

# Black holes with primary scalar hair

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# Breakthrough in observational data concerning compact objects



- There is a multitude of observational data for compact objects
- **GW signals from compact binaries** : for example the GW170817 neutron star merger, GW190814 and the large mass secondary at  $2.59^{+0.08}_{-0.09} M_{\odot}$
- Array of **radio telescopes**, EHT : for example image of M87 black hole with its light ring
- **GRAVITY** VLT: observation of star trajectories orbiting SgrA central black hole : orbit characteristics give us tests of strong gravity which get better as precision increases.
- **X-ray telescopes** and timing observations of pulsars, (eg NICER aiming to measure EoS for neutron stars).
- GR with a tiny cosmological constant is an effective theory that is compatible with data

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- **X-ray telescopes** and timing observations of pulsars, (eg NICER aiming to measure EoS for neutron stars).
- GR with a tiny cosmological constant is an effective theory that is compatible with data
- **Can we find alternatives to GR black holes and neutron stars as precise rulers of departure from GR?**



- Horndeski theories and beyond... as a measurable departure from GR
- Constructing black holes with primary hair → Singularity free black holes and solitons
- Axial perturbations. The notion of effective and background metrics and spontaneous Lorentz symmetry breaking
- Concluding remarks

Key notions :Primary and secondary hair, diffeomorphisms, local and global symmetries, effective and background metric

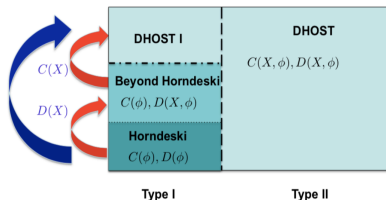
# Scalar tensor theories : a robust measurable departure from GR

Simplest modified gravity theory with a single scalar degree of freedom

BD theory,..., Horndeski (or generalised Galileon [Deffayet, Deser, Esposito-Farèse,...]),..., beyond Horndeski,..., DHOST theories [Achour, Crisostomi, Koyama, Langlois, Noui, Piazza, Vernizzi, et.al.]

- *Nothing fundamental* about ST theories, they are just measurable departures from GR which are robust with a single additional degree of freedom
- They are limits of more complex fundamental theories (massive gravity, braneworld models, EFT from string theory, Lovelock theory etc.)
- Horndeski is parametrized by 4 functions of scalar and its kinetic energy,  $G_i = G_i(\phi, X)$  with  $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ .
- Beyond Horndeski or DHOST are parametrised by two more functions corresponding to conformal and disformal transformations of Horndeski

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi, X)g_{\mu\nu} + D(\phi, X)\nabla_\mu\phi\nabla_\nu\phi$$



[Langlois, 2018]

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Two extra functions are understood via conformal and disformal transformations

In simple terms from Horndeski via disformal we go to Beyond Horndeski via conformal to DHOST

$$S_H = \int d^4x \sqrt{-g} (L_2 + L_3 + L_4 + L_5)$$

$$L_2 = G_2(\phi, X),$$

$$L_3 = -G_3(\phi, X)\square\phi,$$

$$L_4 = G_4(\phi, X)R + G_{4X} \left[ (\square\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right],$$

$$L_5 = G_5(\phi, X)G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{G_{5X}}{6} \left[ (\square\phi)^3 - 3\square\phi(\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3 \right]$$

## Scalar tensor theories

**Example** in Horndeski with  $G_2 = \Lambda + \eta X$ ,  $G_4 = 1 + 2\beta X$

$$S = \int d^4x \sqrt{-g} \left[ R - 2\Lambda_b - X + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right],$$

- Kinetic term is  $X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$  ( $= -\frac{1}{2} g^{\mu\nu} \phi_\mu \phi_\nu$ ).
- The theory has global shift and parity symmetry. Conserved current  $\nabla_\mu J^\mu = 0$  for the scalar field equation
- One simple (stealth) solution reads

$$f = h = 1 - \frac{2M}{r} - \frac{1}{6\beta} r^2$$

$$\phi = qt + \int dr \frac{q}{f} \sqrt{1-f}$$

with secondary hair  $q^2 = \frac{1-2\Lambda_b\beta}{\beta}$  related to the action couplings.

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*Shift symmetry allows for linear time dependence.*

The associated energy-momentum tensor for the scalar must have the same symmetries as the metric, ... **not the scalar field itself**

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- Properties :  
 $X = g^{\mu\nu} \dot{\phi}_\mu \dot{\phi}_\nu = -q^2$  is constant and stealth solutions are generic [Kobayashi, Tanahashi]
- Scalar field is regular at the (future) event horizon for all  $q$ .  $\phi = qv - q \int \frac{dr}{1+\sqrt{1-f}}$ , in advanced EF coordinates where  $dv = dt + \frac{dr}{f}$

- Consider shift and parity symmetric beyond Horndeski theory parametrised  $G_2(X), G_4(X), F_4(X)$  [Gleyzes, Langlois, Piazza, Vernizzi]

- The theory reads,

$$S[g_{\mu\nu}, \phi] = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left\{ G_2(X) + G_4(X) R + G_{4X} [(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}] + F_4(X) \epsilon^{\mu\nu\rho\sigma} \epsilon^{\eta\beta\gamma}{}_{\sigma} \phi_{\mu}\phi_{\eta}\phi_{\nu\beta}\phi_{\rho\gamma} \right\}.$$

- We seek spherically symmetric and static solutions

$$ds^2 = -h(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \quad \phi = qt + \psi(r),$$

- Setting,  $Z(X) = 2XG_{4X} - G_4 + 4X^2F_4$ , we find
- Field Equations are integrable for given BH theory:

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$$2X'Z_X = Z \left( \frac{h'}{h} - \frac{f'}{f} \right)$$

$$r^2(G_2Z)_X + 2(G_4Z)_X \left( 1 - \frac{q^2\gamma^2}{2Z^2X} \right) = 0$$

$$2\gamma^2 \left( hr - \frac{q^2r}{2X} \right)' = -r^2G_2Z - 2G_4Z \left( 1 - \frac{q^2\gamma^2}{2Z^2X} \right) + \frac{q^2\gamma^2X'r}{ZX^2} (2XG_{4X} - G_4)$$

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$$\frac{f}{h} = \frac{\gamma^2}{Z^2}$$

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$$f = h, Z = \gamma$$

$$r^2 G_{2X} + 2G_{4X} \left(1 - \frac{q^2}{2X}\right) = 0$$

$$2\gamma \left( hr - \frac{q^2 r}{2X} \right)' = -r^2 G_2 - 2G_4 \left(1 - \frac{q^2}{2X}\right) + \frac{q^2 X' \gamma r}{X^2} (2XG_{4X} - G_4)$$

# Primary hair black holes: Solving the system

[A. Bakopoulos, C.C. P. Kanti, N. Lecoeur., T. Nakas, 2023]

[Baake, Cisterna, Hassaine, Hernandez-Vera 2024], [Bakopoulos, Chatzifotis, Nakas 2024]

- For  $Z = \gamma$  theories therefore we have  $f = h$  (homogeneous solutions)

- The class of theories

$$G_2(X) = -\frac{2\eta}{\lambda^2} X^p, \quad G_4(X) = 1 - \eta X^p, \quad F_4(X) = \frac{\eta}{4} (2p - 1) X^{p-2}.$$

where  $\eta$  and  $\lambda$  are the coupling constants and  $p$  is integer or half-integer.

- the scalar equation,

$$r^2(G_2)_X + 2(G_4)_X \left(1 - \frac{q^2}{2X}\right) = 0$$

- gives  $X$  which is not constant but regular everywhere and the scalar  $\phi = qt + \psi(r)$  :

$$X = \frac{q^2/2}{1 + (r/\lambda)^2} \quad \psi'(r)^2 = \frac{q^2}{f^2(r)} \left[1 - \frac{f(r)}{1 + (r/\lambda)^2}\right],$$

Finally we solve for the metric,

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$$f(r) = 1 - \frac{2M}{r} - \frac{2\lambda\xi_p}{r} \int du \frac{u^2/\lambda^2}{(1 + (u/\lambda)^2)^p}$$

We have two independent integration constants  $M$  and  $\xi_p = \eta(2p - 1)(q^2/2)^p$

- Noether charge :  $Q = \int_{\Sigma} d^3x \sqrt{\gamma} n_\mu J^\mu \propto q^{2p-1} \left[ \frac{\eta}{2p} \int_0^\infty \frac{r^2}{(1+(r/\lambda)^2)^p} dr \right]$  associated to Noether current  $\nabla_\mu J^\mu = 0$ .

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# Primary hair black holes: Solving the system

[A. Bakopoulos, C.C.P. Kanti, N. Lecoeur., T. Nakas, 2023]

[Baake, Cisterna, Hassaine, Hernandez-Vera 2024], [Bakopoulos, Chatzifotis, Nakas 2024]

- For  $Z = \gamma$  theories therefore we have  $f = h$  (homogeneous solutions)
- The class of theories

$$G_2(X) = -\frac{2\eta}{\lambda^2}X^p, \quad G_4(X) = 1 - \eta X^p, \quad F_4(X) = \frac{\eta}{4}(2p-1)X^{p-2}.$$

where  $\eta$  and  $\lambda$  are the coupling constants and  $p$  is integer or half-integer.

- the scalar equation,

$$r^2(G_2)_X + 2(G_4)_X \left(1 - \frac{q^2}{2X}\right) = 0$$

- gives  $X$  which is not constant but regular everywhere and the scalar  $\phi = qt + \psi(r)$  :

$$X = \frac{q^2/2}{1 + (r/\lambda)^2} \quad \psi'(r)^2 = \frac{q^2}{f^2(r)} \left[1 - \frac{f(r)}{1 + (r/\lambda)^2}\right],$$

Finally we solve for the metric,

- 

$$f(r) = 1 - \frac{2M}{r} - \frac{2\lambda\xi_p}{r} \int du \frac{u^2/\lambda^2}{(1 + (u/\lambda)^2)^p}$$

We have two independent integration constants  $M$  and  $\xi_p = \eta(2p-1)(q^2/2)^p$

- Noether charge :  $Q = \int_{\Sigma} d^3x \sqrt{\gamma} n_{\mu} J^{\mu} \propto q^{2p-1} \left[ \frac{\eta}{2^p} \int_0^{\infty} \frac{r^2}{(1+(r/\lambda)^2)^p} dr \right]$  associated to Noether current  $\nabla_{\mu} J^{\mu} = 0$ .

For generic theories :

$$G_2(X) = -\frac{2\eta}{\lambda^2} X^p, \quad G_4(X) = 1 - \eta X^p, \quad F_4(X) = \frac{\eta}{4} (2p - 1) X^{p-2}.$$

We have primary hair black holes with :

-Scalar charge  $Q(\eta, q, p) < \infty$  finite iff  $p > 3/2$

-Regular scalar for all  $q$  at all future horizons  $X = \frac{q^2/2}{1+(r/\lambda)^2}$   $\psi'(r)^2 = \frac{q^2}{f^2(r)} \left[ 1 - \frac{f(r)}{1+(r/\lambda)^2} \right]$

Examples parametrised by  $M$  and  $\xi_p = \eta(2p - 1) \frac{q^{2p}}{2^p}$

- We have :

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2,$$

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$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2,$$

$p = 1/2$  : Stealth solution as  $f(r) = 1 - \frac{2M}{r}$ . This is the only homogeneous Horndeski case!

For generic theories :

$$G_2(X) = -\frac{2\eta}{\lambda^2} X^p, \quad G_4(X) = 1 - \eta X^p, \quad F_4(X) = \frac{\eta}{4} (2p - 1) X^{p-2}.$$

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Examples parametrised by  $M$  and  $\xi_p = \eta(2p - 1) \frac{q^{2p}}{2^p}$

- We have :

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2,$$

$p = 1$ , Canonical kinetic term :  $f(r) = 1 - 2\xi_1 - \frac{2M}{r} - 2\xi_1 \frac{\pi/2 - \arctan(r/\lambda)}{r/\lambda}$ .

The solution is only locally asymptotically flat.

Similar to gravitational monopole [Bariola, Villenkin]

For generic theories :

$$G_2(X) = -\frac{2\eta}{\lambda^2} X^p, \quad G_4(X) = 1 - \eta X^p, \quad F_4(X) = \frac{\eta}{4} (2p - 1) X^{p-2}.$$

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- We have :

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2,$$

$p = 2$  Asymptotically flat black hole with primary hair,  
 $f(r) = 1 - \frac{2M}{r} + \xi_2 \left( \frac{\pi/2 - \arctan(r/\lambda)}{r/\lambda} + \frac{1}{1+(r/\lambda)^2} \right)$

# Primary hair black hole examples and properties

For generic theories :

$$G_2(X) = -\frac{2\eta}{\lambda^2} X^p, \quad G_4(X) = 1 - \eta X^p, \quad F_4(X) = \frac{\eta}{4} (2p - 1) X^{p-2}.$$

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-Scalar charge  $Q(\eta, q, p) < \infty$  finite iff  $p > 3/2$

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Examples parametrised by  $M$  and  $\xi_p = \eta(2p - 1) \frac{q^{2p}}{2^p}$

- We have :

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2,$$

$p = 5/2$  etc...

$$f(r) = 1 - \frac{2M}{r} + \frac{2\xi_{5/2} \lambda}{3r} \left( 1 - \frac{(r/\lambda)^3}{(1+(r/\lambda)^2)^{3/2}} \right)$$

# Primary hair black hole examples and properties

For generic theories :

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Examples parametrised by  $M$  and  $\xi_p = \eta(2p - 1) \frac{q^{2p}}{2^p}$

- We have :

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2,$$

What is the asymptotic structure? What are the possible horizons?

Take  $p = 2$  and  $f(r) = 1 - \frac{2M}{r} + \xi_2 \left( \frac{\pi/2 - \arctan(r/\lambda)}{r/\lambda} + \frac{1}{1+(r/\lambda)^2} \right)$

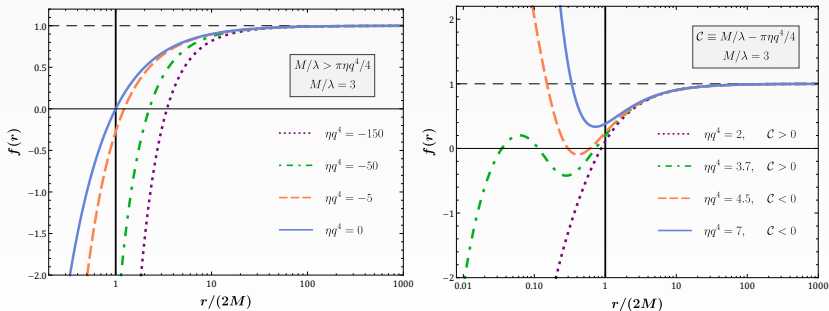
# Horizons and asymptotics

Far away the black hole behaves much like RN but with the scalar playing the role of EM charge

$$f(r) = 1 - \frac{2M}{r} + \xi \left( \frac{\pi/2 - \arctan(r/\lambda)}{r/\lambda} + \frac{1}{1 + (r/\lambda)^2} \right)$$
$$= 1 - \frac{2M}{r} + 2\lambda^2 \frac{\xi}{r^2} + \mathcal{O}\left(\frac{1}{r^4}\right), \quad r \rightarrow \infty$$

while close to the origin we get,

$$f(r) = 1 - \frac{2M - \pi\xi\lambda/2}{r} - \frac{2\xi r^2}{3\lambda^2} + \mathcal{O}(r^4).$$



**Figure:** Left:  $\eta < 0$ , unique horizon greater than the Schwarzschild radius  $r_S = 2M$ . Right:  $\eta > 0$ , one, two, three or zero horizons, horizon smaller than Schwarzschild.

# Regular spacetime (black hole or soliton)

For  $M = \pi \xi_{reg} \lambda / 4$ , the central singularity disappears and all curvature invariants become infinitely regular:

$$f(r) = 1 - \frac{4M}{\pi\lambda} \left( \frac{\arctan(r/\lambda)}{r/\lambda} - \frac{1}{1 + (r/\lambda)^2} \right)$$

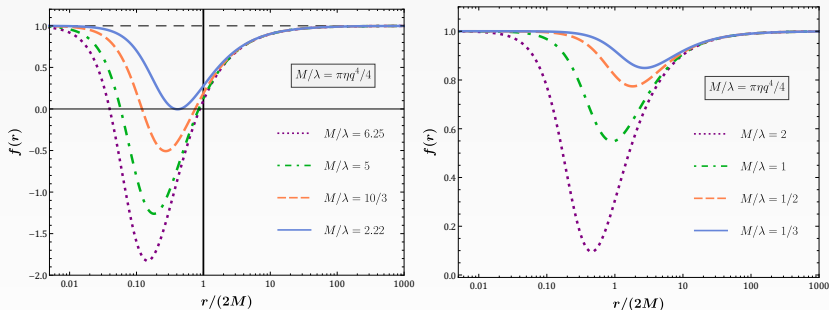


Figure: Left: Regular BH solutions. Right: regular solitonic solutions.

We have a regular black hole for  $M > M_{bh} = \frac{3\sqrt{3}}{4}$  or a regular soliton for  $M < M_{bh}$

For spherical symmetry disformal transformations  $\tilde{g}_{\mu\nu} = g_{\mu\nu} + D(X) \partial_\mu \phi \partial_\nu \phi$

take us from homogeneous ( $f = h$ ) to non homogenous black holes ( $f = \frac{h}{Z^2}$ ).

- $Z(X)$  and  $D(X)$  are mathematically equivalent. We have a class of equivalence defined modulo  $Z$  taking us from homogeneous ( $Z = 1$ ) to non homogeneous solutions ( $Z \neq \text{constant}$ )
- $c_g = c$  frame given by  $\tilde{Z} = -\tilde{G}_4 \Rightarrow D(X) = \frac{\eta}{2} X^{p-1}$ .
- Horndeski frame given by  $\tilde{F}_4 = 0 \Rightarrow D(X) = \eta \frac{2p-1}{2(p-1)X^{p-1}}$

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$p = 2$  homogeneous black hole with  $Z = \gamma$

$$f(r) = 1 - \frac{2M}{r} + \xi_2 \left( \frac{\pi/2 - \arctan(r/\lambda)}{r/\lambda} + \frac{1}{1+(r/\lambda)^2} \right)$$

For spherical symmetry disformal transformations  $\tilde{g}_{\mu\nu} = g_{\mu\nu} + D(X) \partial_\mu \phi \partial_\nu \phi$

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$p = 2$  homogeneous black hole with  $Z = \gamma$

$$f(r) = 1 - \frac{2M}{r} + \xi_2 \left( \frac{\pi/2 - \arctan(r/\lambda)}{r/\lambda} + \frac{1}{1+(r/\lambda)^2} \right)$$

- for  $p = 2$  the  $c_g = c$  frame gives,

$$d\tilde{s}^2 = -\tilde{f}(r) dt_*^2 + \left( 1 - \frac{\xi_2}{3(1 + \frac{r^2}{\lambda^2})} \right) \frac{dr^2}{\tilde{f}(r)} + r^2 d\Omega^2$$

with  $\tilde{f}(r) = 1 - \frac{2M}{r} + \xi_2 \left( \frac{\pi/2 - \arctan(r/\lambda)}{r/\lambda} \right) + \frac{2\xi_2}{3} \frac{1}{1+(r/\lambda)^2}$

For spherical symmetry disformal transformations  $\tilde{g}_{\mu\nu} = g_{\mu\nu} + D(X) \partial_\mu \phi \partial_\nu \phi$

take us from homogeneous ( $f = h$ ) to non homogenous black holes ( $f = \frac{h}{Z^2}$ ).

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$p = 2$  homogeneous black hole with  $Z = \gamma$

$$f(r) = 1 - \frac{2M}{r} + \xi_2 \left( \frac{\pi/2 - \arctan(r/\lambda)}{r/\lambda} + \frac{1}{1+(r/\lambda)^2} \right)$$

- for  $p = 2$  the **Horndeski frame** gives

$$d\tilde{s}^2 = -\tilde{f}(r) dt_\star^2 + \left( 1 - \frac{\xi_2}{(1+(r/\lambda)^2)^2} \right) \frac{dr^2}{\tilde{f}(r)} + r^2 d\Omega^2$$

$$\tilde{f}(r) = 1 - \frac{2M}{r} + \xi_2 \left( \frac{\pi/2 - \arctan(r/\lambda)}{r/\lambda} \right)$$

- In order for disformal transformation to be invertible, we have the constraint  $\xi_p < 2p - 1$

For theories :

$$G_2(X) = -\frac{2\eta}{\lambda^2} X^p, \quad G_4(X) = 1 - \eta X^p, \quad F_4(X) = \frac{\eta}{4} (2p - 1) X^{p-2}.$$

We have primary hair black holes with :

- Homogeneous and non homogeneous solutions modulo disformal transformations controlled by  $Z = -G_4 + 2XG_{4X} + 4X^2F_4$
- Two independent charges, mass  $M$  and scalar charge  $\xi$ . One coupling constant  $\lambda$  sets the units or size of the scalar effect around the gravitational solution.
- Noether charge  $Q(\eta, q, p) < \infty$  finite for  $p > 3/2$ . We then have good asymptotics, horizons for all solutions in the same disformal class
- Regular black hole or soliton at the maximal charge  $\xi_p^{reg} \sim M_p^{reg}$  parametrised by ADM mass
- Regular scalar for all  $q$  at all future horizons  $X = \frac{q^2/2}{1+(r/\lambda)^2}$   $\psi'(r)^2 = \frac{q^2}{f^2(r)} \left[ 1 - \frac{f(r)}{1+(r/\lambda)^2} \right]$
- One can sum sources and mix solutions with different values of  $p$

GW astronomy provides a window to test gravity

-Ringdown phase of a BH merger is described by linear BH perturbation theory

- We go to the frequency domain  $f(t, r) = f(r)e^{-i\omega t}$  and expand in spherical harmonics.
- We have Axial and Polar modes which to linear order are independent for given boundary conditions
- In GR we get two master equations in Schrodinger form. The effective metric where gravitons propagate and the background metric are identical.
- In scalar tensor theories we have an additional polar mode. We have one axial mode but, the effective and background metrics are now different!

For GR Schwarzschild,

- Given spherical symmetry we can expand metric perturbations in spherical harmonics  $(\ell, m)$  and (odd-even) parity
- Given staticity we can separate modes in each given frequency  $\omega$  in the Regge-Wheeler gauge

$$\begin{aligned}
 h_{t\theta} &= \frac{1}{\sin\theta} \sum_{\ell,m} h_0^{\ell m}(t, r) \partial_\varphi Y_{\ell m}(\theta, \varphi), & h_{t\varphi} &= -\sin\theta \sum_{\ell,m} h_0^{\ell m}(t, r) \partial_\theta Y_{\ell m}(\theta, \varphi), \\
 h_{r\theta} &= \frac{1}{\sin\theta} \sum_{\ell,m} h_1^{\ell m}(t, r) \partial_\varphi Y_{\ell m}(\theta, \varphi), & h_{r\varphi} &= -\sin\theta \sum_{\ell,m} h_1^{\ell m}(t, r) \partial_\theta Y_{\ell m}(\theta, \varphi), \quad (1)
 \end{aligned}$$

Essentially we seek  $h_0 = h_0(r)$ ,  $h_1 = h_1(r)$  labelled each by eigenvalues  $\ell, m, \omega$ .

- The analysis boils down to a single second order master equation for one radially dependent function  $\mathcal{Y}(r)$  for each label,  $l, m$

$$-\frac{d^2 \mathcal{Y}}{dr_*^2} + V_\ell \mathcal{Y} = \omega^2 \mathcal{Y}$$

- with outgoing and ingoing boundary conditions for  $dr_* = dr/f_{GR}$  the tortoise coordinate.
- We have  $f_{GR} = 1 - \frac{2M}{r}$  with a positive definite potential

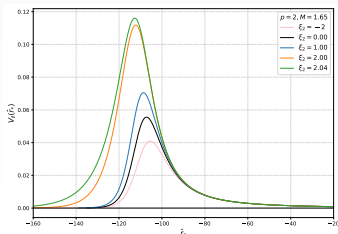
$$V_\ell(r) = \frac{\ell^2 + \ell - 2}{r^2} f_{GR} - \frac{1}{r} f_{GR} f'_{GR} + \frac{2}{r^2} f_{GR}^2,$$

- Using Regge Wheeler gauge we get a **modified Schrodinger potential for all  $p$  and  $\eta$  parametrising the theories** :

$$V_\ell(r) = \frac{\ell^2 + \ell - 2}{r^2} \Phi - \frac{\kappa_1(p, \eta, q)}{r} \Phi \Phi' + \frac{2\kappa_2(p, \eta, q)}{r^2} \Phi^2$$

with  $\kappa_{1,2}$  depending on the theory at hand for example  $\kappa_1 = \frac{1 + \eta(\frac{p}{2} - 1)X^p - \frac{\eta p}{2}X^{p+1}}{(1 - \eta X^p)^2}$

- and effective metric  $\Phi(r) = f(r) - \frac{\eta q^2}{2} X^{p-1}$
- Effective metric where axial gravitons propagate is not the background metric [Langlois, Noui and Roussille]. It is the  $c_g = c$  frame with a conformal factor  $C = 1!$
- Axial gravitons see a different metric than light or matter close to the black hole horizon. Axial gravitons propagate in the  $c_g = c$  frame.
- Axial perturbations are stable in their domain of definition due to the well defined disformal geometries as long as  $\xi_p < 2p - 1$  and  $p > 3/2$ .



# Phase diagram for the background and effective metrics

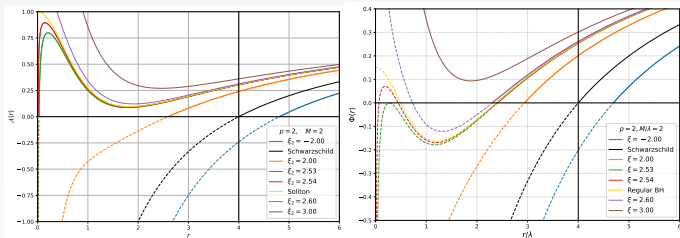
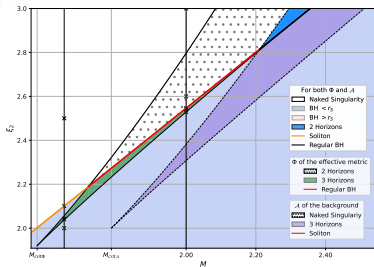
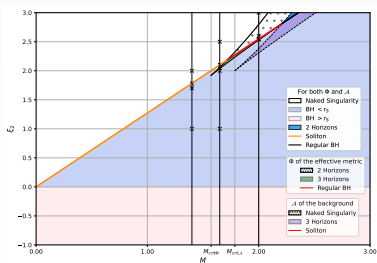


Figure: Plot of the background and effective metrics for  $p = 2$  and mass  $M = 2$  and different values of  $\xi_2$



- We have constructed static black holes with primary hair.
- Key is the presence of a time dependent scalar, that provides an independent scalar charge and renders the solutions regular.
- The solution becomes regular at the origin for maximal scalar charge without any fine tuning of theory parameters. We then have a regular black hole or soliton parametrised by the ADM mass.
- Homogeneous and non homogeneous black holes belong to the same equivalence class related via the theory function  $Z = Z(X)$ . Axial perturbations of all members of this class have identical axial perturbations.
- Axial gravitons propagate at a different background metric than light. This is a clear effect of extra gravitating degrees of freedom.
- Only in the  $c = c_g$  frame the effective and background metrics are identical.
- Axial stability of the solutions is generic modulo within well defined effective metric and essentially the presence of a finite scalar charge.
- Polar perturbations are the crucial step in understanding the stability. Is there a notion of effective metric in this case? What are the monopole perturbations?
- Can we find rotating counterparts to these solutions?