

Searching for emergent spacetime in spin glasses

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based on *arXiv:2510.20902* with Lev Shaposhnik

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National and Kapodistrian University of Athens



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If we can find a relation between their observables, states and dynamics, we can refer to these two different systems as **dual**.

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Example: The Ising model

$$H_1 = -J \sum_j \sigma_j^z \sigma_{j+1}^z - h \sum_j \sigma_j^x \quad \xrightarrow{\text{Jordan-Wigner transformation}} \quad H_2 = iJ \sum_j \gamma_{2j} \gamma_{2j+1} + ih \sum_j \gamma_{2j-1} \gamma_{2j}$$

There is an isometric map between states from $\mathcal{H}_1 \rightarrow \mathcal{H}_2$

There is also a linear map between the Hamiltonians $\Phi(H_1) = \Phi(H_2)$

The two systems describe the same "physics"

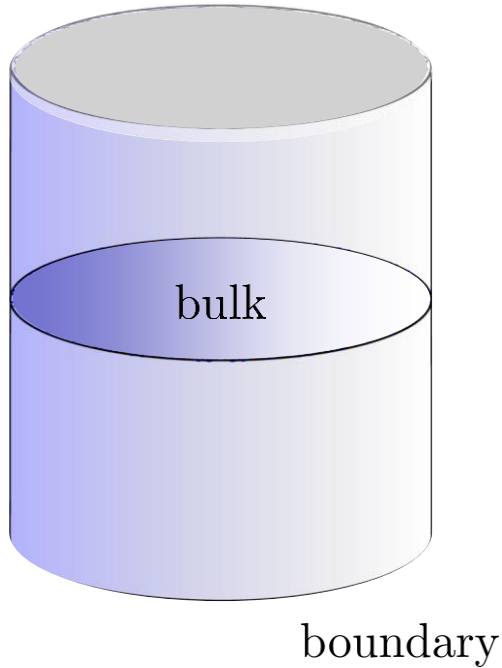
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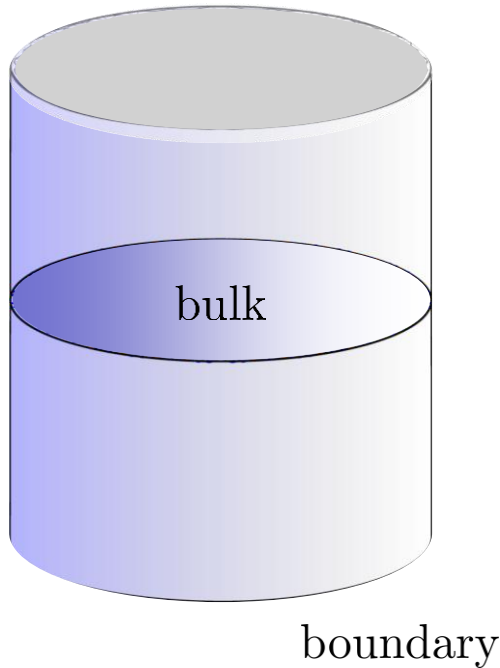


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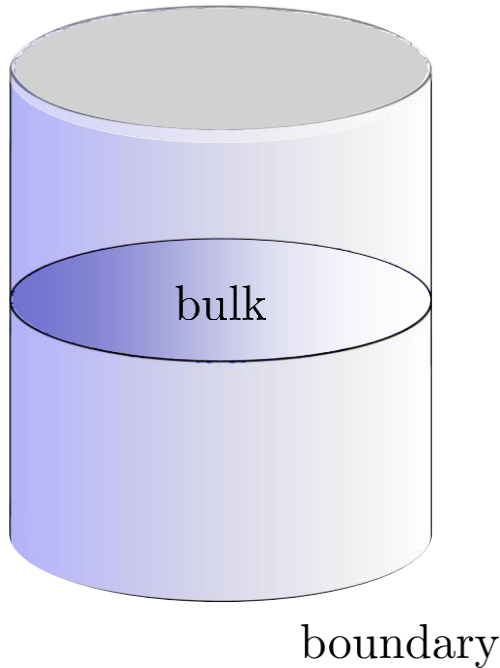
Then we can find an operator \hat{O} which is the dual of ϕ

$$\phi(x) = \int_{\text{bdy}} d^2Y K(x|Y) O(Y) + \mathcal{O}(1/N)$$

where $K(x|Y)$ is a smearing function

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In such a case, we will say **”the bulk theory emerges from the boundary theory”**

Idea #1

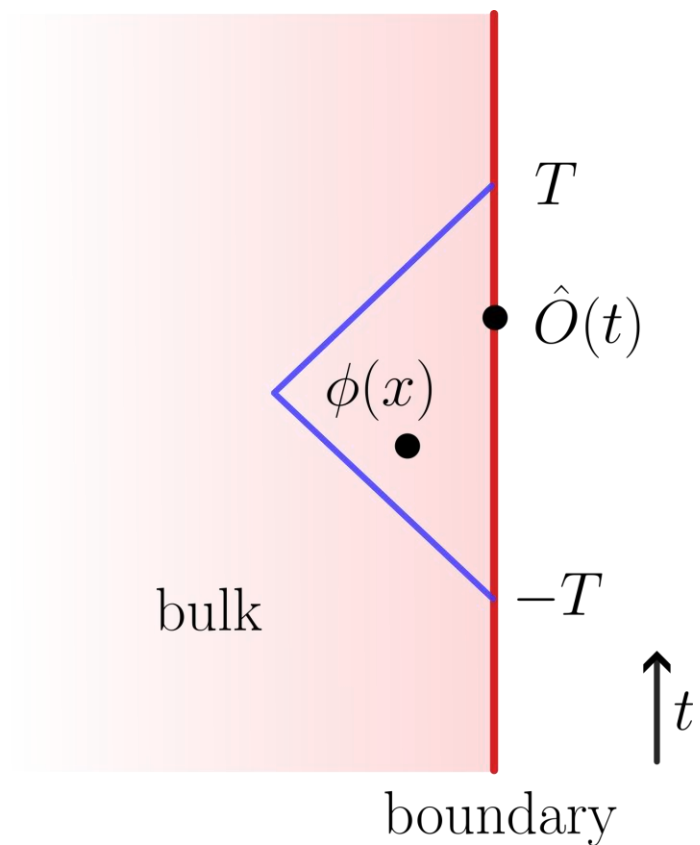
spectral functions and bulk emergence

(based on: Gesteau, Liu, arXiv:2408.12642)

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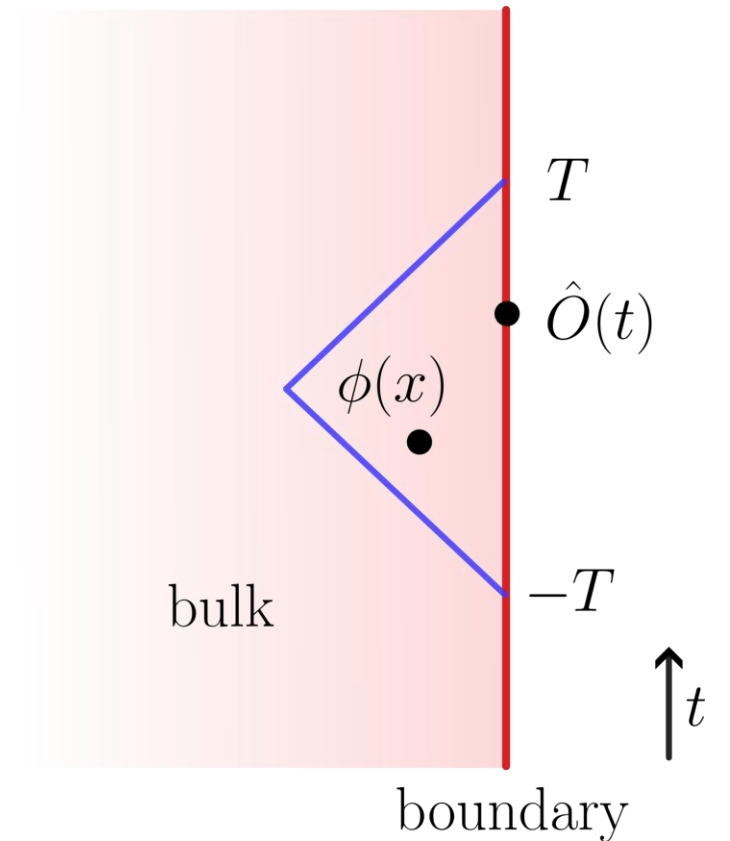
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Leutheusser, Liu, Phys. Rev. D **108**, 086020 (2023)

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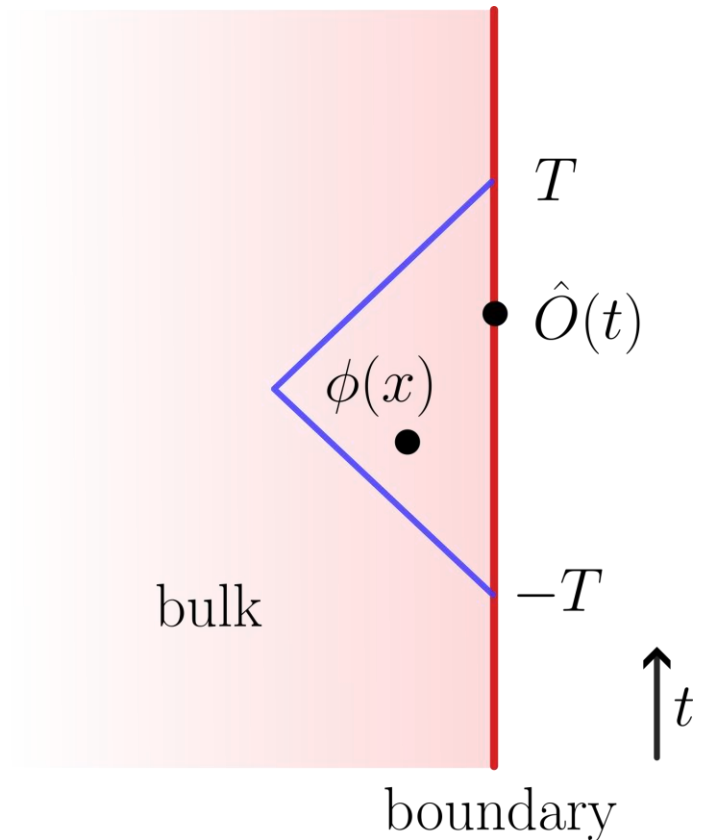
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$\hat{O}(f)$ define an algebra \mathcal{A}_T , and ϕ operators define an algebra \mathcal{Y}_T

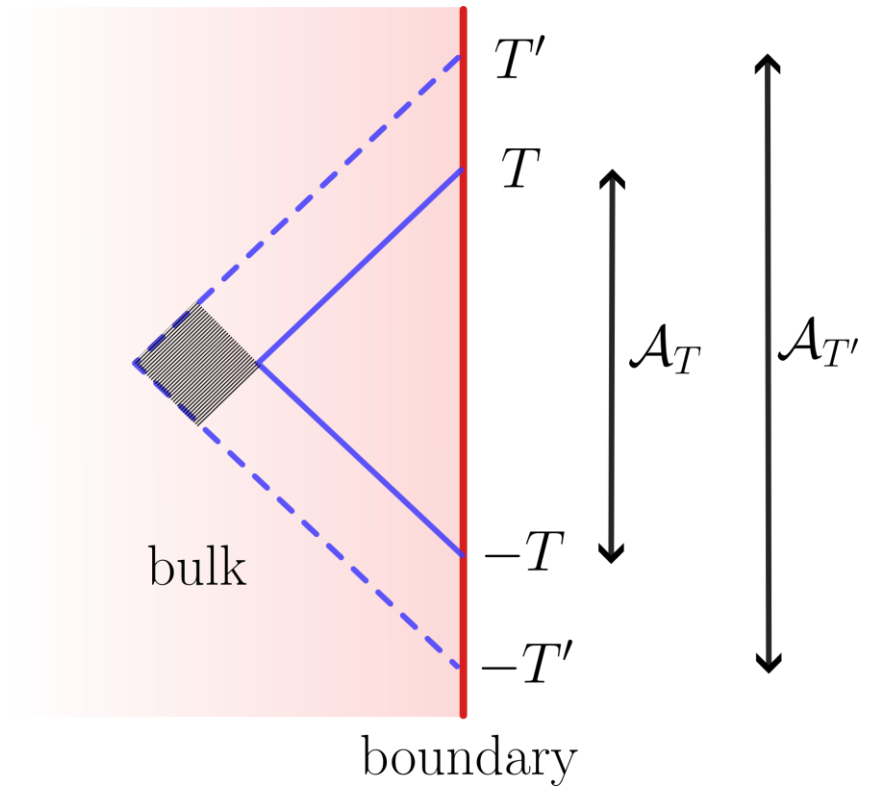
The two algebras carry the same information



Now, let us assume **two** boundary regions.

If there is an emergent bulk as in the picture, the operators in the grey region:

- are in \mathcal{A}'_T , but are **not** generated by the \mathcal{A}_T algebra
- and they commute with every operator in \mathcal{A}_T



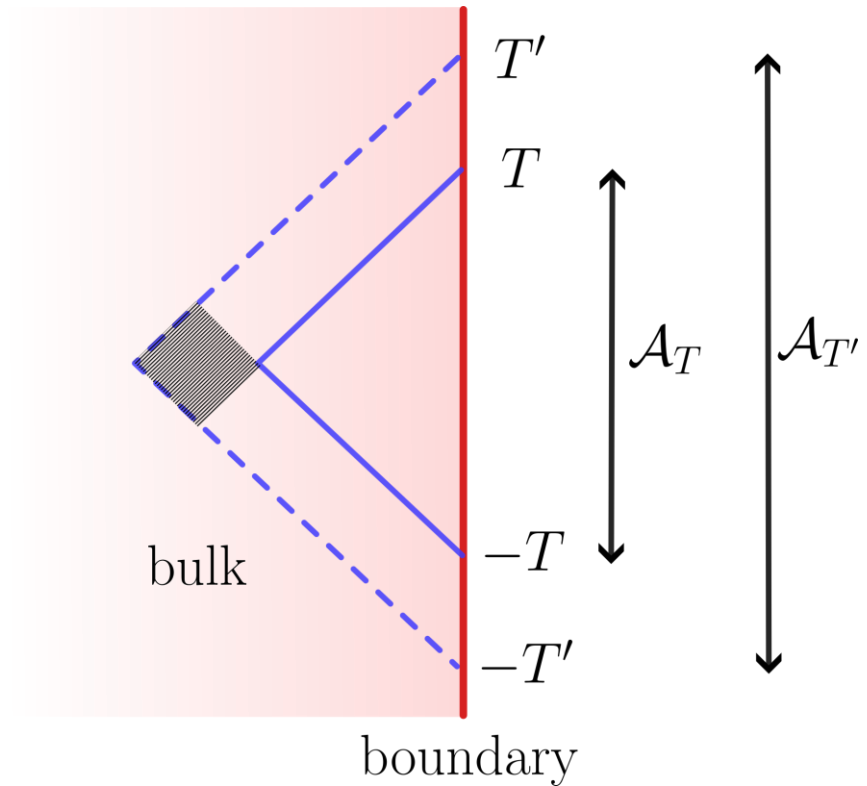
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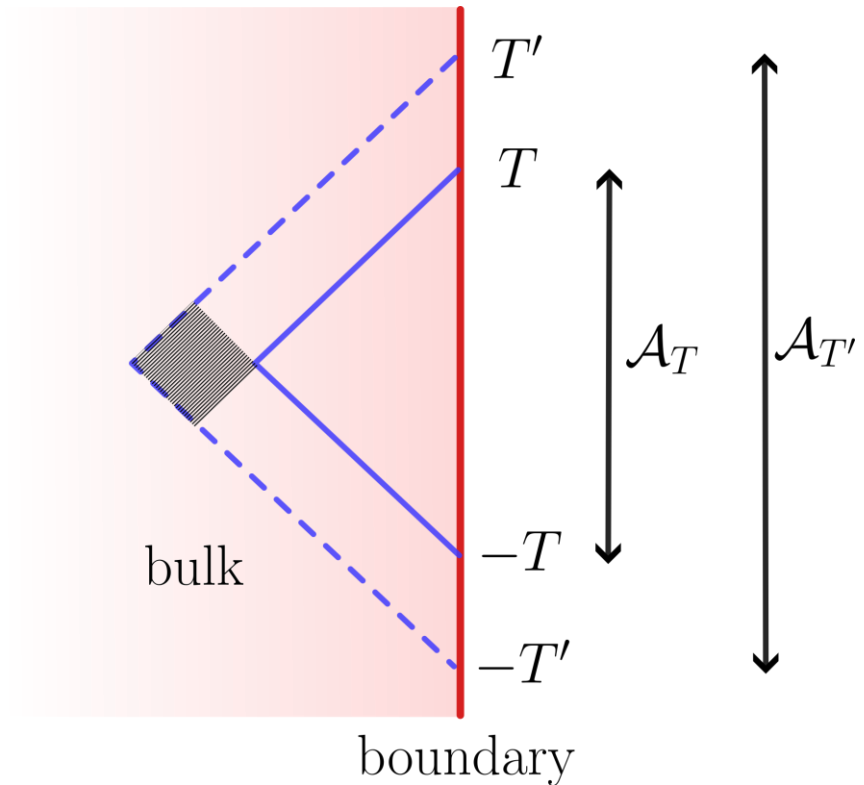
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The question whether a bulk emerges becomes:
Does there exist a function g such that:

$$\int dt dt' f(t) g(t') \underbrace{\langle [\hat{O}(t), \hat{O}(t')] \rangle}_{\rho(t, t')} = 0$$



The Fourier transform of $\rho(t, t')$ is the **spectral function**, $\rho(\omega)$.

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take-home message of idea #1

we can use the behavior of the spectral function as a diagnostic to check whether there could be an emergent bulk

Idea #2

dualities and quantum many-body models

The SYK model in some limits is expected to have a bulk dual. For example, the double-scaled SYK has a bulk description, called “chord Hilbert space”.

$$H = (i)^{q/2} \sum_{1 \leq i_1 < \dots < i_q \leq N} J_{i_1 \dots i_q} \gamma_{i_1} \dots \gamma_{i_q}$$

Lin, Stanford SciPost Phys. 15, 234 (2023)

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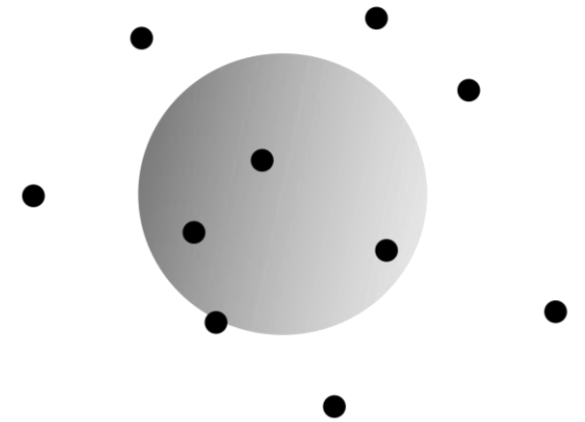
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These are stationary multi-horizon bound states, forming configurations stable under gravitational and electromagnetic forces.

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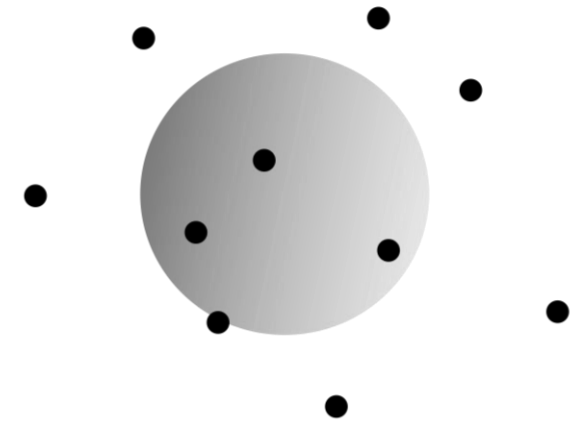
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- Exponentially long relaxation times
- Memory effects, ...



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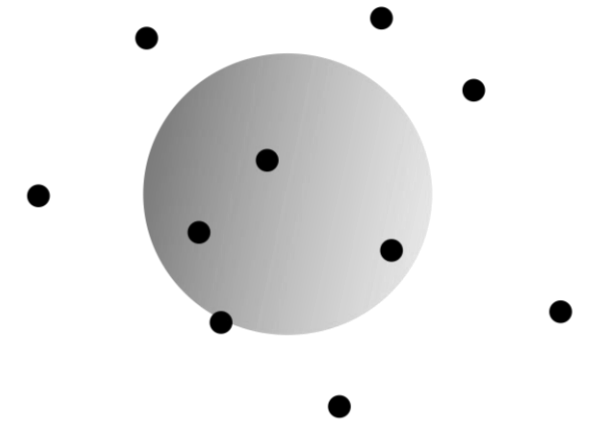
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Proposition for a dual candidate → **spin glasses**

Why spin glasses?

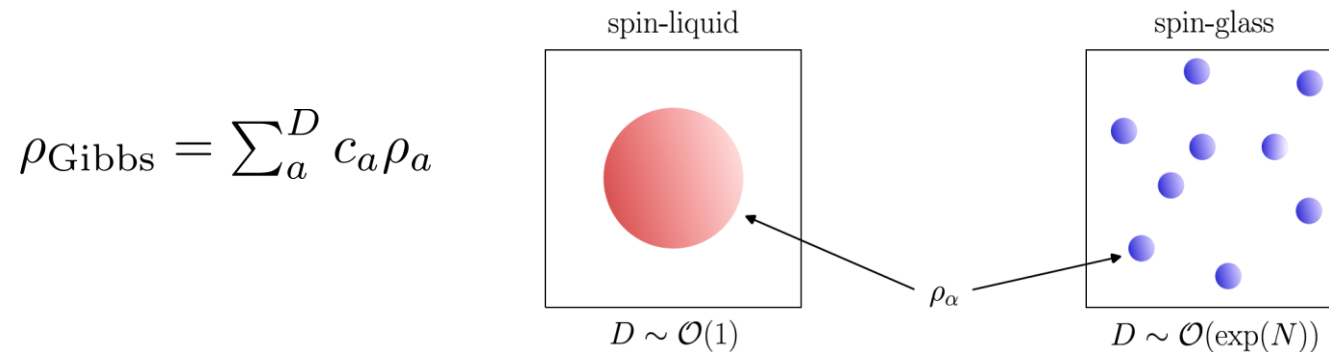
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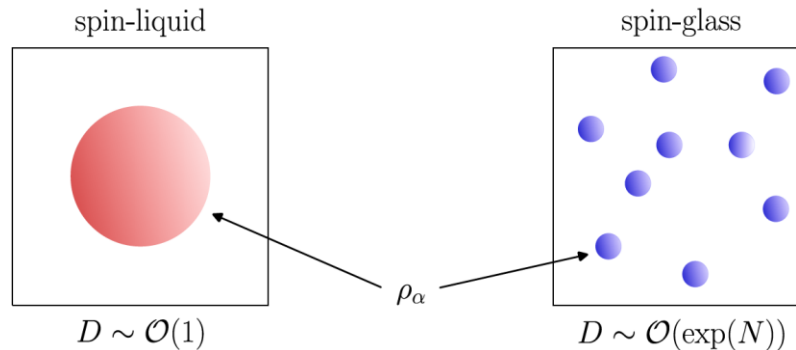
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$$\rho_{\text{Gibbs}} = \sum_a^D c_a \rho_a$$

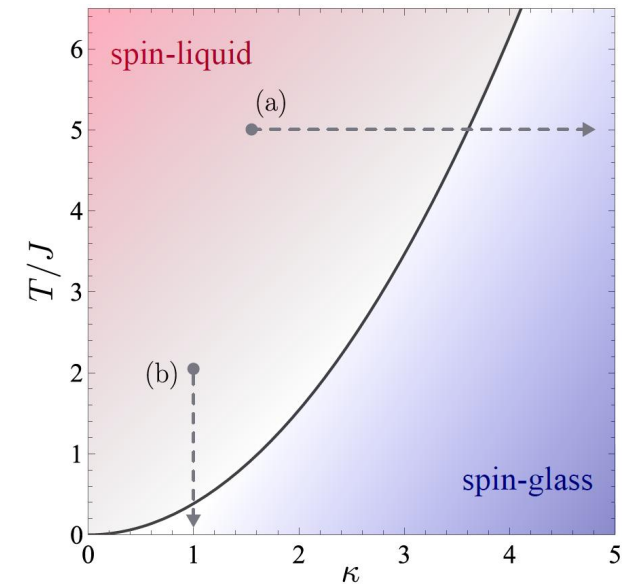


Spin glasses break the **replica symmetry**

SU(M) Heisenberg model

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$$\sum_{\alpha} b_{\alpha}^{\dagger}(i) b^{\alpha}(i) = \kappa M$$



take-home message of idea #1

we can use the behavior of the spectral function as a diagnostic to check whether there could be an emergent bulk

take-home message of idea #2

spin glass models might be dual models to gravitational frameworks of high complexity

What did we do?

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Let us combine idea #1 and idea #2

We study the behaviour of the **spectral functions** of concrete many-body physics models

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1) Apply the diagnostic tool on a model, that is expected to have a **dual**.

Simulate the large N dynamics of the SYK model.

In the large q limit ($v \rightarrow 1$):

$$\rho(\omega) \sim C \omega^{\frac{2}{q-1}-1} e^{-\frac{\beta\omega}{2} \left(\frac{1}{v}-1\right)}$$

The spectral function has **full support** with **exponential** decay (**not polynomial).

As we expect the large q limit of SYK to be holographic, we will consider this behavior as our benchmark.

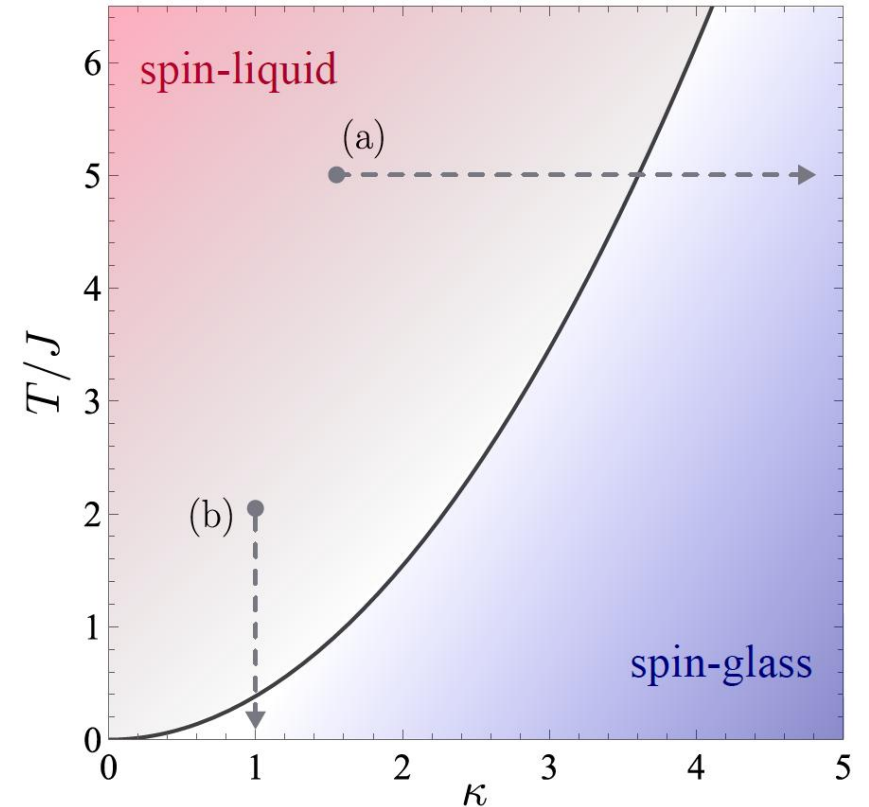
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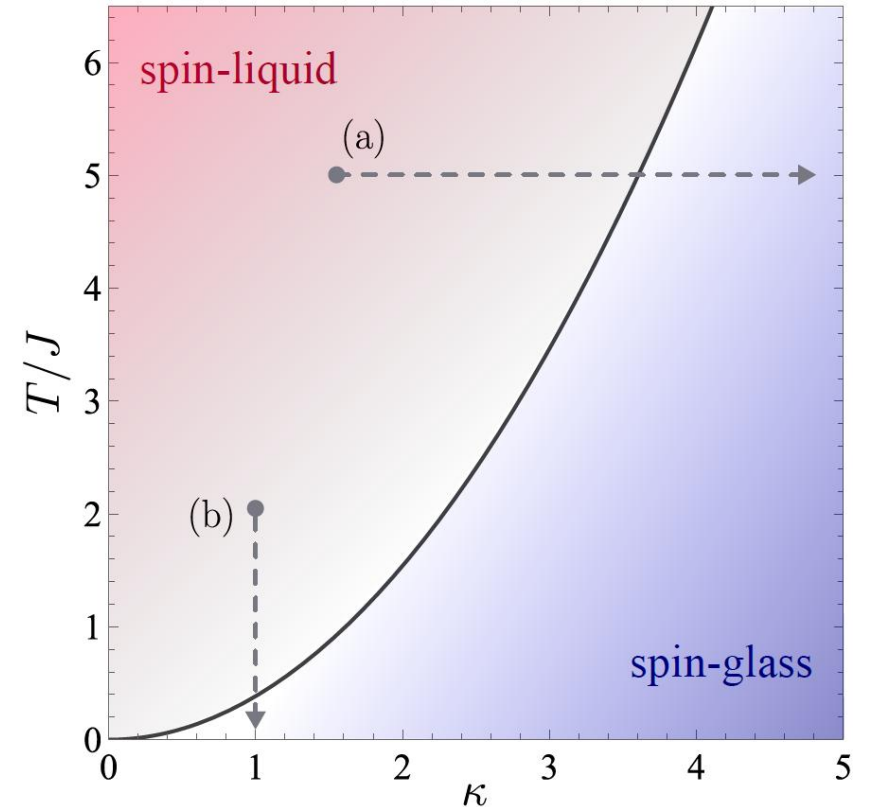
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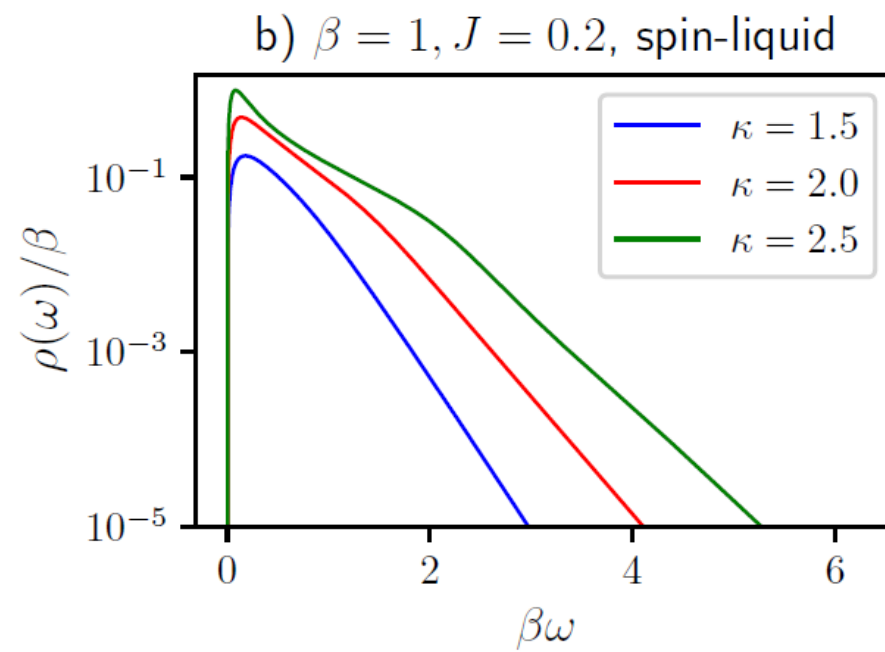
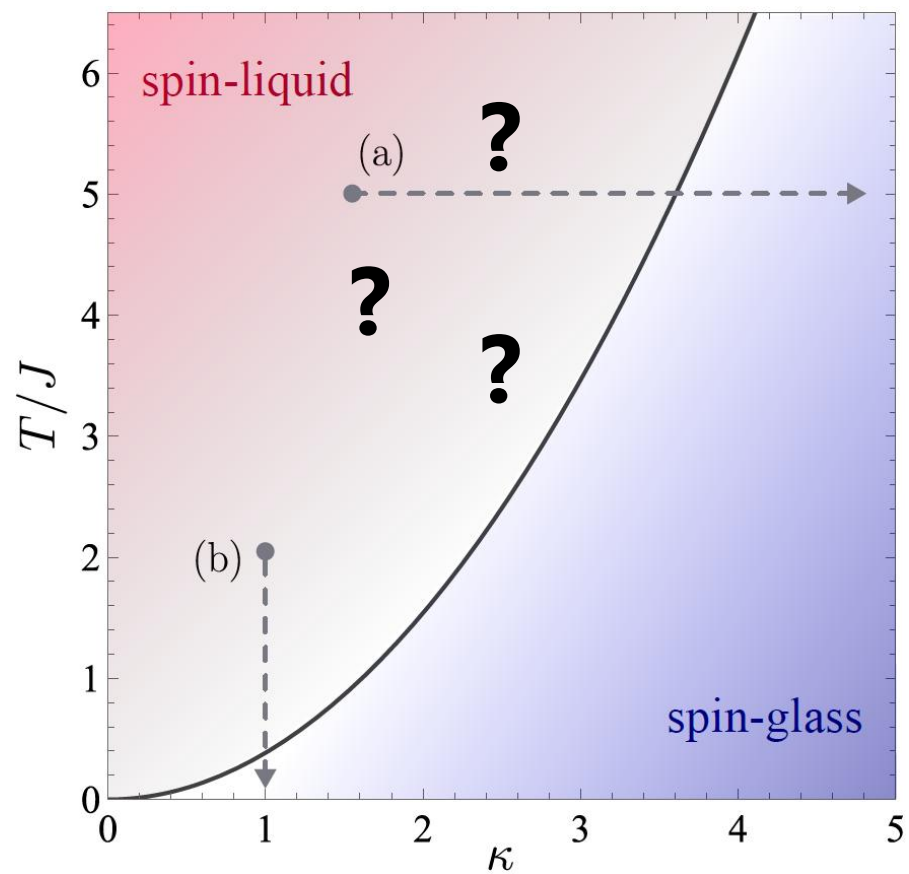
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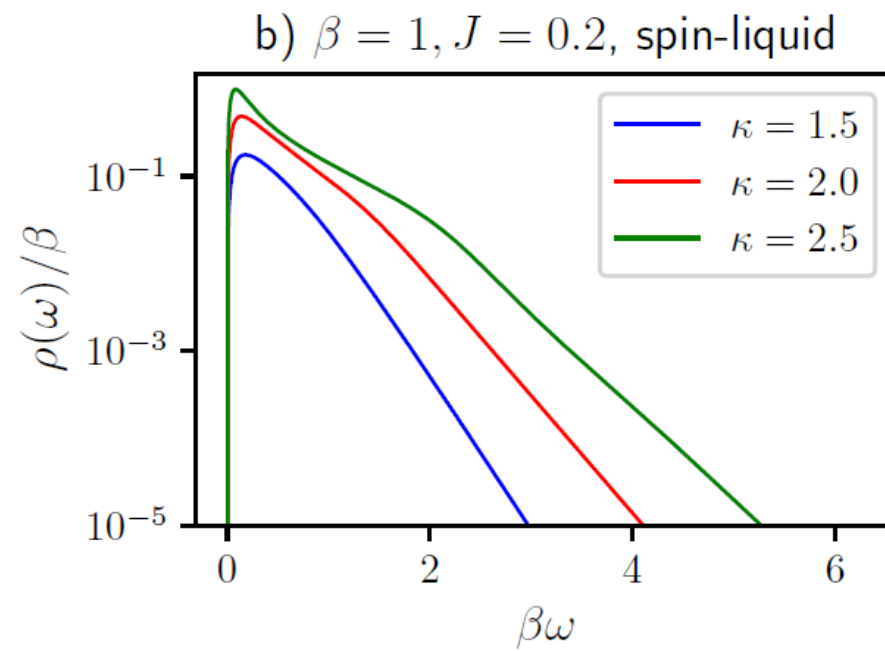
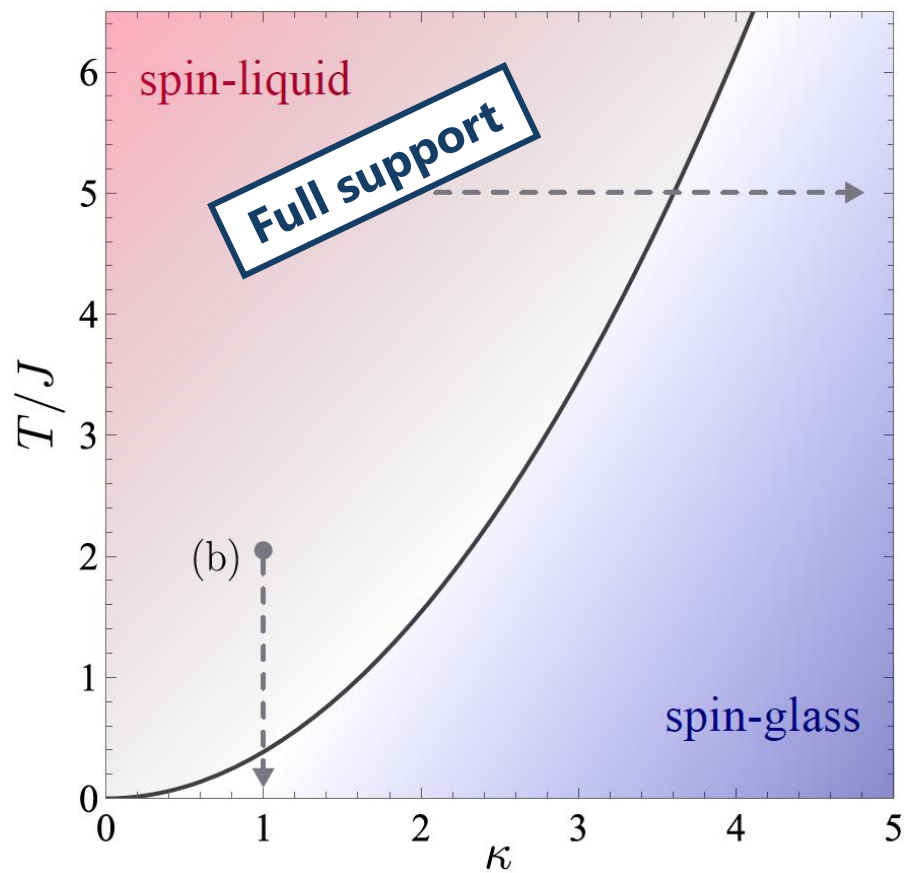
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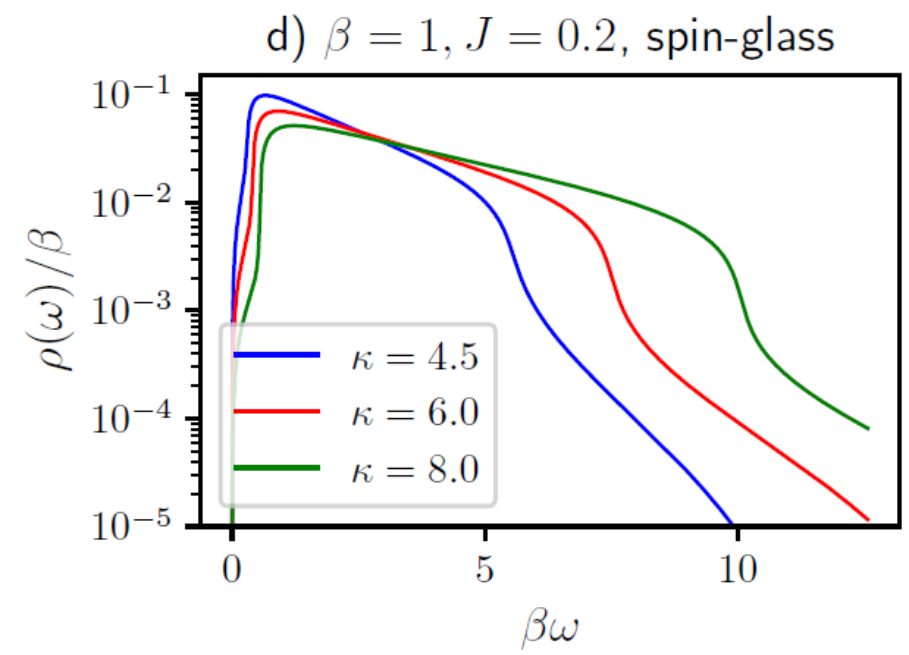
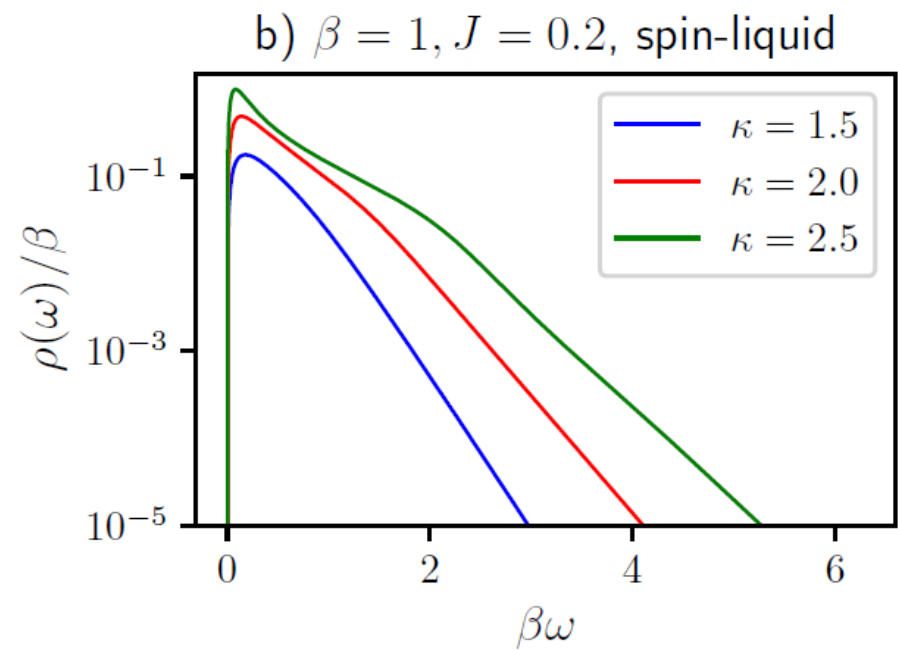
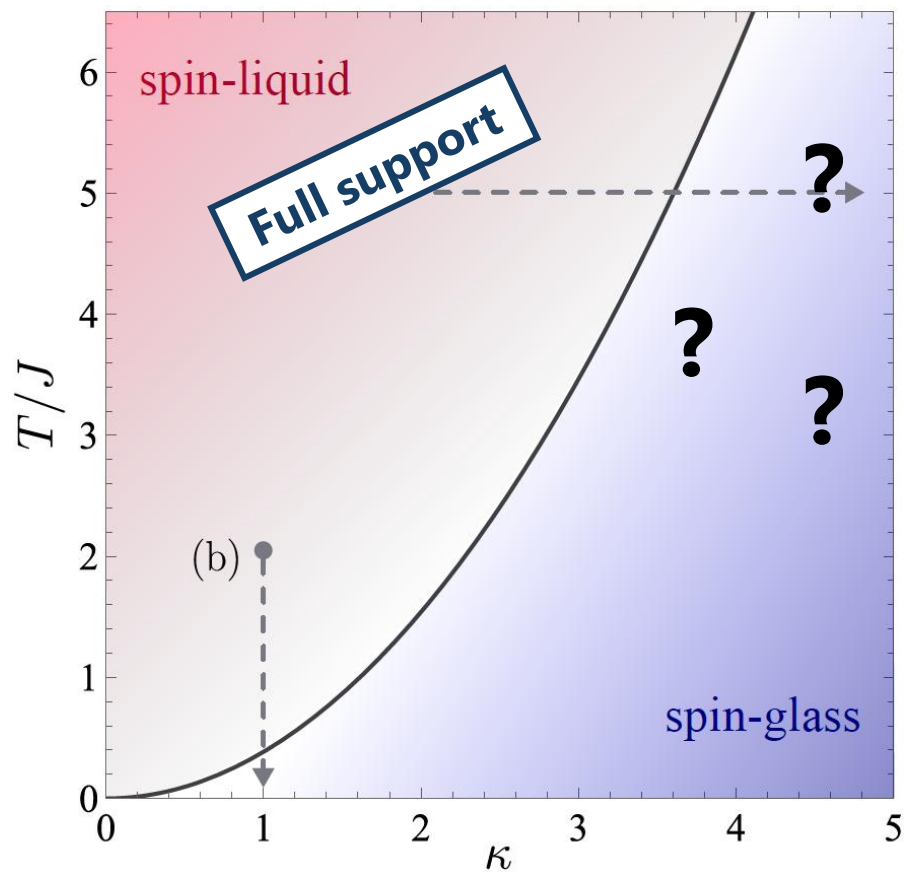
We compute the **Schwinger-Dyson equation** and the **2 equations of motion** for the spin glass parameters.

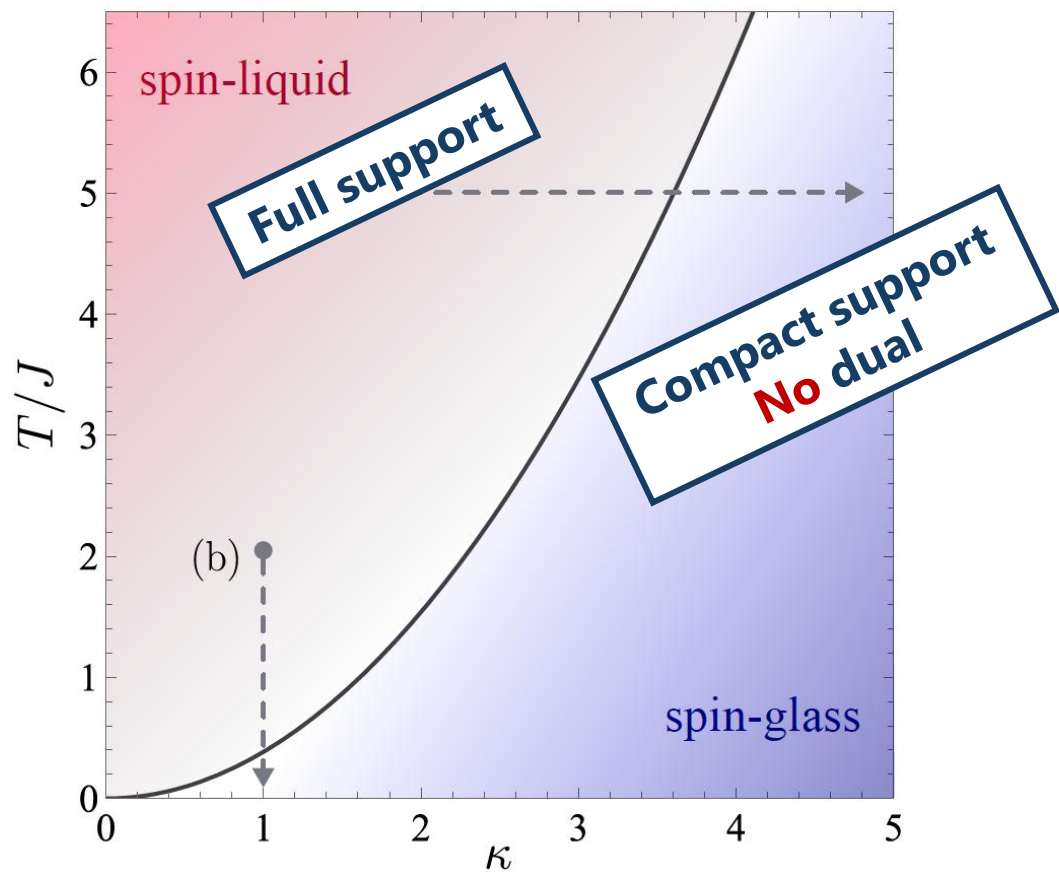
For each point on the phase diagram, we solve the SD and the 2 EOMs.



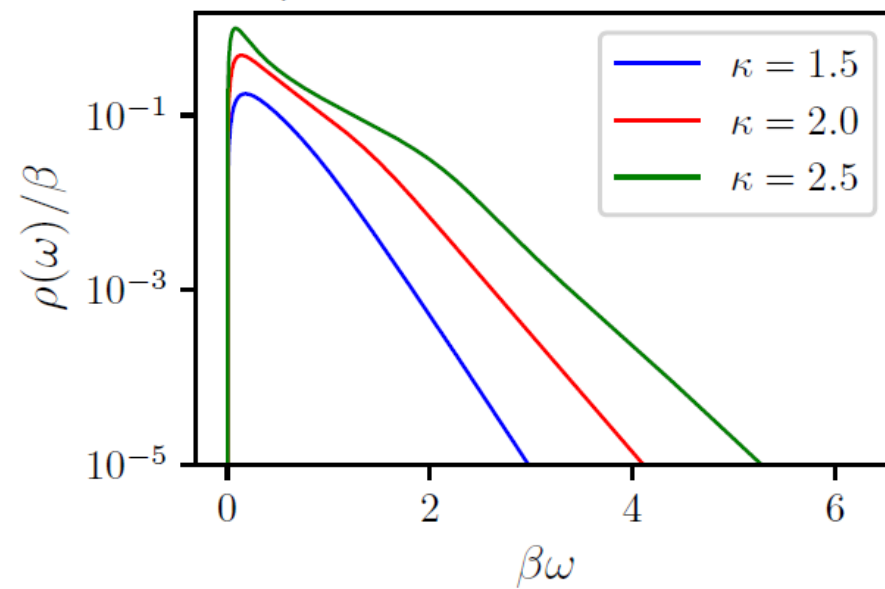




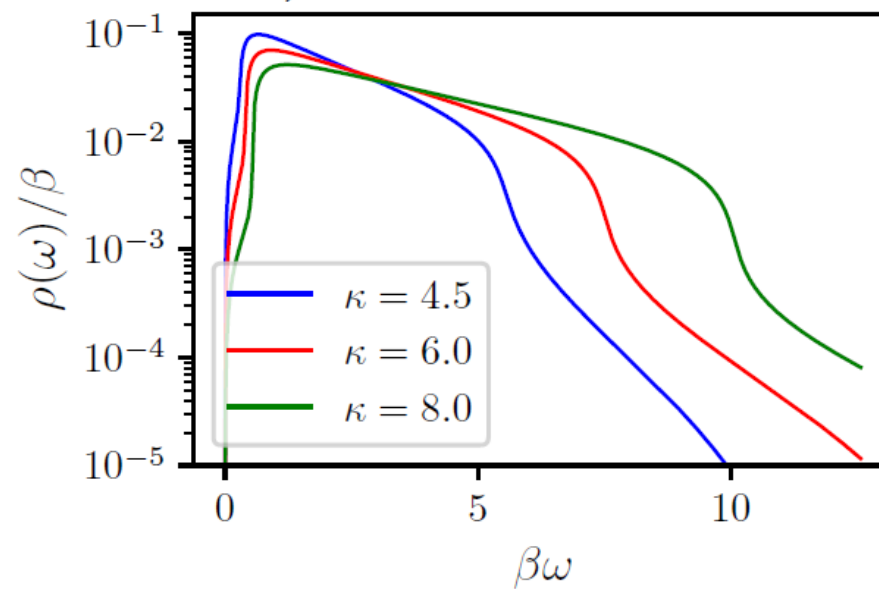


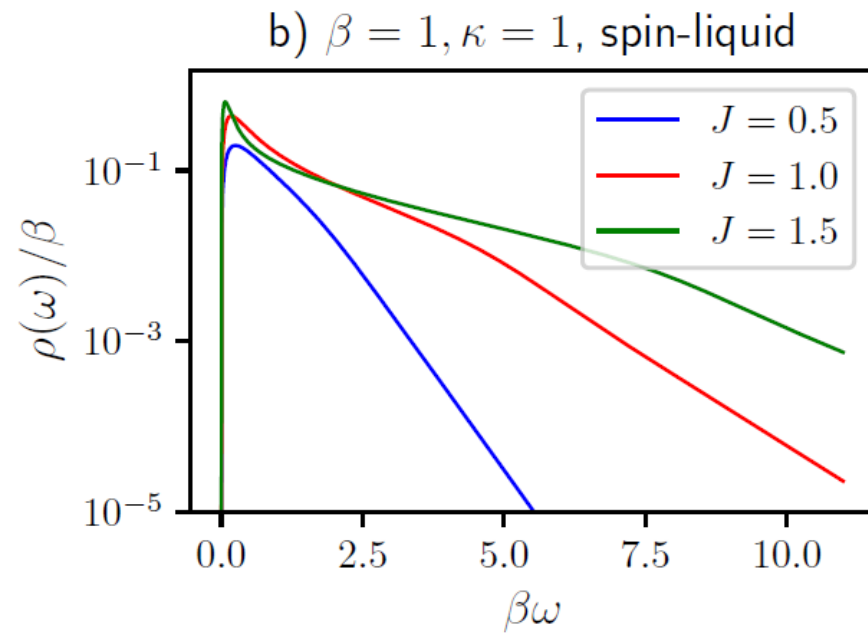
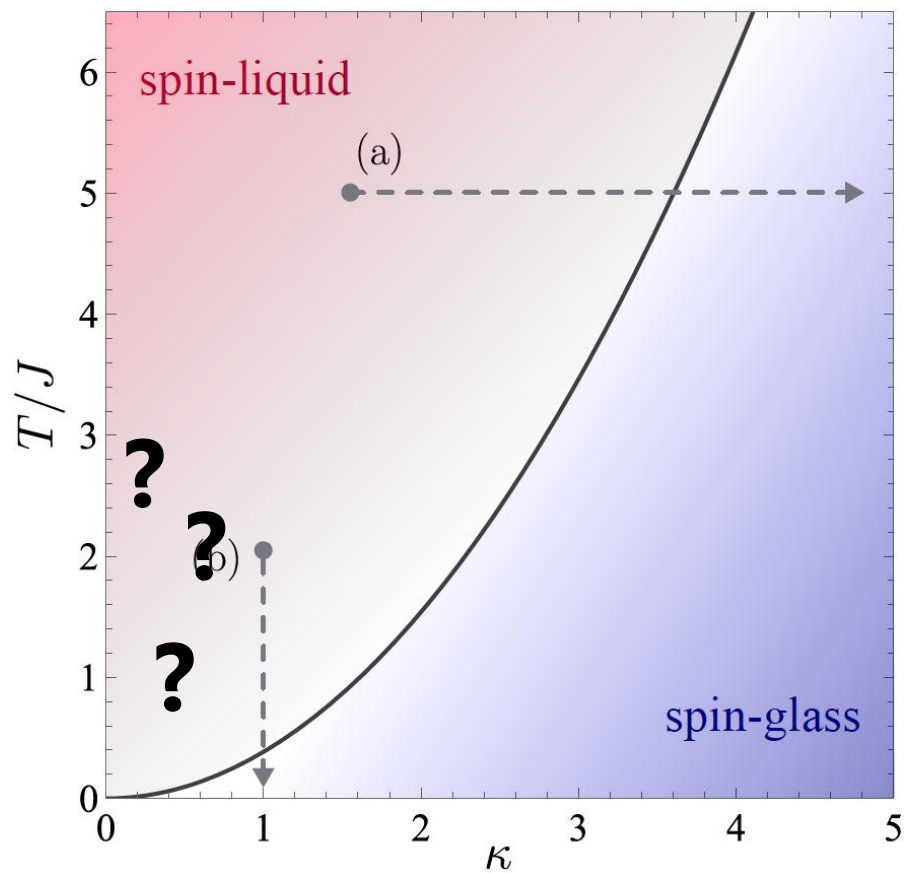


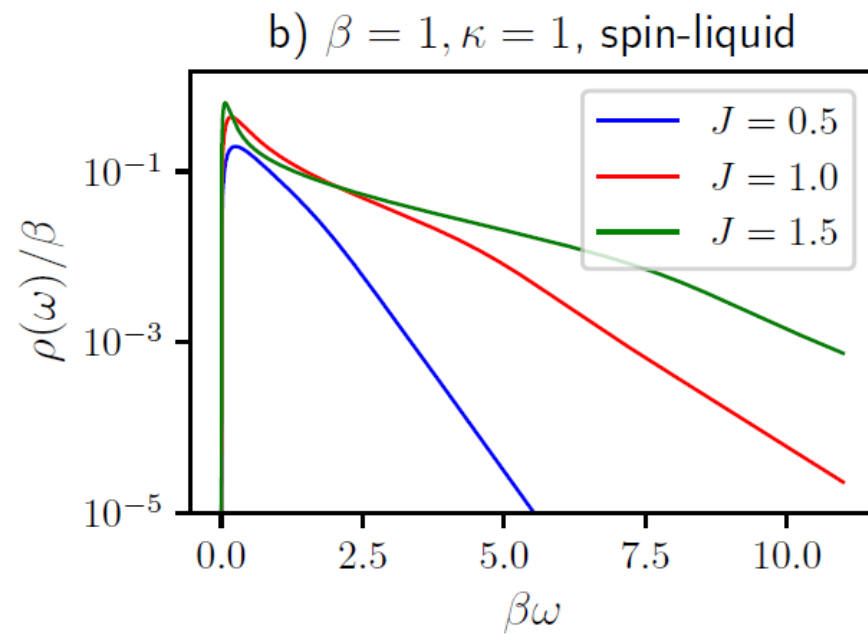
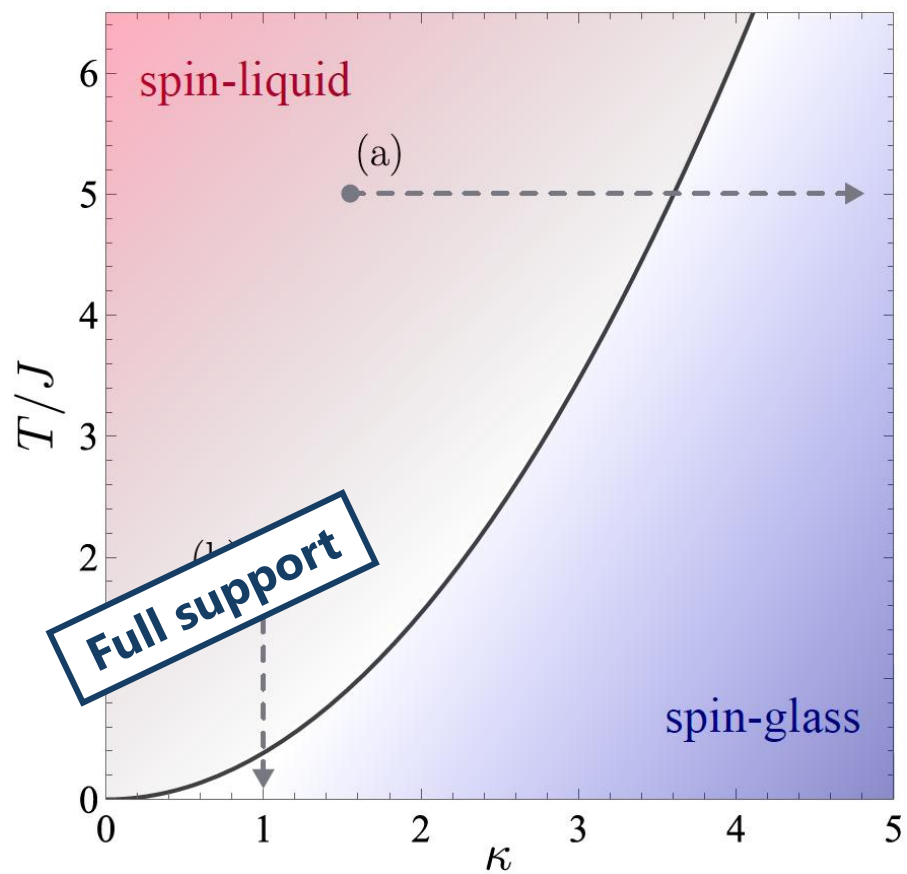
b) $\beta = 1, J = 0.2$, spin-liquid

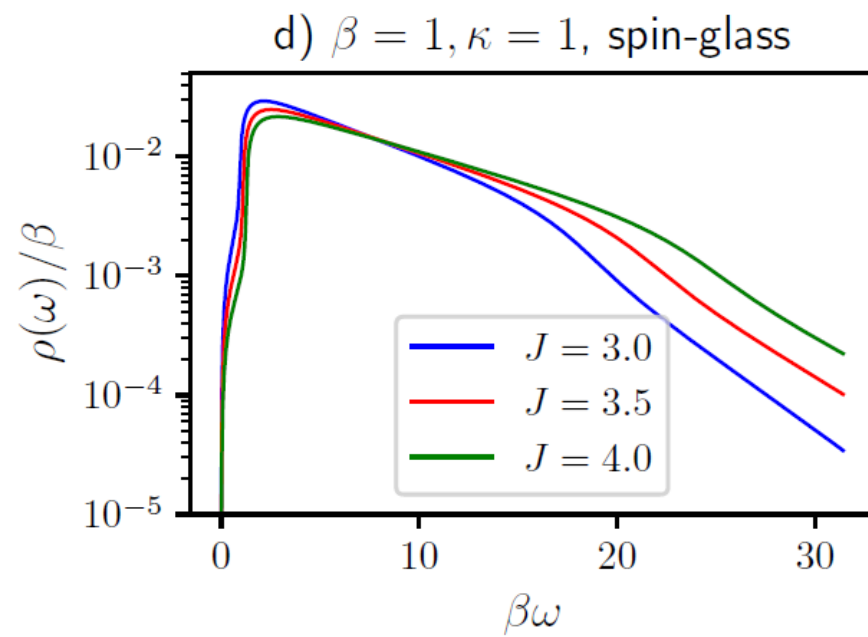
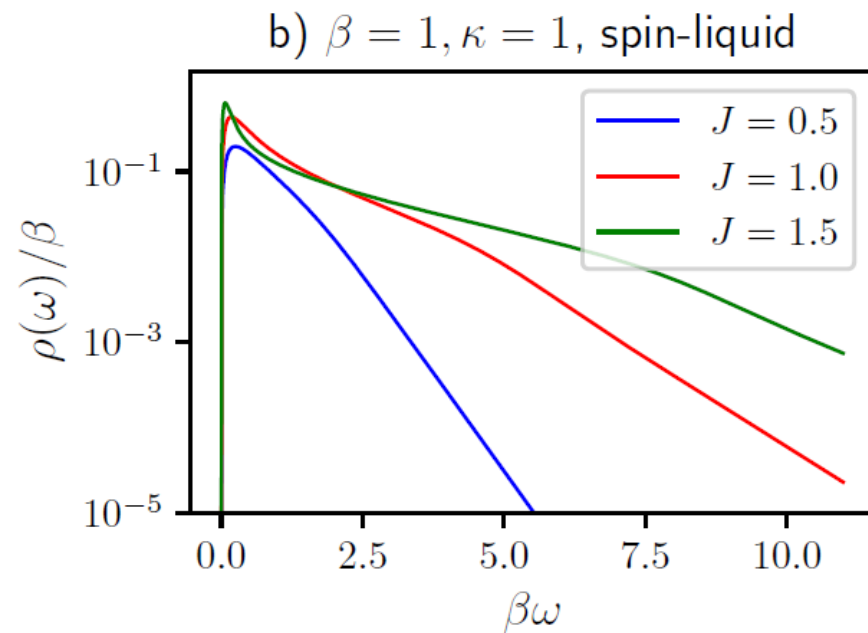
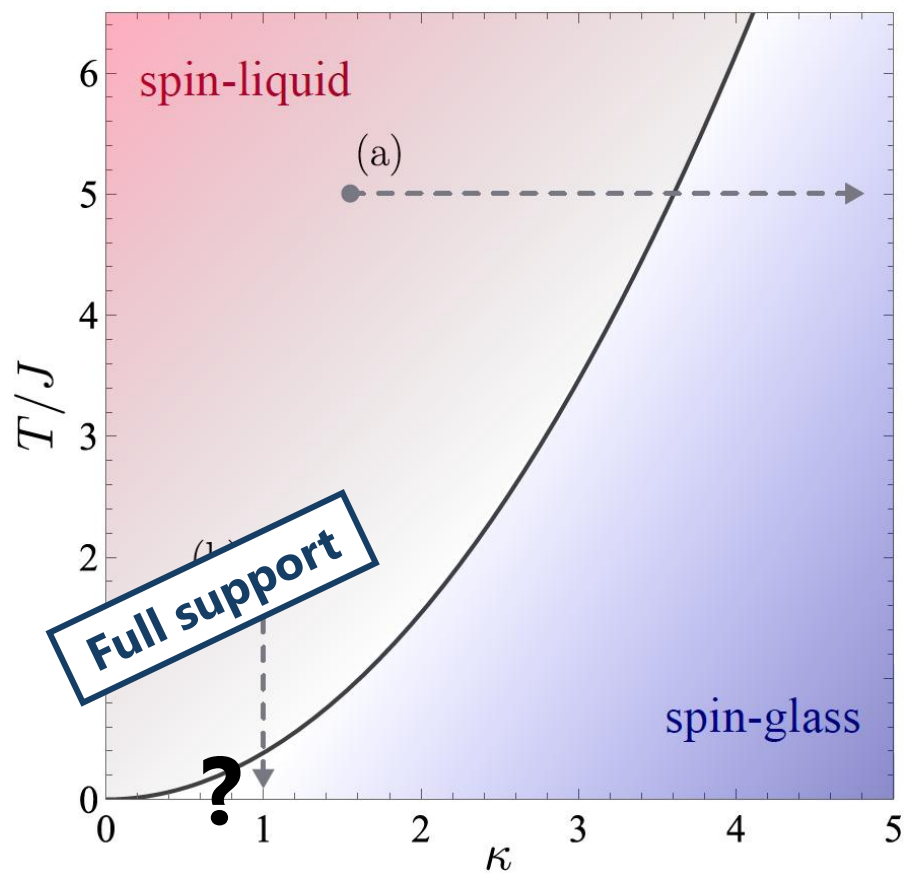


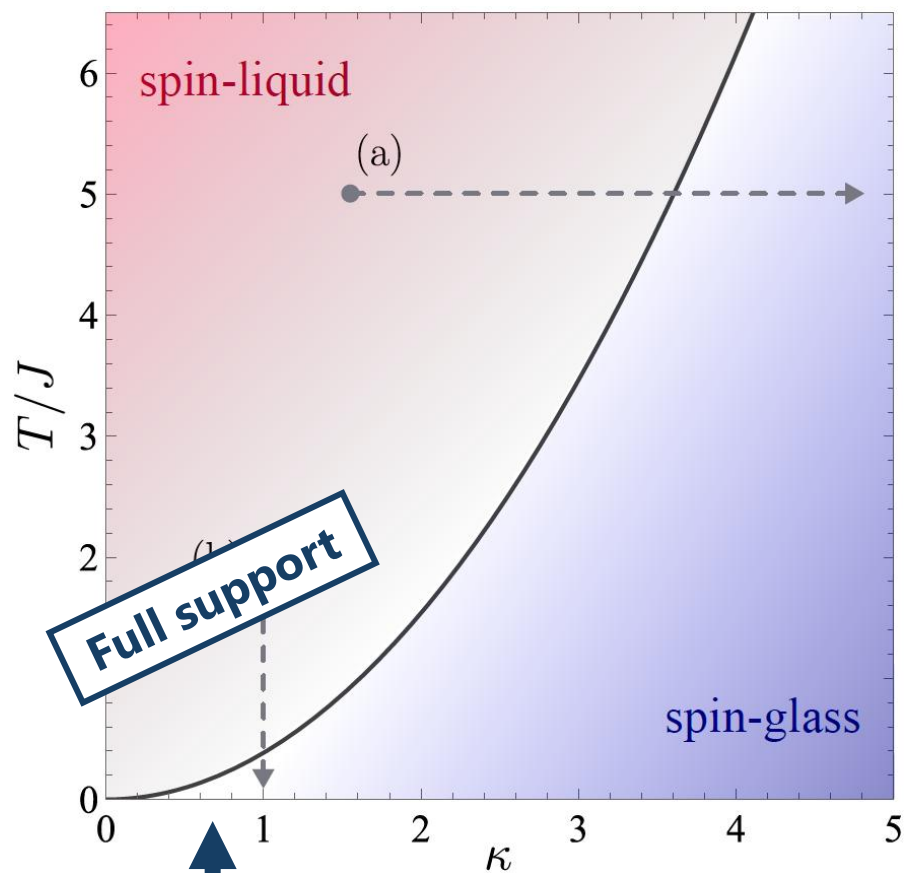
d) $\beta = 1, J = 0.2$, spin-glass



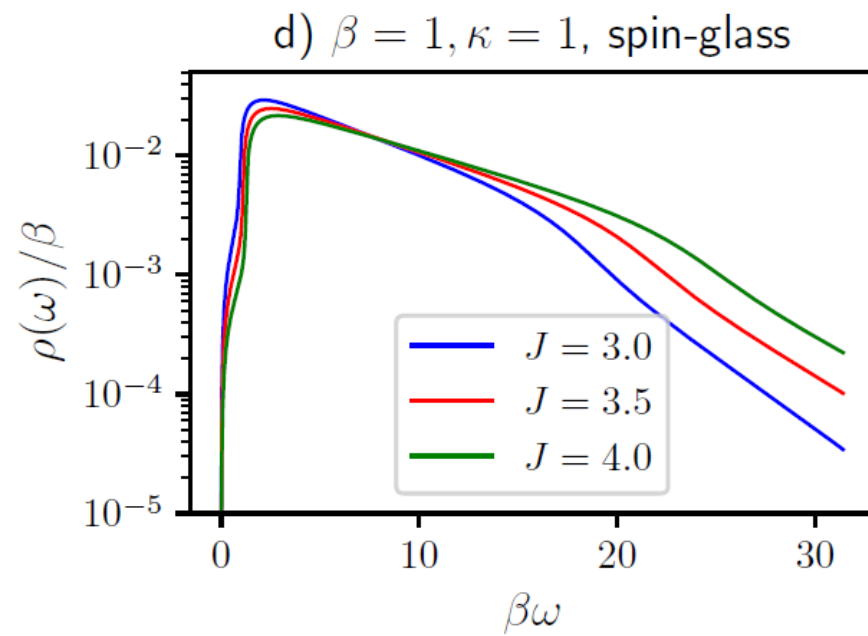
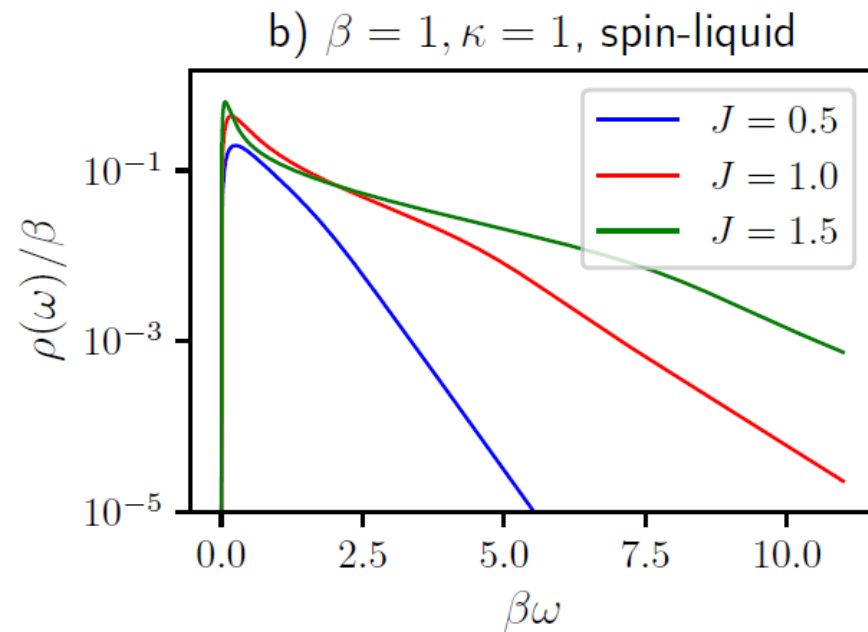








regime with a possible dual!!



Take home messages and future questions

- The large N spectral function $\rho(\omega)$ contains some information about bulk emergence.
- Using it as a diagnostic, could give a first indication whether a quantum many-body system could be holographic in a parameter regime.
- We found a parameter regime of the $SU(M)$ Heisenberg model, whose spectral function shows full support, even in the spin glass phase.

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Thank you for your attention!!

