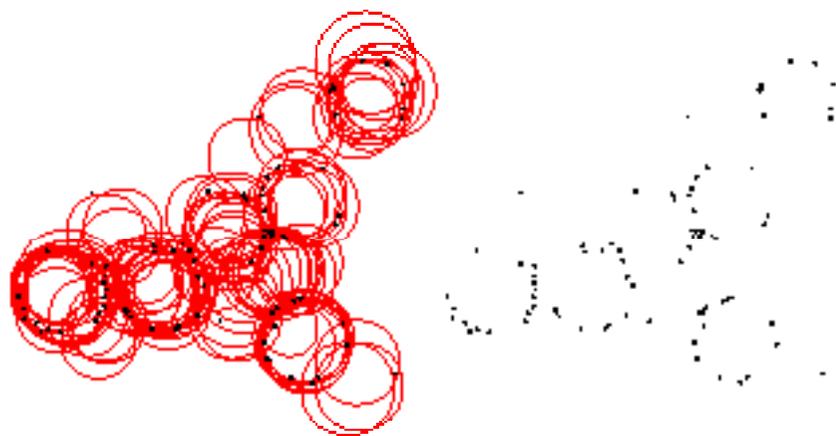
Sample  $p(\mathbf{R}|\mathbf{H})$  with MCMC & study results after certain # of iterations.

# Ring Finding with a Markov Chain

Possibility explored in A. Linka et al., Czech. J Phys. 49/S2 (1999) 161 and C.G. Lester, NIM A560 (2006) 621 (latter discussed here)

Generate and then find rings in standalone, general RICH, simulation:

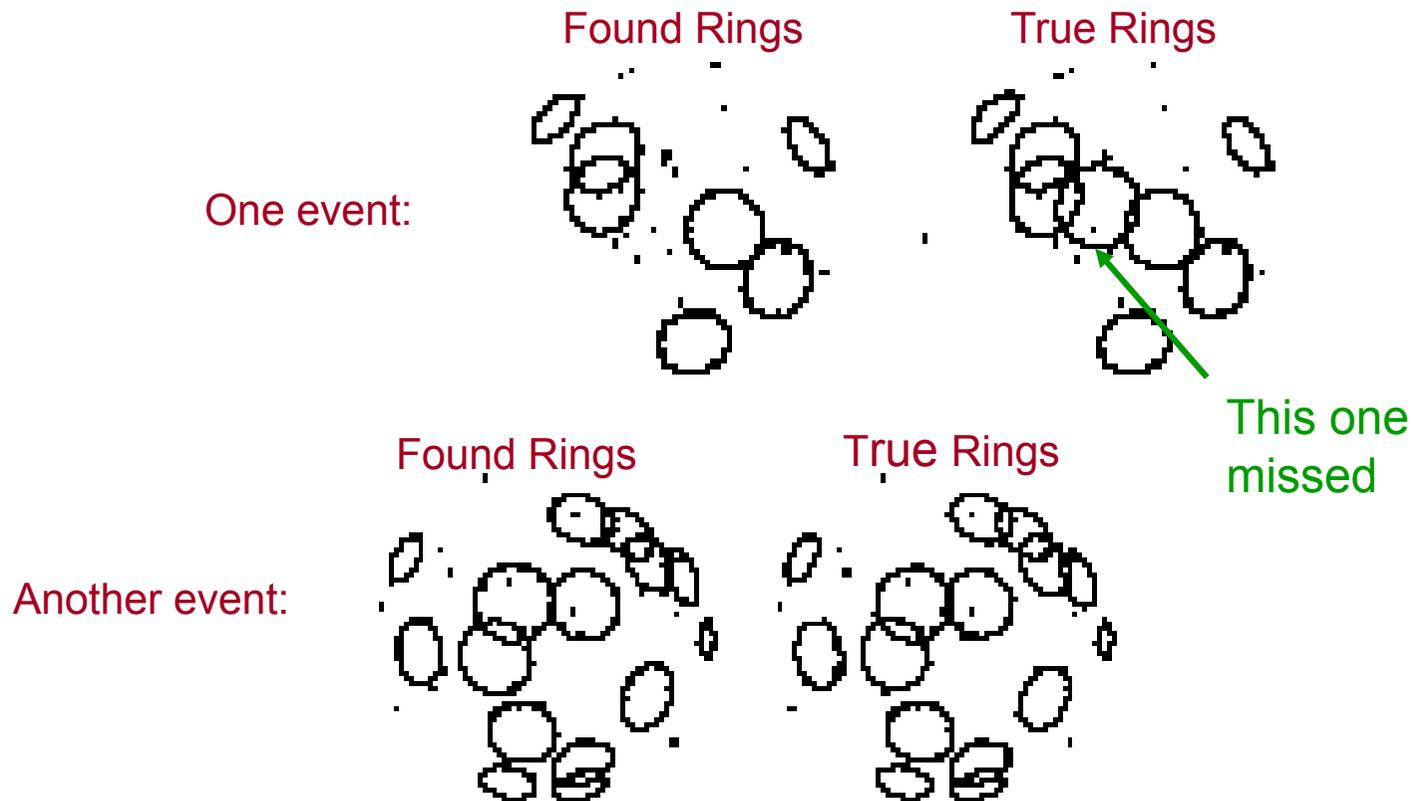
- Proposal distribution allows for creation, modification and removal of rings
- New, proposed rings are seeded by possibilities which pass through three hits



# Markov Chain Ring Finding: Results

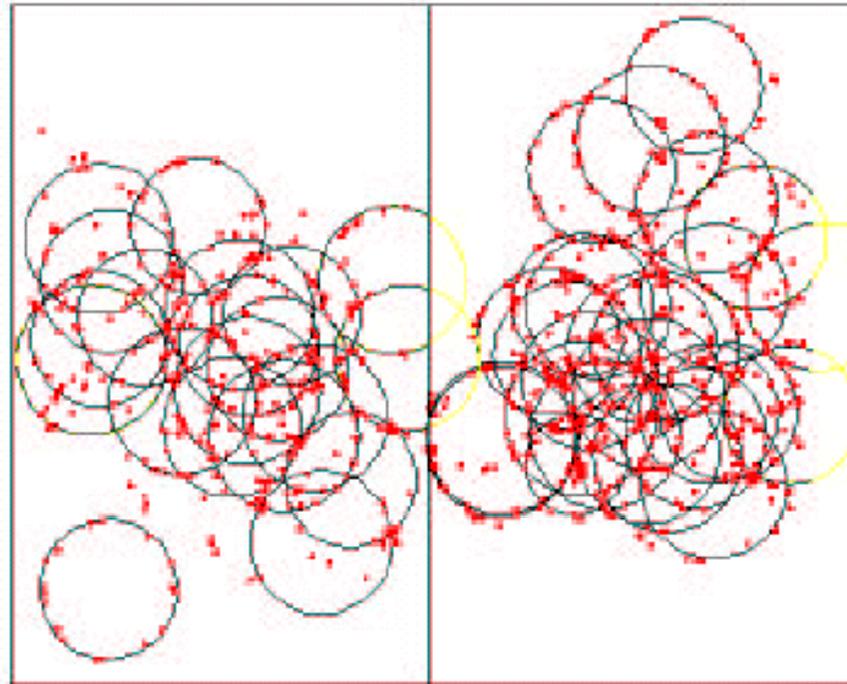
Results in standalone simulation very encouraging:

(hyperbolic projections)



# Markov Chain Ring Finding in LHCb

Ring-finder applied to LHCb RICH 2 simulated events works well...



A. Buckley,  
PhD thesis,  
Cambridge, 2005

...but in existing studies still not competitive in performance or CPU budget with likelihood approach. But it exists as an option – perhaps as a tool to clean up rings not found by track based approach ?

# Summary

My (unproven) suspicion: rather little thought given to the challenges of reconstruction when RICH detectors are being designed.

But people rise to the challenge and in cases reviewed here, impressive performance is obtained.

Likelihood algorithms and Hough Transforms have proven record of making sense of even the most intimidating environments. In general these make significant use of tracking information.

Other approaches exist (eg. Markov chain), but have not yet achieved performance to displace baseline methods.

Will be interesting to see how methods developed on MC for high multiplicity experiments (eg. LHCb, ALICE) cope with real data!