FFA with negative k-value

Shinji Machida

FFA school/workshop 2025

16 September 2025

UKRI Science and Technology Facilities Council
ISIS Neutron and Muon Source

Review of FFA optics

- Scaling FFA
- Linear nonscaling FFA
- Nonlinear nonscaling FFA
- Vertical excursion FFA
- FFA with a negative k

Science and
Technology
Facilities Council

Lattice with negative k-value

FFA with a negative k-value in a literature

- Symon, K., et. al. Phys Rev **103** 6, 1956.

“It is possible in the radial-sector FFAG designs to have k large and **negative**. In this case there is no transition energy, and high-energy orbits lie on the inner radius of the machine. Negative-k designs appear to be not practical with spiral sectors.”

- Kolomensky, A. A., Levedev, A. N., *Theory of Cyclic Accelerators.*

“For n0>0 (the inverse version of the ring-phasotron) the motion takes place in one and the same direction along both orbits. Let us note that the orbits only exist in this case if the average field is large enough: $b_0^2 > 4n_0S^2/N^2$, while the symmetrical machine is not possible when n0>0.”

$$n_0 = -k$$

$$S^2 = 2\sum_{m=1}^{\infty}\frac{|b_m|^2}{m^2}$$

“ring-phasotron” = FFA

“Symmetrical machine” = Two beams FFA

Science and Technology Facilities Council

Momentum compaction factor

- Kolomensky, A. A., Levedev, A. N., *Theory of Cyclic Accelerators.*
  - Momentum compaction factor.

$$\alpha_c = \frac{1}{k+1} < 0$$

  - Higher momentum particle has shorter path length.

$$\frac{\Delta C}{C} = \alpha_c \frac{\Delta p}{p}$$

  - No transition energy

FFA with a negative k-value
= imaginary transition energy lattice

$$\frac{\Delta t}{t} = \left(\alpha_c - \frac{1}{\gamma^2}\right)\frac{\Delta p}{p}$$

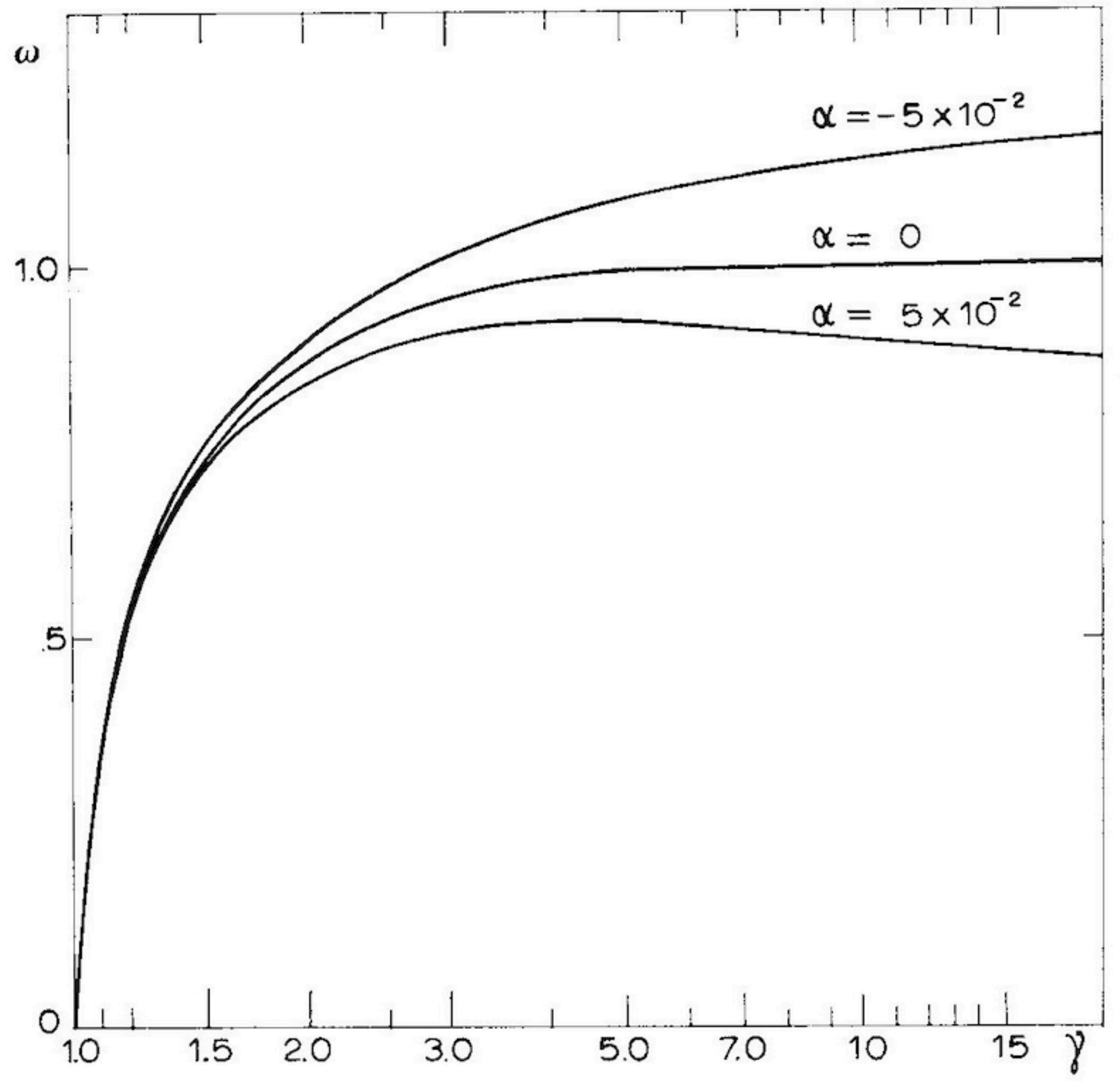


Fig. 80. The curves $\omega = \omega(\gamma)$ where $\gamma = E/m_0c^2$ in an accelerator with scaling orbits: $\alpha = 5\times10^{-2} > 0$, direct version; $\alpha = -5\times10^{-2} < 0$, inverse version.

Science and Technology Facilities Council

Basic principle

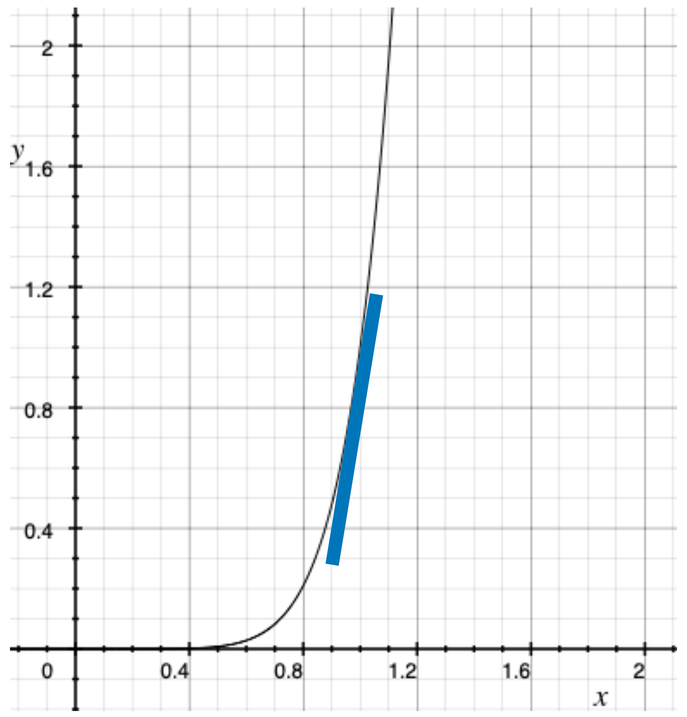
- Acceleration makes the equilibrium orbit goes inside.
- Not as peculiar as a vertical excursion FFA, but still interesting.
- Maybe useful for some applications.

$$\left(\frac{B}{B_0}\right) = \left(\frac{r}{r_0}\right)^k$$

$k = +7$

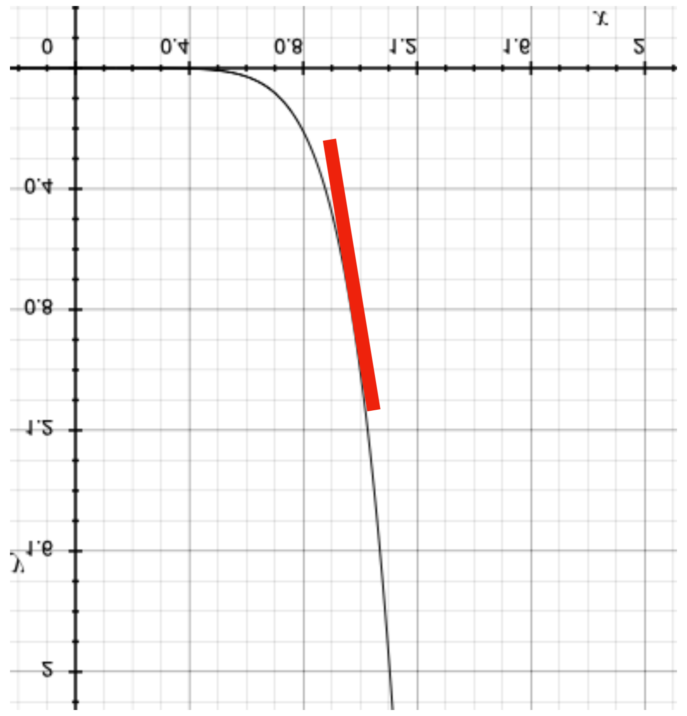
Bf (normal bend)

Focus



Bd (reverse bend)

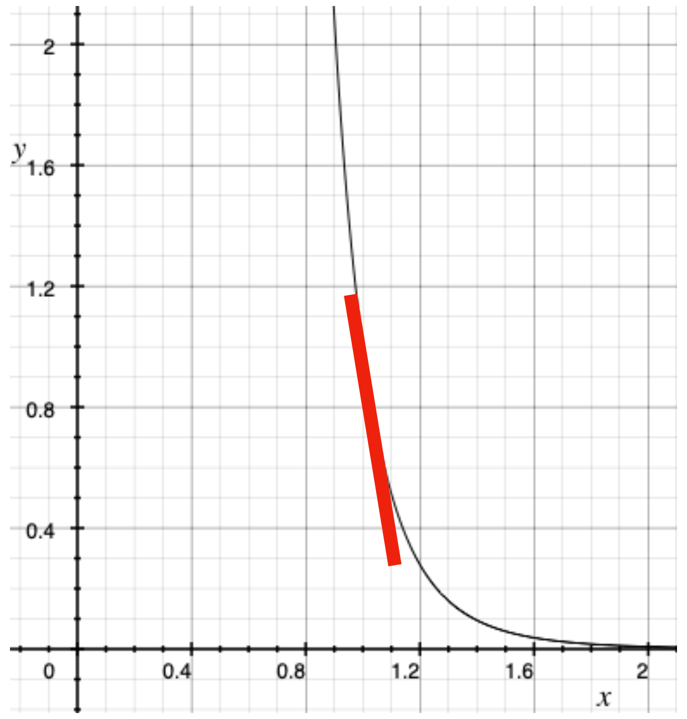
Defocus



$k = -7$

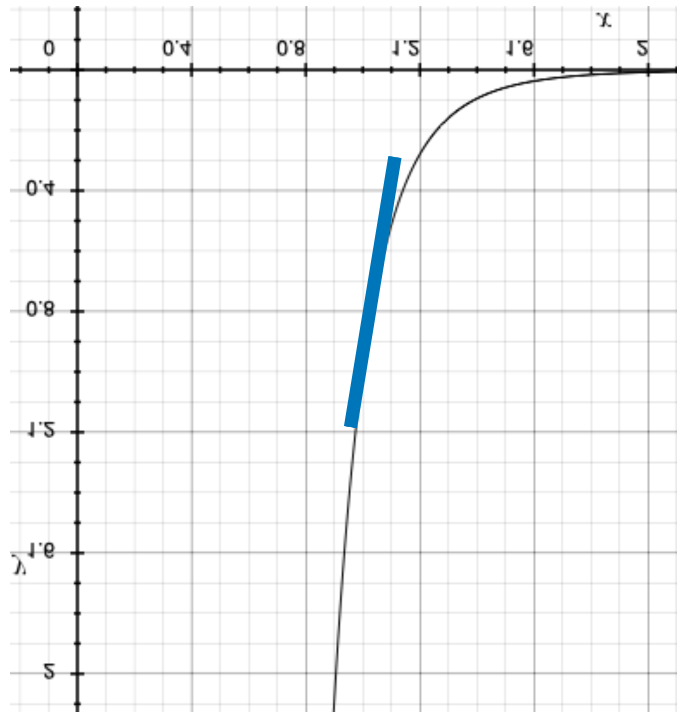
Bf (normal bend)

Defocus



Bd (reverse bend)

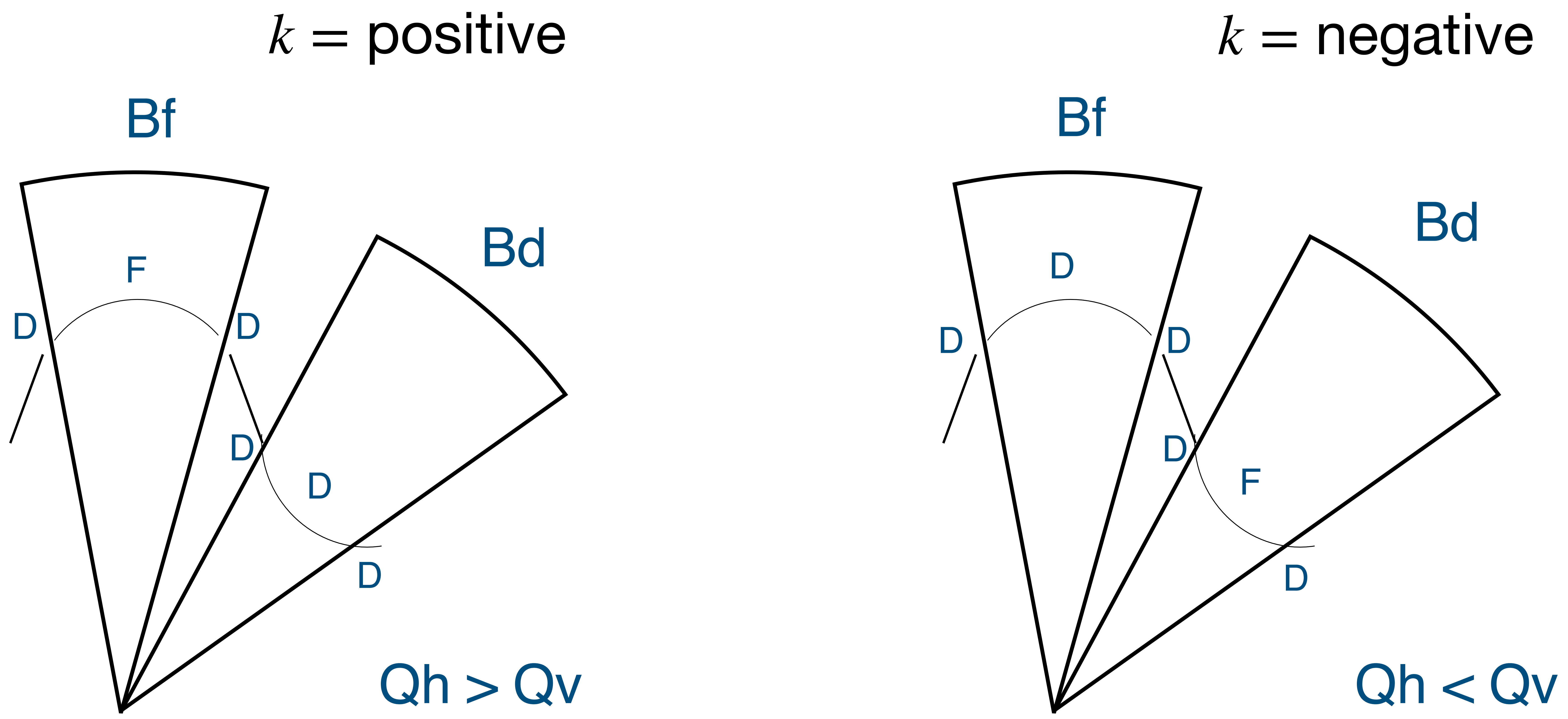
Focus



Science and Technology Facilities Council

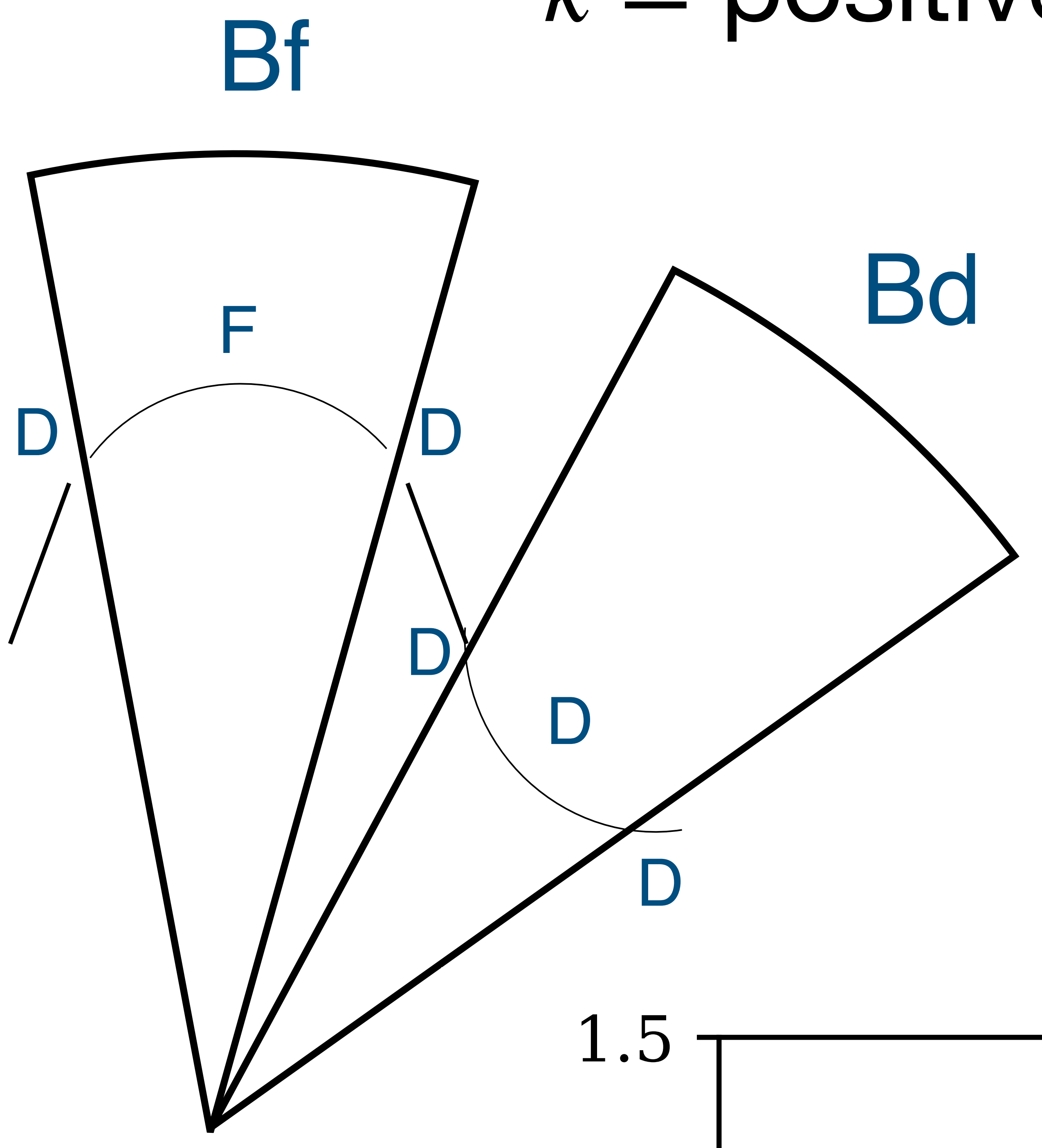
Orbit and optics

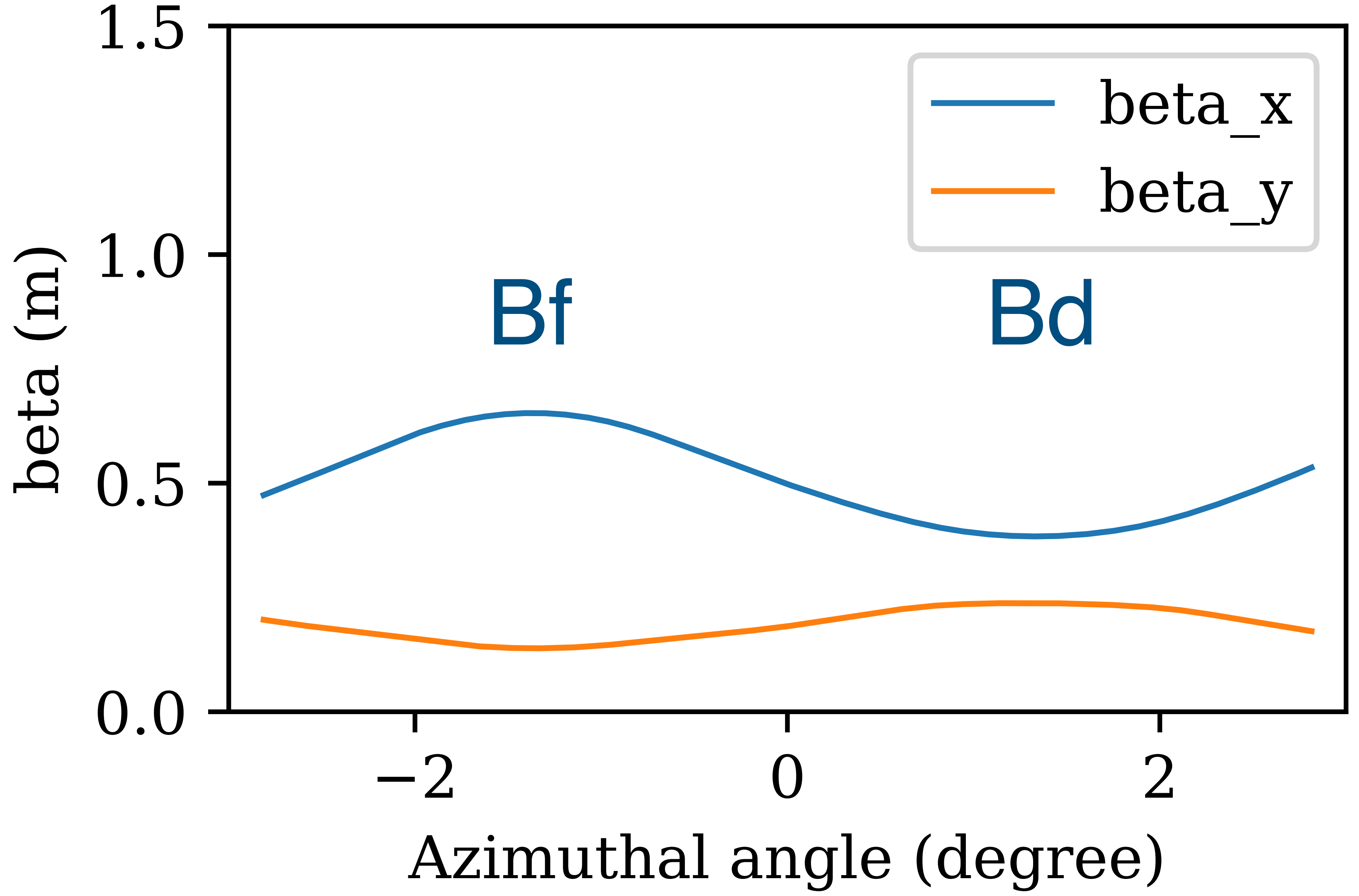
- Defocusing at a normal bending, focusing at a reverse bending: opposite to a positive k scaling FFA.
- In a positive k lattice, Bf is stronger than Bd so the horizontal tune is higher. In a negative k lattice, it is opposite.



Beta function

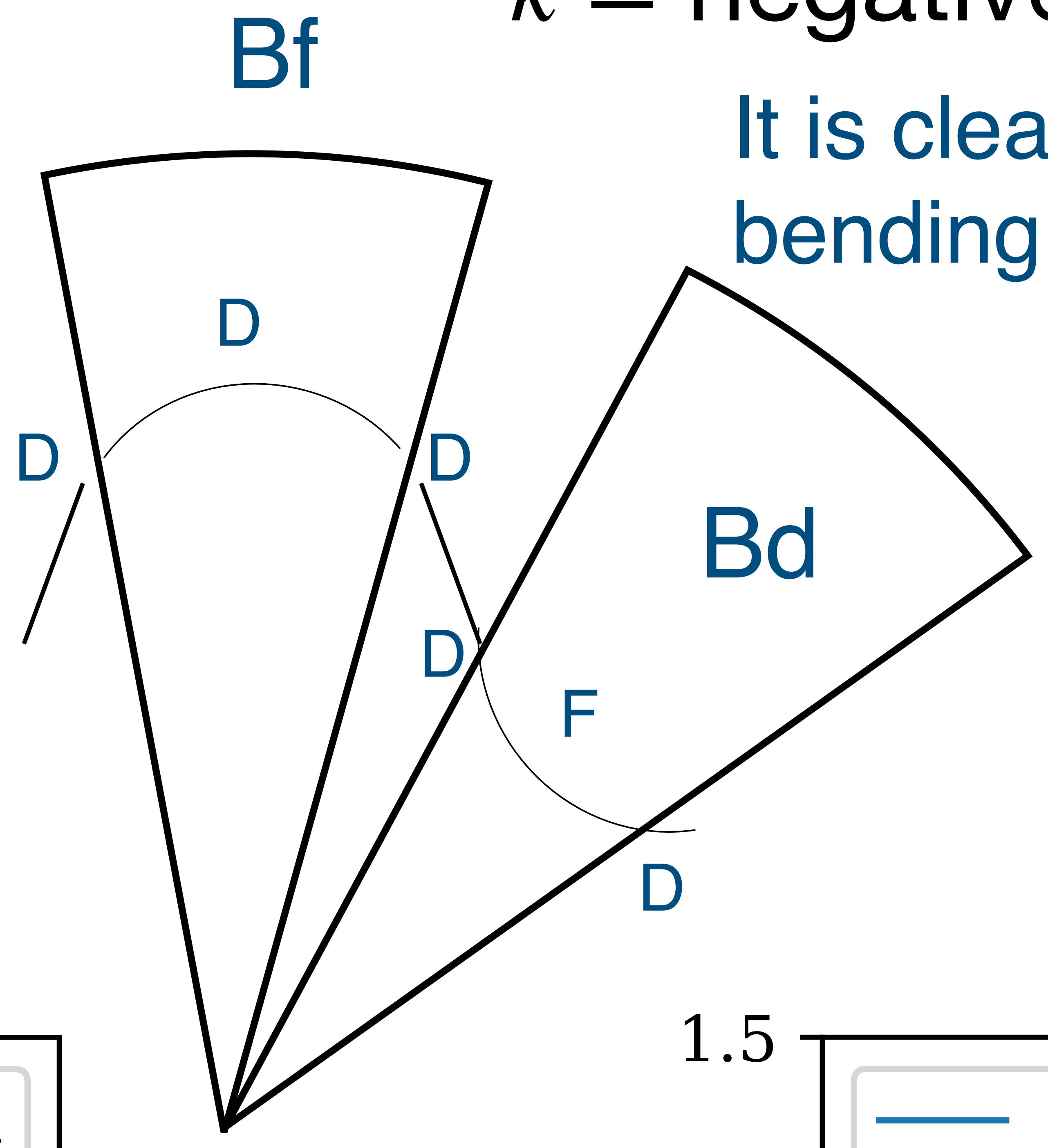
$k =$ positive

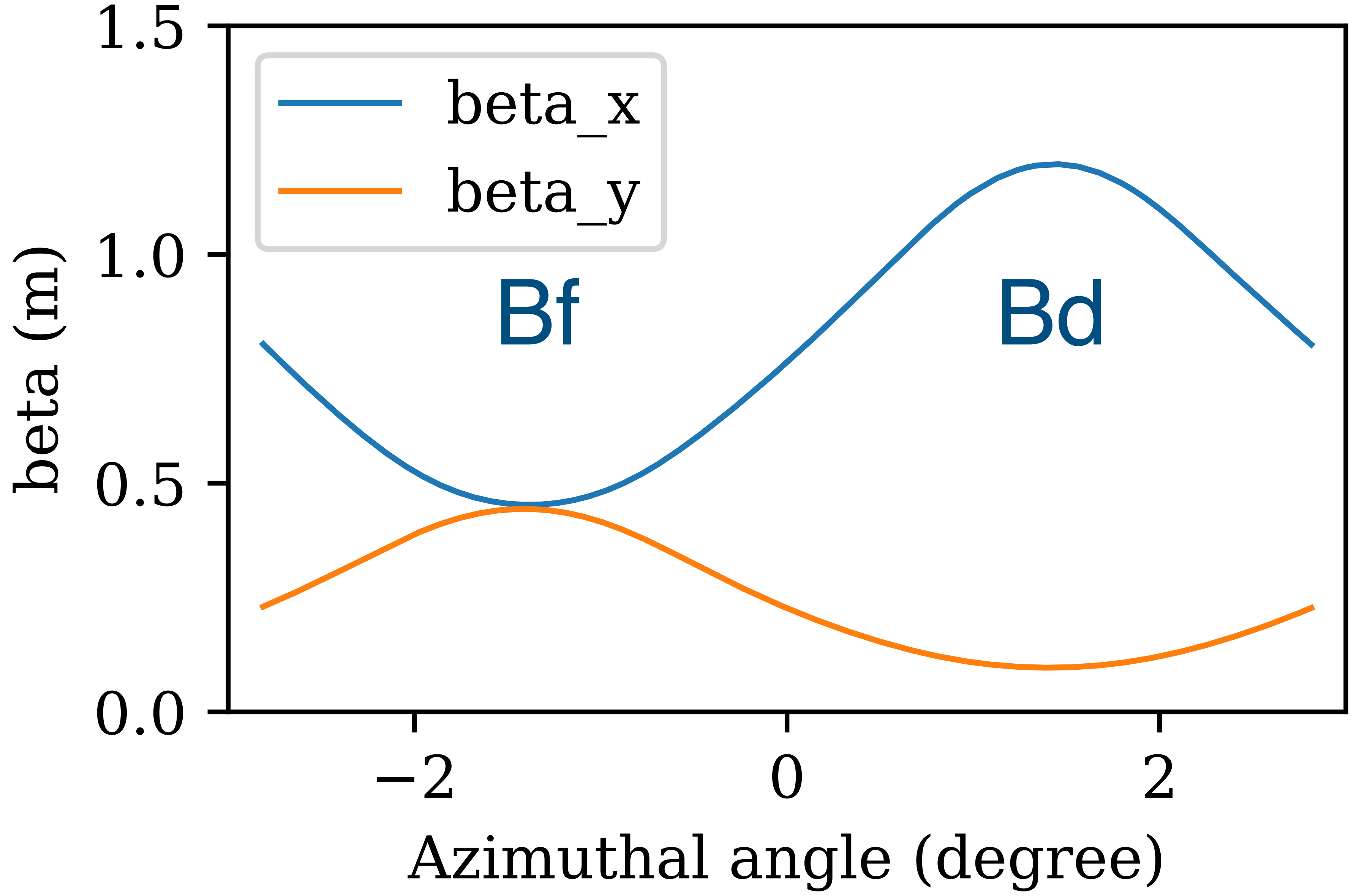




$k =$ negative

It is clear that defocusing at a normal bending, focusing at a reverse bending





Parameter dependence

Review of FFA optics

- Scaling FFA
- Linear nonscaling FFA
- Nonlinear nonscaling FFA
- Vertical excursion FFA
- FFA with a negative k

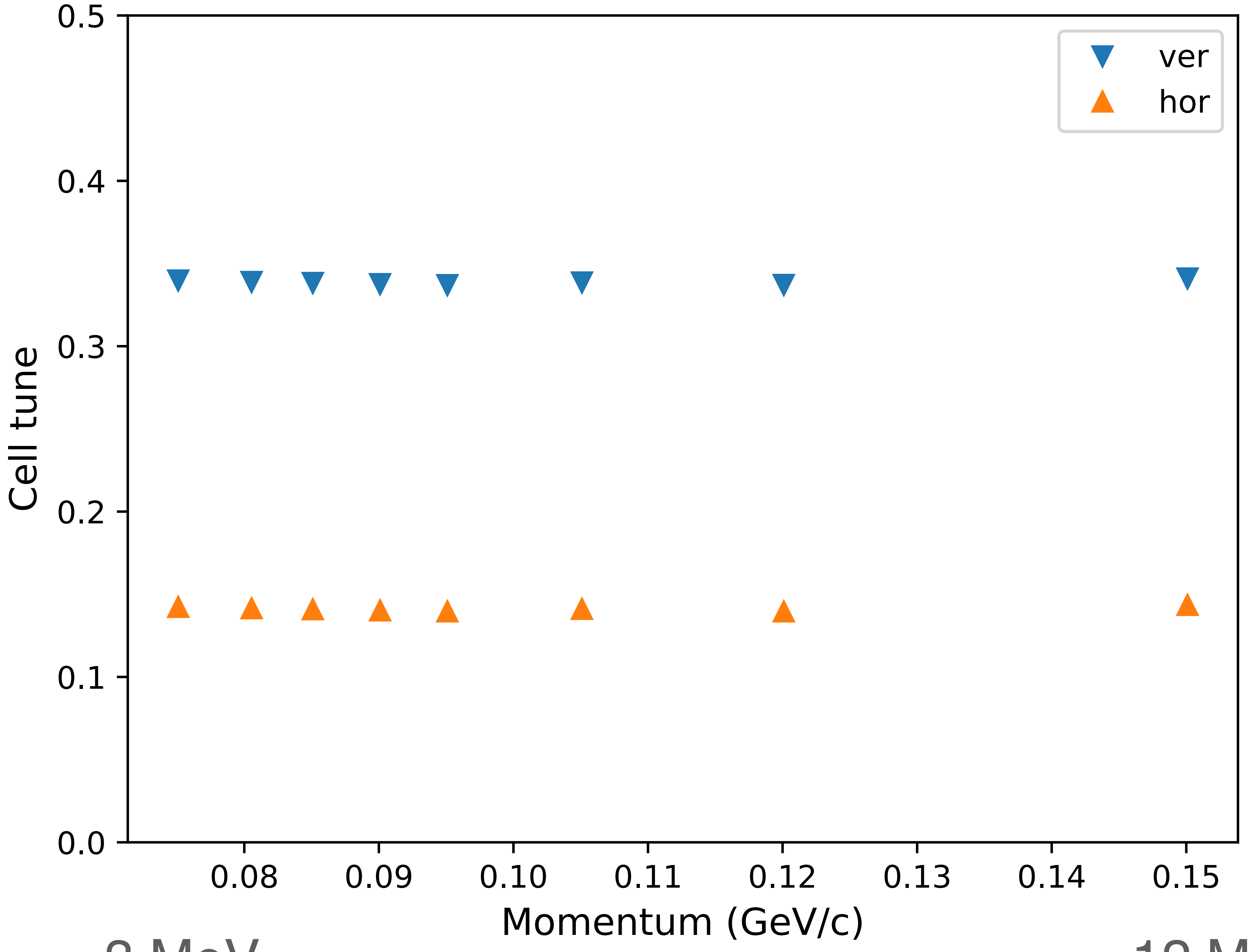
Not many design studies exist on a negative k FFA.

Science and Technology Facilities Council

Tune and radius vs momentum

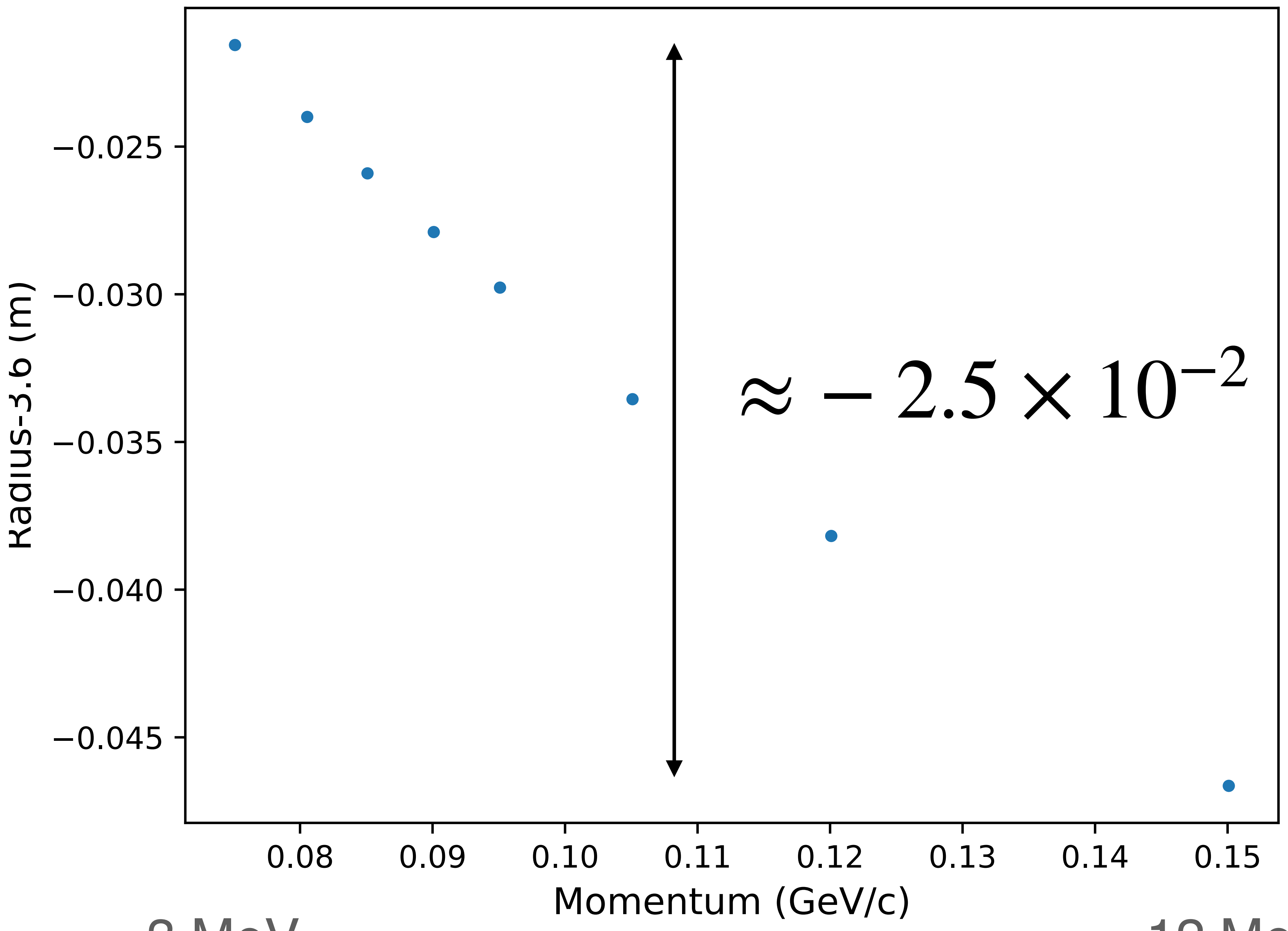
64 cells FODO, k=-100, radius=3.6 m, energy=3-12 MeV

Tune



3 MeV 12 MeV

Radius



3 MeV 12 MeV

when

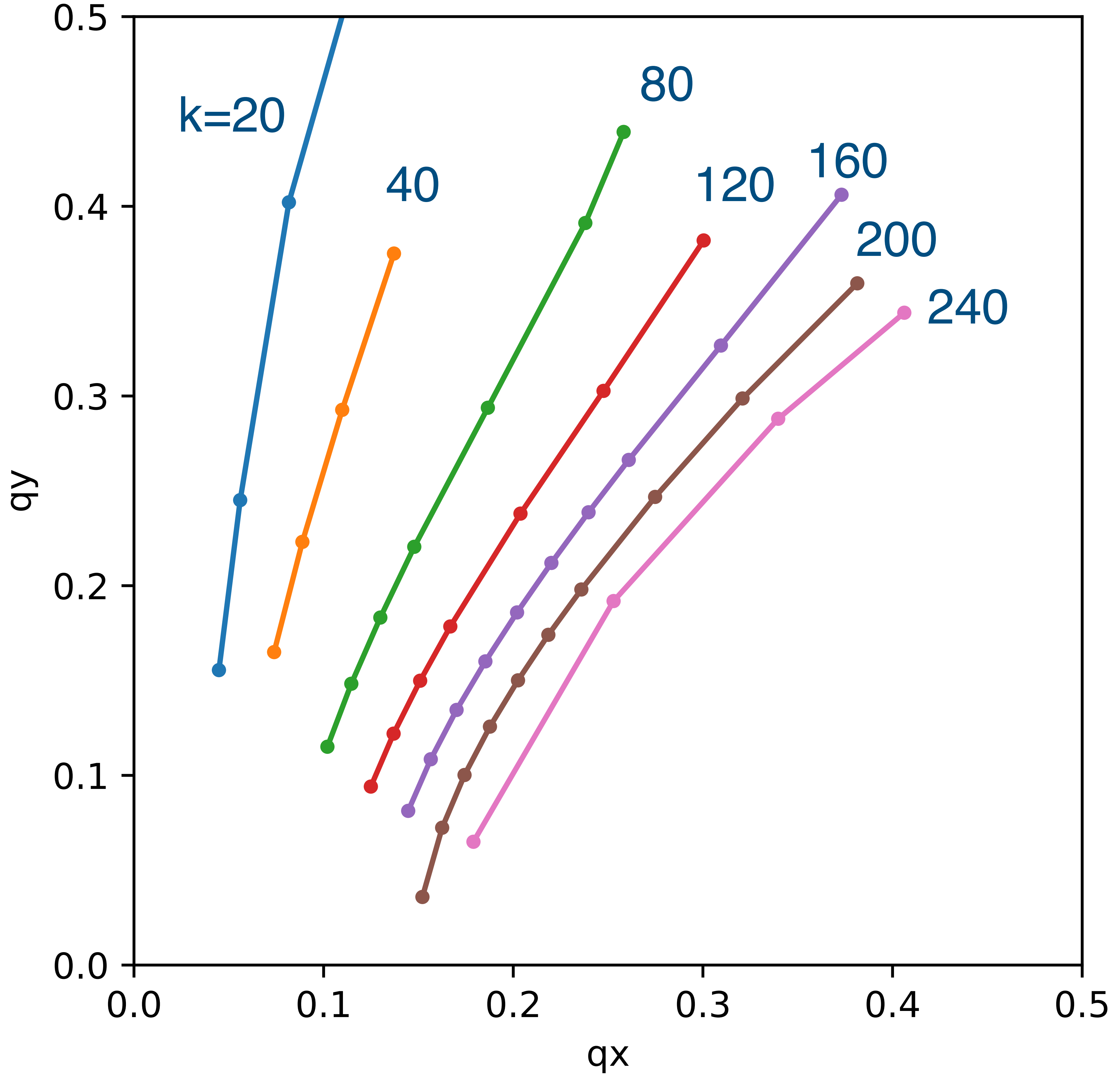
$$k = -100 \quad \frac{p}{p_0} = 2$$

$$\Delta r = \left[\left(\frac{p}{p_0} \right)^{\frac{1}{k+1}} - 1 \right] = -2.51 \times 10^{-2}$$

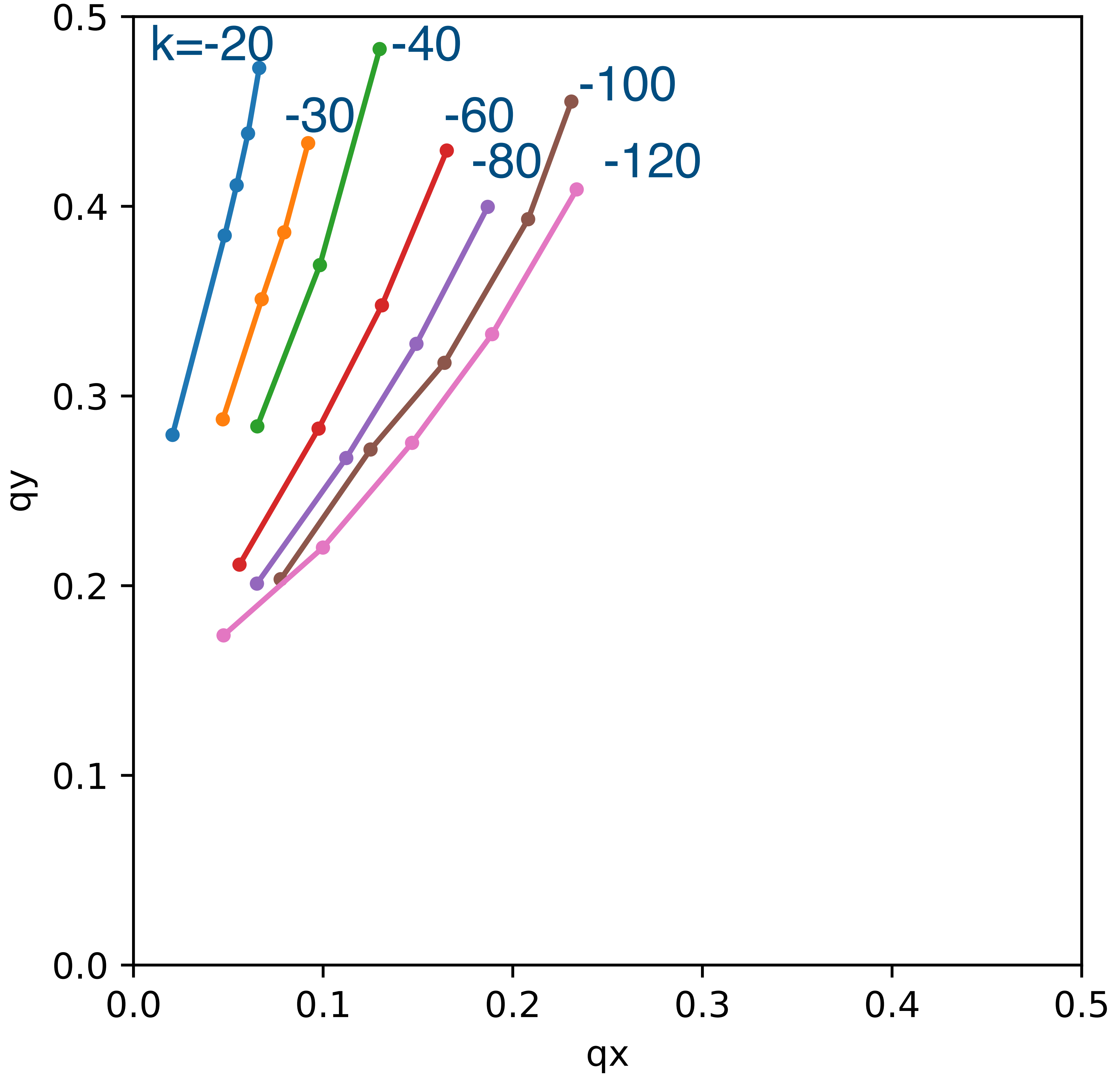
Science and Technology Facilities Council

Tune vs k and Bd/Bf ratio

128 cell, positive k



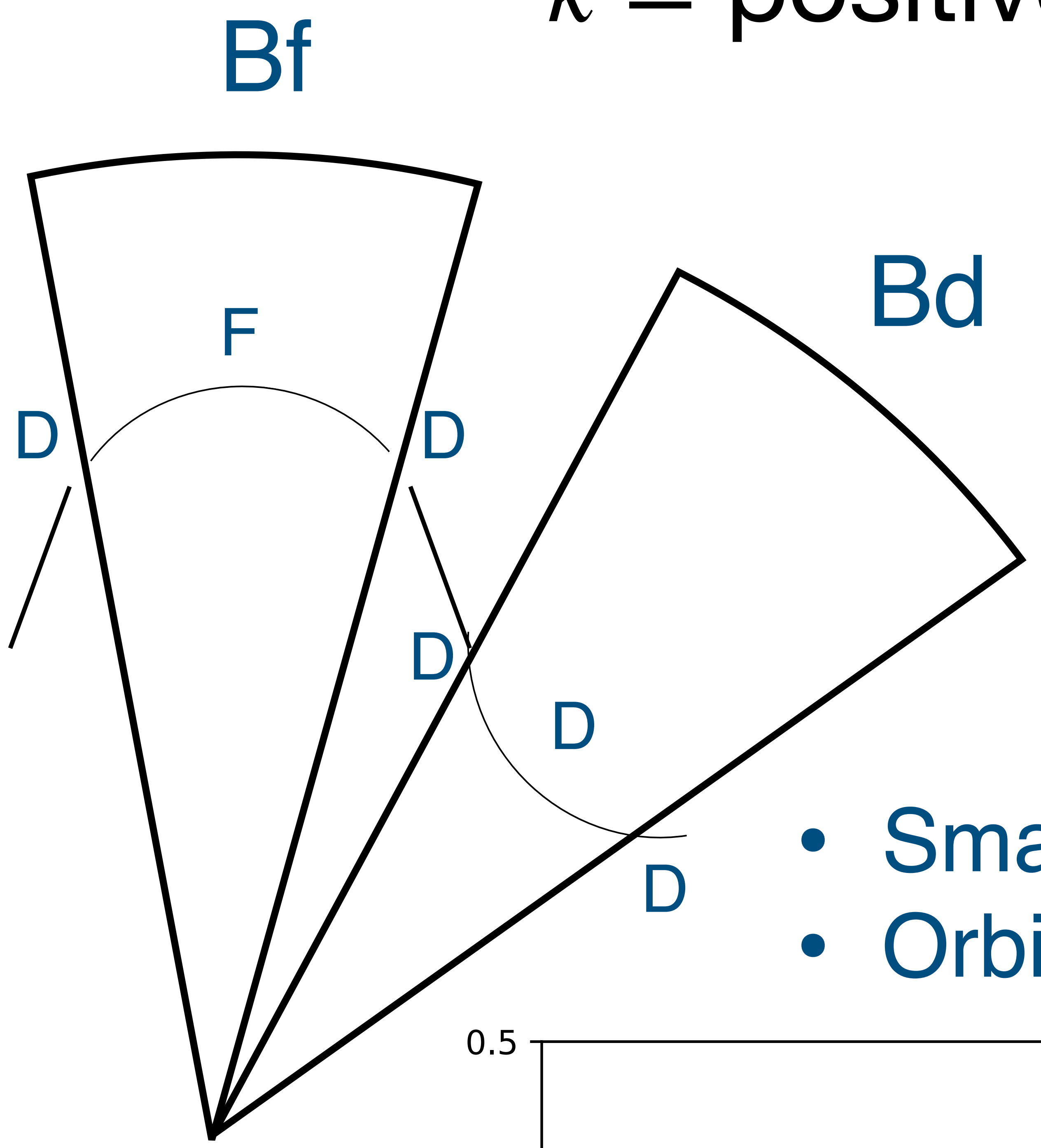
128 cell, negative k



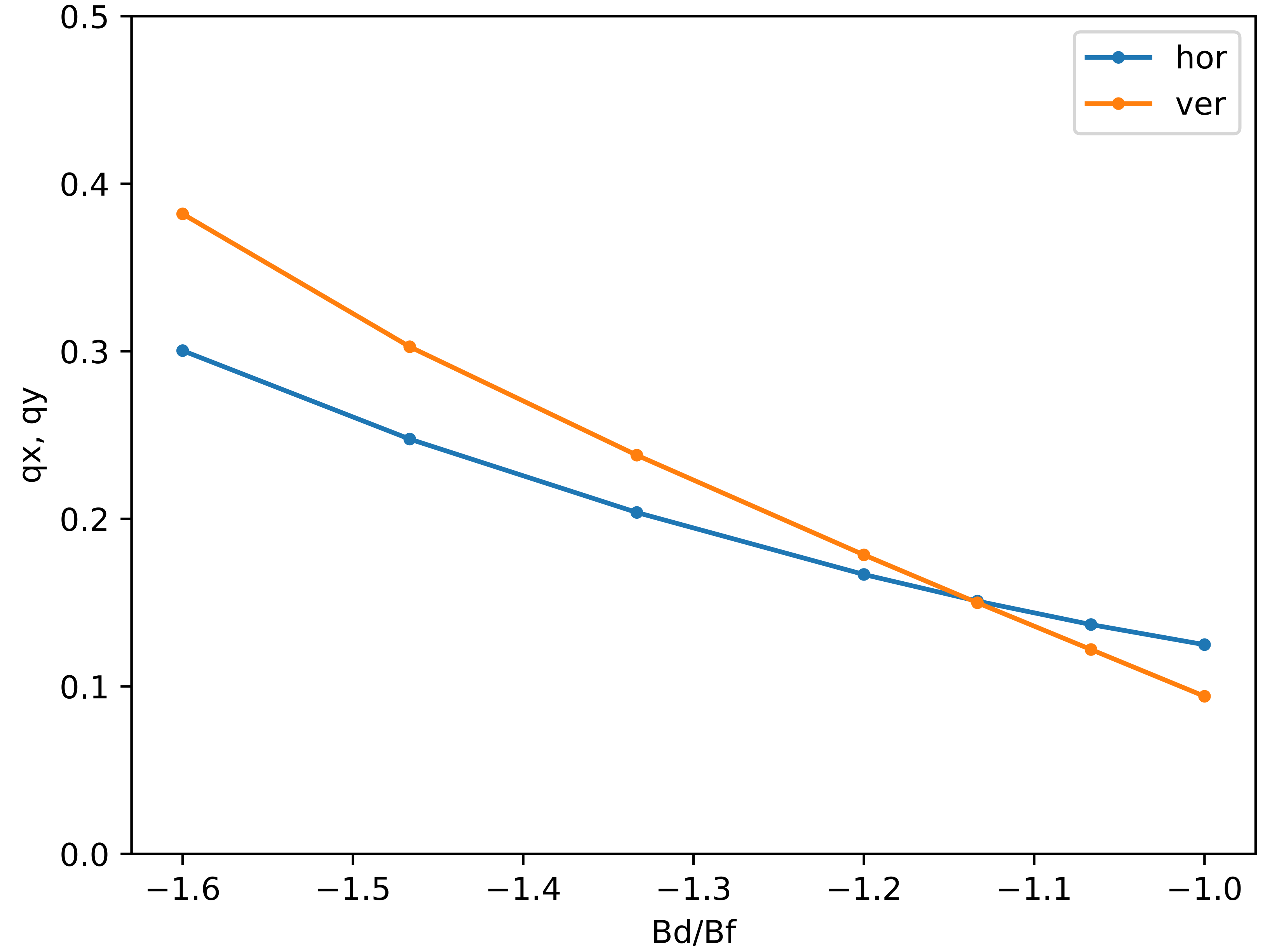
Science and Technology Facilities Council

Bd/Bf ratio vs tune

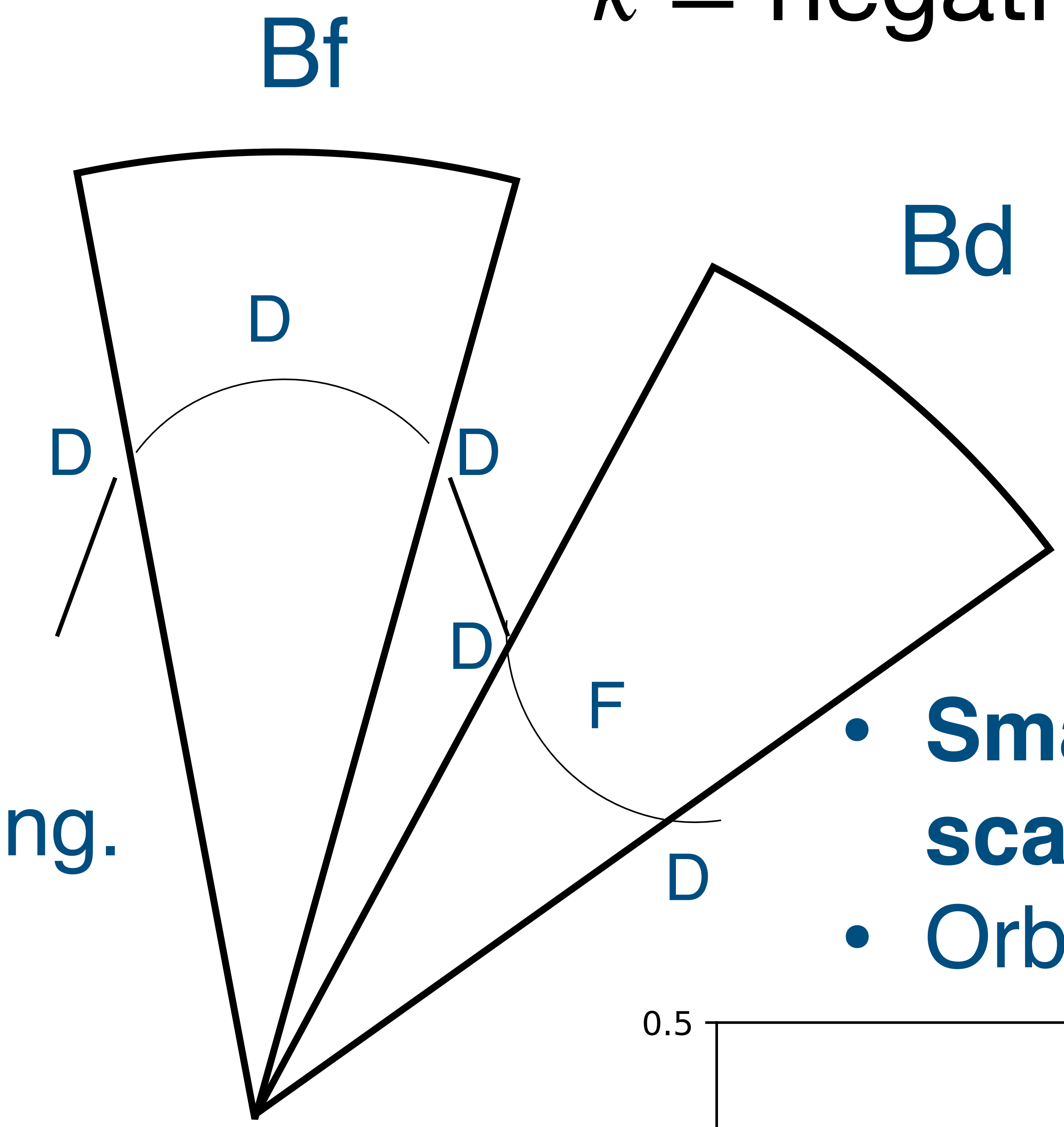
k = positive



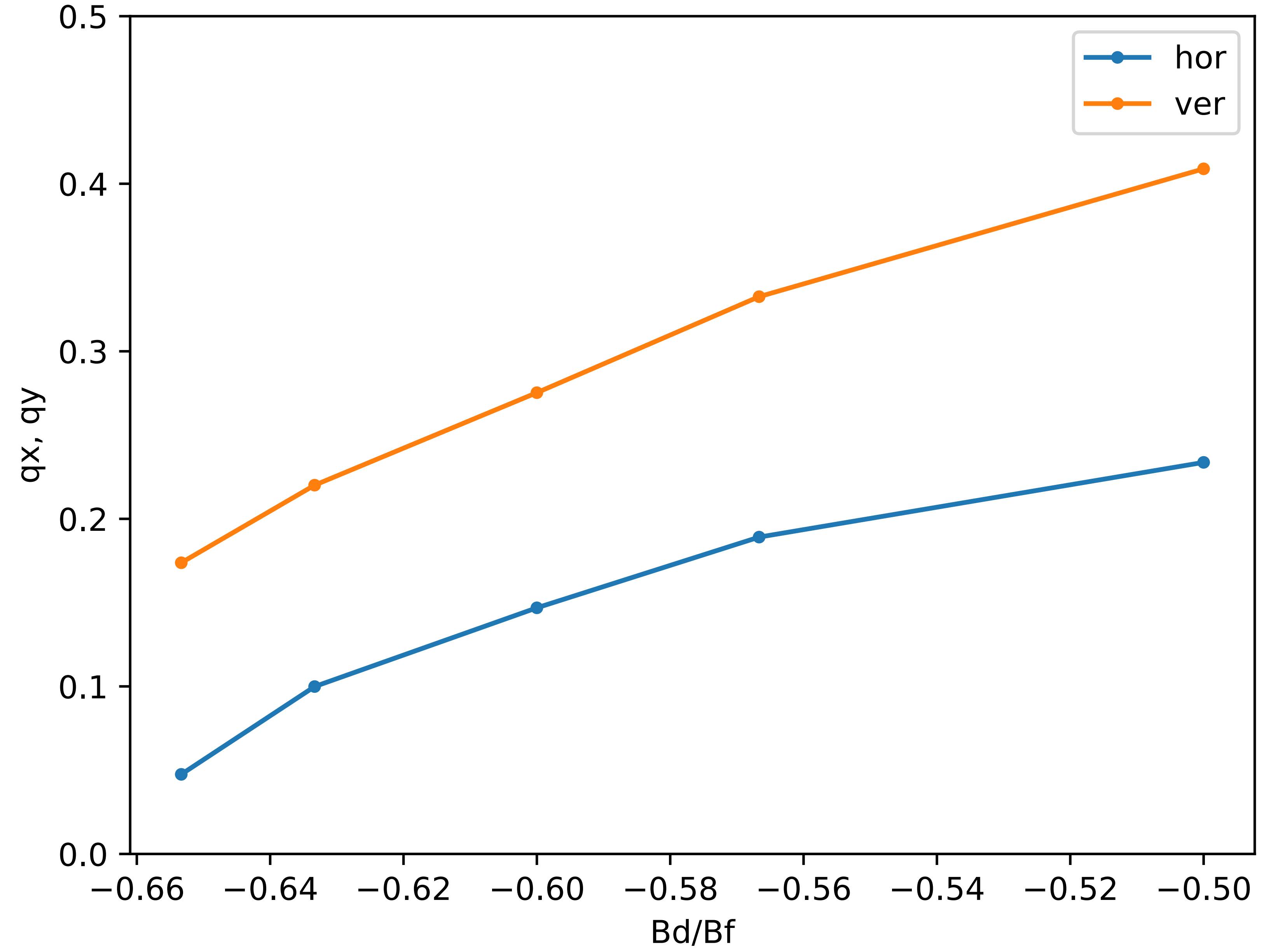
- Smaller Bd reduces scalloping.
- Orbit at Bf shift inner.



k = negative

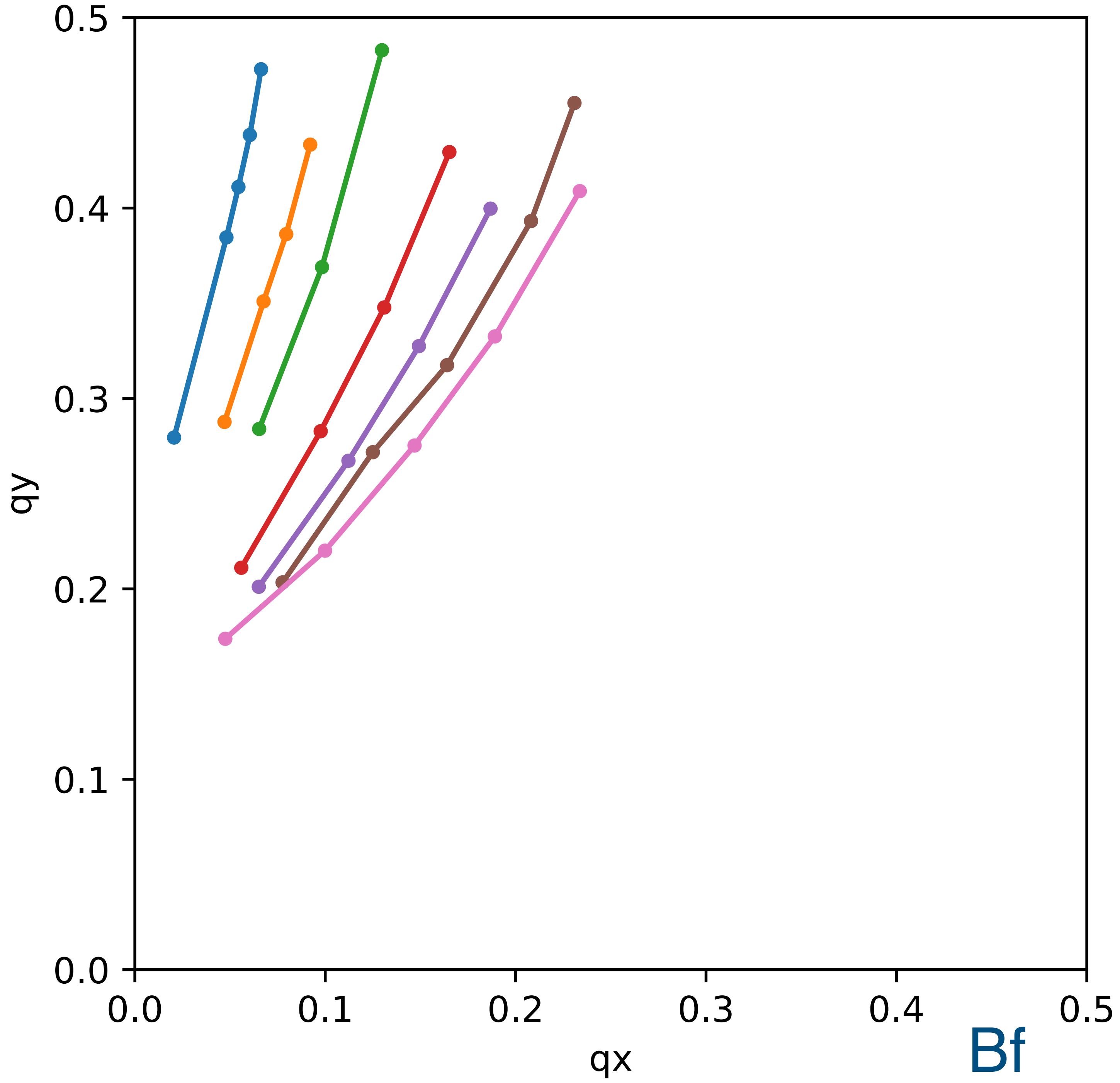


- **Smaller Bd reduces scalloping, then Qv higher?**
- Orbit at Bd shift outer.

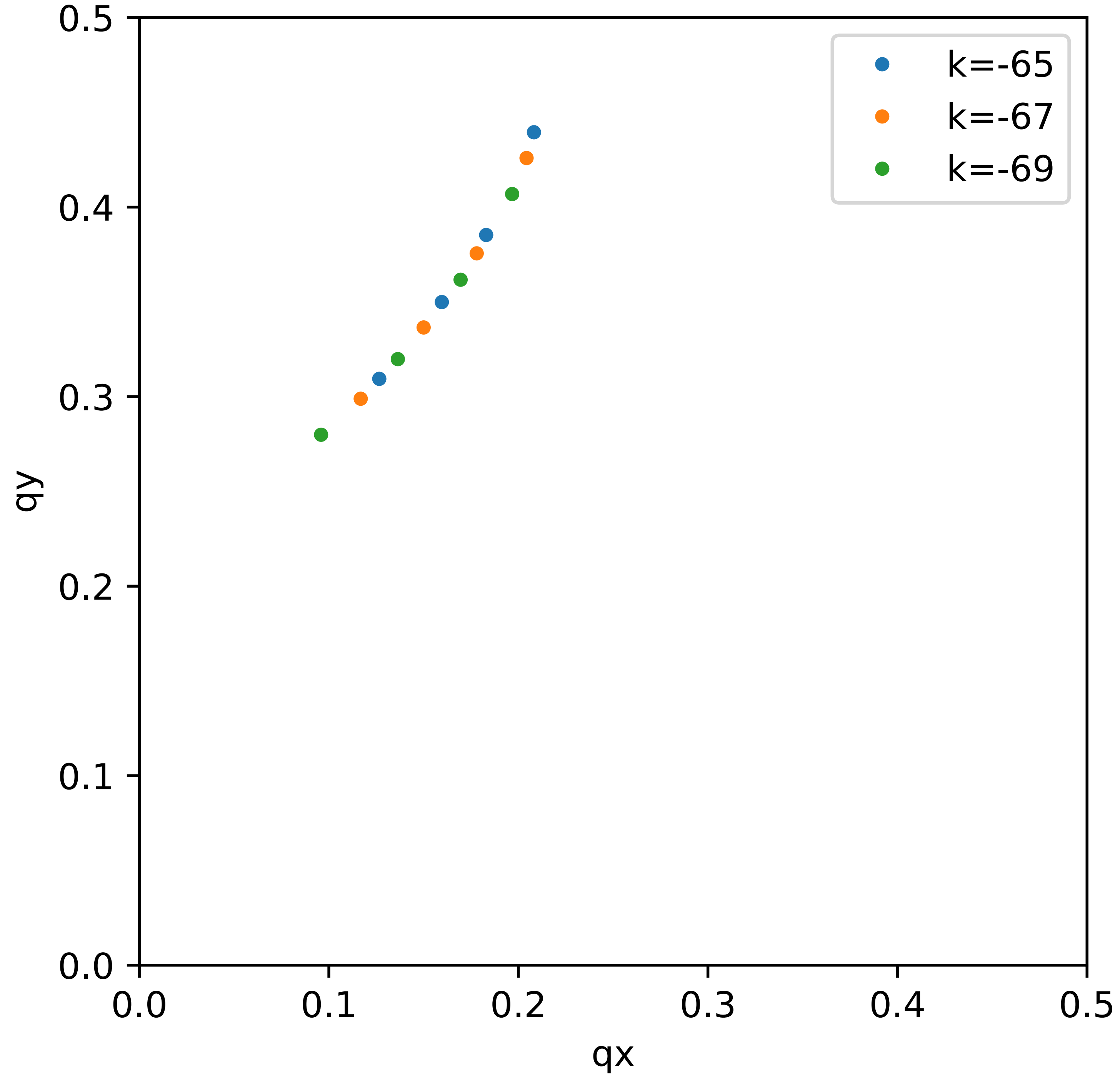


Cell number

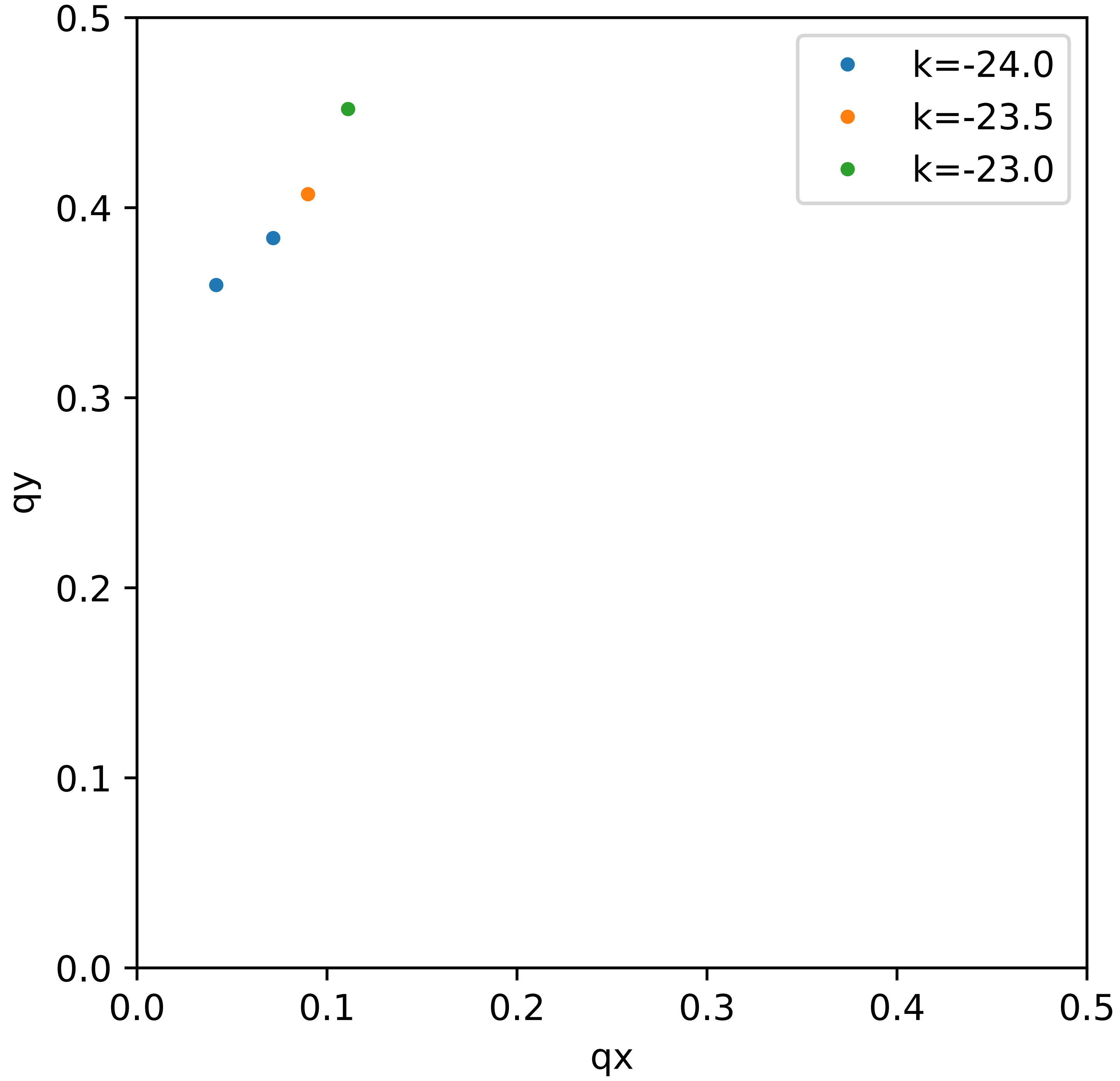
128 cell

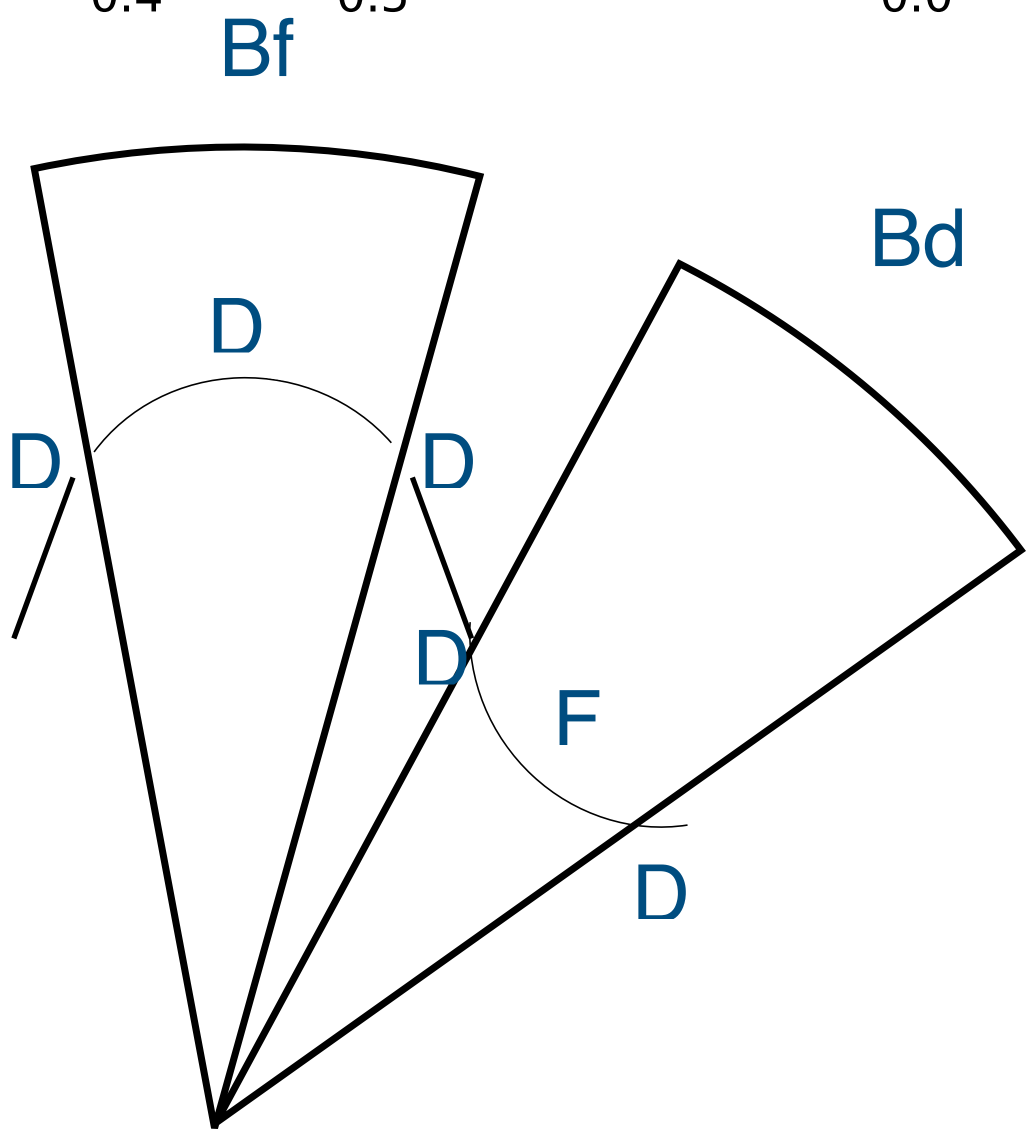


64 cell



32 cell





- The smaller the number of cell, the stronger the vertical focusing from the edge angle.
- Becomes more difficult to find stabile horizontal tune.
- It is more practical for a large machine with the large number of cells.

Science and Technology Facilities Council

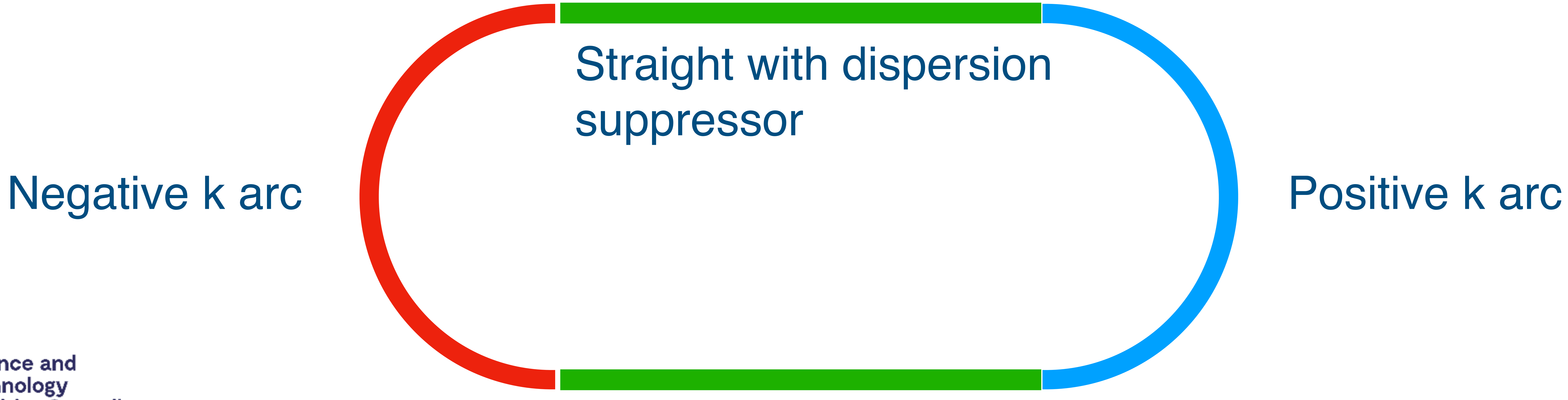
Applications

An isochronous muon accelerator with an hFFA

- Isochronous lattice by combination of positive k and negative k for muon acceleration.

$$\frac{\Delta C}{C} = \alpha_+ \frac{\Delta p}{p} + \alpha_- \frac{\Delta p}{p} = \underset{\approx 0}{\left(\alpha_+ + \alpha_-\right)} \frac{\Delta p}{p}$$

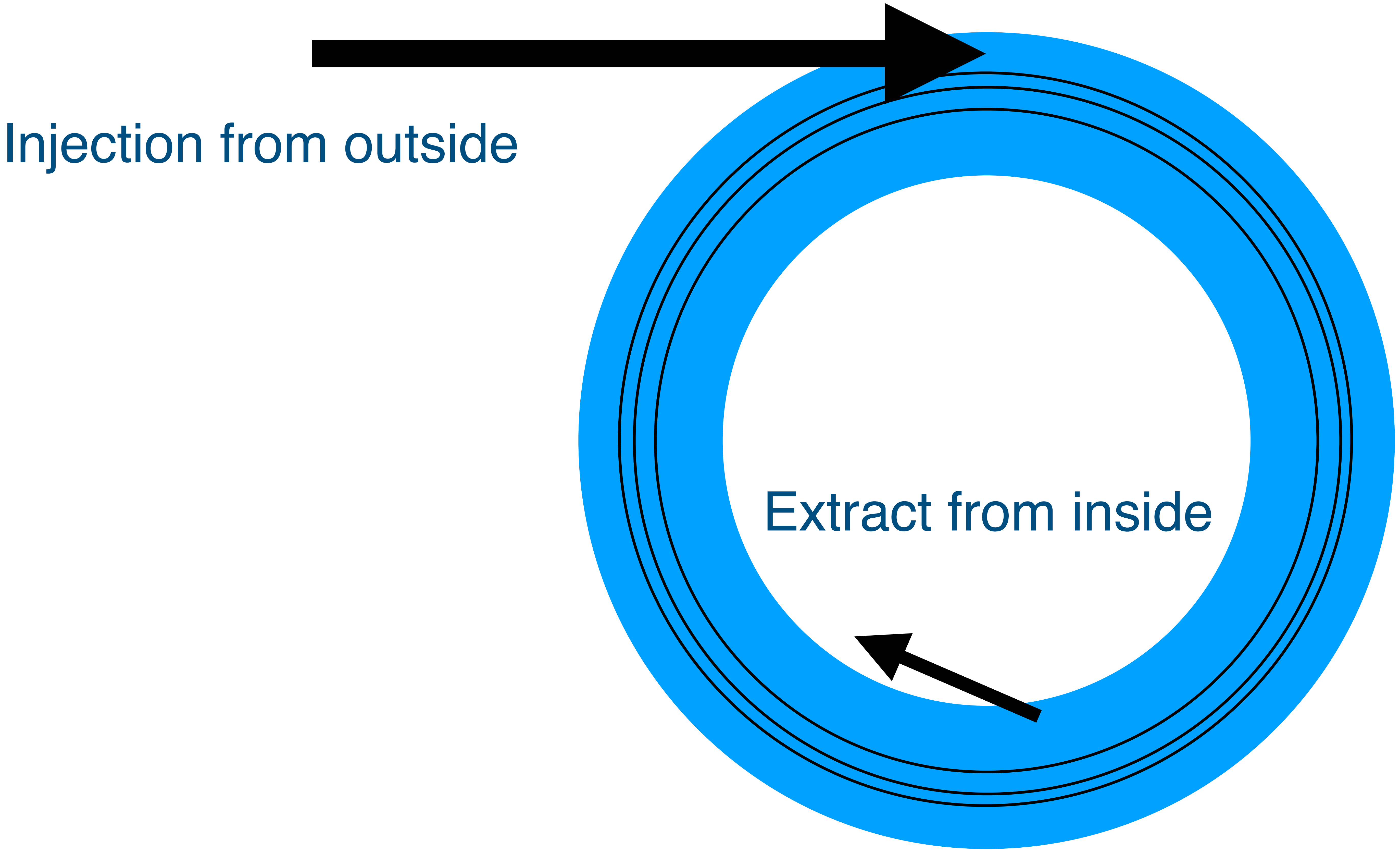
Nearly cancel with both sign of α_c



Science and Technology Facilities Council

Proton therapy driver without gantry

- Inject the beam from the outer radius and extract from inside.



Summary

- To complete covering all the varieties of FFA optics, a negative k optics is examined.
- Stable area may be smaller than that of a positive k FFA.
- Some applications with this unique feature of optics.

Thank you for your attention

Science and
Technology
Facilities Council

Backup