Holography & a codimension-2 defect CFT

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Synopsis

* Motivation - state of the art
* The D5-brane - Embedding & BF bound
* Holographic 1pt function of $T_{\mu\nu}$
* Holographic 1pt function of chiral primary operators
* Field theory approach & 1pt functions at weak coupling
* Comparing strong & weak coupling results
* Conclusions & future directions

Motivation - state of the art

CFT dynamics in the presence of defects describes a plethora of physical systems, e.g. boundaries/interfaces to objects such as WL, strings and branes.

Defect breaks, partially or even completely, the conformal symmetry of the ambient CFT ⇒ renders the calculation of observables much more involved

⬇

Characteristics of the dCFT {
- defect preserves some amount of SUSY?
- BC's associated with the defect are integrable?
- co-dimension of the defect?

Main focus ⇒ codimension-2 defects in the context of gauge/gravity duality

AdS/CFT ⇒ defects correspond to extended objects in the bulk ⇒ e.g. probe branes wrapping appropriate submanifolds of the $AdS_5 \times S^5$

↓ Codimension-1 case: D3/D5 brane intersection

Defect in $\mathcal{N} = 4$ is realized by a stack of D3-branes ending on a D5-brane, preserving N=(4,4) supersymmetry on the defect.

Holographic dual: Probe D5-brane wrapping $AdS_4 \times S^2$ inside $AdS_5 \times S^5$

↓ Codimension-2 case

Natural holographic duals are branes with a worldvolume that contains an AdS_3 factor and wraps internal cycles

↑ encodes the conformal symmetry on the defect

Codimension-2 case: Gukov-Witten defect

Holographic description: a probe D3 wrapping an $AdS_3 \times S^1$ - preserves 1/2 SUSY

FT side: classical solutions with non-zero diagonal vevs for some of the scalar fields

Vevs exhibit a simple pole on the defect ⇒ Spacetime dependence dictated by conformal invariance

Very limited list of examples

The D5-brane

The symmetry of the induced metric is $AdS_3 \times S^1 \times S^2$ & it ends on 2d submanifold on the boundary of AdS

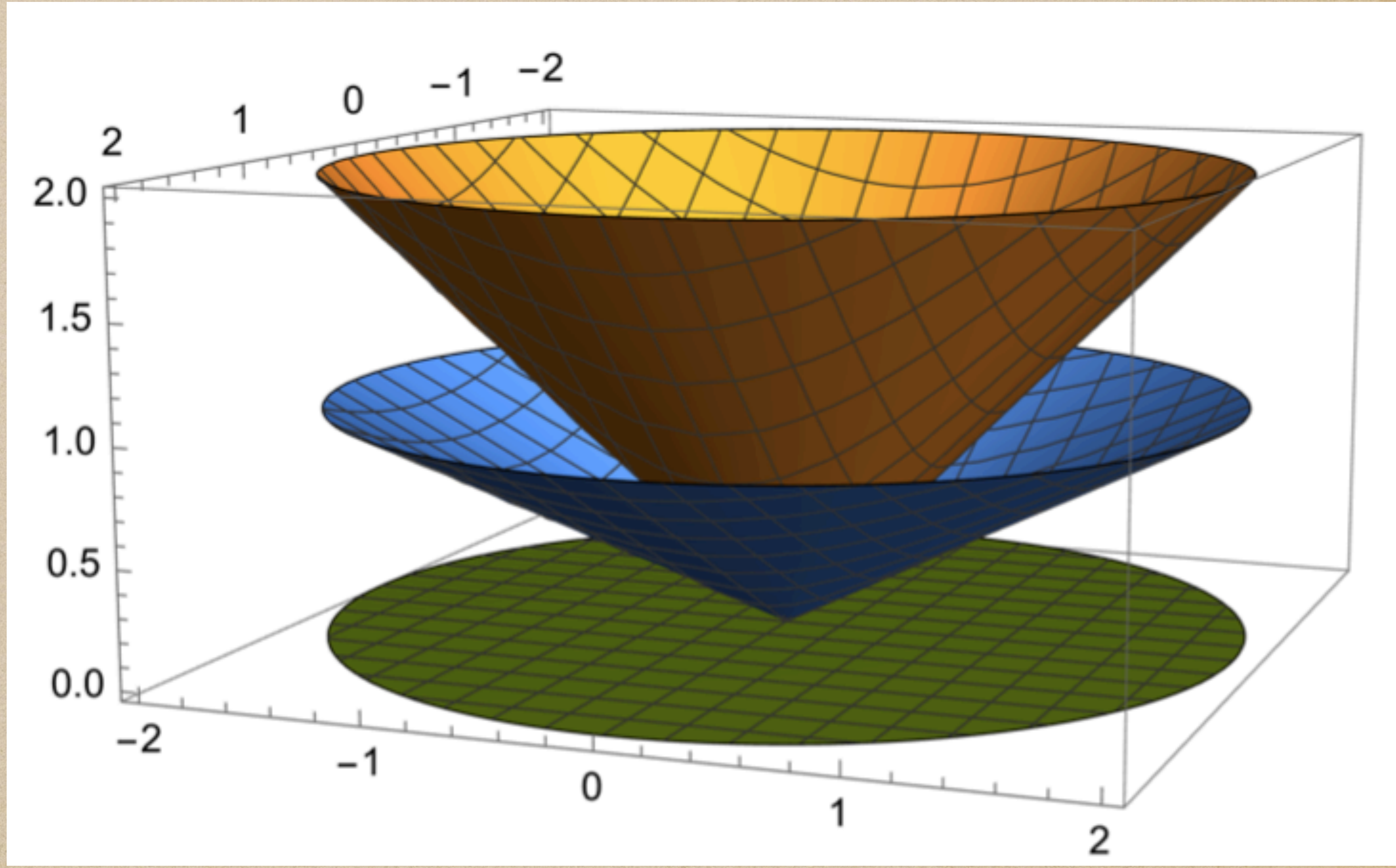
	x_0	x_1	r	ψ	z	$\tilde{\psi}$	$\tilde{\beta}$	$\tilde{\gamma}$	β	γ
D3	•	•	•	•						
D5 probe	•	•	•	•					•	•

Embedding ansatz — Depends on 2 parameters

$$\tilde{\psi} = 0\,, \quad \tilde{\beta} = \frac{\pi}{2}, \quad \tilde{\gamma} = 0 \quad \& \quad z = \sigma r$$

$$A = \frac{\kappa}{2\pi\alpha'} \cos\beta \, d\gamma \quad \text{with} \quad \kappa = -\frac{4+\sigma^2}{\sigma\sqrt{8-\sigma^2}} \quad (0 < \sigma < 2\sqrt{2})$$

Horizontal green plane ⇒ boundary of AdS & vertical axis ⇒ holographic coordinate z.



D5-brane with x_0, x_1, β & γ suppressed

Blue surface ⇒ D5 brane & the orange is the critical surface after which the D5 becomes unstable.

From the figure depicting the D5-brane

D5-brane intersects AdS bdy at z=r=0 ⇒ defect is a 2-dimensional plane $\mathbb{R}^{(1,1)}$ that extends along 2/4 directions of the boundary (x_0 & x_1) ⇒ defect of codim-2

To quantize the flux impose ⇒ $k = -\int_{S^2} \frac{F}{2\pi} \quad \Rightarrow \quad k = \frac{\kappa}{\pi\alpha'} = \frac{\kappa\sqrt{\lambda}}{\pi}$

The induced metric on the D5-brane, after a change of variables

$$ds^2_{ind} = \frac{1+\sigma^2}{\sigma^2}\,\frac{-dx_0^2 + dx_1^2 + d\hat{r}^2}{\hat{r}^2} + \frac{1}{\sigma^2}d\psi^2 + d\beta^2 + \sin^2\beta\, d\gamma^2$$

$AdS_3 \times S^1 \times S^2$ symmetry of the D5 brane becomes apparent

Examine the stability of the solution ⇒ fluctuations of the transverse coordinates acquire masses which are above the BF bound ⇒ $m^2 \geq -1$

From the point of view of the D5-brane these fluctuations behave as scalars propagating on AdS_3

Introduce fluctuations around the D5-brane solution for the world-volume gauge field & for transverse coordinates

Interested in the time evolution of the fluctuations (detect tachyon instabilities) ⇒ dependence only on the time coordinate x_0

Transformation from time x_0 to proper time s

$$\delta z''(s) - \sigma^2\, \delta z(s) = 0 \quad \Rightarrow \quad m_z^2 = -\sigma^2 \quad \Rightarrow \quad \sigma \leq 1$$

puts a stronger constraint than $\sigma \leq 2\sqrt{2}$

EOMs for $\delta\tilde{\beta}$, $\delta\tilde{\gamma}$ and δA are trivially satisfied and EOM for $\delta\tilde{\psi}$ does not put any further constraint on σ

Holographic 1pt function of $T_{\mu\nu}$

important observable

Codimension 2 defect

$$\langle T_{\mu\nu}\rangle = 0 \quad \begin{cases} \text{theory without any defect} \\ \text{Codimension 1 defect} \end{cases}$$

Conformal invariance constrains space-time structure up to a function $\mathbf{h}(g_{YM}, N)$

$\mu, \nu = 0,1$: coordinates along the surface defect R^2 embedded in R^4

$$\langle T_{\mu\nu}\rangle = \mathbf{h}\frac{\eta_{\mu\nu}}{r'^4}, \quad \langle T_{ij}\rangle = \frac{\mathbf{h}}{r'^4}\left(4\, n_i n_j - 3\,\delta_{ij}\right) \quad \& \quad \langle T_{\mu i}\rangle = 0$$

r' ⇒ radial distance of the position of $T_{\mu\nu}$ from the defect

Unit vector normal to the defect

Choosing the position of the operator $T_{\mu\nu}$ at the boundary to be $x^m = \{0,0,0,r'\}$

$$\langle T_{00}\rangle = \langle T_{11}\rangle = \langle T_{33}\rangle = \frac{\mathbf{h}}{r'^4} \quad \& \quad \langle T_{22}\rangle = -3\frac{\mathbf{h}}{r'^4} \quad \Rightarrow \quad \langle T^m{}_m\rangle = 0$$

h contains the particularities of the specific defect CFT ⇒ Calculation at strong coupling using holography

⬇

T_{ij} is dual to the fluctuations of the AdS metric $g_{mn} = \hat{g}_{mn} + \delta g_{mn}$

1pt function of the SE tensor in the presence of a "heavy" object (Dp-brane) is

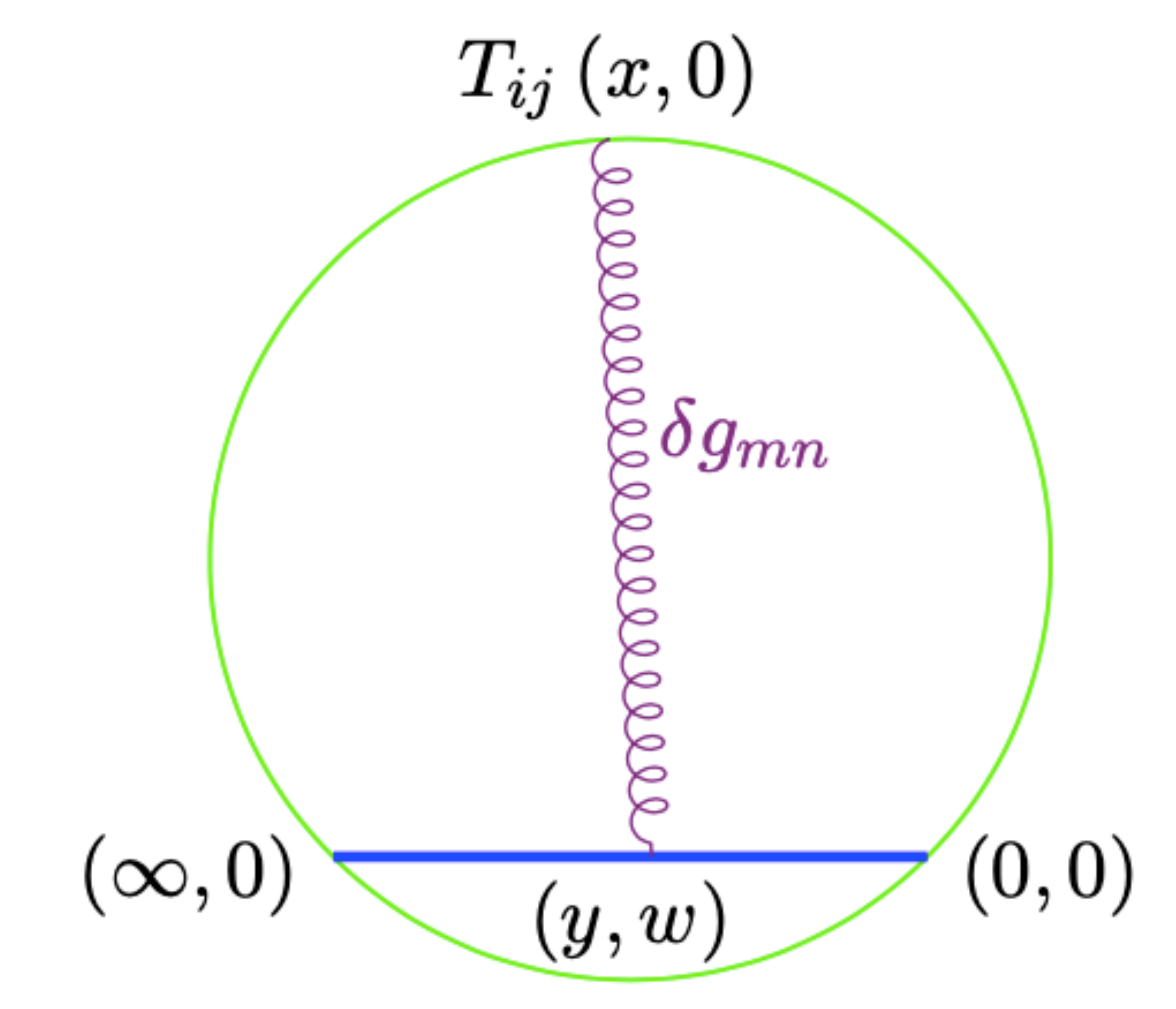
$$\left\langle T_{ij}(x) \right\rangle_{\text{brane}} = \lim_{z \to 0} \left\langle \delta g_{ij}(x,z) \cdot \frac{1}{Z_{\text{brane}}} \int D\mathbb{Y} e^{-S_{\text{brane}}[\mathbb{Y}]} \right\rangle_{\text{bulk}}$$

⬇

At strong coupling, path integral is dominated by a saddle point ⇒ corresponds to the classical solutions $\mathbb{Y}_{\text{cl}}$ describing the embedding of the Dp-brane

Expanding the Dp-brane action around the classical solution, cancels the partition function and the 1pt function becomes

$$\left\langle T_{ij}(x)\right\rangle_{\text{brane}} = -\lim_{z\to 0}\left\langle \delta g_{ij}(x,z)\cdot\left(\left.\frac{\partial S_{\text{brane}}\left[\mathbb{Y}_{\text{cl}}\right]}{\partial\delta g_{mn}}\right|_{\delta g_{mn}=0}\cdot\delta g_{mn}(y,w)\right)\right\rangle_{\text{bulk}}$$



$T_{\mu\nu}$ sits at the point $(x,0)$ on the boundary of AdS & (y,w) is a point on the Dp-brane

Generic variation of a probe Dp-brane action with respect to g_{mn}

$$\frac{\partial S_{\text{brane}}\left[\mathbb{Y}\right]}{\partial\delta g_{mn}} = \frac{T_p}{2g_s}\int d^{p+1}\zeta\sqrt{h}\, h^{ab}\partial_a\mathbb{Y}^m\partial_b\mathbb{Y}^n$$

Substitute everything in the expression for the 1pt func of $T_{\mu\nu}$ ⇒ use the classical solution for the Dp brane & the bulk-to-boundary propagator for the graviton

perform a 6-dimensional integral

$$\langle T_{22}\rangle = -3\,\langle T_{00}\rangle = -3\,\langle T_{11}\rangle = -3\,\langle T_{33}\rangle = -3\left[-\frac{\lambda^{3/2}\left(2\sigma^2+1\right)}{6\pi^3\sigma^3\sqrt{8-\sigma^2 g_{YM}^2}}\right]\frac{1}{r'^4}$$

$\mathbf{h}^{\text{strong}}$

This result has precisely the form that is dictated by conformal invariance

Holographic 1pt function of chiral primary operators

The 1pt function is non-zero only when the CPO respects the $SO(3) \times SO(3)$ symmetry of the defect brane ⇒ choose the proper spherical harmonic

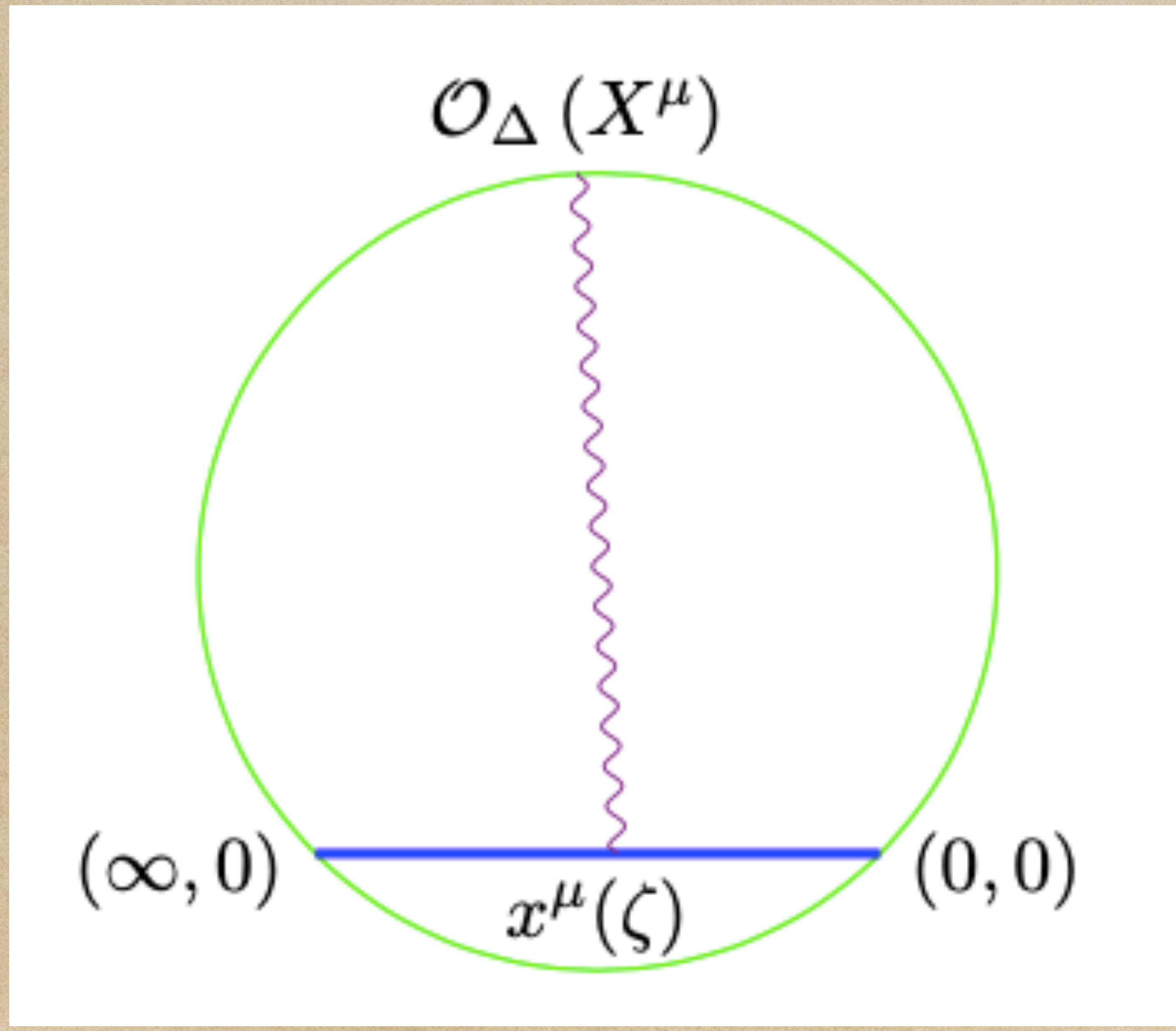
one of the most important data in defect CFTs

perturb the Euclidean action

$$\mathscr{L}^{(1)}_{DBI} = \frac{1}{2}\sqrt{\det H}\,\left(H^{-1}_{sym}\right)^{ab}\partial_a X^M \partial_b X^N h_{MN} \quad \& \quad \frac{\mathscr{L}^{(1)}_{WZ}}{2\pi\alpha'} = F_{\beta\gamma}\frac{1}{4!}\epsilon^{abcd}(Pa)_{abcd}$$

induced metric on the D5

$h_{\mu\nu}$ & a_4: fluctuations of the metric & of the RR form in terms of the SUGRA field which is dual to the CPO



CPO is located at the boundary point

$$X^\mu = (X_0, X_1, r', \psi', 0)$$

$x^\mu(\zeta)$ denotes an arbitrary point on the D5 brane

Wavy line represents the fluctuations of the metric & of the 4-form.

1st order fluctuation of the DBI term, for the specific D5-brane embedding ansatz

$$\mathscr{L}^{(1)}_{DBI} = \frac{-\Delta\, s \sin\beta}{2\, r^3 \sigma^3 \sqrt{8-\sigma^2}} \Bigg[40 + \frac{16\,\sigma^2}{K^2} \Big[(x_0 - X_0)^2 + (x_1 - X_1)^2 + r^2(1+\sigma^2) + r'^2 \Big]^2$$

$$+ \frac{32}{\sigma^2} + 17\,\sigma^2 - \frac{32\,(1+\sigma^2)}{K} \Big[(x_0 - X_0)^2 + (x_1 - X_1)^2 + r^2(1+\sigma^2) + r'^2 \Big] \Bigg]$$

SUGRA field s which is dual to the CPO

$$K = z^2 + (x_0 - X_0)^2 + (x_1 - X_1)^2 + r^2 + r'^2 - 2\, r\, r' \cos(\psi - \psi')$$

Substitute the value of s & start integrating with respect to x_0, x_1, r, ψ, β & γ

$$\mathscr{L}^{Contribution}_{DBI} = + \frac{1}{r'^{\Delta}} \frac{2^{4-\Delta}\, \pi^3}{(1+\Delta)\,\sigma\sqrt{8-\sigma^2}} \Big[\Xi_1 \Psi_1 + \Xi_2 \Psi_2 + \Xi_3 \Psi_3 \Big] c_\Delta\, Y_\Delta(0)$$

$$\Xi_1 = \frac{8\left(5\Delta^2+\Delta-4\right)\sigma^2+\Delta(17\Delta-15)\sigma^4+32\Delta(\Delta+1)}{(1-\Delta)\sigma^2}$$

$$\Xi_2 = 32\left(\Delta\sigma^2+\Delta+1\right)\,, \quad \Xi_3 = -\,16\,\Delta\,\sigma^2$$

$$\Xi_4 = (1-\Delta)\,\sigma^2\,, \quad \Xi_5 = \Delta - \Xi_4$$

$$\Psi_1 = \int_0^{\frac{\pi}{2}} d\chi\, \sin^{\Delta-3}\chi\, P^{(0)}_{\Delta-2}\left(\sqrt{1+\frac{1}{\sigma^2}\sin^2\chi}\right)$$

$$\Psi_2 = \int_0^{\frac{\pi}{2}} d\chi\sqrt{1+\frac{1}{\sigma^2}\sin^2\chi}\,\sin^{\Delta-3}\chi\, P^{(0)}_{\Delta-1}\left(\sqrt{1+\frac{1}{\sigma^2}\sin^2\chi}\right)$$

$$\Psi_3 = \int_0^{\frac{\pi}{2}} d\chi\left[1+\frac{1}{\sigma^2}\sin^2\chi\right]\sin^{\Delta-3}\chi\, P^{(0)}_{\Delta}\left(\sqrt{1+\frac{1}{\sigma^2}\sin^2\chi}\right)$$

$P^{|k|}_{\delta}(x)$: associated Legendre polynomials

Ψ_1, Ψ_2 & Ψ_3 cannot be calculated for generic values of Δ; however, for specific values of Δ the integrals can be performed

WZ contribution

$$\mathscr{L}^{(1)}_{WZ} = 4\,i\,\kappa\,\sin\beta\sqrt{g^{AdS}}\left[g^{zz}\partial_z - \sigma\, g^{rr}\partial_r\right]s$$

$$\mathscr{L}^{Contribution}_{WZ} = \frac{1}{r'^{\Delta}}\frac{2^{7-\Delta}\,\pi^3\,(4+\sigma^2)}{(\Delta-1)\,\sigma^3\sqrt{8-\sigma^2}}\left[\Xi_4\,\Psi_2 + \Xi_5\,\Psi_1\right]c_\Delta\,Y_\Delta(0)$$

$$\langle \mathcal{O}_\Delta(r') \rangle^{(strong)} = \frac{i^\Delta 2^{-\Delta-\frac{3}{2}}}{\pi\sqrt{\Delta}} \sqrt{\frac{\Delta+2}{\Delta+1}} \frac{\sigma}{\sqrt{8-\sigma^2}} \frac{\sqrt{\lambda}}{\Delta-1} \frac{1}{r'^\Delta} \times \left[\left(9\Delta^2 - 15\Delta + 8\right) \Psi_1 - 8(\Delta-1)(3\Delta-1)\Psi_2 + 16(\Delta-1)\Delta\Psi_3 \right]$$

For $\Delta = 4$

$$\langle \mathcal{O}_4(r') \rangle^{(strong)} = \frac{\sqrt{\lambda}}{16\sqrt{15}\,\pi} \frac{1}{r'^4} \frac{1+\sigma^2}{\sigma^5\sqrt{8-\sigma^2}} \left(5\sigma^4 + 56\sigma^2 + 96\right)$$

straightforward to evaluate the result for CPOs of arbitrary large Δ

Field theory approach & 1pt functions at weak coupling

What is the dual to the D5-brane defect CFT?

$$\mathcal{L}_{\mathcal{N}=4} = \frac{2}{g_{\rm YM}^2}\,\mathrm{tr}\Big\{ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\left(D_\mu\varphi_i\right)^2 + \frac{i}{2}\,\bar{\psi}\not{D}\,\psi + \frac{1}{2}\,\bar{\psi}\,\Gamma^{i+3}\,[\varphi_i,\psi] + \frac{1}{4}\,[\varphi_i,\varphi_j]^2 \Big\}$$

EOMs of the bosonic fields with $A_\mu = 0$

$$[\partial_\nu\varphi_i,\varphi_i] = 0 \quad \& \quad \partial^\mu\partial_\mu\varphi_i = \Big[\varphi_j,[\varphi_j,\varphi_i]\Big] \quad \xRightarrow{\varphi_i(r')} \quad \frac{d^2\varphi_i}{dr'^2} + \frac{1}{r'}\frac{d\varphi_i}{dr'} = [\varphi_j,[\varphi_j,\varphi_i]]$$

Ansatz that solves the EOMs

FT dual of the D5-probe

$$\varphi_i = 0 \quad \& \quad \varphi_{i+3} = \varphi_{i+3}^{\rm cl}\left(r'\right) = \frac{1}{r'}\cdot\left[\begin{array}{cc} (t_i)_{k\times k} & 0_{k\times(N-k)} \\ 0_{(N-k)\times k} & 0_{(N-k)\times(N-k)} \end{array}\right]_{N\times N} \quad \text{with} \quad i = 1,2,3$$

Comments

- The choice of the solution is dictated by the fact the D5-brane sits at $\tilde{\psi} = 0$

  $\Rightarrow$ implies that only the 3 scalars related to $S^2(\beta, \gamma)$ should have non-zero vevs

  $\Rightarrow \varphi_4, \varphi_5$ & φ_6
- t_i realise a k-dimensional irreducible representation of $\mathfrak{su}\,(2)$
- In the dual gravity description, k corresponds to the integer flux through the $S^2(\beta, \gamma) \subset S^5$
- Solution scales as $\dfrac{1}{r'} \Rightarrow$ Manifestation of the conformal symmetry
- the symmetry of the brane is $AdS_3 \times S^1 \Rightarrow$ existence of S^1- related to the angle $\psi \Rightarrow$ expectation value of the scalar fields should not depend on ψ

1pt function of $T_{\mu\nu}$ at weak coupling

The relevant part of $T_{\mu\nu}$ containing only scalars

$$T_{\mu\nu}^{(\text{scalars})} = \frac{2}{g_{\text{YM}}^2} \cdot \text{tr}\left\{\frac{2}{3}\left(\partial_\mu\varphi_i\right)\left(\partial_\nu\varphi_i\right) - \frac{1}{3}\varphi_i\left(\partial_\mu\partial_\nu\varphi_i\right) - \frac{1}{6}\eta_{\mu\nu}\left[\left(\partial_\varrho\varphi_i\right)^2 + \frac{1}{2}\left[\varphi_i,\varphi_j\right]^2\right]\right\}$$

$i = 1,2,\ldots,6$

Substituting the classical solution we obtain the form that is dictated from conformal invariance

$$\mathbf{h}^{(weak)} = -\frac{1}{48g_{YM}^2}(k^2-1)k$$

1pt function of chiral primary operators at weak coupling

Symmetry of D5-brane ⇒ only operators respecting the $SO(3) \times SO(3)$ symmetry can have non-zero 1pf

The most generic CPO of $\mathcal{N} = 4$ SYM can be written as

$$\mathcal{O}_{\Delta I}(x) \equiv \frac{(8\pi^2)^{\frac{\Delta}{2}}}{\lambda^{\frac{\Delta}{2}}\sqrt{\Delta}} C_I^{i_1 i_2 \ldots i_\Delta} \,\mathrm{Tr}\left[\varphi_{i_1}(x)\varphi_{i_2}(x)\ldots\varphi_{i_\Delta}(x)\right]$$

Scalars of the theory

$$\langle \mathcal{O}_{\Delta_1 I_1}(x)\mathcal{O}_{\Delta_2 I_2}(y)\rangle = \frac{\delta_{I_1 I_2}\delta_{\Delta_1\Delta_2}}{|x-y|^{2\Delta_1}} \quad \Leftarrow \quad \sum_{i_1\ldots i_\Delta=1}^{6} C_{I_1}^{i_1 i_2 \ldots i_\Delta} C_{I_2}^{i_1 i_2 \ldots i_\Delta} = \delta_{I_1 I_2}$$

Tensors $C_I^{i_1 i_2 \ldots i_\Delta}$ are symmetric and traceless in the indices $i_1 \ldots i_\Delta$

Normalization for $C_I^{i_1 i_2 \ldots i_\Delta}$ implies the normalization of 2pf of the operators

Spherical harmonics of S^5 can be written as $\Rightarrow$ $Y_{\Delta I} = C_I^{i_1 i_2 \dots i_\Delta} \hat{x}_{i_1} \hat{x}_{i_2} \dots \hat{x}_{i_\Delta}$

$\hat{x}_i$'s are components of a unit vector

$\hat{x}_1^2 + \hat{x}_2^2 + \hat{x}_3^2 = s_{\tilde{\psi}}^2$

$\hat{x}_3^2 + \hat{x}_4^2 + \hat{x}_4^2 = c_{\tilde{\psi}}^2$

$$\sum_{i=1}^{6} \hat{x}_i^2 = 1$$

Substitute the classical solution

For each even $\Delta = 2l$ there is a unique $SO(3) \times SO(3)$ symmetric CPO $\Rightarrow$ $\mathcal{O}_\Delta(x')$

$$\langle \mathcal{O}_\Delta(x') \rangle^{(weak)} = \frac{(8\pi^2)^{\frac{\Delta}{2}}}{\lambda^{\frac{\Delta}{2}} \sqrt{\Delta}} \frac{(k^2 - 1)^{\Delta/2} k}{(2\sqrt{2} r')^{\Delta}} Y_\Delta(0)$$

Comparing strong & weak coupling results

Agreement between the leading term in the expressions for the weak & strong coupling in the limit $\frac{k}{\sqrt{\lambda}} = \frac{\kappa}{\pi} \gg 1$

non-trivial test of the correspondence

Similar agreement for the CPOs in the codim-1 SUSY D3-D5 & non-SUSY D3-D7

Situation reminiscent of the BMN limit ⇒ engineer a quantity (here: $\frac{\lambda}{k^2}$, BMN: $\frac{\lambda}{J^2}$) which can be taken to be small at weak/strong coupling

Then one compares the observables order by order in the small quantity

1pt function of $T_{\mu\nu}$

$$\frac{k}{\sqrt{\lambda}} = \frac{\kappa}{\pi} \gg 1 \quad \xrightarrow[\kappa=\frac{4+\sigma^2}{\sigma\sqrt{8-\sigma^2}}]{} \quad \sigma \to 0 \quad \& \quad k = \frac{\sqrt{2\lambda}}{\pi\sigma} \quad \Rightarrow \quad \mathbf{h}^{(weak)} = \mathbf{h}^{(strong)} = -\frac{\sqrt{2}}{24 g_{YM}^2}\frac{\lambda^{3/2}}{\pi^3\sigma^3}$$

1pt function of CPO's

Expanding weak & strong coupling expressions near $\sigma = 0$, we find agreement!!!

For $\Delta = 4$

$$\langle \mathcal{O}_4(r') \rangle^{(weak)} = \langle \mathcal{O}_4(r') \rangle^{(strong)} = \sqrt{\frac{3}{10}}\,\frac{\sqrt{\lambda}}{\pi\sigma^5}\,\frac{1}{r'^4} + \cdots$$

Comments:

- dots denote subleading terms in the large $\frac{k}{\sqrt{\lambda}} \sim \frac{1}{\sigma}$ expansion - generically do not agree between weak and strong coupling
- we have checked that the agreement persists at least up to $\Delta = 40$

the leading term of the 1pf for CPOs of generic dimension $\Delta = 2l$ is

$$\langle \mathcal{O}_\Delta(r') \rangle^{(weak)} = \langle \mathcal{O}_\Delta(r') \rangle^{(strong)} = (-1)^{\frac{\Delta}{2}} \sqrt{\frac{\Delta+2}{\Delta\,(\Delta+1)}}\, \frac{\sqrt{\lambda}}{\pi\,\sigma^{\Delta+1}}\, \frac{1}{r'^{\Delta}} = (-1)^{\frac{\Delta}{2}} \sqrt{\frac{\Delta+2}{\Delta\,(\Delta+1)}}\, \frac{\pi^{\Delta}\, k^{\Delta+1}}{2^{(\Delta+1)/2} \lambda^{\Delta/2}}\, \frac{1}{r'^{\Delta}}$$

Expression derived by expanding the weak coupling result

Conclusions & future directions

New holographic duality: a non-SUSY defect CFT & its gravity dual

FT side: defect is a flat 2d plane $\Rightarrow \mathbb{R}^{(1,1)}$ or $\mathbb{R}^2$

Gravity side: novel solution of a D5 brane depending on a parameter
$0 < \sigma \leq 1 \Rightarrow$ wraps an $S^2 \subset S^5 \Rightarrow$ carries k units of flux $\Rightarrow$ induced metric
symmetry $AdS_3 \times S^1 \times S^2 \Rightarrow$ brane ends on $\mathbb{R}^{(1,1)} \Rightarrow$ codim-2 defect

Holographically calculated the 1pf function of $T_{\mu\nu}$ & CPO

Weak coupling: Calculations using the classical solution that is dual to D5 brane

Appropriate limit $\Rightarrow$ agreement between the weak & strong coupling results

Agreement provides strong evidence in favor of the proposed duality

In our case $\langle T^m{}_m \rangle = 0 \Leftarrow$ both the spacetime & the defect itself are flat

Generically

2d defect $(y^a, a = 1,2)$ embedded in a D-dim space $(X^m(y), m = 0,1,\cdots D-1) \Rightarrow$

Weyl anomaly associated with the defect is

$$\langle T^m{}_m \rangle_{\text{def}} = -\frac{1}{24\pi}\left(b\,\mathcal{R}_{def} + d_1\, Y^\mu_{ab} Y^{ab}_\mu - d_2\, W_{ab}{}^{ab} \right)$$

defect's intrinsic scalar curvature

traceless part of the second fundamental form

pullback of the Weyl tensor to the defect

b, d_1 & $d_2 \Rightarrow$ <u>defect central charges</u>

$$\mathbf{h} = -\frac{1}{2\pi}\frac{1}{3\mathrm{vol}(\mathbb{S}^{D-3})}\frac{D-3}{D-1}d_2$$

Strong/weak coupling agreement for d_2 Weyl anomaly coefficient

D is the # of dimensions that the defect lives $\Rightarrow$ D=4

Future directions

Calculate the remaining anomaly coefficients - b & d_1

Include a new type of defect D5 brane that interpolates between the D3/D3 & D3/D5 intersections symmetry of the D5 brane $\Rightarrow AdS_3 \times S^1 \times S^2$

brane solution depends on <u>2 parameters</u> σ & ρ $\Rightarrow$ a combination of σ & ρ determines the value of an angle $\tilde{\psi}_0 \Rightarrow$ radius of S^2/S^1 is $\cos\tilde{\psi}_0/\sin\tilde{\psi}_0$

as the radius of S^2 increases the radius of the S^1 decreases

two endpoints

$$\rho = \sqrt{\frac{8\sigma^2 + 8}{8 - \sigma^2}}$$

$\cos\tilde{\psi}_0 = 1 \Rightarrow$ radius of the S^2 becomes maximal & S^1 shrinks to a point

$$ds^2_{ind} = \frac{1}{r^2\sigma^2}\left(-dx_0^2 + dx_1^2 + (1+\sigma^2)dr^2\right) + \frac{1}{\sigma^2}d\psi^2 + (d\beta^2 + \sin^2\beta\, d\gamma^2)$$

$\rho \to 1 \Rightarrow \cos\tilde{\psi}_0 = 0 \Rightarrow$ size of the S^2 goes to zero $\Rightarrow$ 2 of the directions of the D5-brane shrink to zero $\Rightarrow$ D3-brane

$$ds^2_{ind} = \frac{1}{r^2\sigma^2}\left(-dx_0^2 + dx_1^2 + (1+\sigma^2)dr^2\right) + \left(1 + \frac{1}{\sigma^2}\right)d\tilde{\gamma}^2$$

SUSY D3 brane solution which realizes the gravity dual of the Gukov-Witten surface operators

Thank you

Supersymmetry

Study Poincare & conformal SUSY's preserved by the solution

Study the SUSY variation of the gaugino

# of independent spinors for which the SUSY variation is zero is the # of the preserved (super)-conformal symmetries

CFT metric: $ds^2 = -(dx^0)^2 + (dx^1)^2 + dr^2 + r^2$ with $\delta\psi = \left(\frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu} + D_\mu\varphi_i\Gamma^{\mu\, i+3} - \frac{i}{2}[\varphi_i, \varphi_j]\Gamma^{i+3\, j+3}\right)\epsilon_1$

Poincare SUSY variation

Inserting the classical solution

μ runs from 0 to 3 while i runs from 1 to 6

$$\delta\psi = -\frac{x_2'}{r'^3}\Gamma^{2,i+6}\epsilon_1 \otimes \mathcal{T}_i - \frac{x_3'}{r'^3}\Gamma^{3,i+6}\epsilon_1 \otimes \mathcal{T}_i + \frac{1}{2\sqrt{2}\, r'^2}\epsilon_{ijl}\Gamma^{i+6,j+6}\epsilon_1 \otimes \mathcal{T}_l \quad \text{with} \quad \mathcal{T}_i = \begin{bmatrix} (t_i)_{k\times k} & 0_{k\times(N-k)} \\ 0_{(N-k)\times k} & 0_{(N-k)\times(N-k)} \end{bmatrix}_{N\times N}$$

focusing on the term proportional to

$$\frac{x'^2}{r'^3} \Rightarrow \delta\psi = 0 \Rightarrow \Gamma^{2,i+6}\epsilon_1 = 0 \Rightarrow \epsilon_1 = 0$$

None of the 16 SUSY's of the background is preserved in the presence of the defect

Analysis of the superconformal SUSY's transformations ⇒ all superconformal symmetries are broken