Features of Gravitini and Supersymmetry on de Sitter Spacetime

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Based on 2503.04515 [A. Higuchi and V. A. Letsios]
Also: 2303.00420 [V. A. Letsios], 2206.09851 [V. A. Letsios], 2510.19652 [D. Anninos, C. Baracco, V. A. Letsios, G. A. Silva]

Motivation to study field theory in de Sitter spacetime

- 4-dimensional de Sitter spacetime (dS_4) $\rightarrow$ maximally symmetric solution of vacuum Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 0$$

  with positive cosmological constant $\Lambda = \frac{3}{\mathscr{R}^2}$
- $\mathscr{R}$ is the 'de Sitter radius'
- Relevant to inflationary cosmology and to the accelerated expansion of the current Universe.

Background material: dS spacetime as a hyperboloid

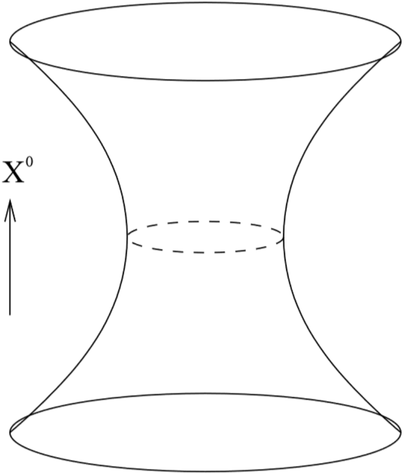
- dS_4 can be represented as one-sheeted hyperboloid embedded in $\mathbb{R}^{4,1}$

$$-(X^0)^2 + (X^1)^2 + ... + (X^4)^2 = \mathscr{R}^2$$

- Often work in units with $\mathscr{R} = 1$.
- We can draw dS_2 embedded in 3D Minkowski

2D dS manifold embedded in 3D Minkowski

2-dimensional dS embedded in 2+1 Minkowski



dS symmetries $SO(4,1)$

- Elementary "particles" on dS_4 should be identified with UIRs of the dS group $SO(4,1)$.
- Let us denote the dS algebra as $so(4,1)$
- dS spacetime **does not have time translation symmetry** $\rightarrow$ no notion of (global) conserved energy
- Unitarity is well-defined group theoretically $\rightarrow$ has nothing to do with positivity of energy
- A convenient coordinate system, which covers the whole dS manifold, is

$$ds^2 = -dt^2 + \cosh^2 t \, d\Omega_3^2, \ \ (S^3 \text{ spatial slices})$$

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dS spacetime vs Supersymmetry (SUSY)

- Unlike in AdS and Minkowski spacetimes, formulating supersymmetric theories in dS presents fundamental challenges.

Two main obstacles for unitary, unbroken dS SUSY

- **1) Unitarity problems of global SUSY on fixed dS.** Follows from abstract rep. theory [1]: Supersymmetrise dS algebra $so(4,1)$ with spinorial supercharges $Q_A \to$ find $\sum_A \{Q_A, Q^{A\dagger}\} = 0 \to$ does **not** admit unitary representations. Expected because of absence of dS energy. **(Unitarity no-go theorem)**.
- **2) Unitarity problems of dS_4 Supergravity.** $N=2$ Supergravity action with a positive Λ exists (real vierbein, real photon, and two symplectic Majorana gravitini). Although action is SUSY invariant, the photon kinetic term has the wrong sign.

[1] [Lukierski, Nowicki; Pilch, van Nieuwenhuizen, Sohnius]

Avoiding the no-go theorem for global SUSY in dS_4

There is a known way out for unitary global SUSY in dS_4.

- Enlarge $so(4,1)$ to the conformal algebra $so(4,2) \supset so(4,1)$.
  Anti-commutator of supercharges closes on $so(4,2)$ $\rightarrow \sum_A \{Q_A,\ Q^{A\dagger}\} \geq 0$
- Unitary representations exist $\rightarrow$ realised in superconformal QFTs on fixed dS_4 [2].
- **NEW RESULT:** We used this observation to obtain a unitary supersymmetric QFT, but not in the superconformal context $\sim so(4,2)$ realised via conformal-like transformations

[2] Anous, Freedman and Maloney 2014

New result

New unitary QFT on fixed dS_4 that includes (a version of) the fields of the SUGRA multiplet [Higuchi, Letsios]:

The free supersymmetric theory of the chiral graviton and chiral gravitino fields.

Plan of the talk

- **1) Discuss separately the ingredients of the supersymmetric theory:**
  - **1A) Free gravitino (spin-3/2)**
  - **1B) Free graviton (spin-2)**
- **2) Introduce SUSY transformations mixing the fields among themselves:** Demonstrate group-theoretic unitarity.

1A) The free gravitino field on dS_4

Massless spin-3/2 field (gravitino) on dS_4 - EOM

- On dS_4, the Rarita-Schwinger equations are:

$$\gamma^{\mu\rho\sigma}\left(\nabla_\rho + \frac{i}{2}\gamma_\rho\right)\Psi_\sigma = 0.$$

- Ψ_μ is a vector-spinor gauge potential with gauge symmetry $\delta\Psi_\mu = (\nabla_\mu + \frac{i}{2}\gamma_\mu)\lambda$.
- Imaginary mass parameter! There is **no** Majorana reality condition. We will use a complex 'Dirac' field Ψ_μ.
- In the TT gauge, the equations take the Dirac-like form

$$\not\nabla\Psi^{(TT)}_\mu = -i\Psi^{(TT)}_\mu, \quad \gamma^\mu\Psi^{(TT)}_\mu = \nabla^\mu\Psi^{(TT)}_\mu = 0.$$

- The gauge-invariant field strength is $F_{\mu\nu} = (\nabla_\mu + \frac{i}{2}\gamma_\mu)\Psi_\nu - \mu \leftrightarrow \nu$.

Quantisation of the gravitino field, and the chirality constraint

• Solution space of EOM carries a representation of the dS algebra $so(4,1)$.
• To understand this, find mode solutions, in coord. system

$$ds^2 = -dt^2 + \cosh^2 t \; d\Omega^2$$

in the Coulomb-like gauge: $\not\nabla \Psi_j^{(TT)} = -i\Psi_j^{(TT)}, \quad \gamma^j \Psi_j^{(TT)} = \nabla^j \Psi_j^{(TT)} = 0, \quad \Psi_t^{(TT)} = 0$.

Quantisation the gravitino field, and the chirality constraint

• As is customary (in Minkowski and dS), there are **two 'chiral' irreducible representations** of the dS algebra, formed by mode solutions:

$$\{ \underbrace{\psi^{(-\ell,m)}}_{\text{helicity -3/2}} \}_{\ell=1,\ldots} \quad \text{and} \quad \{ \underbrace{\psi^{(+\ell,m)}}_{\text{helicity +3/2}} \}_{\ell=1,\ldots}$$

• $\ell = 1, \ldots$ is the angular momentum on the S^3 spatial slice. (m: other quantum numbers)

• **Irreducibility:** Let ξ be a dS Killing vector ∼ spinorial Lie derivative $\mathscr{L}_\xi$ generates dS transformations. Irreducibility means

$$\mathscr{L}_\xi \psi_j^{(\sigma\ell,m)} \equiv \xi^\rho \nabla_\rho \psi_j^{(\sigma\ell,m)} + \cdots = \sum_{\ell',m'} c_{\ell',m'}^{(\ell,m)} \, \psi_j^{(\sigma\ell',m')}, \quad \sigma = \pm.$$

Quantisation of the gravitino field, and the chirality constraint

• **Unitary irrep** $\equiv$: **positive-definite** inner product + generators (Lie derivatives) are **anti-hermitian**, a.k.a. dS invariant inner product.

• Using the dS invariant inner product, one would expect positivity, but...

$$\left(\psi^{(\sigma\ell,m)}|\psi^{(\sigma'\ell',m')}\right)_{axial}$$
$$= \int_{S^3} \sqrt{-g}\, d^3\boldsymbol{\theta}\, \left(\psi_j^{(\sigma\ell,m)}(t,\boldsymbol{\theta})\right)^{\dagger} \gamma^5 \psi^{(\sigma'\ell',m')j}(t,\boldsymbol{\theta})$$
$$= -\sigma\, \delta_{\sigma\sigma'}\delta_{\ell\ell'}\delta_{mm'}, \quad \text{where } \sigma = \pm. \text{ Indefinite norm!}$$

• Unlike Minkowski and AdS spacetime, the positivity/negativity of norm depends on helicity.

Quantisation of the gravitino field, and the chirality constraint

• How can we achieve unitarity in the QFT Fock space?

• Impose on-shell **anti-self-duality constraint** on the field strength

$$\widetilde{F}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma} = -i\ F_{\mu\nu}$$

• **Removes negative norm states** (half of the helicities) from the QFT Fock space.

The chiral gravitino gauge potential and its unitarity

• The **quantum chiral gravitino potential** that is consistent with the anti-self-duality constraint is expanded as:

$$\Psi_j^{(TT)-}(t,\boldsymbol{\theta}) =$$

$$\sum_{\ell,m}\left(a^{-}_{\ell,m}\underset{\text{-3/2}}{\psi_j^{(-\ell,m)}}(t,\boldsymbol{\theta}) + b^{+\,\dagger}_{\ell,m}\ \underset{+3/2}{\psi_j^{(+\ell,m)C}}(t,\boldsymbol{\theta})\right),$$

where $\{a^{-}_{\ell,m}, a^{-\dagger}_{\ell',m'}\} = \{b^{+}_{\ell,m}, b^{+\dagger}_{\ell',m'}\} = \delta_{\ell\ell'}\delta_{mm'}$.

Gravitino summary

To sum up:
• The chiral gravitino on dS_4 describes a unitary QFT (also microcausal).
• Half of the helicities were removed with chirality constraint, unlike in AdS and Minkowski.
• $so(4,1)$ UIRs in the Hilbert space.
• Alternative quantisation, without assuming existence of local Lagrangian, bypasses the indefinite norm problem [Anninos, Baracco, Letsios, Silva]

1B) The free graviton field on dS_4

The real graviton

- The graviton is well-studied on dS_4. It corresponds to a direct sum of UIRs of $so(4,1)$.
- Real symmetric spin-2 gauge potential $h_{\mu\nu}$ that satisfies the lin. Einstein equations.
- Gauge invariance $\delta h_{\mu\nu} = \nabla_{(\mu} K_{\nu)}$ (linearised diffeomorphisms).
- Describes two real propagating helicities ± 2.

The complex, chiral, graviton

- For a supersymmetric theory, we need the same number of fermionic and bosonic dof.
- Thus, **consider the chiral graviton field** $\mathfrak{h}^-_{\mu\nu}$.
- Field strength satisfies the **anti-self-duality constraint**

$$\widetilde{U}_{\mu\nu\rho\sigma} \equiv \frac{1}{2}\varepsilon_{\mu\nu}{}^{\alpha\beta}U_{\alpha\beta\rho\sigma} = -iU_{\mu\nu\rho\sigma}$$

- $U_{\mu\nu\rho\sigma}$ is the linearised Weyl tensor, i.e. the gauge-invariant field strength of the graviton

$$U_{\mu\nu\rho\sigma} = \left(\nabla_\rho\nabla_{[\mu}\mathfrak{h}_{\nu]\sigma} + g_{\rho[\mu}\mathfrak{h}_{\nu]\sigma}\right) - \rho \leftrightarrow \sigma$$

Quantising the chiral graviton

- We can quantise it as in the case of the chiral gravitino
- Fixing the Coulomb-like gauge $\mathfrak{h}_{tt}^{(TT)-} = \mathfrak{h}_{tj}^{(TT)-} = 0$, $g^{ij}\mathfrak{h}_{ij}^{(TT)-} = 0$, $\nabla^j \mathfrak{h}_{ji}^{(TT)-} = 0$:

$$\mathfrak{h}_{ij}^{(TT)-}(t,\boldsymbol{\theta}) = \sum_{L,M} \left(c_{L,M}^{-} \underset{-2}{\mathfrak{h}_{ij}^{(-L,M)}}(t,\boldsymbol{\theta}) + d_{L,M}^{+\,\dagger} \underset{+2}{\mathfrak{h}_{ij}^{(+L,M)*}}(t,\boldsymbol{\theta}) \right)$$

- $L = 2, 3, \ldots$ is the angular momentum on S^3.
- $[c_{L,M}^{-}, c_{L',M'}^{-\,\dagger}] = \delta_{LL'}\delta_{MM'}$, $[d_{L,M}^{+}, d_{L',M'}^{+\,\dagger}] = \delta_{LL'}\delta_{MM'}$

Chiral graviton summary

• Chiral graviton corresponds to a unitary QFT (also microcausal).

• Single-particle Hilbert space furnishes two UIRs of $so(4,1)$ with helicities -2 and $+2$ (same as in the case of real graviton).

• Real graviton and chiral graviton are related as $h_{\mu\nu} = \mathfrak{h}^{-}_{\mu\nu} + {\mathfrak{h}^{-}_{\mu\nu}}^{\dagger}$

2) Unitary SUSY for the chiral graviton and chiral gravitino

Looking for SUSY

• Now that we have a bosonic and a fermionic field with the same dof, can we find SUSY between them?

schematically: $\delta^{susy}\mathfrak{h}^{-}_{\mu\nu} \to \Psi^{-}_{\mu}, \qquad \delta^{susy}\Psi^{-}_{\mu} \to \mathfrak{h}^{-}_{\mu\nu}$

• i.e. does the solution space of the **chiral** fields carry a representation of SUSY?

• So far, we know that the pair $(\mathfrak{h}^{-}_{\mu\nu}, \Psi^{-}_{\mu})$ carries $so(4,1)$ and $so(4,2)$ UIRs in the **total single-particle Hilbert space** $\mathscr{H} = \mathscr{H}_{boson} \bigoplus \mathscr{H}_{fermion}$. The **pos. definite and invariant inner products** are $(\cdot|\cdot)_{KG}$ (Klein-Gordon inner product) and $(\cdot|\cdot)_{axial}$, respectively.

SUSY transformations for the Chiral graviton and Chiral gravitino on dS_4

- The SUSY transformations are:

$$\delta^{susy}(\epsilon)\Psi^{-}_{\mu} = \frac{1}{4}\left(i\,\mathfrak{h}^{-}_{\mu\sigma}\gamma^{\sigma} + \nabla_{\lambda}\mathfrak{h}^{-}_{\mu\sigma}\gamma^{\sigma\lambda}\right)\epsilon,$$

$$\delta^{susy}(\epsilon)\mathfrak{h}^{-}_{\mu\nu} = \bar{\epsilon}\gamma^{5}\gamma_{(\mu}\Psi^{-}_{\nu)},$$

where the parameters ϵ are complex Killing spinors.

- They map chiral graviton solutions to gravitino ones and vice versa $\rightarrow$ they generate SUSY representation.
- **Superalgebra:** Commutator between two SUSY transformations closes on $so(4,2) \bigoplus u(1)$, not $so(4,1)$:

$$[\delta^{susy}(\epsilon_2), \delta^{susy}(\epsilon_1)]\mathfrak{h}^{-}_{\mu\nu} = -\underbrace{\mathscr{L}_{\xi}}_{isometry}\mathfrak{h}^{-}_{\mu\nu} + \underbrace{\mathscr{T}_{V}}_{conf-like}\mathfrak{h}^{-}_{\mu\nu} - \underbrace{i\;a}_{u(1)}\mathfrak{h}^{-}_{\mu\nu}$$

SUSY transformations for the Chiral graviton and Chiral gravitino on dS_4 - Unitarity?

- New symmetry appeared. SUSY transformations close on $so(4,2)\bigoplus u(1)$, the **unitarity no-go theorem is avoided**.
- **Unitarity must be checked explicitly**. A representation of **SUSY is unitary if**[3]:
  - There are **positive-definite inner products** for both $\mathscr{H}_{boson}$ and $\mathscr{H}_{fermion}$, which are **invariant under the even subalgebra** $(so(4,2)\bigoplus u(1))$ ✓
  - The inner products are SUSY-invariant, in the sense that, for any two solutions $\mathfrak{h}^{-}_{\mu\nu}$ and Ψ^{-}_{μ}, we have:

$$\left(\delta^{susy}(\epsilon)\mathfrak{h}^{-}|\mathfrak{h}^{-}\right)_{KG} = \left(\Psi^{-}|\delta^{susy}(\epsilon)\Psi^{-}\right)_{axial}$$

[3] Furutsu, Hirai 1988

SUSY for the Chiral graviton and Chiral gravitino on dS_4: unitarity ✓

• After long computations, we have proved that both conditions are satisfied.

• We have also shown that the anti-commutator of quantum supercharges is positive: $\sum_A \{Q_A, Q^{A\dagger}\} \geq 0$

• We thus arrive at the main result:

The 'chiral graviton - chiral gravitino ' supermultiplet forms a unitary representation of SUSY.

Final remarks and future directions

- We showed that, contrary to common expectations, it is possible to have a unitary supersymmetric QFT, involving graviton and gravitini on fixed dS_4.
- The main field variables are chiral (and thus complex) gauge potentials.
- The theory is formulated in terms of EOM because of the anti-self-duality constraint.

Final remarks and future directions

- **Open question:** Can we introduce interactions that preserve SUSY? i.e. is there a ‘ non-linear completion’ of our free theory? (supergravity-like theory with local SUSY)

Thank you!

Thank you!

Bonus slides

Conformal-like symmetry for the chiral gravitino gauge potential

• The gravitino also enjoys a larger $so(4,2)$ conformal-like symmetry.[4]

• New **conformal-like transformations** are generated by the 5 non-Killing conf. Killing vectors V^μ of dS_4

•**Not** conventional infinitesimal conf. transformations.

[4] Letsios 2023

Conformal-like symmetry for the chiral gravitino gauge potential

- Conformal-like transformations have the following form:

$$\mathscr{T}_V \Psi_\mu^{(TT)} = \gamma^5 \Big(V^\rho \nabla_\rho \Psi_\mu^{(TT)} + i\, V^\rho \gamma_\rho \Psi_\mu^{(TT)} - i\, V^\rho \gamma_\mu \Psi_\rho^{(TT)} + \frac{3}{2} \frac{\nabla_\rho V^\rho}{4} \Psi_\mu^{(TT)} \Big) - \frac{2}{3} \left(\nabla_\mu + \frac{i}{2} \gamma_\mu \right) \gamma^5 \Psi_\rho^{(TT)} V^\rho,$$

- **Not** of the form: Lie derivative$+$ conf. weight term.
- 15 Generators in total: 5 $\mathscr{T}_V$'s $+$ 10 isometries $\mathscr{L}_\xi \rightarrow$ generate $so(4,2) \rightarrow$ two UIRs with helicities $-3/2$ and $+3/2$.
- Plays an important role in the supersymmetric theory.

Conformal-like symmetry for the chiral graviton

• Conf-like transformations are generated by the 5 non-Killing conf. Killing vectors V^μ of dS_4.

• The conformal-like transformations are given by

$$\mathscr{T}_V \mathfrak{h}_{\mu\nu} = i\, V^\rho \varepsilon_{\rho\sigma\lambda(\mu} \nabla^\sigma \mathfrak{h}^\lambda{}_{\nu)}$$

• **15 Generators in total:** 5 $\mathscr{T}_V$'s + 10 isometries $\mathscr{L}_\xi \to$ generate $so(4,2) \to$ two UIRs with helicities -2 and $+2$.

• The conformal-like symmetry plays an important role in the supersymmetric theory.