

# Analytical Approach to Neutron Yield Estimation in FFA-ERIT-Based Systems

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# Outline

- ✓ Introduction
- ✓ Status of this study
- ✓ Summary

# Background

There has been a notable rise in the requirements for accelerator secondary particle sources.

- Boron Neutron Capture Therapy(BNCT)
- Nuclear physics (e.g. nuclear data)
- Production of medical radioisotopes

Accelerator neutron sources

【In a traditional method to product neutron by accelerator 】

Beam interact with the target once.



A few beam particles contribute to neutron production.

→ Low neutron production efficiency

# Status of FFA-ERIT system

Low energy ERIT (11 MeV)

(1): K. Okabe, Y. Mori, et al.: Proc. EPAC08, (2008), 3512-3514.

The principle was validated by Kyoto University<sup>(1)</sup>.

- Neutron production when  $6.25 \times 10^{12}$  proton particles are injected to W-184 target. This was calculated with PHITS.<sup>(2)</sup>

Beam energy [MeV]	Neutron flux with the conventional method [ $1/\text{cm}^2/\text{s}$ ] s
10.0	$3.93 \times 10^6$
100	$5.95 \times 10^9$

Higher beam energy leads to increased neutron production and spread of energy spectrum.

It is expected to

- ✓ Shorten the neutron irradiation time (BNCT)
- ✓ Use it in a wider range of application fields (nuclear physics)

# Development of Simulation Code for Basic Design

The parameters used in the design of the FFA-ERIT accelerator are to meet the requirements for secondary particle production and energy spectrum.

## Using simulation code

Setting the magnetic field distribution	the 3D magnetic field calculation code (OPERA-3D)
Orbit calculation	the Runge-Kutta method
Interaction between beam and target	transport calculation code (PHITS)

perform high-precision calculations.

It takes a long time to use it at the basic design stage.

# Development of Simulation Code for Basic Design

Easy to handle for use in the basic design stage

Setting the magnetic field distribution	<ul style="list-style-type: none"><li>➤ approximation by Maxwell's equations</li><li>➤ a linear edge model</li></ul>
Orbit calculation	the Runge-Kutta method
Interaction between beam and target	semi-empirical formulas

Simulation code can be expected to shorten the calculation time but it is necessary to evaluate the calculation accuracy.

In this presentation, we focus on the interaction between the beam and the target.

# Purpose

Evaluate the validity of the calculation accuracy of the simulation code for use in the basic design stage.

## 【evaluation method】

- The increase energy straggling were evaluated using **probability distributions** such as the Gaussian distribution, Landau distribution, and Vavilov distribution.
- The multiple scattering angle distribution was calculated using Lynch's formula from Molière theory.



The results were compared with the rate equation using the results of the semi-empirical equation.

# Outline

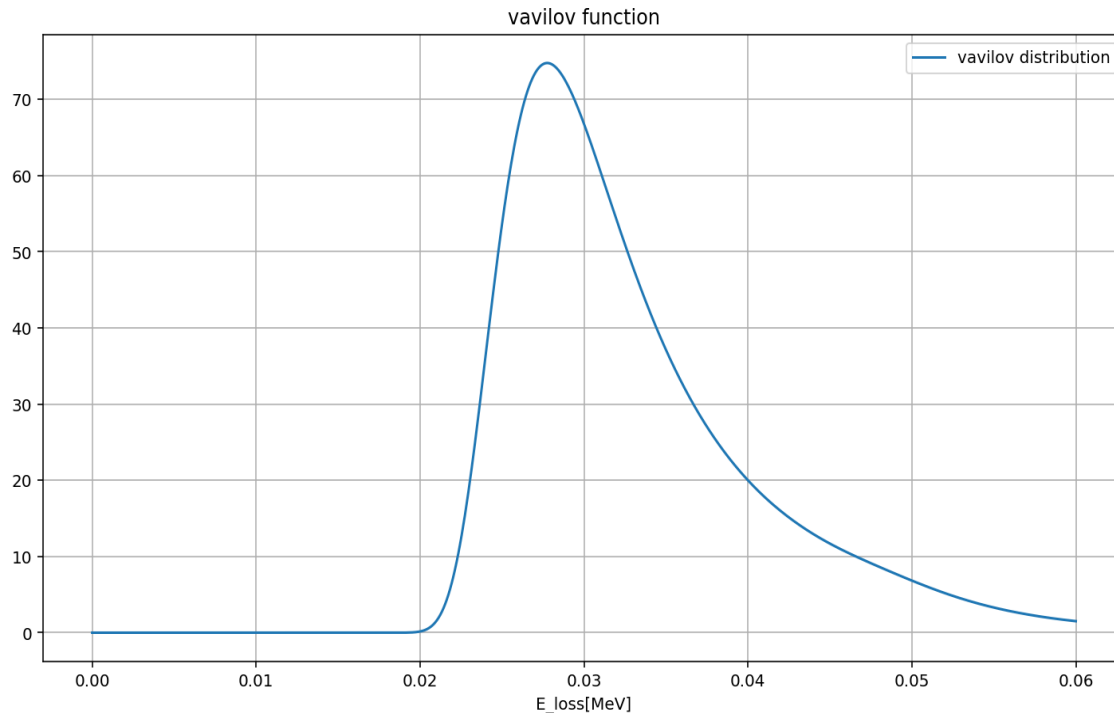
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# Energy loss and Energy straggling

Energy loss and energy straggling at the target were calculated using the Vavilov distribution.[3]

【 Vavilov distribution 】

$$f(x, \Delta) = \frac{1}{\pi\xi} \chi \exp(\chi(1 + \beta^2 C)) \int_0^\infty \exp(\chi f_1) \cos(y\lambda_1 + \chi f_2) dy$$



$$f_1 = \beta^2(\ln y - Ci(y)) - \cos(y) - ySi(y)$$

$$f_2 = y(\ln y - Ci(y)) + \sin(y) + \beta^2 Si(y)$$

$$\lambda_1 = \frac{\Delta - \alpha\chi}{\epsilon_{max}} - \chi(1 + \beta^2 - C)$$

$$\alpha\chi = \xi \left[ \ln \frac{2m_e c^2 \beta^2 \epsilon_{max}}{(1 - \beta^2) I^2(Z)} - 2\beta^2 \right]$$

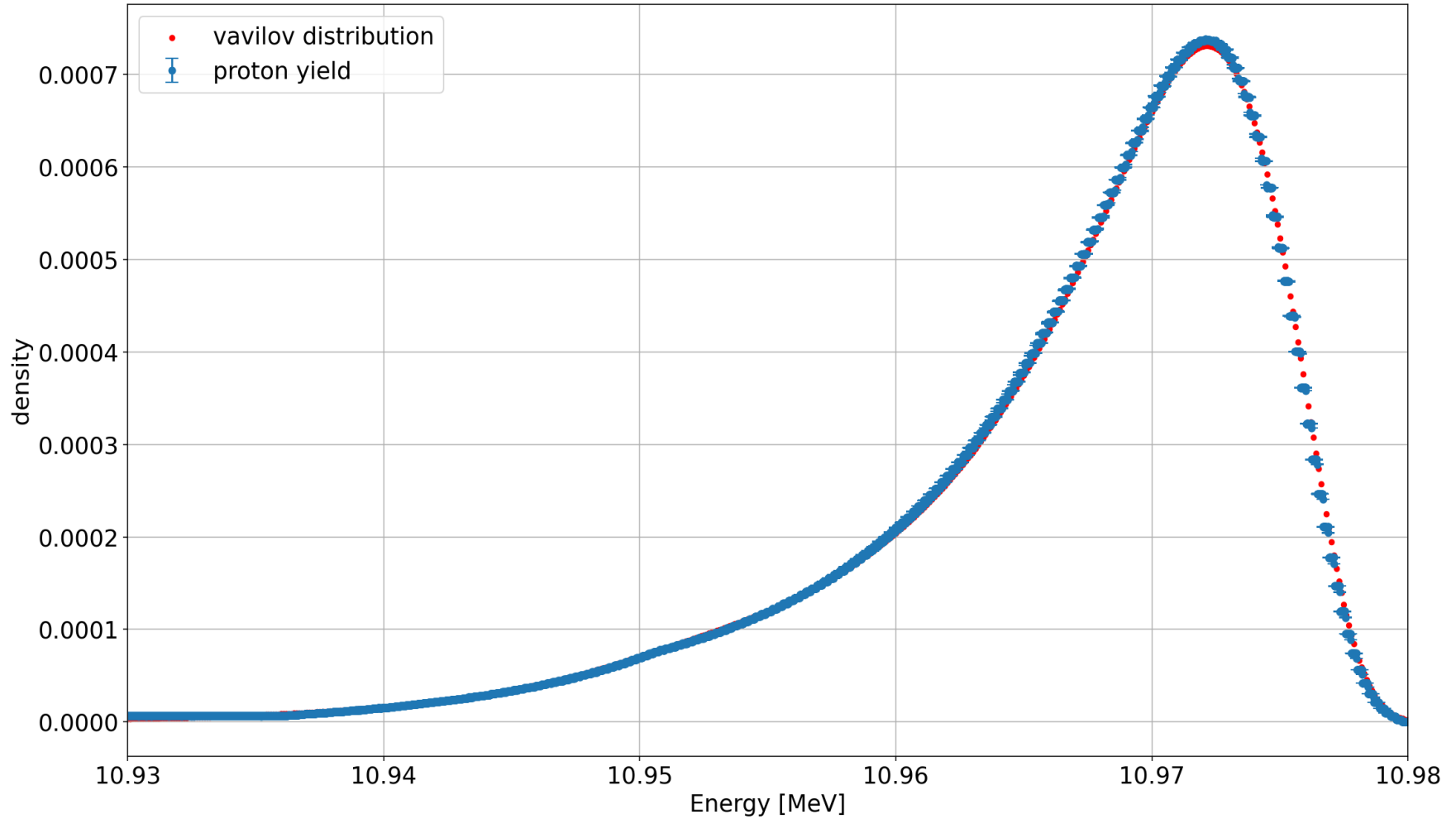
$$\xi = 0.300x \left( \frac{m_e c^2}{\beta^2} \right) Z/A$$

$$\epsilon_{max} = 2m_e c^2 \beta^2 / (1 - \beta^2)$$

# Energy loss and energy straggling

- Vavilov distribution vs PHITS

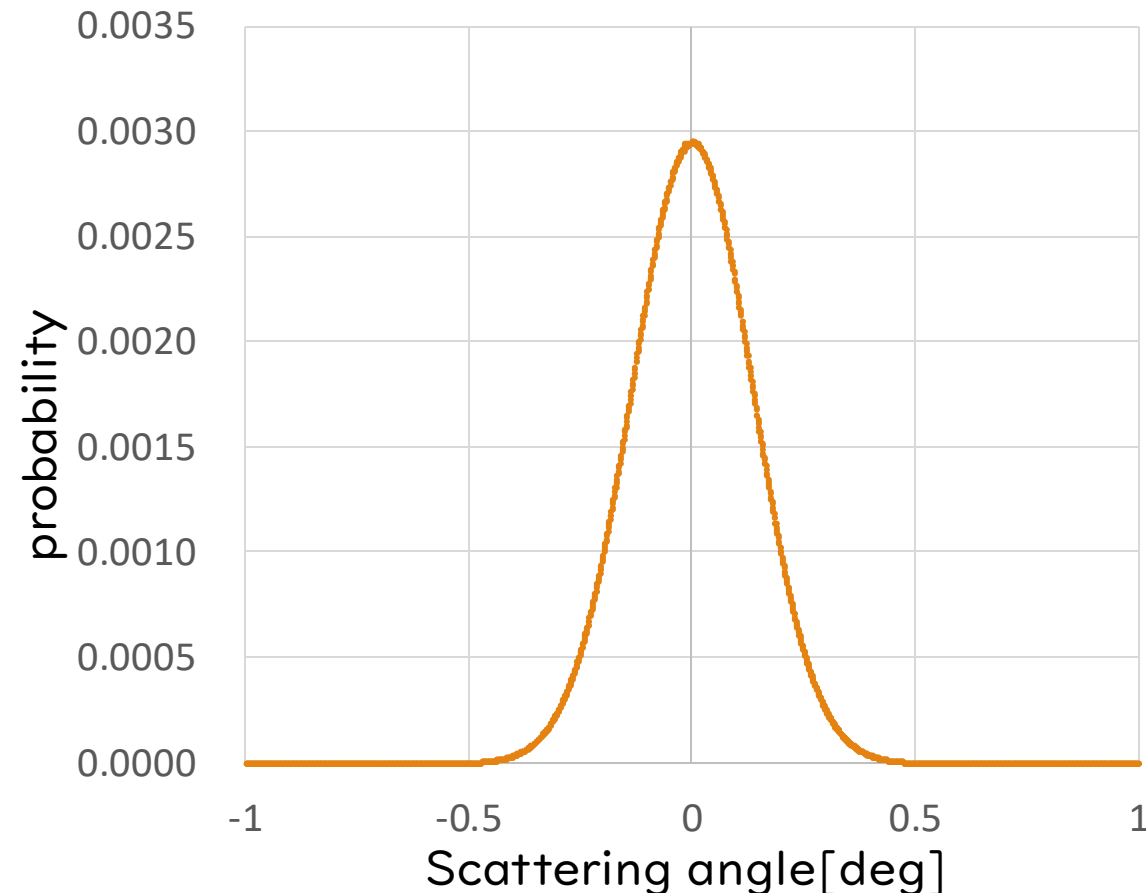
11 MeV proton Be 5  $\mu$ m



# Multiple scattering

The increase in momentum in the beam's vertical direction was calculated using Lynch's equation from the Molière theory.

11 MeV proton Be 5  $\mu$  m



【 Molière theory.】

$$P(\theta) = \frac{1}{\sqrt{2\pi\theta_0^2}} \exp\left(-\frac{\theta^2}{2\theta_0^2}\right) d\theta$$

$$\theta_0 = \frac{13.6}{\beta c p} z \sqrt{\frac{x}{x_0}}$$

$X$ : (thickness)  $\times$  (density)

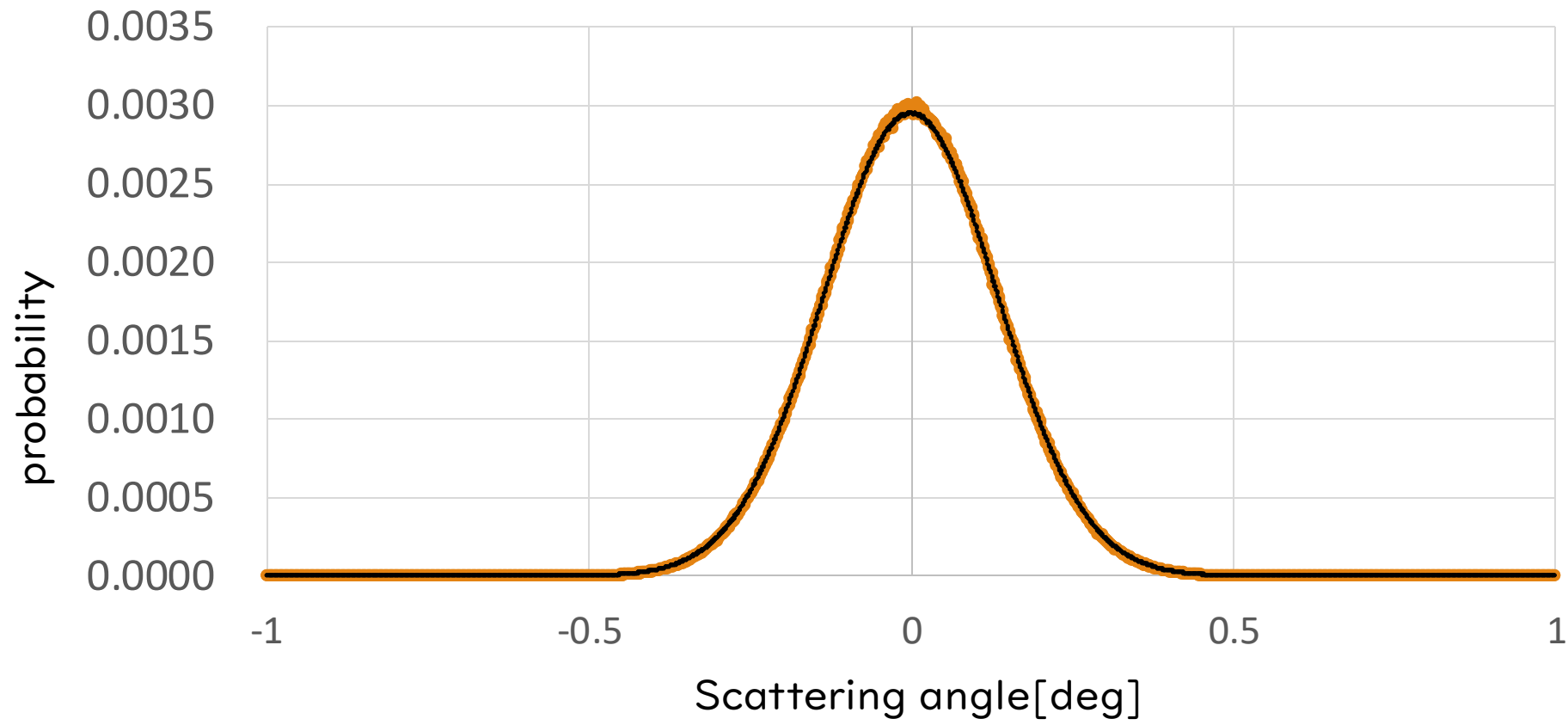
$X_0$ : (radiation length)  $\times$  (density)

$s$ : thickness

# Multiple scattering

- Molière vs PHITS

1 | 1 MeV proton Be 5  $\mu$  m



# Evaluation of 11 MeV FFA-ERIT proof-of-principle ring<sup>(5)</sup>

## 11 MeV FFA-ERIT proof-of-principle ring:

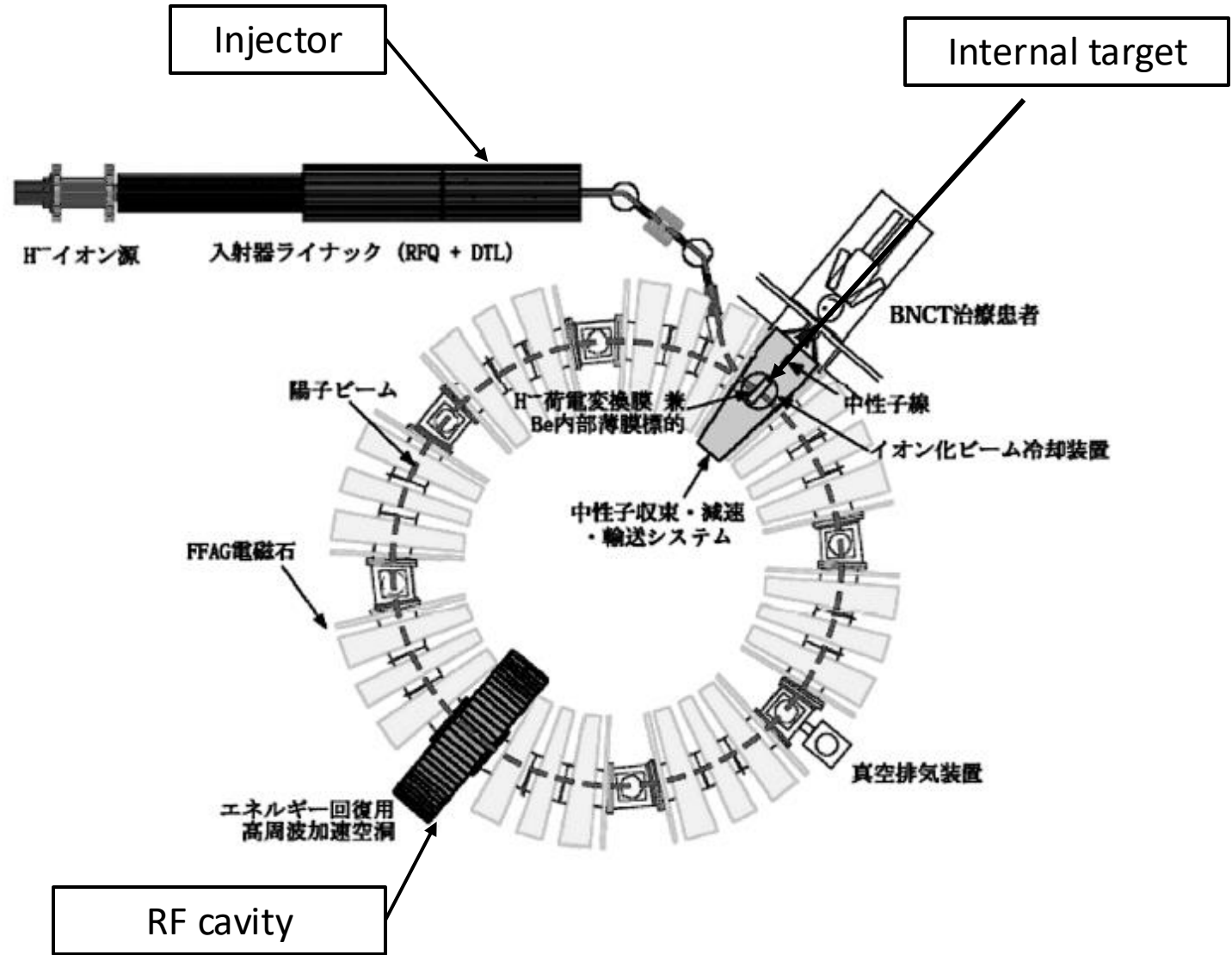
- Stores 11 MeV proton beam.
- Lattice of the storage ring is radial-sector FDF-triplet.
- Physical aperture : (mean radius)  $\pm 0.195\text{m}$ . (r-direction)  
: (median plane)  $\pm 0.06\text{m}$ . (z-direction)

Parameters of 11 MeV FFA-ERIT	
Beam energy	11 MeV
mean radius	2.35 m
number of cells	8
k	1.92
FD ratio	$\sim 3$
$\beta_x, \beta_y$ @target	1.36, 0.79 m

Parameters of RF cavity	
RF voltage	200 kV
frequency	18.3 MHz
Harmonic number	6

Internal target	
material	Be
thickness	5 $\mu\text{m}$

# Components of FFA-ERIT scheme<sup>(6)</sup>



(6): Y. Mori, J. Part. Acc. Soc. Jpn, Vol.5, 2008, 27-35.

# Evaluation of rate equation<sup>(7)</sup>

The following equations are used to estimate by ionization cooling.

【 transverse emittance 】

$$\frac{d\varepsilon_N}{ds} = -\frac{1}{\beta^2 E} \frac{dE}{ds} \varepsilon_N + \frac{\beta\gamma\beta_{\perp}}{2} \frac{d\langle\theta_{rms}^2\rangle}{ds}$$

$$\frac{d\langle\theta_{rms}^2\rangle}{ds} = \frac{\theta^2}{s}$$

$$\theta = \frac{13.6}{\beta c p} z \sqrt{\frac{x}{X_0}}$$

$X$ : (thickness)  $\times$  (density)

$X_0$ : (radiation length)  $\times$  (density)

$s$ : thickness

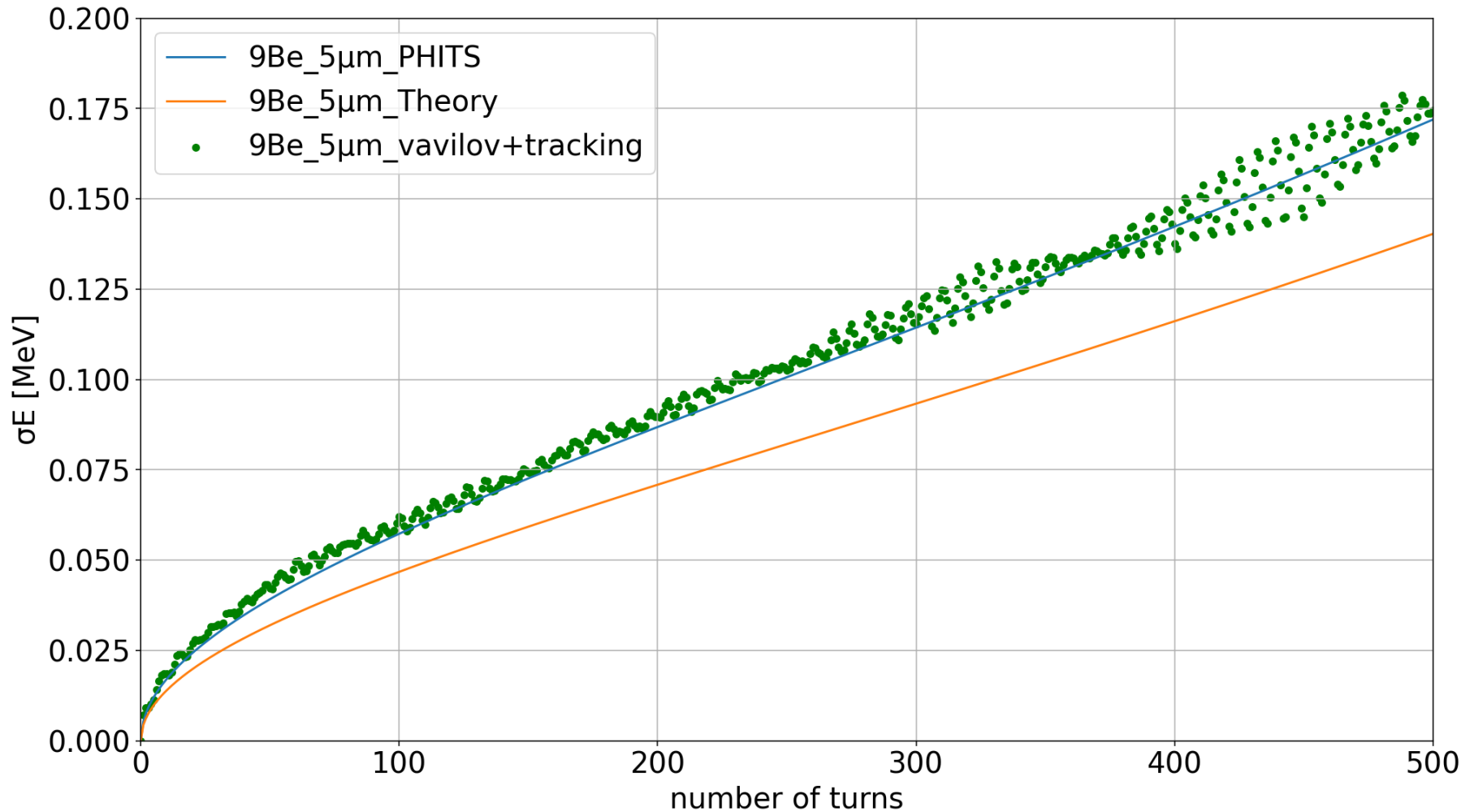
【 longitudinal emittance 】

$$\frac{d\sigma_E^2}{ds} = -2 \frac{\partial}{\partial E} \frac{\partial E}{\partial s} \sigma_E^2 + \frac{d\langle\Delta E_{rms}^2\rangle}{ds}$$

$$\frac{d\langle\Delta E_{rms}^2\rangle}{ds} = 4\pi(r_e m_e c^2)^2 n_e \gamma^2 \left(1 - \frac{\beta^2}{2}\right)$$

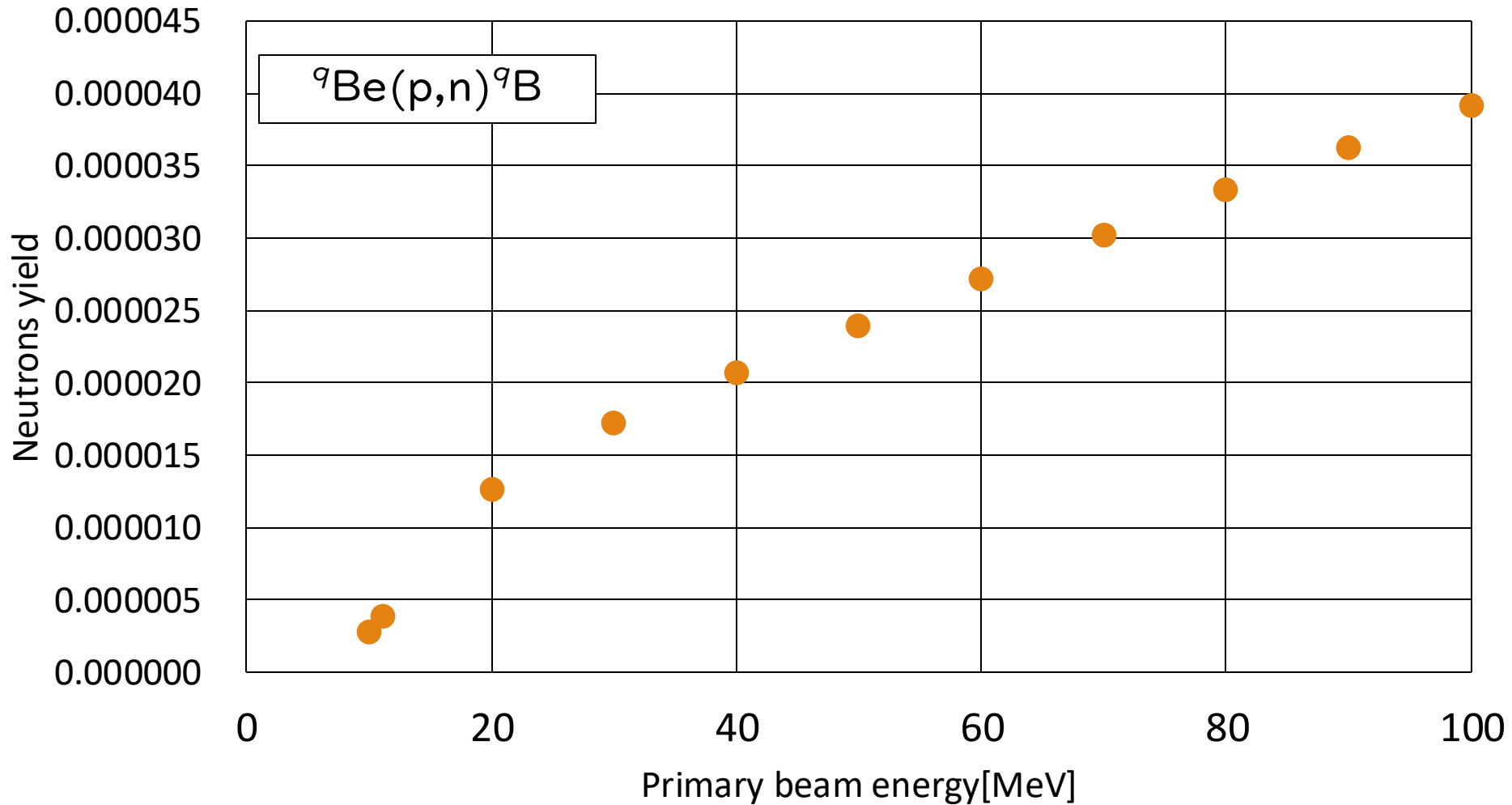
# Comparison of rate equation(11 MeV)

Showing the results of longitudinal emittance for 11 MeV FFA-ERIT proof-of-principle ring.



# Neutron yield

Neutron yield when primary beam energy is increased.



# Increasing primary beam energy

- Stores 25,50 MeV proton beam.
- Lattice of the storage ring is radial-sector FDF-triplet.
- Physical aperture : (mean radius)  $\pm 0.195\text{m}$ . (r-direction)  
: (median plane)  $\pm 0.06\text{m}$ . (z-direction)

## Parameters of FFA-ERIT

Beam energy	25,50 MeV
mean radius	2.35m
number of cells	8
k	1.92
FD ratio	$\sim 3$
Bf/Bd	1.20/1.05 , 1.73/1.26

## Parameters of RF cavity

RF voltage	200 kV
frequency	18.3 MHz
Harmonic number	4, 3

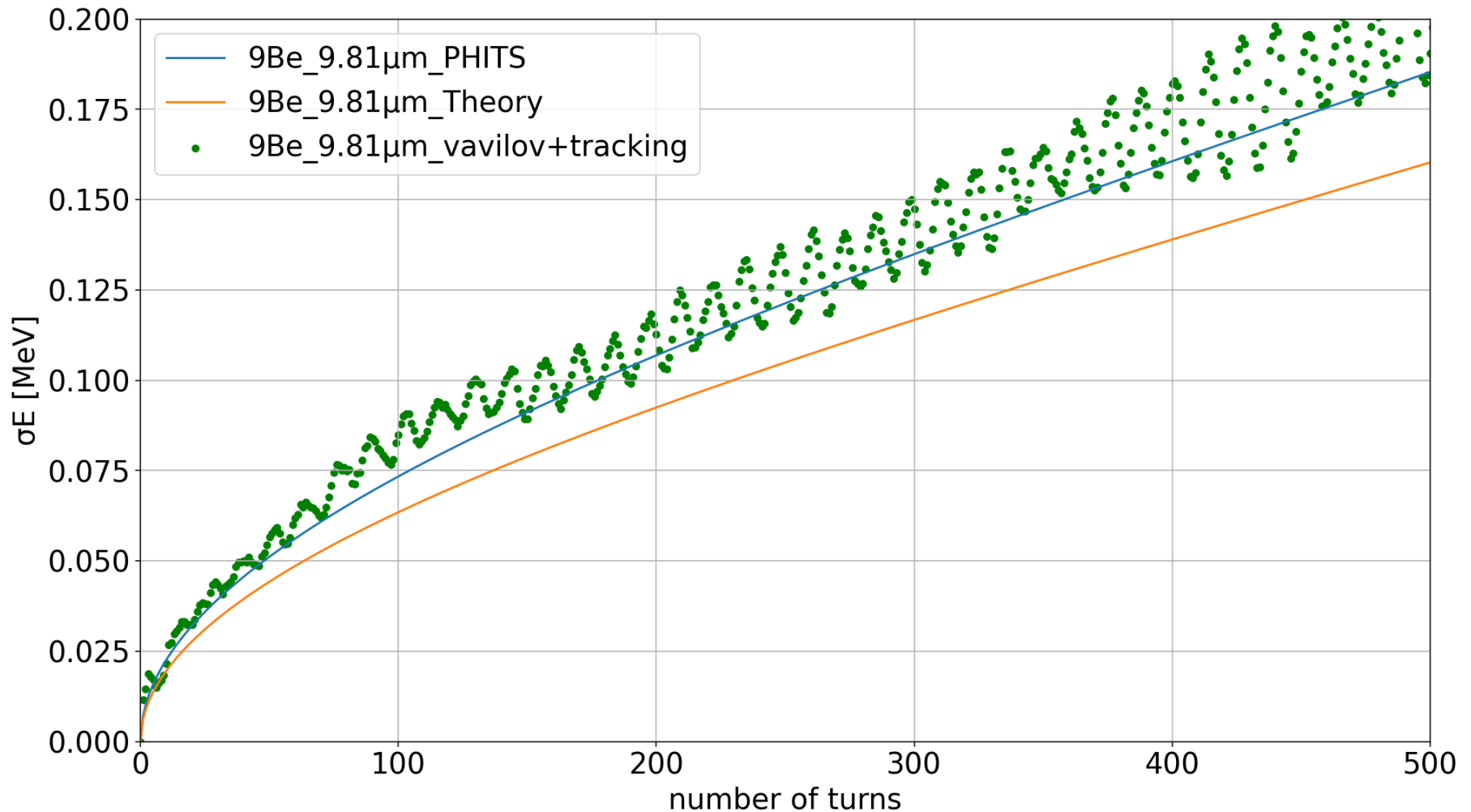
## Internal target

material	Be
Energy loss	32.5keV

- In this presentation, a Be target with a thickness of  $9.81\ \mu\text{m}$  /  $17.24\ \mu\text{m}$  (Energy loss = 32.5 keV) is shown.

# Comparison of rate equation(25MeV)

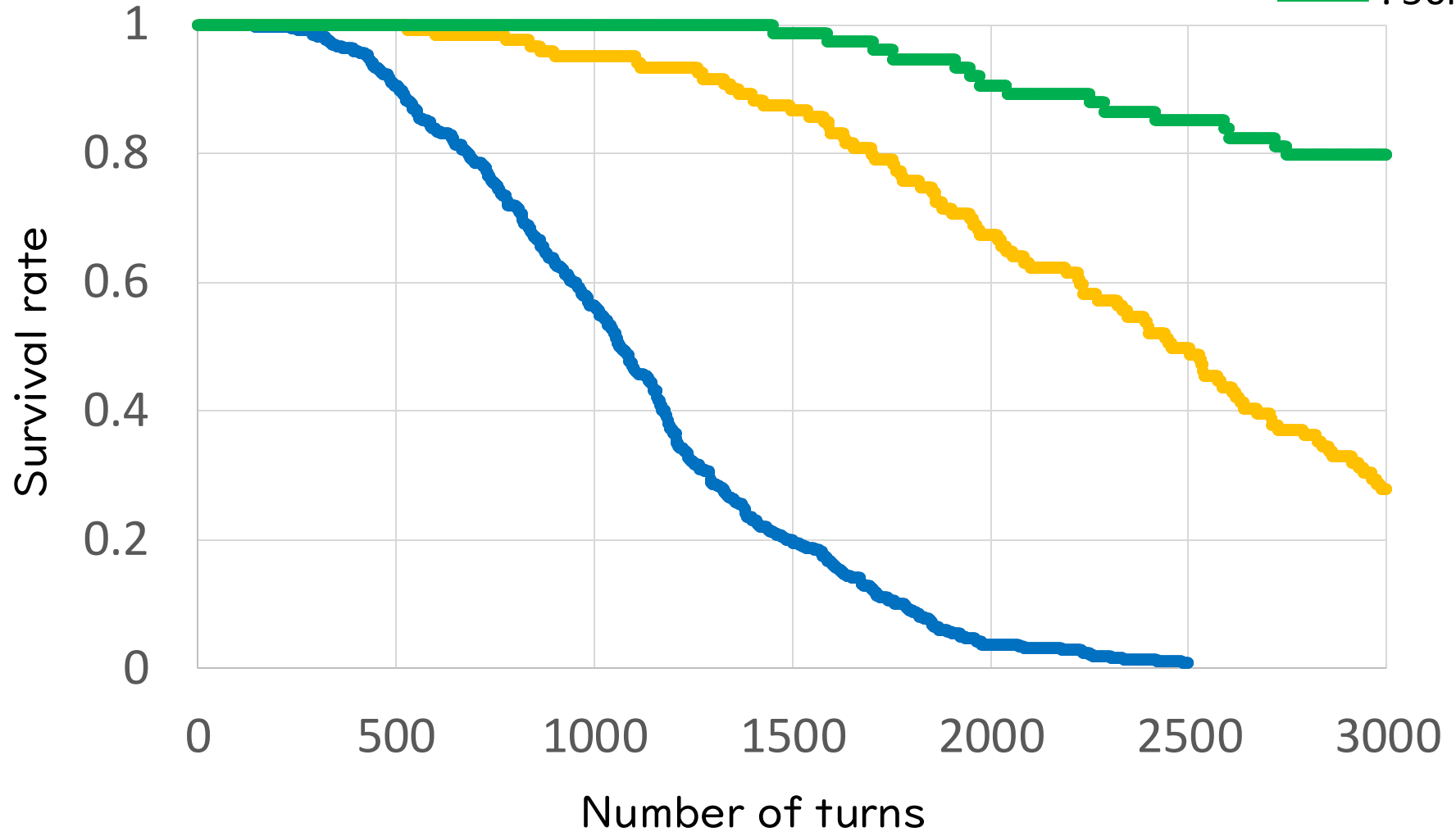
Showing the results of longitudinal emittance for 25 MeV FFA-ERIT accelerator.



# Survival rate for proton

Calculate using developed simulation code for number of turns.

— : 11MeV  
— : 25MeV  
— : 50MeV



# Neutron yield

The rate of increase in neutron yield was calculated from the simulation results.

Beam energy	Neutron yield / turn	Number of turns (half the particles)
11MeV	0.3866E-05	1062
25MeV	1.4962E-05	2456
50MeV	2.3921E-05	4000✂

✂Estimated from calculations up to 3000 turns.

- From 11 MeV to 25 MeV, we can expect about a 9 times increase in neutron yield.
- From 11 MeV to 50 MeV, we can expect about a 23 times increase in neutron yield.✂

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# Summary

A simulation code was developed for the basic design phase of FFA-ERIT systems.

【the calculation accuracy of the simulation code】

- ✓ It calculates energy loss, energy spread, and angular scattering using theoretical formulas, reducing computation time.
- ✓ Results were compared with ionization cooling theory to verify validity.
- ✓ Even with the FFA-ERIT system, it was found that neutron yield increased with increasing primary beam energy.

Thank you for listening.

# Appendix