Mirrors at the edge of spacetime

Null infinity as an inverted extremal horizon

Based on: [2506.15526],[2512.xxxxx]*

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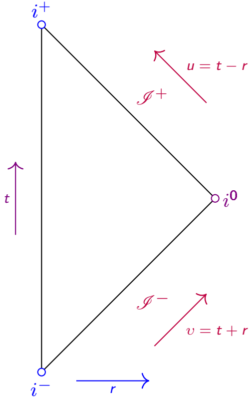
Penrose diagrams and conformal infinities

- Penrose diagrams compactify the entire spacetime while *preserving the causal structure*.
- Example: 4-dim. Minkowski spacetime ($d\Omega_2^2 := d\theta^2 + \sin^2\theta\, d\phi^2$)

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_2^2 = \Omega^2 \left(-dT^2 + dR^2 + \sin^2 R\, d\Omega_2^2\right) .$$



Conformal infinities

- Spatial infinity (i^0): $r \to \infty$ with constant (t, θ, ϕ);
- Future/past timelike infinity ($i^\pm$): $t \to \pm\infty$ with constant (r, θ, ϕ);
- Future/past null infinity ($\mathscr{I}^\pm$): $r \to \infty$ with constant $(t \mp r, \theta, \phi)$;

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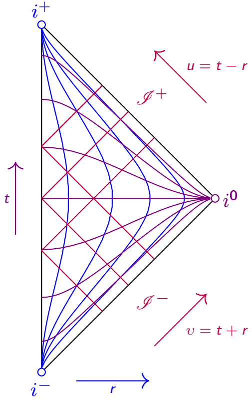


Conformal infinities

- Spatial infinity (i^0): $r \to \infty$ with constant (t, θ, ϕ); "*where spacelike slices end*".
- Future/past timelike infinity ($i^\pm$): $t \to \pm\infty$ with constant (r, θ, ϕ); "*where massive bodies end up/come from*".
- Future/past null infinity ($\mathscr{I}^\pm$): $r \to \infty$ with constant $(t \mp r, \theta, \phi)$; "*where outgoing/ingoing radiation ends up/comes from*".

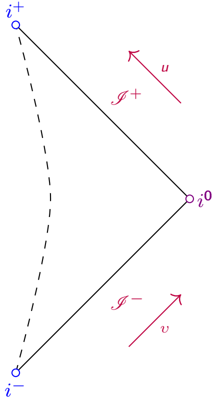
Asymptotic flatness

Bondi & van der Burg & Metzner (1962)
Sachs (1962) , Penrose (1963)
Geroch (1972)

- Does GR support gravitational waves? $\supset$ What is an asymptotically flat spacetime near null infinity?
- Null Gaussian coordinates (u, r, x^A) adapted to outgoing null geodesics ($A = 1, 2$), a.k.a. (partial) Bondi-Sachs coordinates.

$$ds^2_{\mathscr{I}^+} = -du^2 - 2dudr + r^2 q_{AB}(x^C)dx^A dx^B + \frac{2m_B\left(u, x^A\right)}{r} du^2 + rC_{AB}(u, x^C)dx^A dx^B + \ldots$$

u: retarded time , x^A: transverse "angular" coordinates ,
r: "radial" coordinate (varies along null rays).



Gravitational radiation

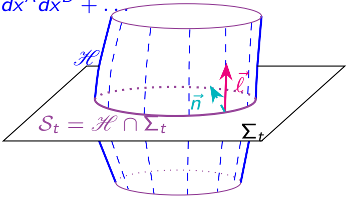
- $C_{\langle AB\rangle}\left(u, x^C\right)$ is free data for a characteristic initial value problem $\rightsquigarrow$ gravitational shear that contains the radiative d.o.f.
- E.o.m. $\Rightarrow \mathscr{I}^+$ is a null hypersurface!

Near-horizon geometry

Gourghoulhon & Jaramillo [gr-qc/0503113]
Booth [1207.6955]
Donnay & Marteau [1903.09654]

$$ds^2_{\mathscr{H}^+} = -2\rho\,\kappa\,d\upsilon^2 + 2d\upsilon d\rho + 2\rho\,\vartheta_A d\upsilon dx^A + (\Omega_{AB} + \rho\lambda_{AB})\,dx^A dx^B + \dots$$

- Gaussian null coordinates: $g_{\upsilon\rho} = +1$, $g_{\rho\rho} = 0$, $g_{\rho A} = 0$.
- Null normal to $\mathscr{H}^+$: $\vec{\ell} = \partial_\upsilon - \rho\,\theta^A\partial_A + \frac{1}{2}\rho^2\mathcal{F}\,\partial_\rho \mathrel{\hat{=}} \partial_\upsilon$.
- Ingoing null ray vector/1-form: $\vec{n} = -\partial_\rho$, $n = -d\upsilon$.
- Intrinsic metric: $q_{\mu\nu} = g_{\mu\nu} + 2\ell_{(\mu}n_{\nu)} \mathrel{\hat{=}} \delta^A_\mu\delta^B_\nu\Omega_{AB}$



Extrinsic geometry

	Name	Evolution equation
Ω_{AB}	horizon metric	-
κ	surface gravity	-
Θ	expansion of $\vec{\ell}$	$\ell^\mu\ell^\nu R_{\mu\nu}$ (Null Raychaudhuri eq. Raychaudhuri (1955) Sachs (1962))
ω_A	twist 1-form	${q_\mu}^\nu\ell^\sigma R_{\nu\sigma}$ (Damour eq. Damour (1979,1982))
σ_{AB}	longitudinal shear	$\ell^\alpha {q_\mu}^\beta\ell^\gamma {q_\nu}^\delta C_{\alpha\beta\gamma\delta}$ (Tidal force eq. Sachs (1962) Adamo et al. [0906.2155])
λ_{AB}	transversal "shear"	${q_\nu}^\sigma {q_\nu}^\lambda R_{\sigma\lambda}$ (Gourghoulhon & Jaramillo [gr-qc/0503113])

- Non-affinity coefficient: $\ell^\nu\nabla_\nu\ell^\mu = \tilde{\kappa}\ell^\mu \Rightarrow \tilde{\kappa} \mathrel{\hat{=}} \kappa$.
- Longitudinal shear: $\sigma_{AB} \mathrel{\hat{=}} \frac{1}{2}\partial_\upsilon\Omega_{AB} - \frac{1}{d-2}\Omega_{AB}\Theta$
- Expansion (of ℓ): $\Theta \mathrel{\hat{=}} \partial_\upsilon \ln\sqrt{\Omega}$
- Twist (Hájíček) 1-form: $\omega_\mu = -{q_\mu}^\nu n_\sigma\nabla_\nu\ell^\sigma \Rightarrow \omega_A \mathrel{\hat{=}} -\frac{1}{2}\vartheta_A$.
- Transversal deformation rate: $\Xi_{AB} \mathrel{\hat{=}} -\frac{1}{2}\lambda_{AB}$

$\mathscr{H}\,/\,\mathscr{I}$ correspondence

Godazgar & Godazgar & Pope [1707.09804]
Agrawal & PC & Donnay [2506.15526]

Conformally completed AFS $\simeq$ Extremal black hole

Under the spatial inversion

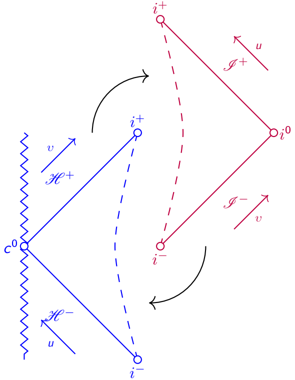
$$\boxed{r \mapsto \frac{\alpha^2}{\rho}\,, \quad u \mapsto \upsilon \quad \Rightarrow \quad d\tilde{s}^2_{\mathscr{I}^+} \mapsto ds^2_{\mathscr{H}^+}}\,,$$

with:

$$\mathcal{F}(\upsilon,\rho,x^A) = \alpha^{-2} F\left(u\mapsto\upsilon, r\mapsto\frac{\alpha^2}{\rho}, x^A\right)\,,$$

$$\theta^A(\upsilon,\rho,x^B) = -\alpha^{-4}\rho U^A\left(u\mapsto\upsilon, r\mapsto\frac{\alpha^2}{\rho}, x^B\right)\,,$$

$$g_{AB}(\upsilon,\rho,x^C) = \alpha^2\mathcal{H}_{AB}\left(u\mapsto\upsilon, r\mapsto\frac{\alpha^2}{\rho}, x^C\right)\,.$$



$$ds^2_{\mathscr{H}^+} = -\rho^2\mathcal{F}d\upsilon^2 + 2d\upsilon d\rho + g_{AB}(dx^A + \rho\,\theta^A d\upsilon)(dx^B + \rho\,\theta^B d\upsilon)\,,$$

$$ds^2_{\mathscr{I}^+} = -Fdu^2 - 2dudr + r^2\mathcal{H}_{AB}\left(dx^A - \frac{U^A}{r^2}du\right)\left(dx^B - \frac{U^B}{r^2}du\right) = \left(\frac{\alpha}{r}\right)^2 d\tilde{s}^2_{\mathscr{I}^+}\,.$$

$\mathscr{H}\,/\,\mathscr{I}$ correspondence

Godazgar & Godazgar & Pope [1707.09804]
Agrawal & PC & Donnay [2506.15526]

$\mathscr{H}\,/\,\mathscr{I}$ dictionary

$\mathscr{H}$	Name	Evolution equation	$\mathscr{I}$
κ	surface gravity	-	0
Θ	expansion	Null Raychaudhuri eq.	0
ω_A	twist	Damour eq.	0
σ_{AB}	longitudinal shear	Tidal force eq.	0
Ω_{AB}	horizon metric	-	q_{AB} (fixed)
λ_{AB}	transversal shear	Trans. deform. rate ev. eq.	C_{AB} (free data)

$$ds^2_{\mathscr{H}^+} = -2\rho\,\kappa\,d\upsilon^2 + 2d\upsilon d\rho + 2\rho\,\vartheta_A d\upsilon dx^A + (\Omega_{AB} + \rho\lambda_{AB})\,dx^A dx^B + \ldots\,,$$

$$ds^2_{\mathscr{I}^+} = -F_0\,du^2 - 2dudr - 2u_A dudx^A + \left(r^2 q_{AB} + rC_{AB}\right) dx^A dx^B + \ldots\,.$$

⚠ ***The AFS and its "dual" extremal non-rotating black hole do not in general live in the same spacetime.***

The canonical example: Extremal Reissner-Nordström black hole and Couch-Torrence symmetry

$S = \int d^4x \sqrt{-g}\left[\frac{R}{16\pi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$

Extremal Reissner-Nordström (ERN) black hole ($d = 4$, $G = c = 1$):

$$|Q| = M \;\Rightarrow\; ds^2_{\text{ERN}} = -\frac{\Delta(r)}{r^2}dt^2 + \frac{r^2 dr^2}{\Delta(r)} + r^2 d\Omega_2^2\,, \quad \Delta(r) = (r-M)^2\;.$$

Couch-Torrence inversion

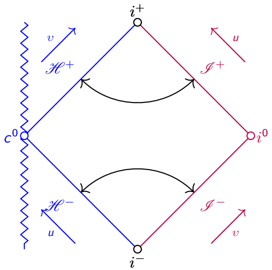
Couch & Torrence (1984)

$$\boxed{r \xrightarrow{\text{CT}} \tilde{r} = \frac{Mr}{r-M} \;:\; \textbf{Isometry of } r^{-2}ds^2_{\text{ERN}}}\,,$$

CT inversion = Reflection of tortoise coord. that preserves $r_{\text{ph}} = 2M$:

$$r_* = r - M - \frac{M^2}{r-M} + 2M\ln\left|\frac{r-M}{M}\right| \xrightarrow{\text{CT}} -r_*$$

$$\Rightarrow (v, r, x^A) \overset{\text{CT}}{\longleftrightarrow} \left(u, \frac{Mr}{r-M}, x^A\right) \Leftrightarrow \boxed{\mathscr{H}^\pm \overset{\text{CT}}{\longleftrightarrow} \mathscr{I}^\pm}\,.$$



Physical implications: Matching of near-$\mathscr{H}$ and near-$\mathscr{I}$ charges

Bizon & Friedrich [1212.0729], Lucietti & Murata & Reall & Tanahashi [1212.2557]
Bhattacharjee & Chakrabarty & Chow & Paul & Virmani [1805.10655]
Fernandes & Ghosh & Virmani [2008.04365], Borthwick & Gourgoulhon & Nicolas [2303.14574]
Agrawal & PC & Donnay [2506.15526]

Massless (minimally coupled) scalar perturbations of ERN black hole:

$$\Box^{(0)}_{\text{ERN}}\Phi = 0 \Rightarrow \begin{cases} \Box^{(0)}_{\mathscr{I}^+}\Phi = 0\,, & \Phi = \Phi(u,r,x^A) = \frac{1}{r}\sum_{n=0}^{\infty}\Phi^{(n)}(u,x^A)\frac{1}{r^n}\,; \\ \Box^{(0)}_{\mathscr{H}^+}\Phi = 0\,, & \Phi = \hat{\Phi}(v,\rho,x^A) = \sum_{n=0}^{\infty}\hat{\Phi}^{(n)}(v,x^A)\left(\frac{\rho}{M}\right)^n\,; \end{cases} .$$

$$r^2\Box^{(0)}_{\mathscr{I}^+} = \partial_r\left(r-M\right)^2\partial_r - 2r\partial_u\partial_r\, r + \Delta_{\mathbb{S}^2}\,,\quad \left(M+\rho\right)^2\Box^{(0)}_{\mathscr{H}^+} = \partial_\rho\,\rho^2\partial_\rho + 2\left(M+\rho\right)\partial_v\partial_\rho\left(M+\rho\right) + \Delta_{\mathbb{S}^2}\,.$$

Newman-Penrose constants

Newman & Penrose (1965, 1968)
Exton & Newman & Penrose (1969)

$$N_{\ell m} = \frac{(-1)^{\ell+1}}{(\ell+1)!}\lim_{r\to\infty}\int_{\mathbb{S}^2}\bar{Y}_{\ell m}\left[\left(r-M\right)^2\partial_r\right]^{\ell}\left[r\left(r-M\right)\partial_r\left(r\Phi\right)\right] \quad\rightsquigarrow\quad \partial_u N_{\ell m} = 0\,.$$

Aretakis constants

Aretakis[1110.2007,1110.2009] [1206.6598]

$$A_{\ell m} = \frac{M^{\ell-1}}{(\ell+1)!}\lim_{r\to M}\int_{\mathbb{S}^2}\bar{Y}_{\ell m}\partial_r^{\ell}\left[r\partial_r\left(r\Phi\right)\right] \quad\rightsquigarrow\quad \partial_v A_{\ell m} = 0\,.$$

CT inversion $\rightsquigarrow N_{\ell m} = A_{\ell m}$

Bizon & Friedrich [1212.0729]
Lucietti & Murata & Reall & Tanahashi [1212.2557]
Bhattacharjee&Chakrabarty& Chow&Paul&Virmani [1805.10655]

- Scalar wave operator transforms homogeneously under CT inversion:

$$\Box^{(0)}_{\mathscr{H}^+} \xrightarrow{\text{CT}} \tilde{\Box}^{(0)}_{\mathscr{H}^+} = \Omega^{+1}\Box^{(0)}_{\mathscr{I}^+}\Omega^{+1}\,, \quad \Omega = \tfrac{r-M}{M}\,.$$

- Action of CT inversion on scalar field:

$$\hat{\Phi}(\upsilon,\rho,x^A) \xrightarrow{\text{CT}} \tilde{\Phi}(u,r,x^A) = \Omega^{-1}\hat{\Phi}\left(\upsilon\mapsto u, \rho\mapsto \tfrac{M^2}{r-M}, x^A\right).$$

$\rightsquigarrow$ If $\Box^{(0)}_{\mathscr{H}^+}\hat{\Phi}(\upsilon,\rho,x^A)=0$, then $\Box^{(0)}_{\mathscr{I}^+}\tilde{\Phi}(u,r,x^A)=0 \xrightarrow{b.c.'s} \Phi(u,r,x^A)=\tilde{\Phi}(u,r,x^A)$.

$$\therefore \boxed{\underline{\text{Matching condition:}} \quad \Phi(u,r,x^A) = \frac{M}{r-M}\hat{\Phi}\left(\upsilon\mapsto u, \rho\mapsto \frac{M^2}{r-M}, x^A\right)}\,,$$

$$\Rightarrow \cdots \Rightarrow \boxed{\boxed{N_{\ell m} = M^{\ell+2}A_{\ell m}}}\,.$$

CT inversion $\rightsquigarrow {}_sN_{\ell m} = {}_sA_{\ell m}$

Fernandes & Ghosh & Virmani [2008.04365]
Agrawal & PC & Donnay [2506.15526]

$$\psi_s = \begin{cases} \Phi & \text{for } s = 0 \rightarrow \text{Massless scalar perturbations}\,; \\ \phi_0 & \text{for } s = +1 \rightarrow \text{Electromagnetic perturbations (frozen gr.)}\,; \\ \Psi_0 & \text{for } s = +2 \rightarrow \text{Gravitational perturbations (constrained em.)}\,; \end{cases} .$$

NP constants: ${}_sN_{\ell m} = \frac{(-1)^{\ell-s+1}}{(\ell-s+1)!} \left\{ \int_{\mathbb{S}^2} {}_s\bar{Y}_{\ell m} \left[(r-M)^2 \partial_r \right]^{\ell-s} \left[\frac{(r-M)^{2s+1}}{r^{2s-1}} \partial_r \left(r^{2s+1} \psi_s \right) \right] \right\} \Big|_{r\to\infty} \rightsquigarrow \partial_u \, {}_sN_{\ell m} = 0$,

Aretakis constants: ${}_sA_{\ell m} = \frac{M^{\ell-s-1}}{(\ell-s+1)!} \left\{ \int_{\mathbb{S}^2} {}_s\bar{Y}_{\ell m} \partial_r^{\ell-s} \left[\frac{1}{r^{2s-1}} \partial_r \left(r^{2s+1} \psi_s \right) \right] \right\} \Big|_{r\to M} \rightsquigarrow \partial_v \, {}_sA_{\ell m} = 0$.

Matching condition: $\psi_s(u, r, x^A) = \left(\frac{M}{r-M} \right)^{2s+1} \hat{\psi}_s \left(v \mapsto u, \rho \mapsto \frac{M^2}{r-M}, x^A \right)$,

$$\Rightarrow \cdots \Rightarrow \boxed{\boxed{{}_sN_{\ell m} = M^{\ell+s+2} \, {}_sA_{\ell m}}} .$$

CT for twisting horizons: Extremal Kerr-Newman black holes

Couch & Torrence (1984)
Agrawal & PC & Donnay [2506.15526]

$$ds^2_{\rm KN} = -\frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta\, d\phi\right)^2 + \frac{\sin^2\theta}{\Sigma}\left(a\, dt - \left(r^2 + a^2\right) d\phi\right)^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma\, d\theta^2\,,$$
$$\Delta = r^2 - 2Mr + a^2 + Q^2\,, \quad \Sigma = r^2 + a^2\cos^2\theta\,.$$

- At *extremality*: $a^2 + Q^2 = M^2 \quad \Rightarrow \quad \Delta = (r-M)^2$.
- Spin-weighted wave equation is conformally invariant under:

$$r - M \mapsto \frac{M^2 + a^2 + \frac{a\partial_\phi}{\partial_t}}{r - M} \approx \frac{M^2 + a^2 - \frac{ma}{\omega}}{r-M}$$

- Exchanges $r = M$ (event horizon) with $r \to \infty$ (null infinity) surfaces

$$\Rightarrow \cdots \Rightarrow {}_sN_{\ell,m=0} = M^{\ell+s+2}\, {}_sA_{\ell,m=0}\,.$$

CT at non-extremality: Lukewarm black holes

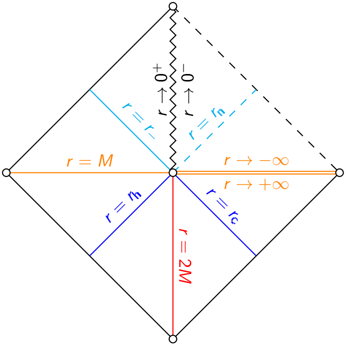
Brannlund [gr-qc/0306074]
Runarsson [1209.3441]
PC & Donnay & Lupsasca [2512.xxxxx]

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2\,, \qquad f(r) = -k\frac{r^2}{\ell^2} + \left(1 - \frac{M}{r}\right)^2 .$$

- $|Q| = M$ electrovacuum solution in asymptotically $\begin{cases} \text{flat} & (k=0) \\ \text{AdS} & (k=-1) \\ \text{dS} & (k=+1) \end{cases}$.
- For flat: ERN. For AdS: naked singularity.
- For dS: *non-extremal* black hole; 4 horizons at $-\infty < r_n < 0 < r_- < r_h < r_c < \infty$.

CT inversion: $r - M \mapsto \frac{M^2}{r-M}$

$$\begin{aligned} r_h &\leftrightarrow r_c\,, \\ r_- &\leftrightarrow r_n\,, \\ r = M &\leftrightarrow r \to \pm\infty\,, \\ r = 2M &\leftrightarrow r = 2M\,. \end{aligned}$$



Summary

- $\mathscr{I}$ is an extremal and non-twisting NEH of the conformally completed AFS. Ashtekar & Speziale [2401.15618,2402.17977]
- The conformally completed AFS is a spatially inverted extremal and non-rotating black hole geometry. Godazgar & Godazgar & Pope [1707.09804] Agrawal & PC & Donnay [2506.15526]
- The extremal and non-rotating black hole dual to an AFS lives in general in a different spacetime. Agrawal & PC & Donnay [2506.15526]
- Special case: the four-dimensional ERN is self-inverted (Couch-Torrence inversion). Couch & Torrence (1984)
- Physical implications of existence of such conformal mappings: near-$\mathscr{I}$ Newman-Penrose constants $\simeq$ near-$\mathscr{H}$ Aretakis constants for scalar (Bizon & Friedrich [1212.0729], Lucietti & Murata & Reall & Tanahashi [1212.2557] Bhattacharjee & Chakrabarty & Chow & Paul & Virmani [1805.10655]), electromagnetic (Fernandes & Ghosh & Virmani [2008.04365]) and gravitational (Agrawal & PC & Donnay [2506.15526]) perturbations.

Outlook

- Matching of asymptotic symmetries — Donnay & Giribet & González & Pino [1511.08687], [1607.05703]; Mao & Wu & Zhang [1606.03226]
- Generalized Couch-Torrence inversion symmetry — Cvetič & Pope & Saha [2008.04944], [2102.02826]; Bianchi & Russo [2110.09579], [2203.14900]
- Full symmetry structure of AFS and $w_{1+\infty}$ for black holes — Strominger [2105.14346]; Freidel & Pranzetti & Raclariu [2112.15573]; Geiller [2403.05195]; Agrawal & PC & Donnay [2412.01647], [25xx.xxxxx]; Ruzziconi & Zwikel [2504.08027]
- Rotating black holes as dual to weakly AFS — Godazgar & Godazgar & Pope [1707.09804]
- $\mathscr{H}/\mathscr{I}$ correspondence at non-extremality? — Runarsson [1209.3441]; PC & Donnay & Lupsasca [2512.xxxxx]
- NP/Aretakis constants from symmetries? — Godazgar & Godazgar & Pope [1809.09076], [1812.06935]; Macaulay [2206.11097]; Gralla & Zimmerman [1711.00855]; Chen & Kovács [2507.12529]
- *Celestial holography Vs Kerr/CFT?*

"Black holes are the hydrogen atom of the 21st century"

't Hooft (2016), EHT (April 10, 2019)

Thank you