Characterisation and Mitigation of RF knockout

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UKRI Science and Technology Facilities Council
ISIS Neutron and Muon Source

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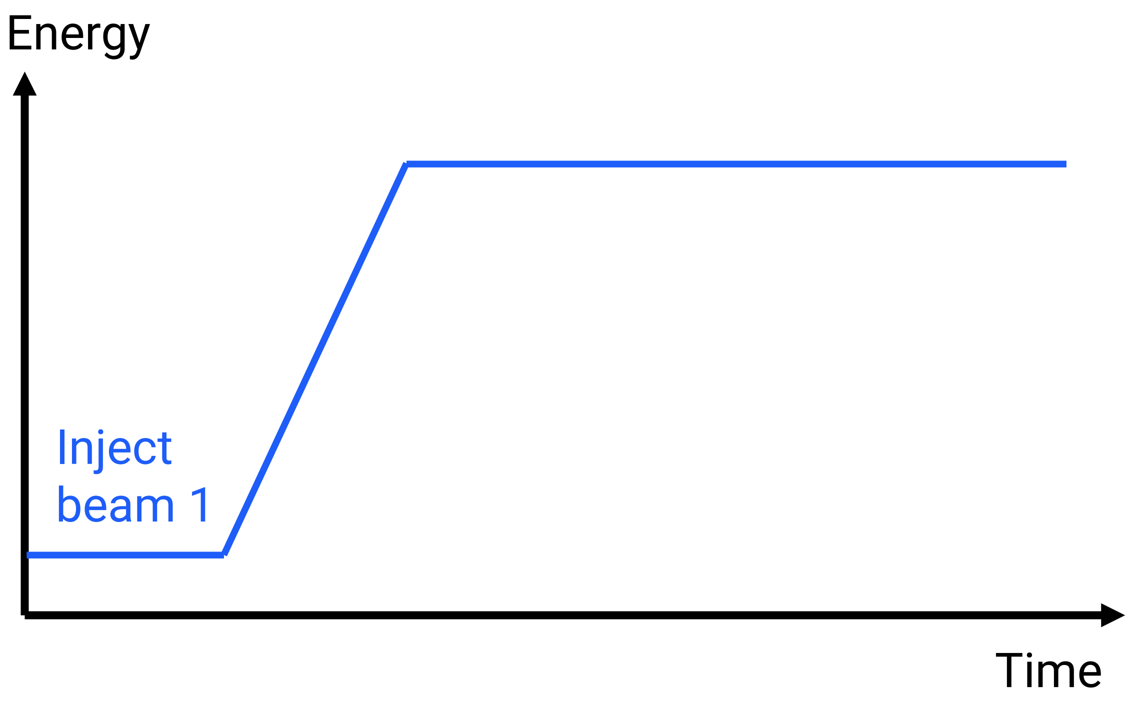

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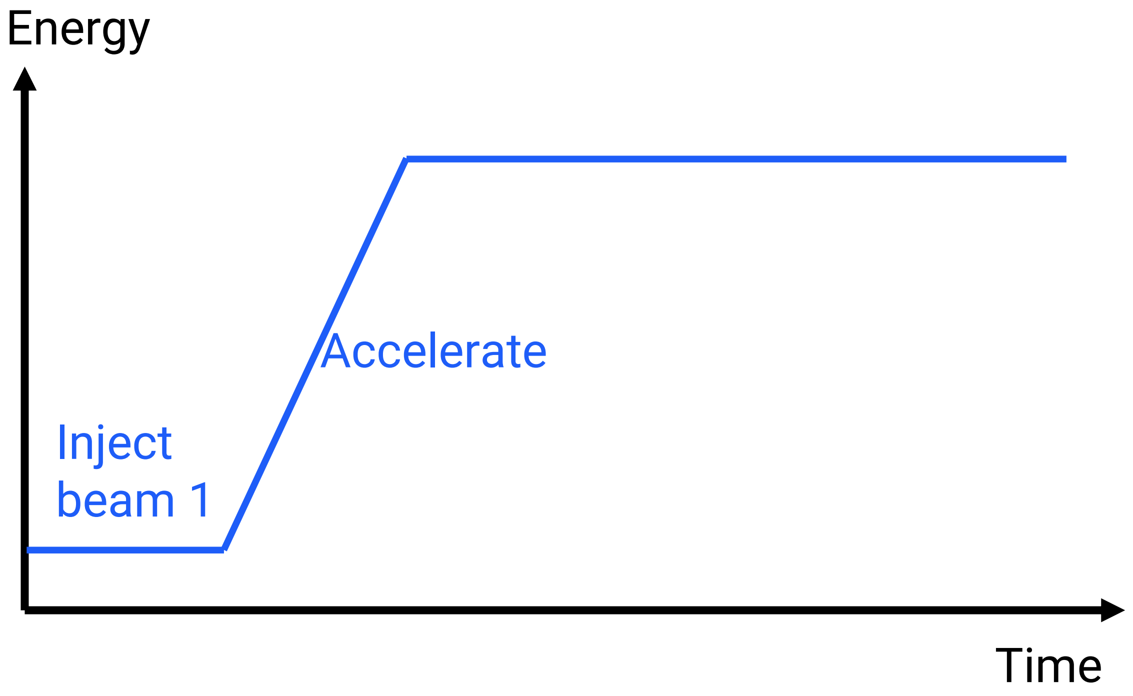

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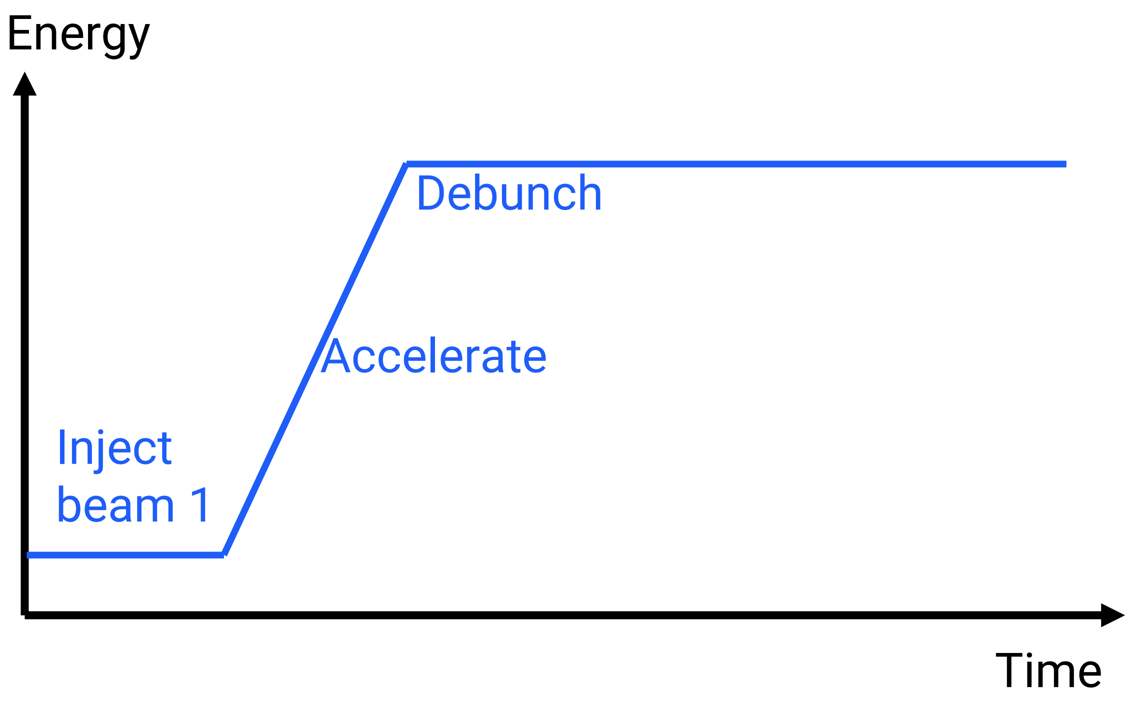

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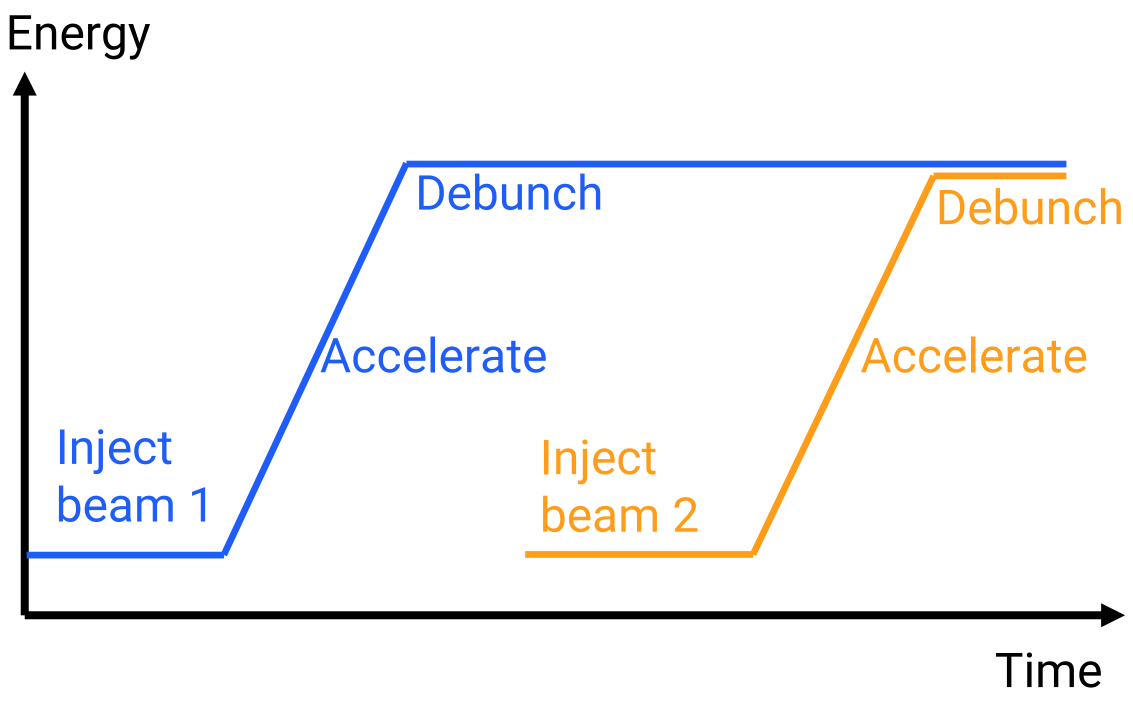

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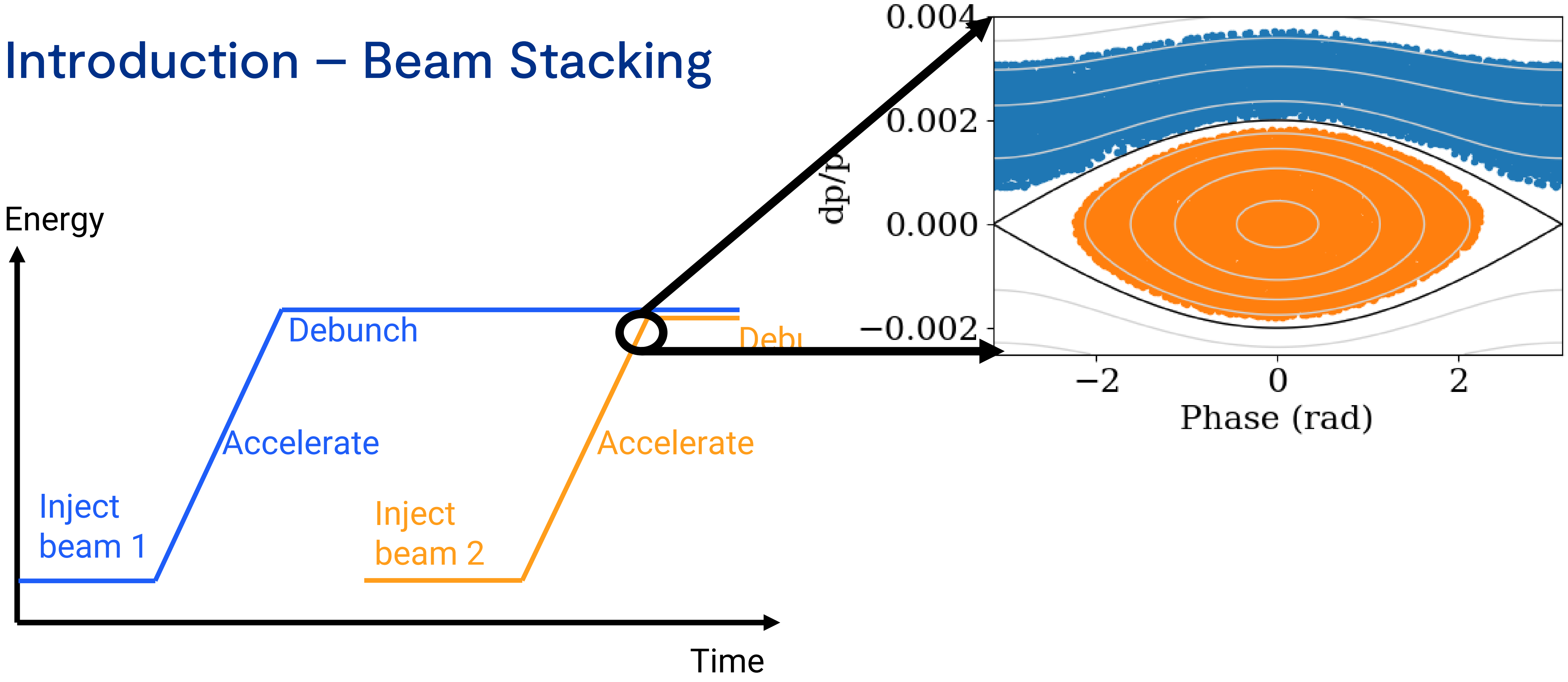

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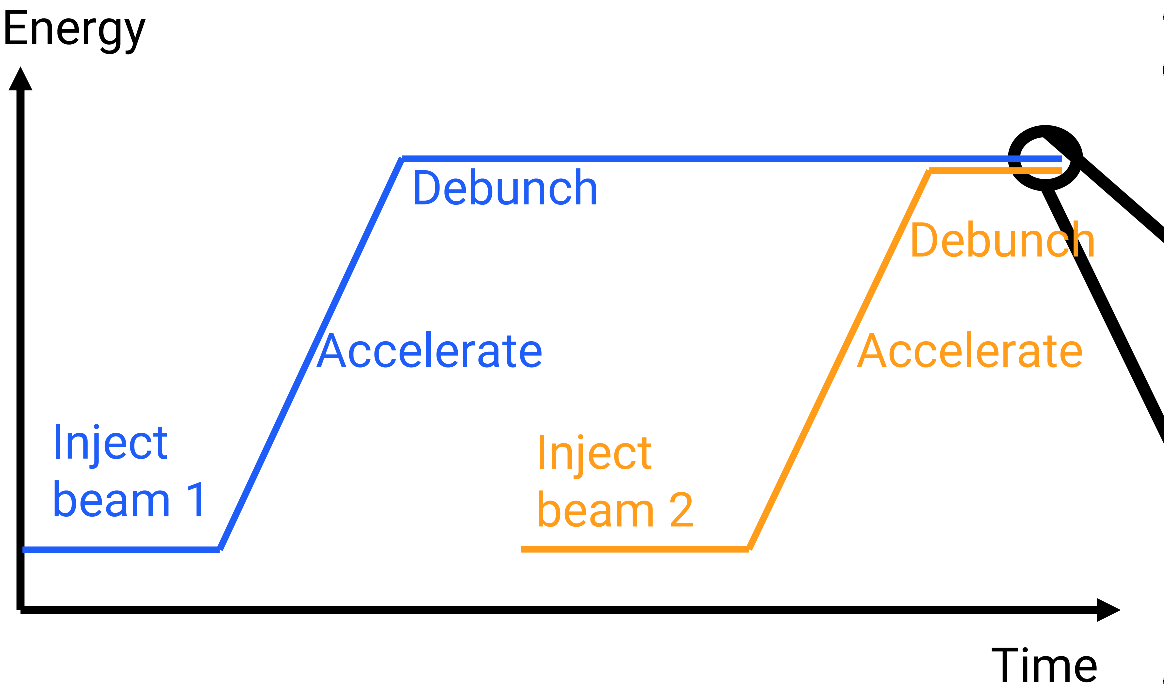
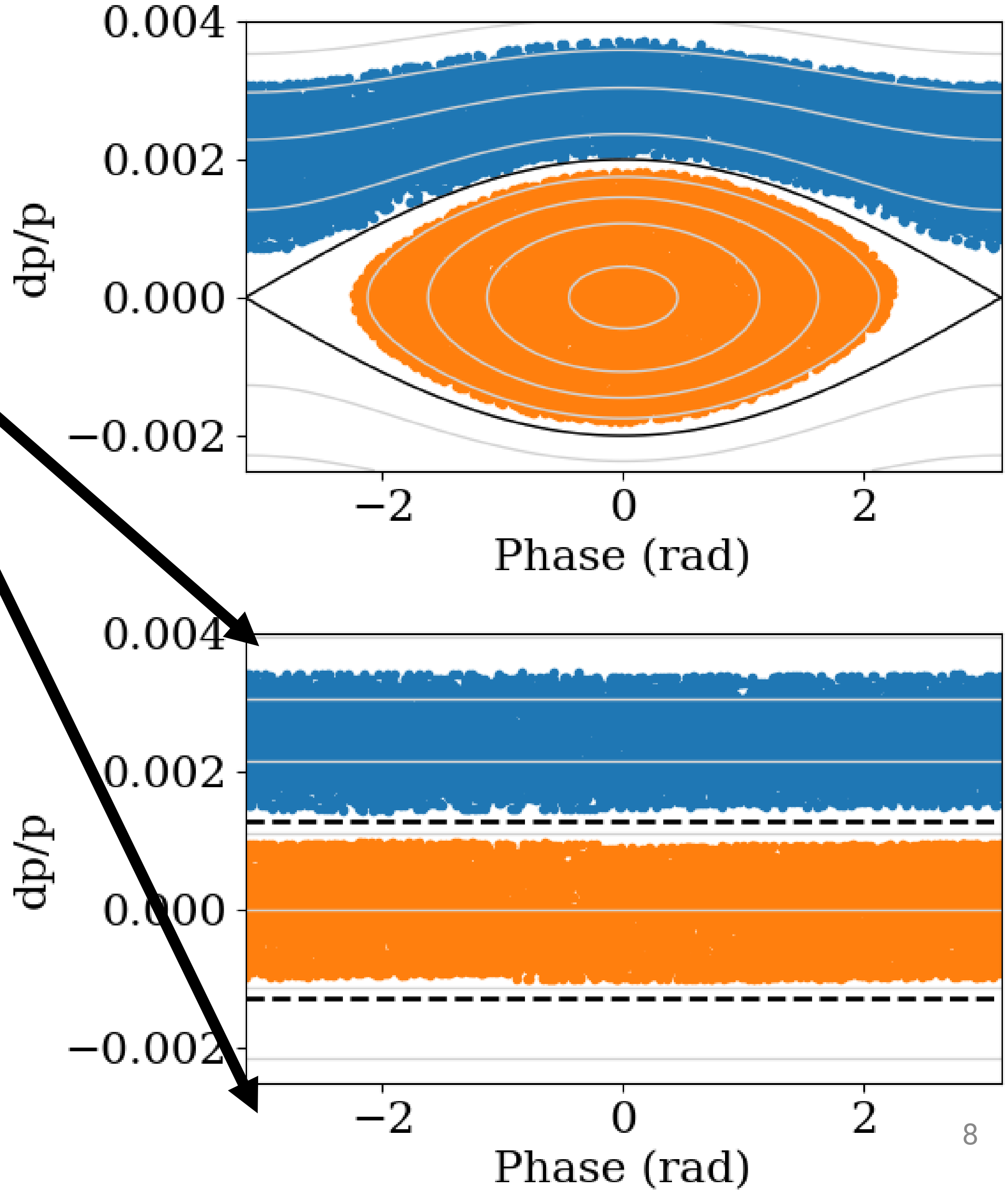
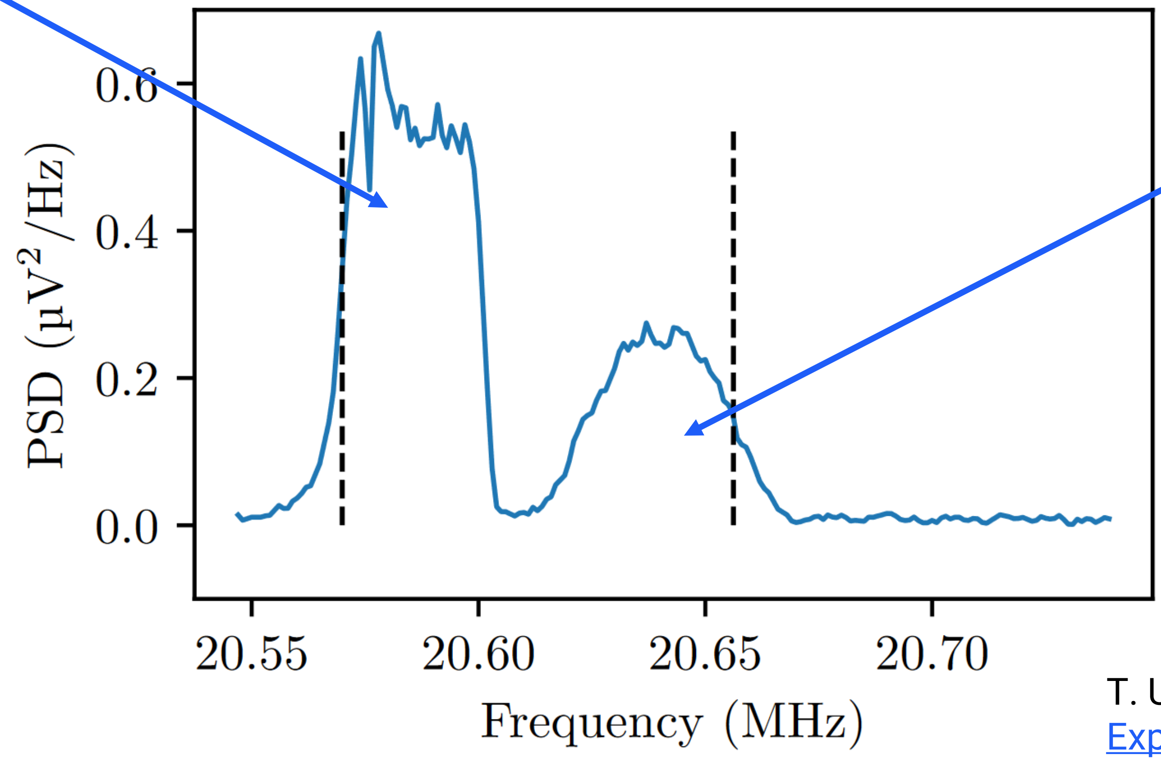




Results from the beam stacking experiment at KURNS in 2023

Schottky signal from
2 stacked beams

Second beam



First beam, the area of the peak is smaller as some of the beam has been lost.

T. Uesugi et al. - [Beam Stacking Experiment at a Fixed Field Alternating Gradient Accelerator]

RF knockout mechanism

Proposed loss mechanism → RF knockout

- Generally, the orbit in an FFA is dispersive. ➡ $D_x = \frac{r_0}{k+1}$

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- Every time the particle gains energy from the cavity, the equilibrium orbit moves because of the finite dispersion.
  → Equivalent to an **instantaneous, horizontal displacement when passing the cavity.**

$$\delta_x = -\frac{D_x}{1+\gamma}\frac{\delta E}{E}$$

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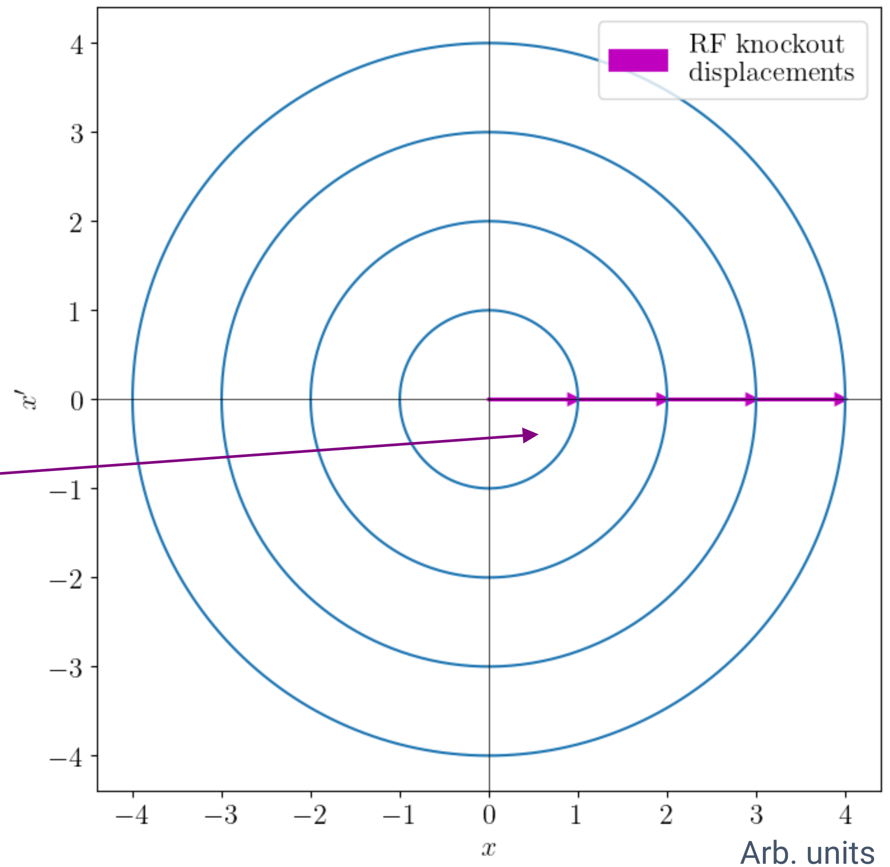
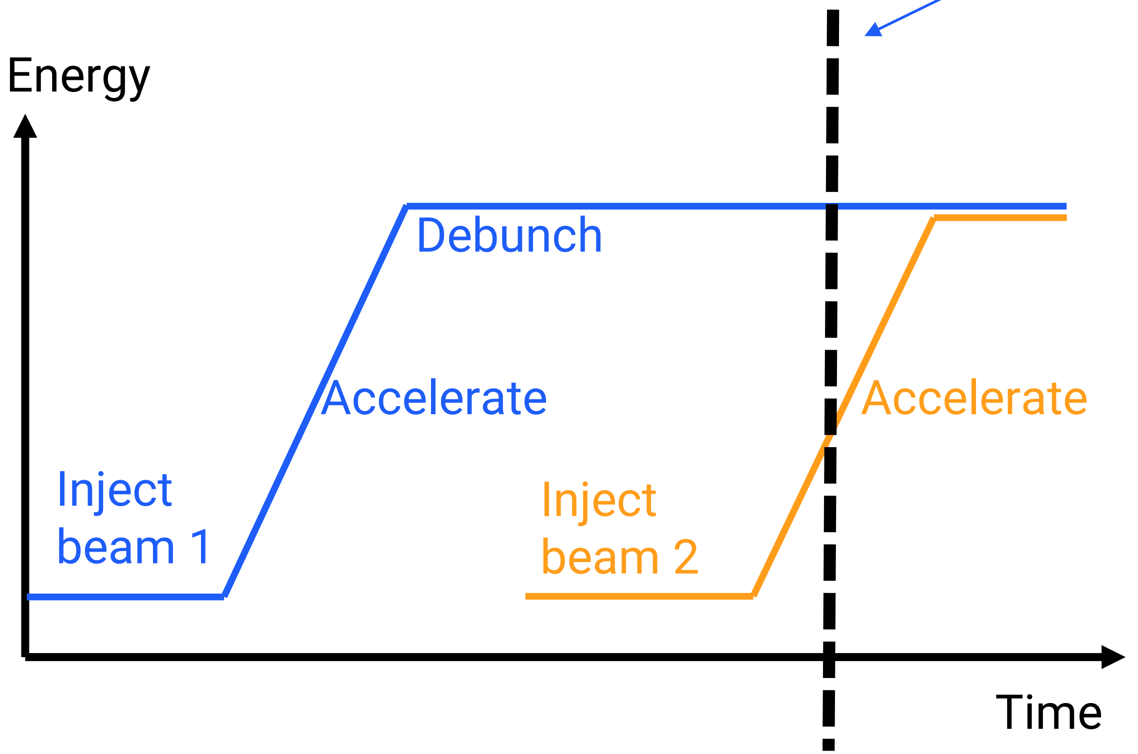
- If these displacements **synchronise with the horizontal tune** then they could build up turn by turn.

Amplitude growth from RF knockout



RF knockout

RF knockout occurs in the **stored beam** when the **accelerating RF** (of the second beam) meets the RFKO condition(s) on the previous slide.



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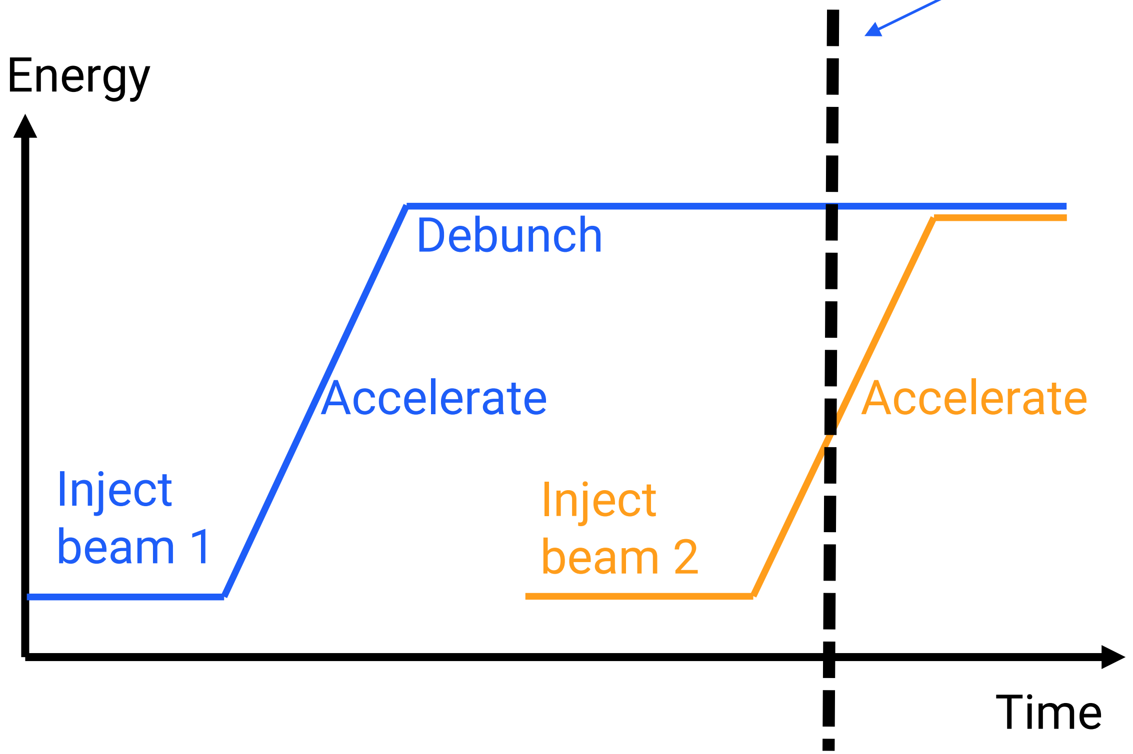

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Particles in the **stored beam** experience displacements from RF knockout every time they pass the cavity

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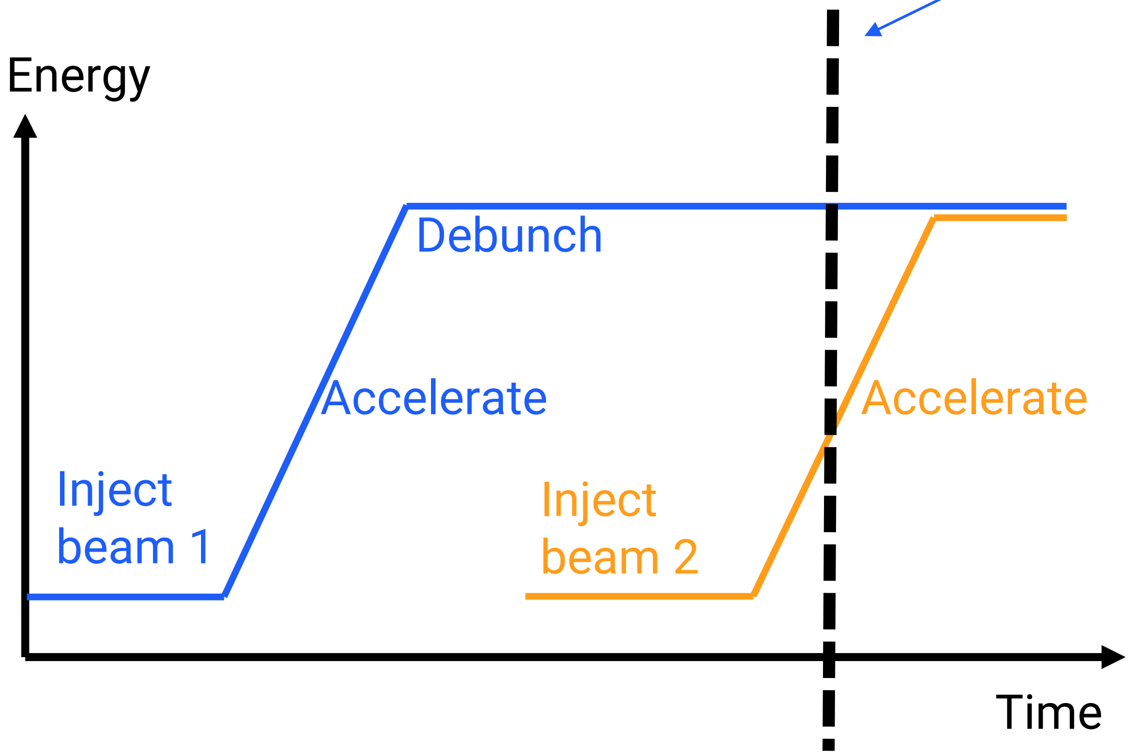

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$$\delta x = -\frac{D_x}{1+\gamma}\frac{\delta E}{E}$$

The total effect is a sum of delta function displacements:

$$\delta x\left(t\right) = -\frac{D_{\mathrm{x}}}{1+\gamma}\frac{\delta E}{E}\sum_{j=-\infty}^{\infty}\delta\left(t - jT_{\mathrm{rev}}\right)$$

RF knockout: Beam as a driven harmonic oscillator

Including the RF waveform:

$$\frac{d^2x}{dt^2} + \omega_\beta^2 x = \frac{D_x}{1+\gamma}\frac{V_{RF}\sin(\omega_{RF}t+\phi)}{E}\sum_j \delta(t - jT_{rev})$$

Re-write sum over j as sum of exponentials.

$$\frac{1}{T_{rev}}\sum_n e^{i(n\omega_{rev}t)}$$

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Re-write sum over j as sum of exponentials.

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Re-writing the $\sin(\omega_{RF}t + \varphi)$ as a complex exponential:

$$\frac{d^2x}{dt^2} + \omega_\beta^2 x = \frac{D_\mathrm{x}}{1+\gamma}\frac{V_\mathrm{RF}}{E}\sum_n e^{i(\omega_\mathrm{RF}+n\omega_\mathrm{rev})t+i\phi}$$

The RF knockout resonance condition is:

$$\pm\omega_\beta = \omega_\mathrm{RF} + n\omega_\mathrm{rev}$$

ISIS synchrotron as a tool for FFAs

ISIS synchrotron as a tool for FFAs

- The ISIS Rapid Cycling Synchrotron usually operates on a 50Hz.
- However, we can set the main magnets to a “storage ring mode” and store a 70MeV beam.
- True beam stacking is not possible in ISIS but we can still study RF knockout.
  - ISIS h=2 RF cavities.

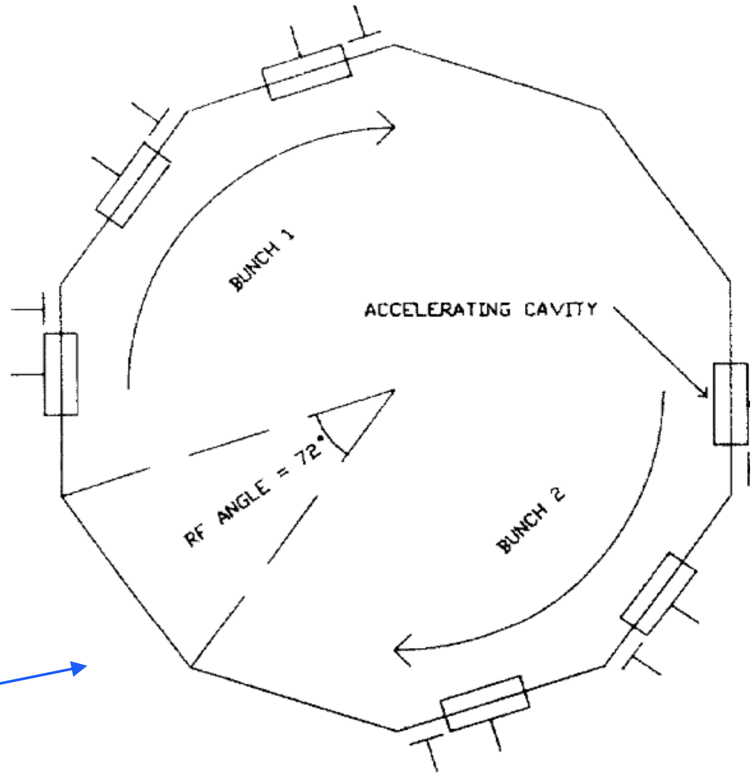


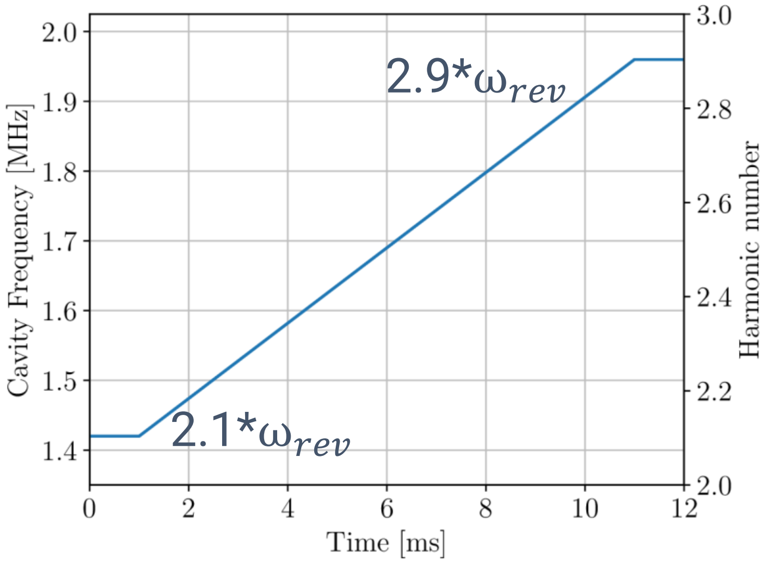
Image: RF system and beam loading compensation on the ISIS synchrotron. EPAC 1990

Experimental confirmation of RF knockout

Procedure:

1. Inject 70 MeV **coasting** beam.

2. Run this RF program to **imitate** a Beam Stacking RF program.



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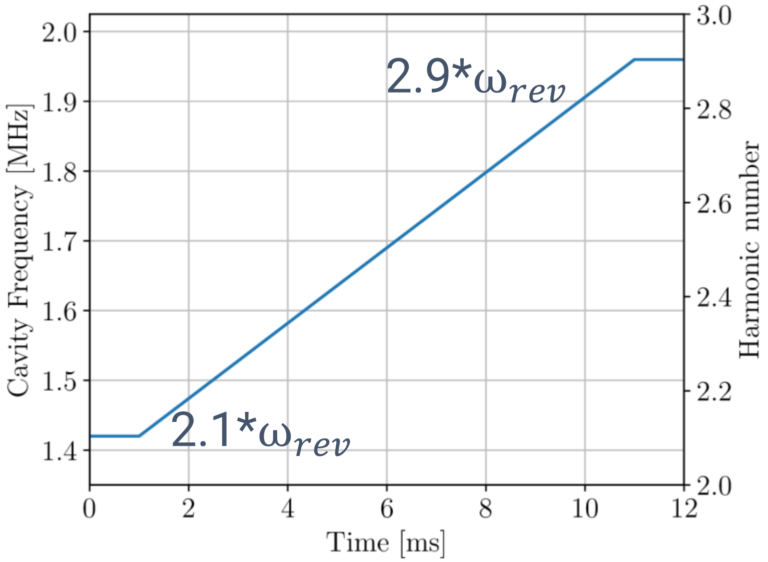
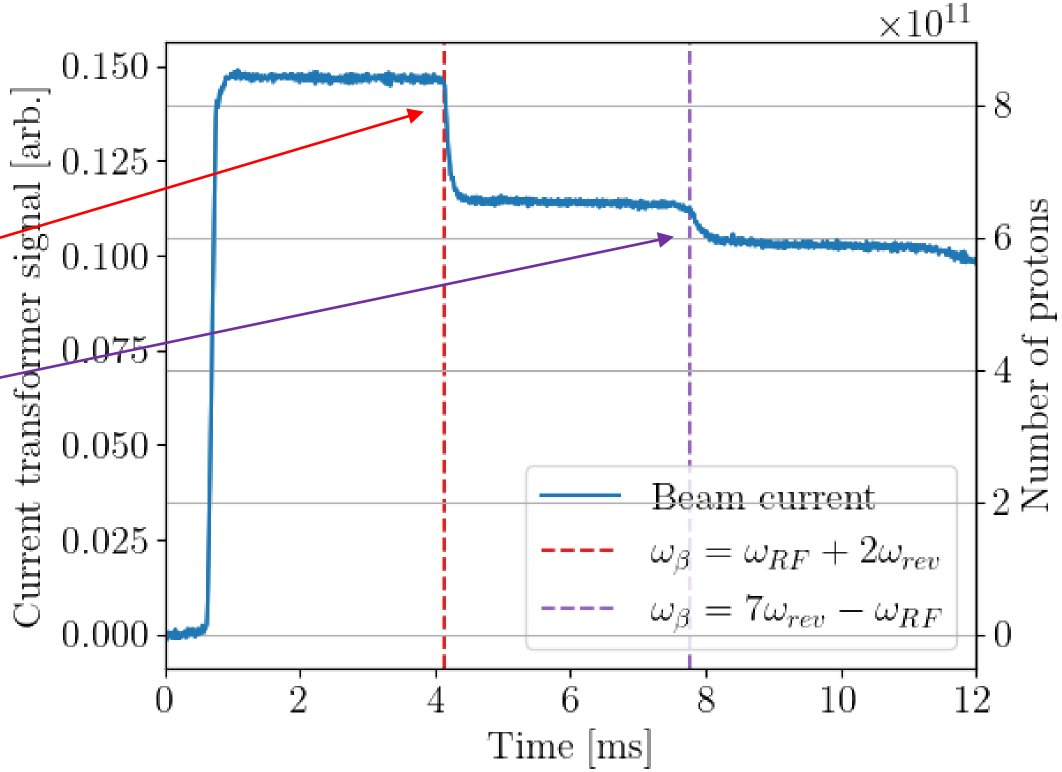


Beam loss when the RF knockout conditions are met.



Characterisation of RF knockout

RF knockout amplitude growth

- Using a similar approach to that for an integer resonance crossing …
  - The amplitude growth from RF knockout is derived by integrating over the displacements to the beam as the frequency changes.

$$\left| \int_{-\infty}^{\infty} \delta x\,(t)\,dt \right| = \frac{f\,D_{\mathrm{x}} V_{\mathrm{RF}}}{2\,(1+\gamma)\,E T_{\mathrm{rev}}} \sqrt{\frac{2\pi}{\alpha}}$$

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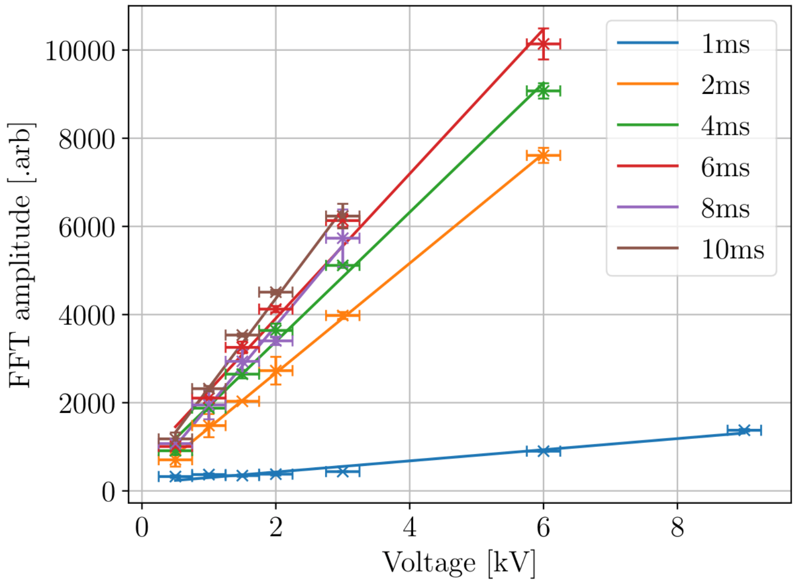
Experimentally, we measure the amplitude growth by recording the betatron oscillation with a position monitor and take the FFT to quantify the oscillation amplitude.

RF knockout characterisation

Expect the amplitude to vary linear with RF voltage.

$$\left| \int_{-\infty}^{\infty} \delta x\,(t)\,dt \right| \propto V_{\mathrm{RF}}$$



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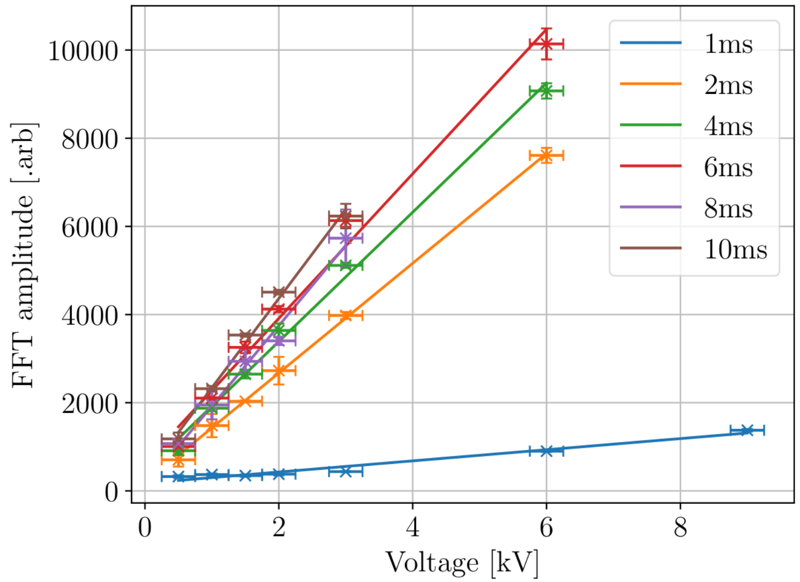
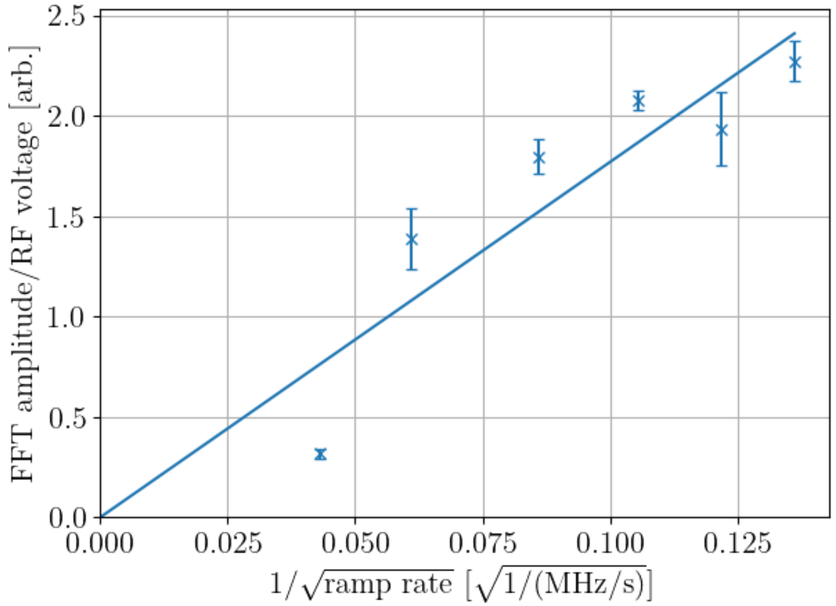


… and with the square root of ramp rate.
The data should lie on a straight line. More repeats are needed here to say for sure.

$$\left| \int_{-\infty}^{\infty} \delta x\left(t\right) dt \right| \propto \sqrt{\frac{1}{\alpha}}$$

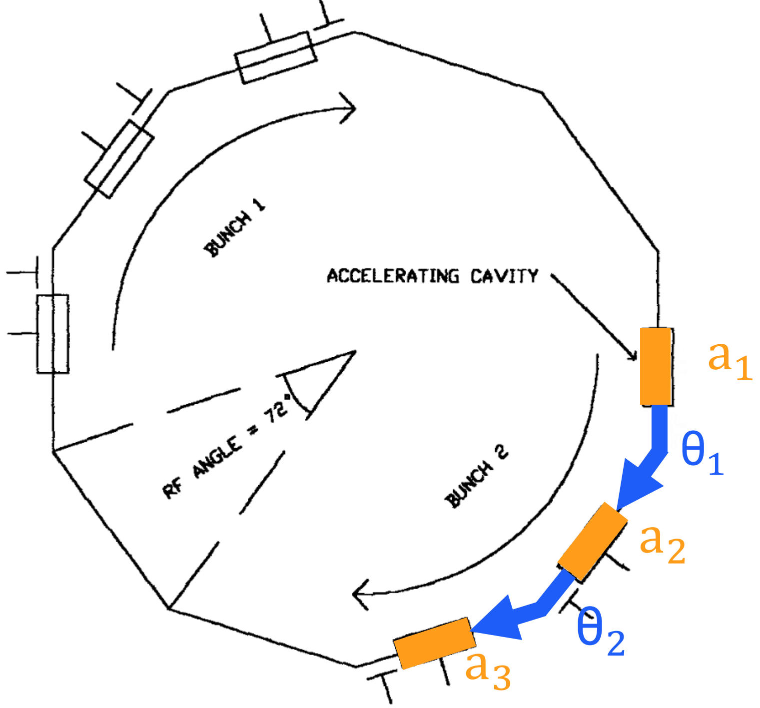


Mitigating RF knockout

Local cancellation

Use consecutive cavities to cancel the kicks:

Fundamental RF cavity positions



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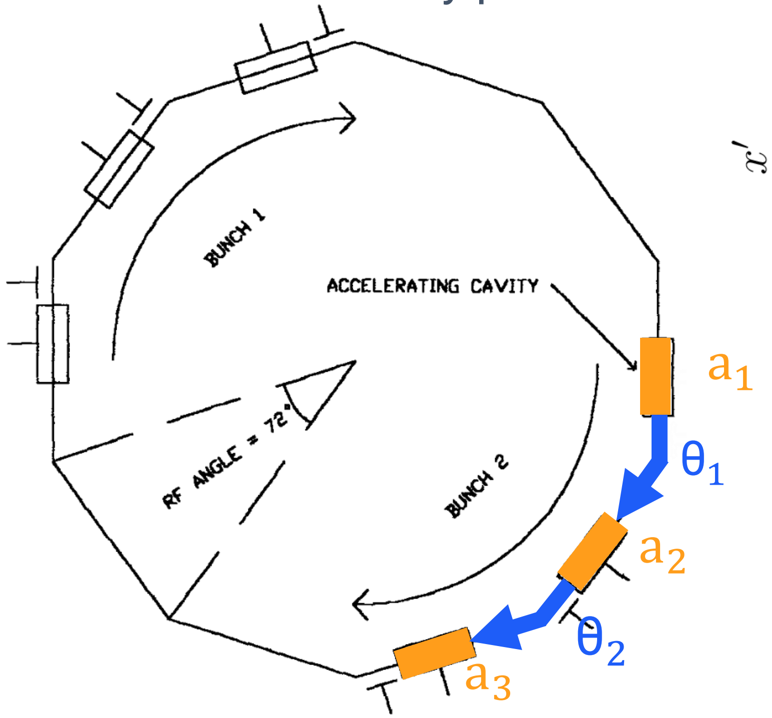
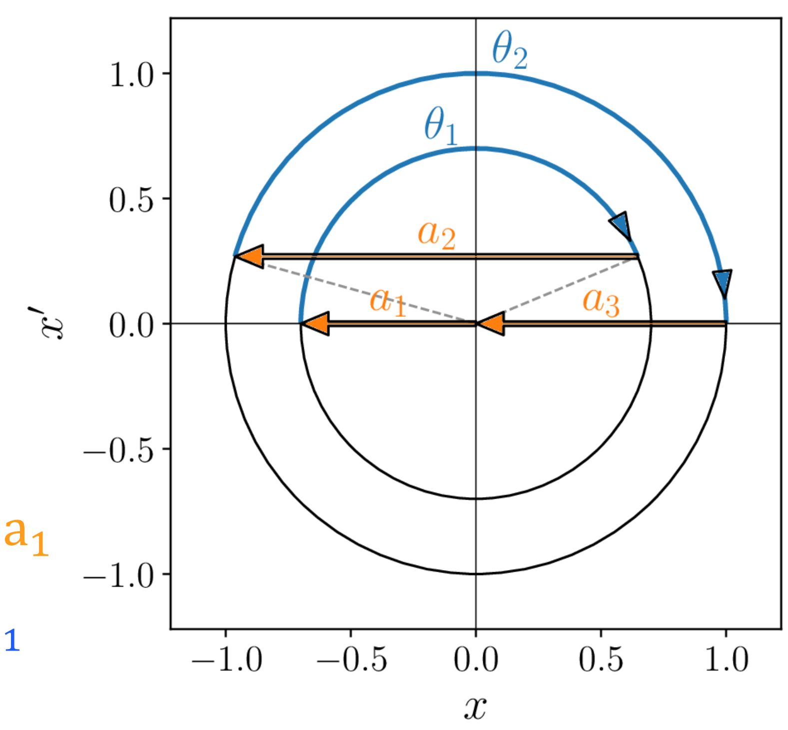




Schematic diagram, not to scale.

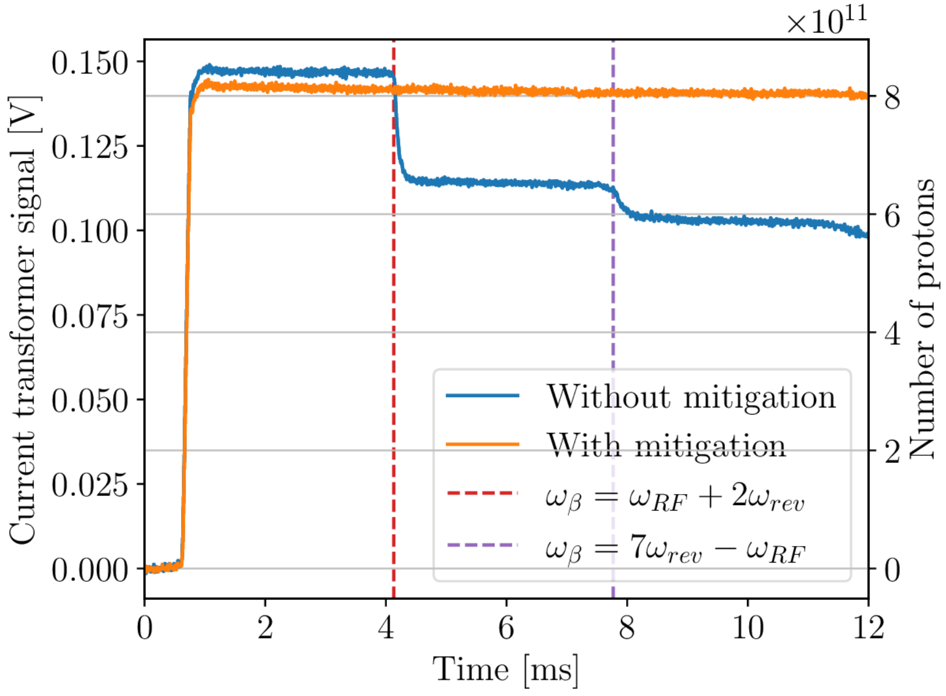
$\theta_1 = \theta_2$ = The phase advance between cavities.

$a_1, a_2, a_3 =$ relative cavity amplitudes, ie the size of the transverse displacement.

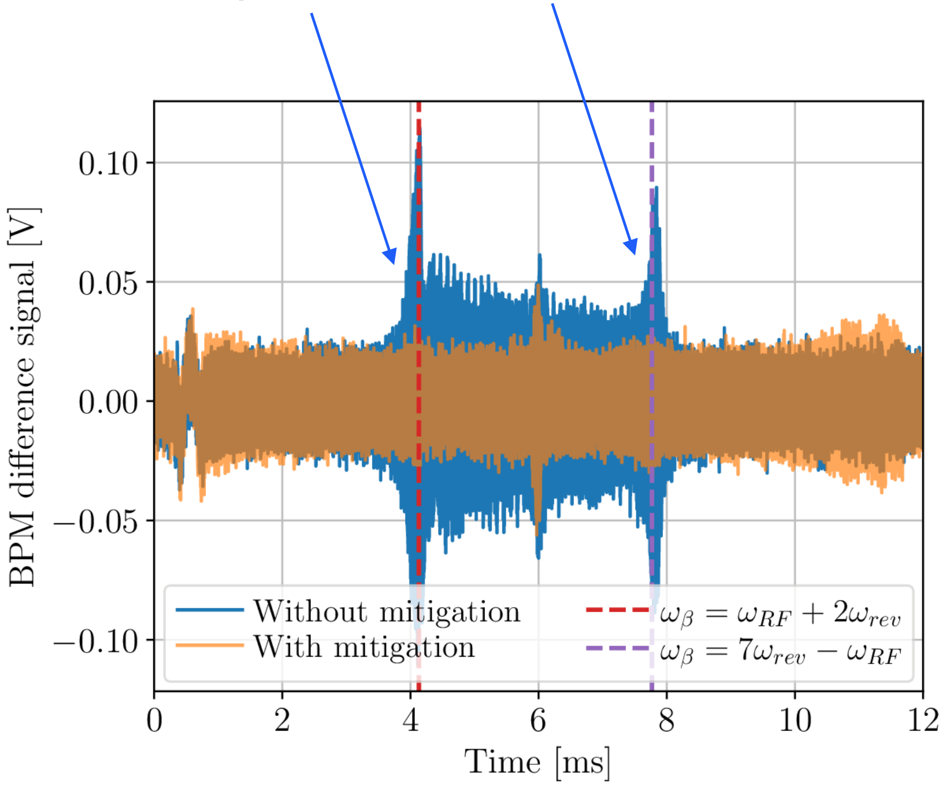
Voltage ratio – 0.55 : 1.0 : 0.55

Local cancellation – experimental results

- RFKO displacements cancelled over one turn by the 3 successive cavities.



Oscillation stet off by the RF knockout displacements



Global cancellation

- The local cancellation method cancels the displacements but it compromises the acceleration of the second beam during stacking …

Global cancellation

- Using the symmetry of the lattice, it is possible to cancel some of the RF knockout resonances…

$$F(t) = A \underbrace{\sum_{n=-\infty}^{\infty} \left(e^{i(\omega_{RF} + n\omega_{rev})t} \right)}$$

Driving force from one cavity only.

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Add term for k cavities

$$\sum_{k} e^{(i2\pi(h+n)L_k/C)}$$

$L_k = k^{th}$ cavity position in the ring.
C = ring circumference

i.e L_k/C = angular cavity position.

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For two cavities on opposite side of the ring: $\frac{L_k}{C} = \frac{1}{2}$

$$1 + e^{i2\pi\frac{1}{2}(h+n)}$$

Global cancellation with two opposite cavities

- Then the total force becomes:

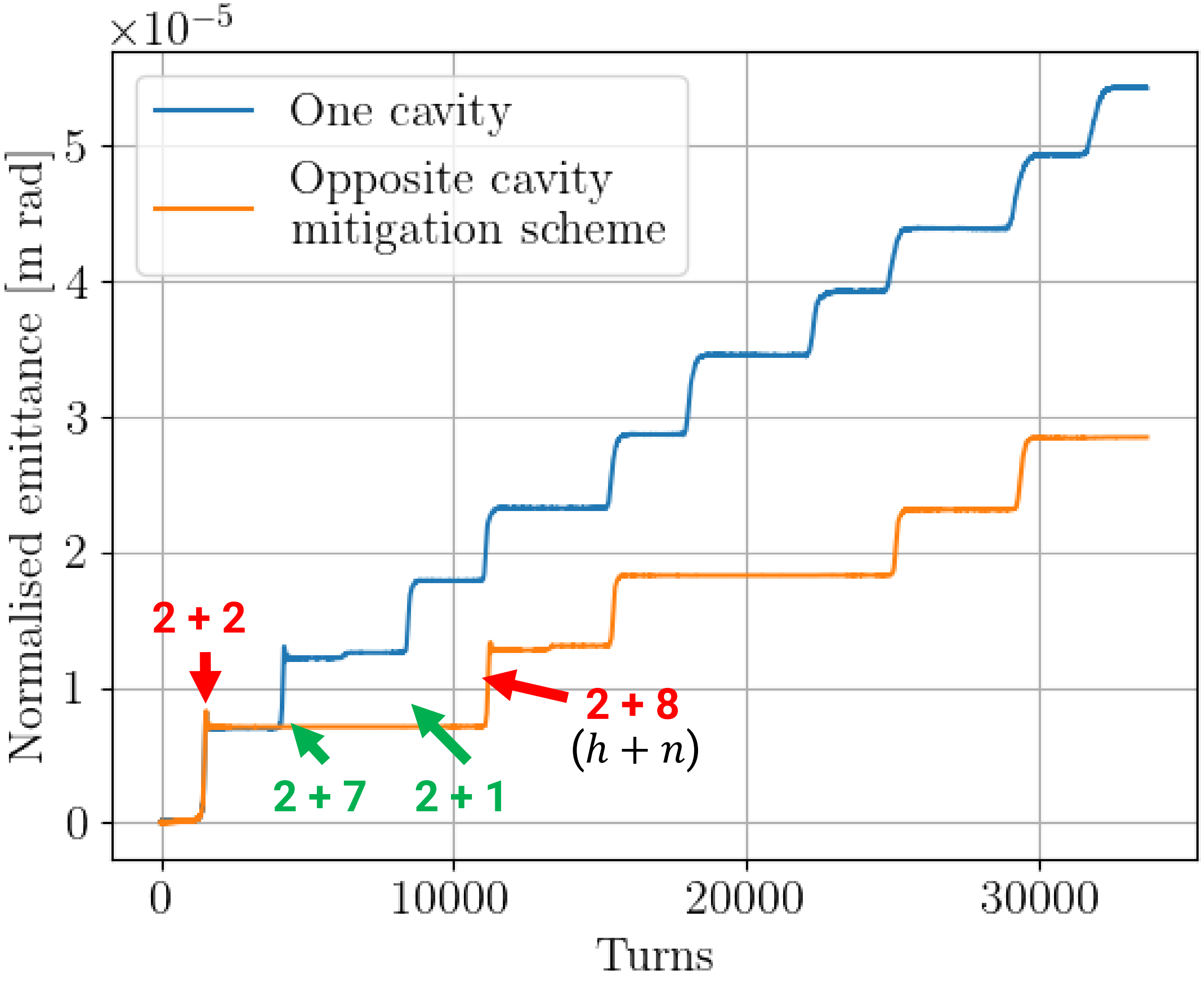
$$F(t) = Ae^{i(\omega_{RF}+n\omega_{rev})t}\left(1 + e^{-i\pi(h+n)}\right)$$

When $h + n$ **is an odd number the RF knockout driving force is zero.**

Opposite cavity mitigation – simulation

- In this case $h \quad = 2.$

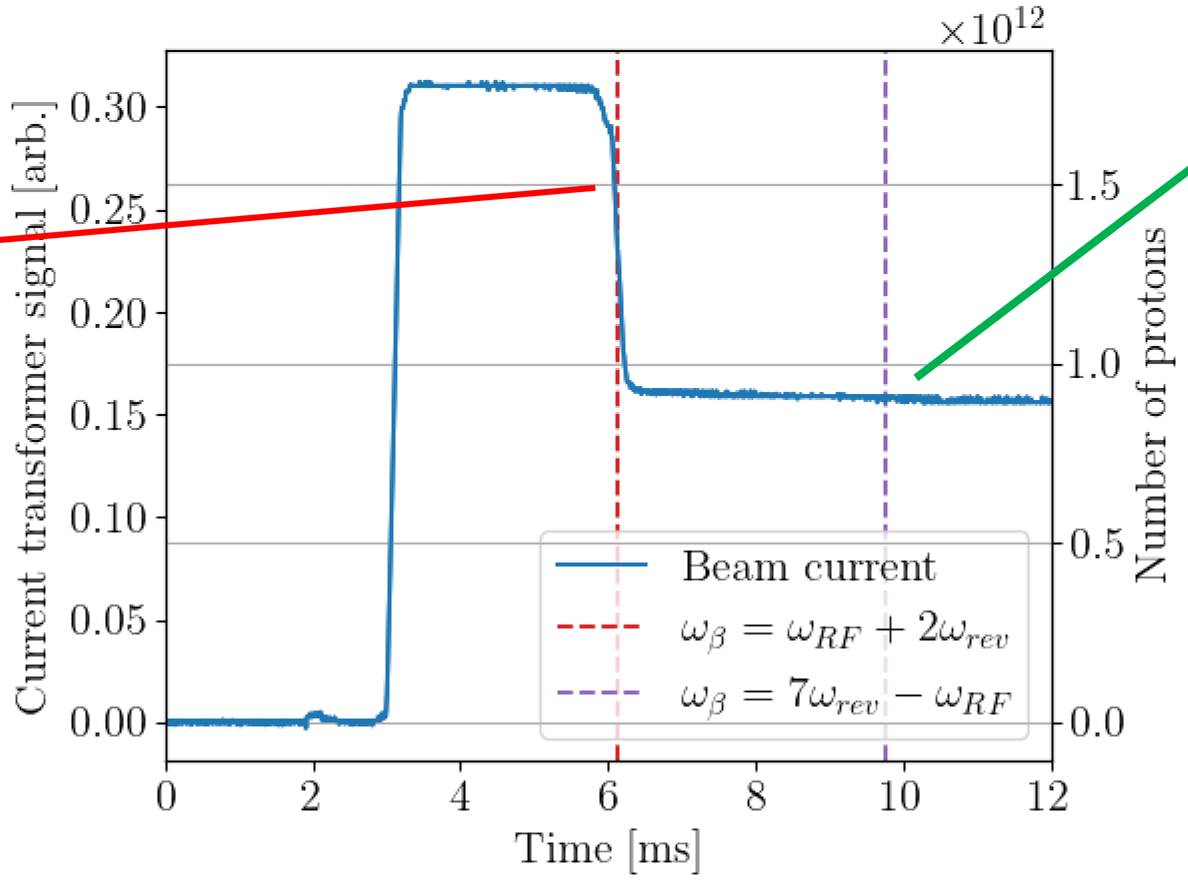
- This simulation sweeps through a large frequency range to show many RF knockout mode numbers.



Global cancellation results – raw data

2 RF cavities on exact opposite sides of the ring

This RFKO condition is not cancelled.



This one is cancelled

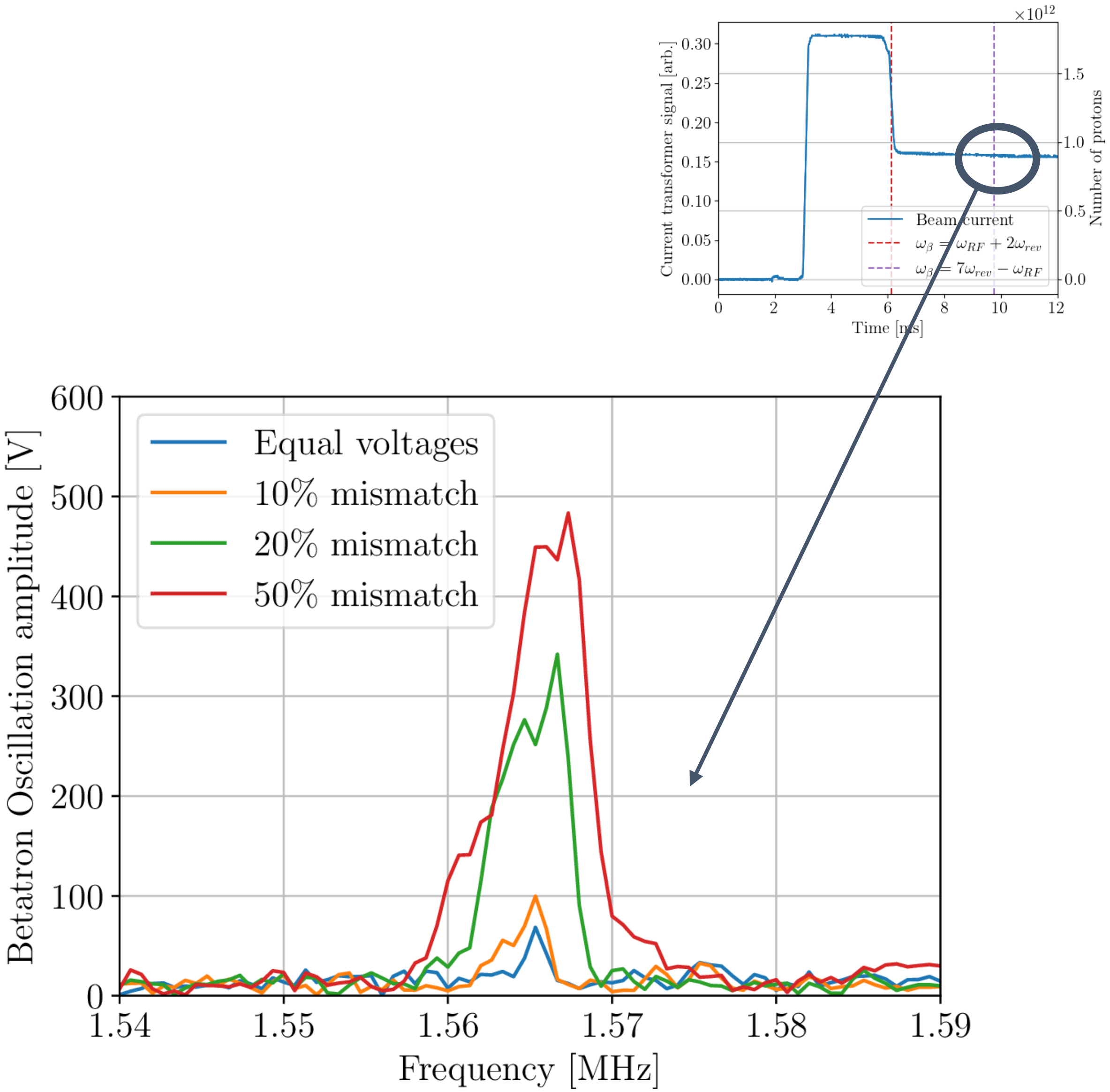
Global cancellation results

- As the two voltage from the two cavities becomes more unequal, the RF knockout displacements return.

    - Zooming into the mitigated condition.
        - Still a small kick even with equal voltages → probably because of the error in the gap volts.

Global cancellation results

- As the two voltage from the two cavities becomes more unequal, the RF knockout displacements return.
  - Zooming into the mitigated condition.
    - Still a small kick even with equal voltages → probably because of the error in the gap volts.
  - Unlike the local cancellation this method does not effect the acceleration of the second beam.
    - → But it does not cancel every condition.



Mitigation methods: side by side comparison

Local cancellation

- Effected by the phase advance of the particles.
- Cancels all of the RF knockout conditions.
- Compromised the acceleration of the incoming beam during stacking.

Global cancellation

- Unaffected by the phase advance of the particles → ie unaffected by a space charge tune spread.
- Does not compromise the acceleration of the incoming beam during stacking.
- Cavity arrangement for global cancellation needs to be considered at design.
- Does not cancel all of the RF knockout conditions.*

* More conditions are cancelled with more cavities.

Conclusions

- RF knockout caused beam loss during our beam stacking experiment at KURNS in 2023.

- Through a series of experiments at the ISIS synchrotron, we've shown that the beam loss from RF knockout can be avoided.

- The global cancellation method could enable lossless beam stacking at a high intensity FFA for ISIS-II.