Landau damping in scaling FFAs

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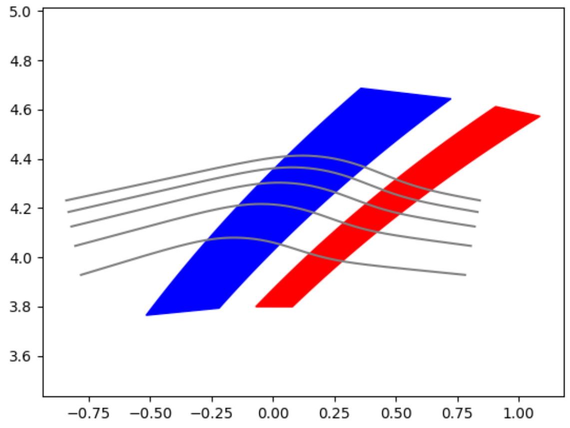
Instabilities in scaling FFAs - general points

Feature	Implication
Zero chromaticity	Removes one source of frequency spread for Landau damping.
Nonlinear fields ($B \sim r^k$)	**Tune shift with amplitude could provide Landau damping.**
Wide aperture magnet	Parallel plate aperture can often be assumed. $Z_V = 2*Z_H$ for resistive wall.
Orbit moves radially during acceleration	The impedance varies with the outward movement of the orbit e..g the beam may see the impedance of the extraction kicker only when close to the extraction orbit.
Stacking	Coasting beam instabilities, even with low growth rates, may be significant.



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The Vaccaro Stability Diagram

- Assume a beam oscillating with coherent tune Q_{coh}

$$x(t) \propto e^{j2\pi f_0 Q_{coh}}$$

- A resistive/reactive impedance leads an imaginary/real coherent tune shift

$$\Delta Q_{coh} = \mathrm{Re}[\Delta Q_{coh}] + j\mathrm{Im}[\Delta Q_{coh}]$$

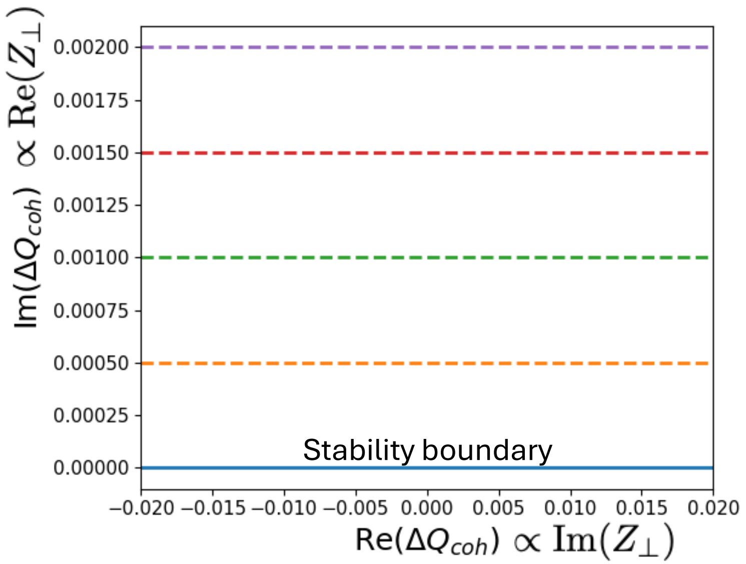
$$x(t) \propto e^{-j2\pi f_0 [Q_0 + \mathrm{Re}(\Delta Q_{coh})]t} e^{2\pi f_0 \mathrm{Im}(\Delta Q_{coh})t}$$

- The Vacarro stability diagram shows curves of constant growth rates in the complex tune shift (or complex impedance) plane. Unstable if $\mathrm{Im}(\Delta Q_{coh}) > 0$ with growth rate $2\pi f_0 \mathrm{Im}(\Delta Q_{coh})$.

- However, the response of the beam can be modified by introducing tune spread.

Transverse coasting beam case

$$\Delta Q_{coh} = -\frac{q^2 \beta c N}{2\gamma m_0 C^2 \omega_\beta \omega_0} j Z_y[(n - Q_y)\omega_0]$$

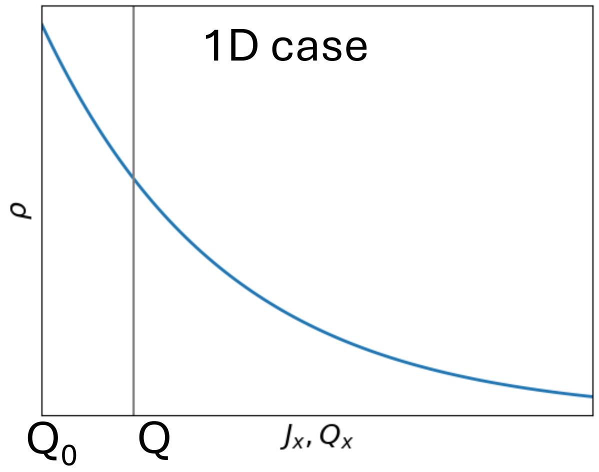


Landau damping due to octupolar tune shift

- Dispersion equation in the presence of tune shift with amplitude

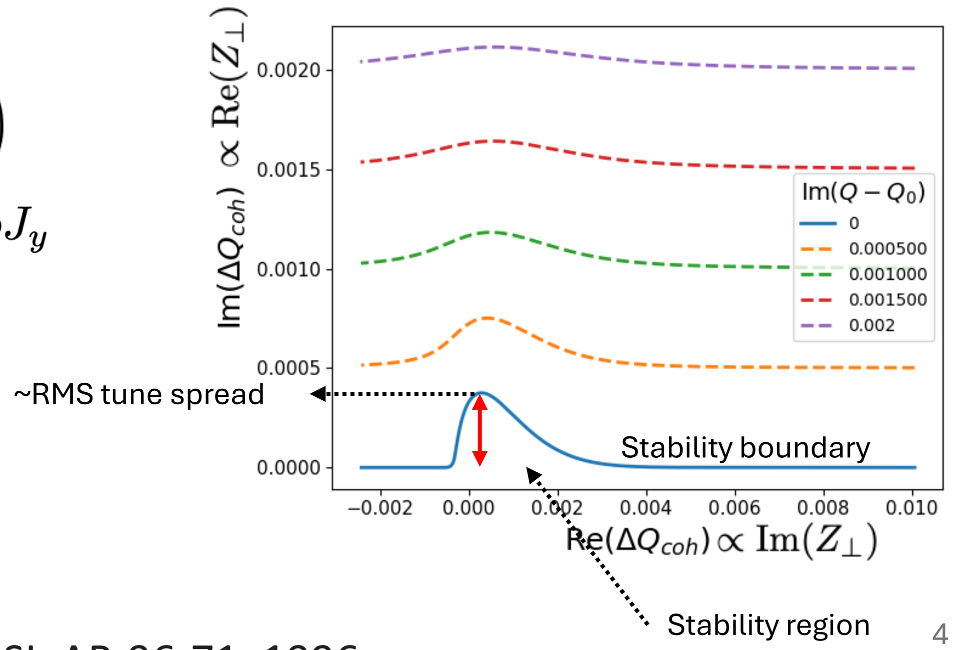
$$1 = -\Delta Q_{coh} \int_0^\infty dJ_x \int_0^\infty dJ_y \frac{J_x \frac{\partial \rho(J_x, J_y)}{\partial J_x}}{Q - Q_x(J_x, J_y)}$$

- Q is the coherent tune, $\rho(J_x, J_y)$ is the density distribution and $Q_x(Jx, Jy)$ is the amplitude dependent tune.
- Including just linear tune shift with amplitude and assuming a Gaussian distribution



$$\rho(J_x, J_y) = \frac{1}{\sigma^4} \exp\left(-\frac{J_x + J_y}{\sigma^2}\right)$$

$$Q_x(J_x, J_y) = Q_0 + aJ_x + bJ_y$$



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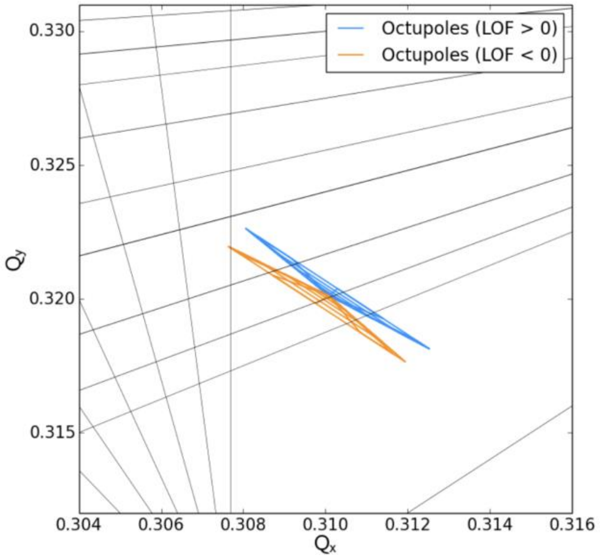
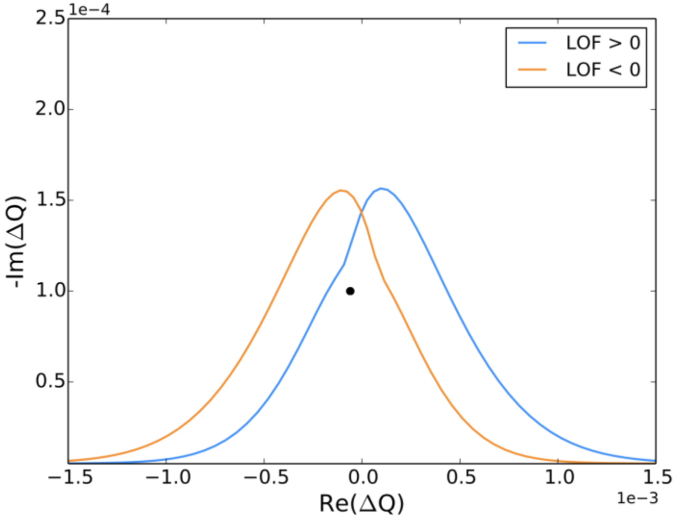
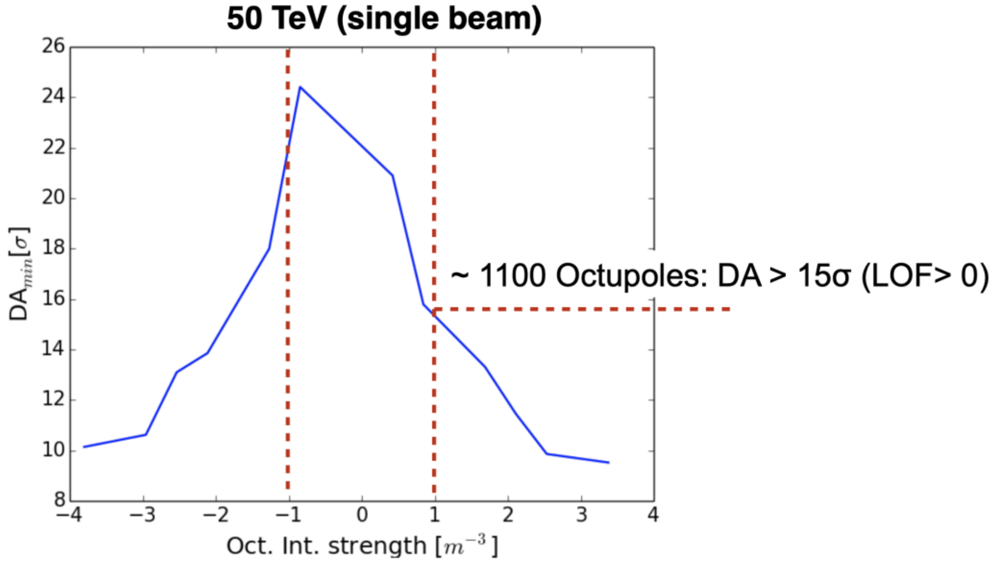
J.S Berg, F. Ruggiero, CERN report SL-AP-96-71, 1996

Landau octupoles – FCC example



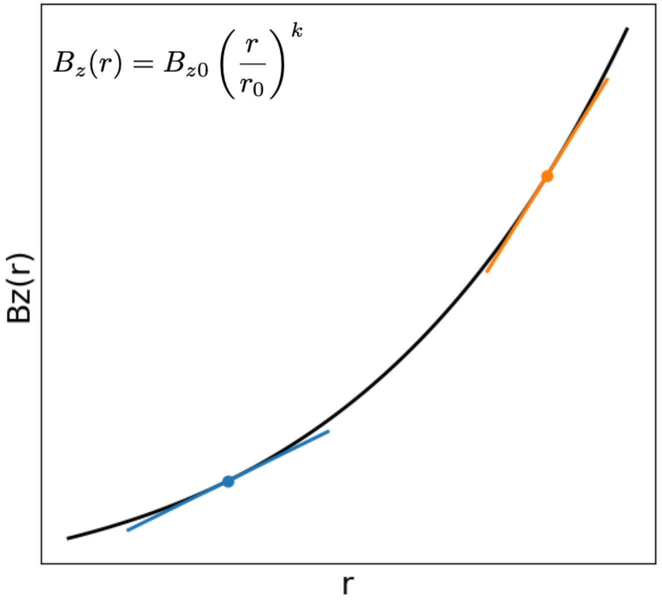




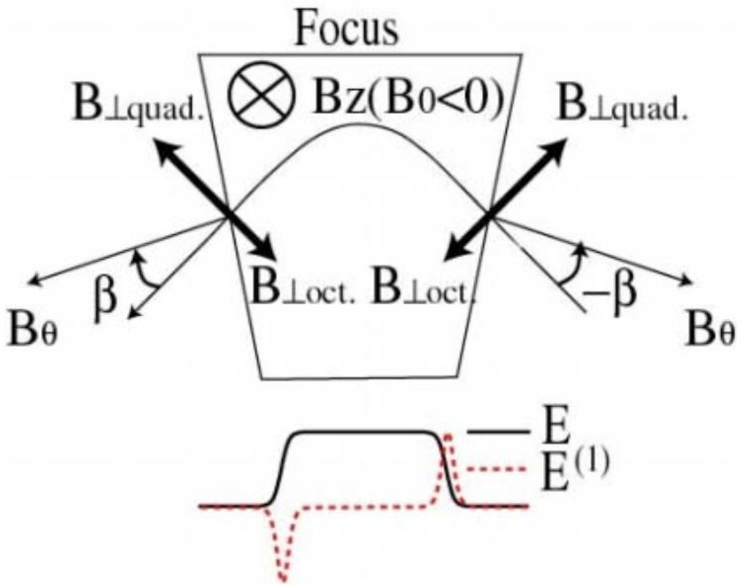
- Octupoles can provide stability but there is a trade-off with dynamic aperture
- Figures show how the octupole polarity affects the (left) the tunes spread, (centre) the stability region and (right) the DA vs octupole strength
- Octupole current 720A, ~1100 octupoles

J. Barranco et al, "Beam-beam effects, octupoles and Landau damping", EuroCirCol Meeting, 9-10 October 2017

Source of octupole in scaling FFAs



High order multipoles in scaling field.



Effect of edge

- Derivative of fringe field results in off-midplane $B_\perp$, B_θ with octupole component.
- $B_\perp$ increases with spiral angle.

M. Aiba Proc. EPAC 2002, Paris, France

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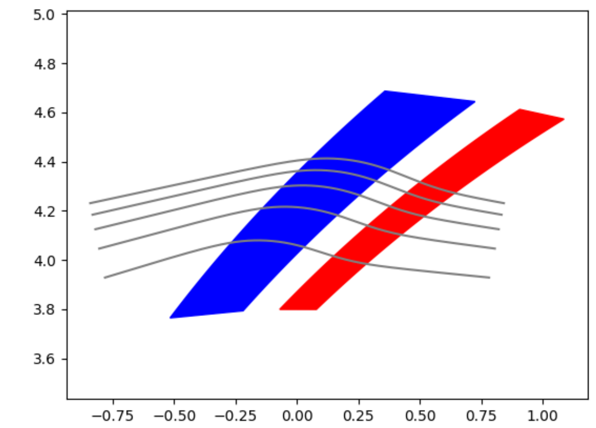
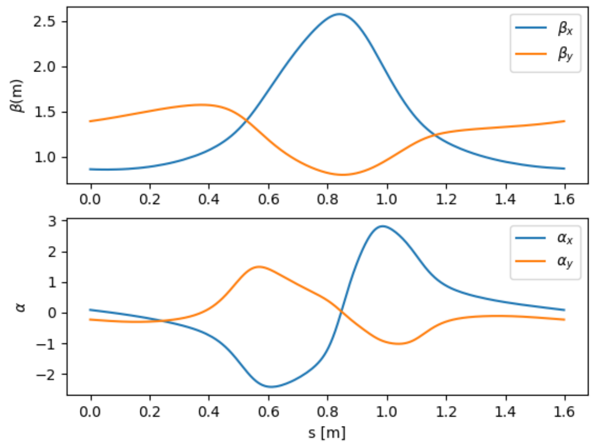
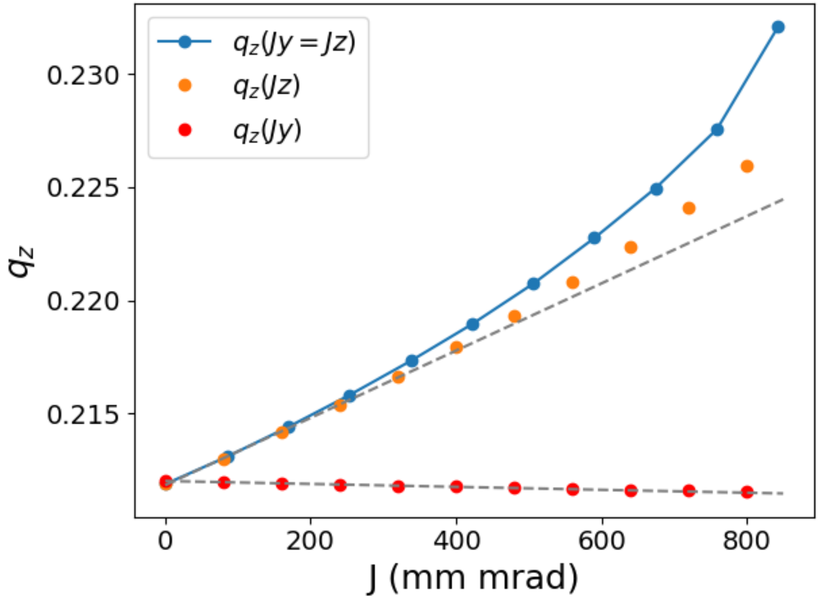
FETS-FFA case (benchmark lattice)





Key Parameters	
Lattice type	DFspiral
Spiral angle	45°
Field index	7.715
Cells	16
Fringe field extent	0.085 m
Radius at injection	4 m
Beam emittance after painting	10 π mm mrad normalized (125 π mm mrad geometric at injection)

Calculate tune shift with amplitude



- Track particles at each amplitude using Zgoubi.
- Tune found at each amplitude using the NAFF algorithm.
- By linear fit find for case of vertical tune **a = 14.86**, **b=-0.65** in the tune shift with amplitude equation

$$q_y(J_x, J_y) = q_{y0} + aJ_y + bJ_x$$

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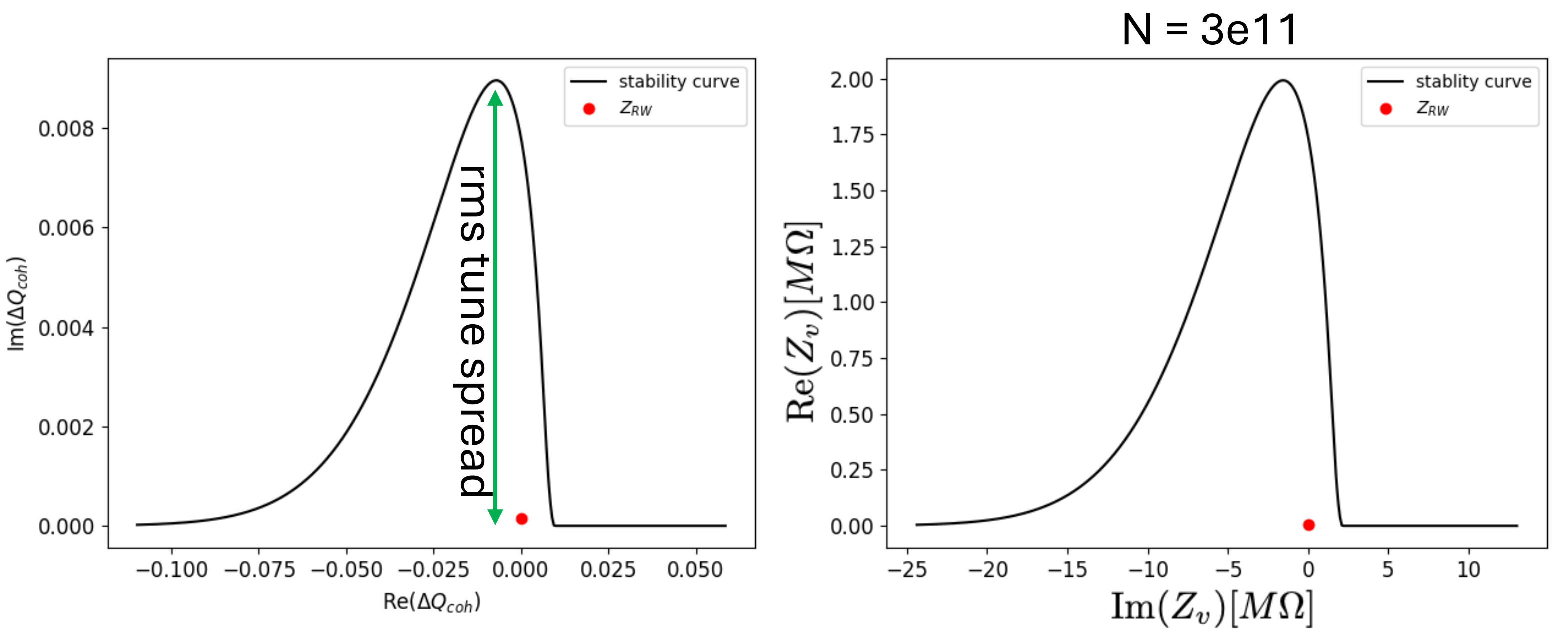
Stability diagram calculation



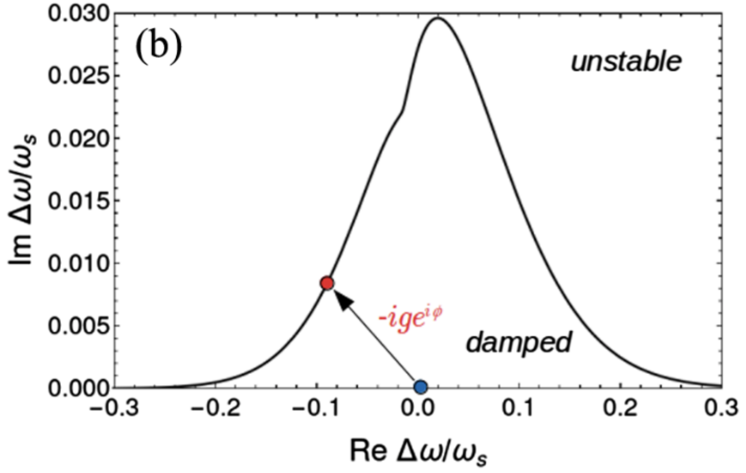
- Assuming a uniformly filled beam core emittance 2J = 125 π mm mrad, then $\sigma = 125/(2\sqrt{3})$ π mm mrad.
- Ring tune spread at the rms emittance is 9e-3 in vertical, 5.5e-4 in horizontal.

$$\Delta Q_{coh} = -\frac{q^2 \beta c N}{2\gamma m_0 C^2 \omega_\beta \omega_0} j Z_y[(n - Q_y)\omega_0]$$

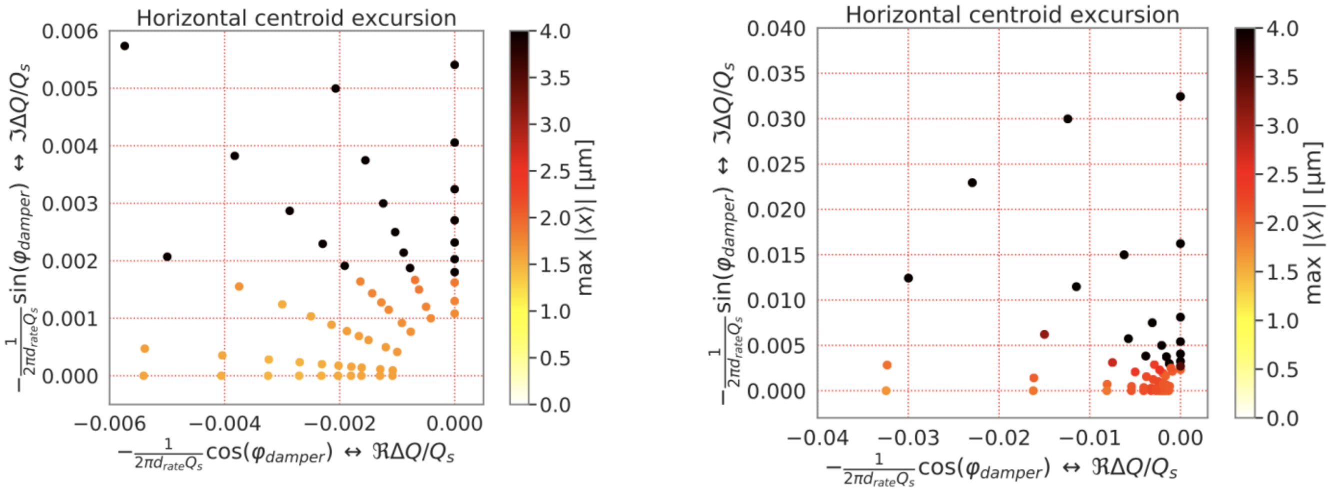
Probing the stability region with an anti-damper

- The antidamper with gain g and phase ϕ acts as a source of impedance which shifts frequency of collective mode by

$$\Delta\omega \propto -ige^{i\phi} \qquad x'_f = x'_i - g\left(\langle x'\rangle \sin\phi + \frac{\langle x\rangle}{\beta}\cos\phi\right)$$



S. A. Antipov et al, PRL 126, 164801 (2021)

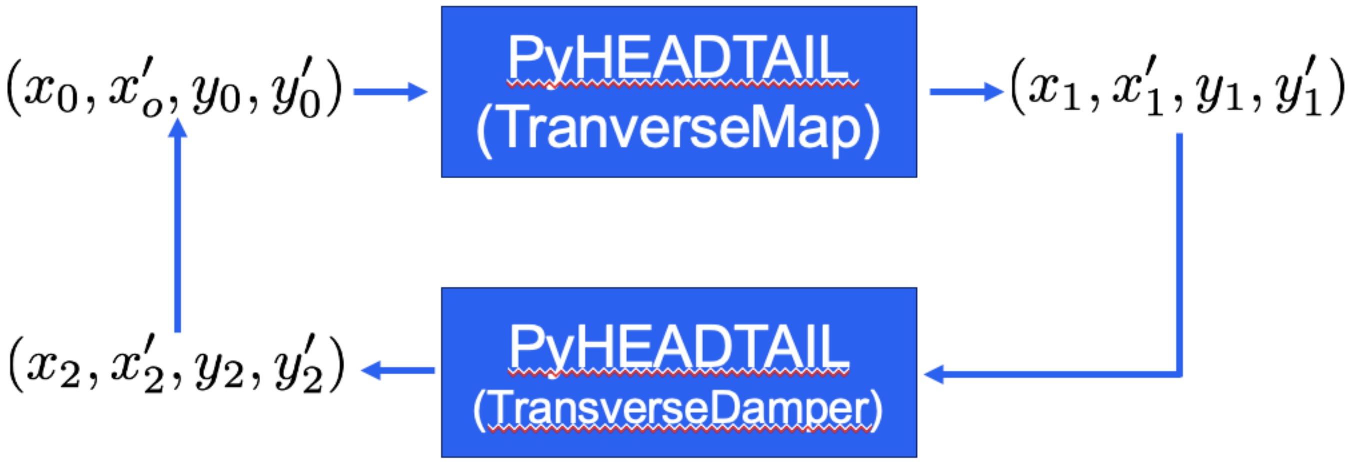


LHC case (left) with natural octupole only (right) with Landau octupoles.

A. Oeftiger

A. Oeftiger

Simulation setup



- The anti-damper with gain g and phase ϕ applies the following kick to the bunch centroid after each cell pass

$$x'_f = x'_i - g\left(\langle x'\rangle \sin\phi + \frac{\langle x\rangle}{\beta}\cos\phi\right) \qquad g = 2\mathrm{Im}(\Delta Q_{coh})$$

- Create transfer map with amplitude detuning term (found by fit to Zgoubi tune results)

```
ampl_det = AmplitudeDetuning(app_x, app_y, app_xy)
trans_map = TransverseMap(s_a,  alpha_x_a, beta_x_a, D_x_a, alpha_y_a, beta_y_a, D_y_a, Q_x, Q_y, [ampl_det])
```

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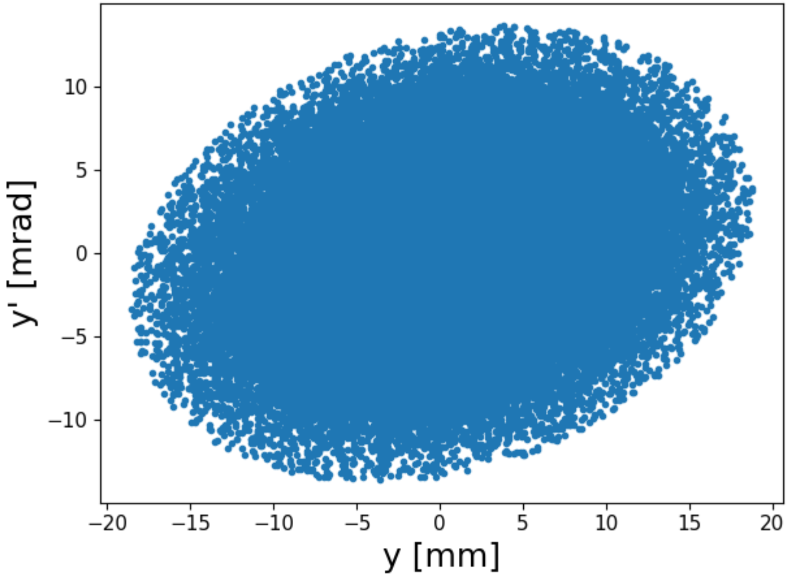
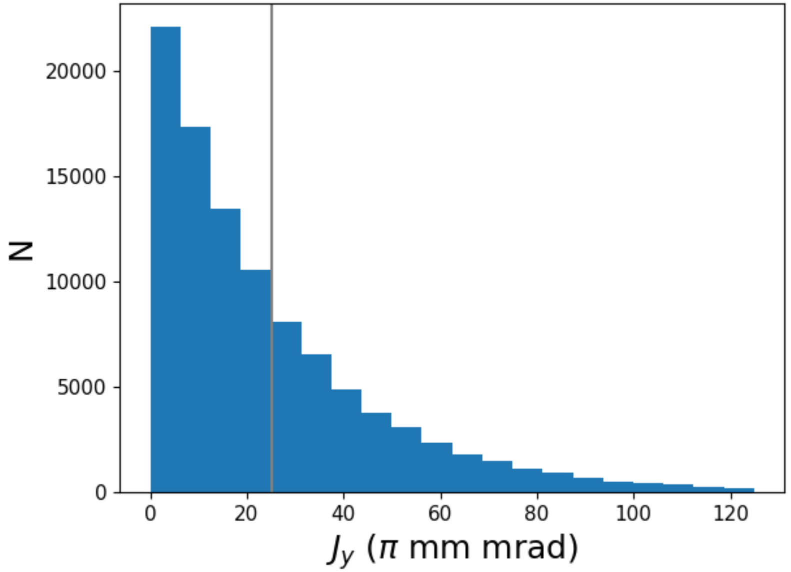
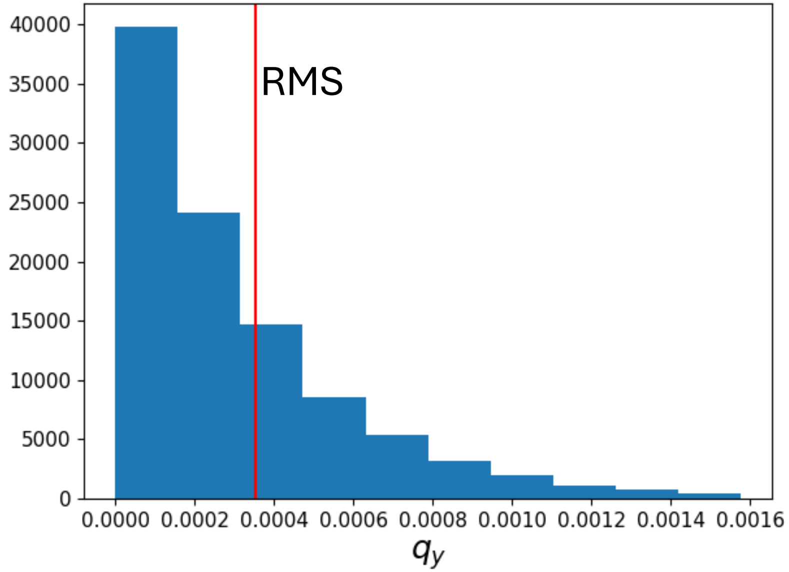
Initial bunch distribution







- 1e5 macroparticles
- Gaussian distribution (exponential in J)
- RMS tune spread 3.5e-4

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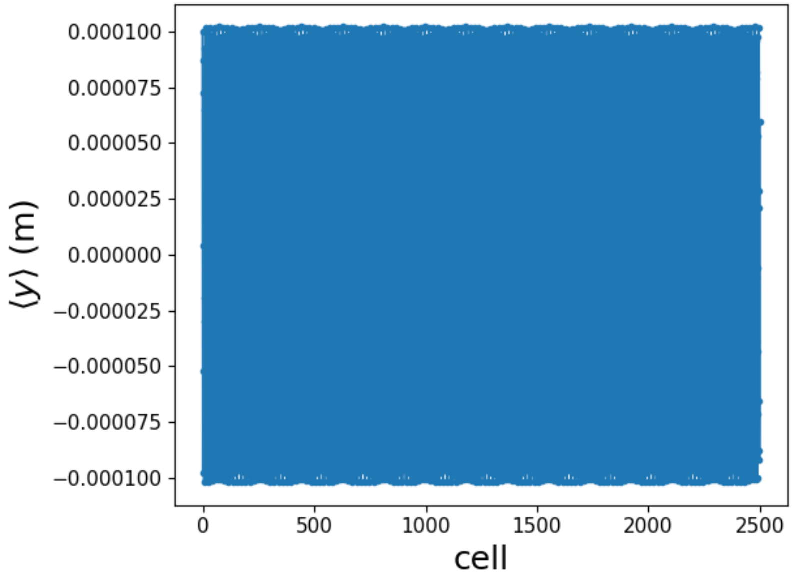
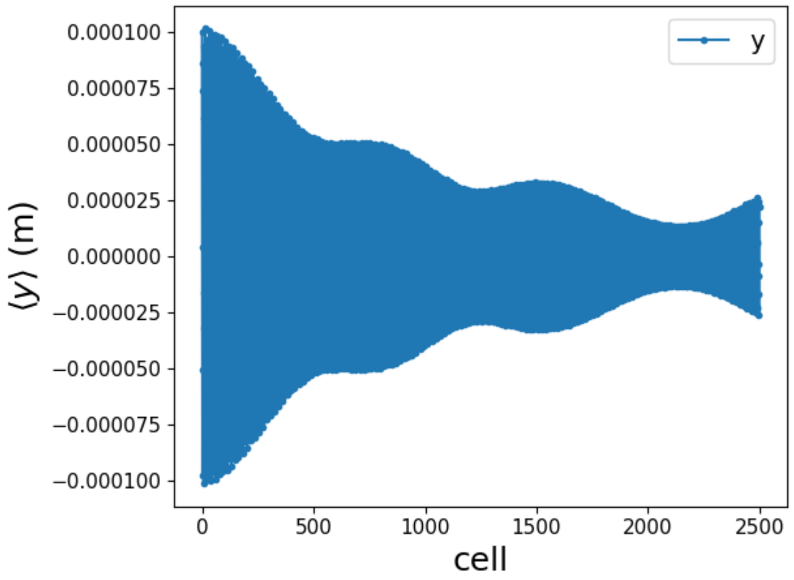
Bunch decoherence

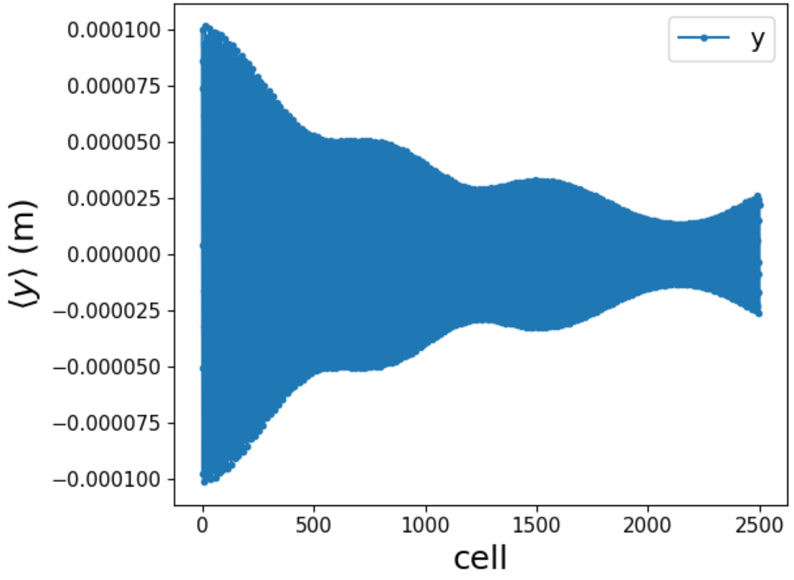
1e5 particles (no tune spread)



1e5 particles (with tune spread)



5e5 particles (with tune spread)



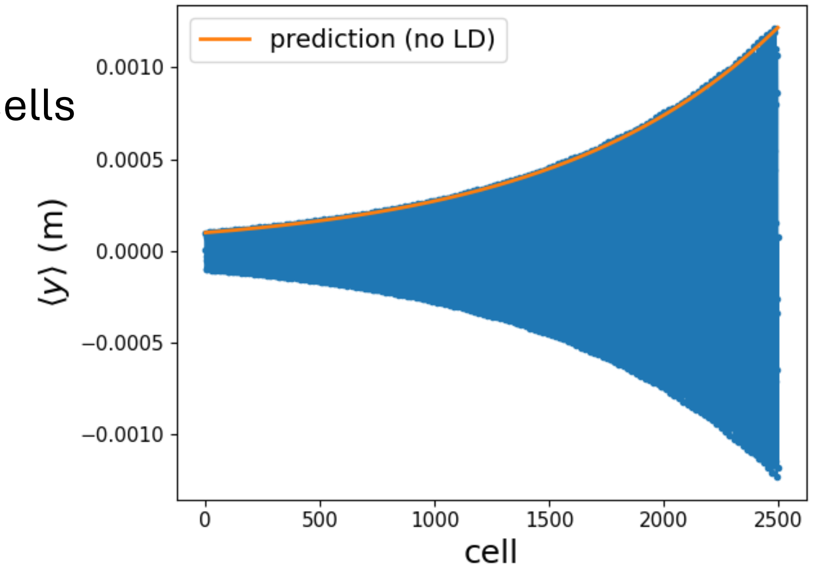
No anti-damper applied.
Initial bunch offset by 100 microns.
With tune spread the initial bunch offset decoheres.

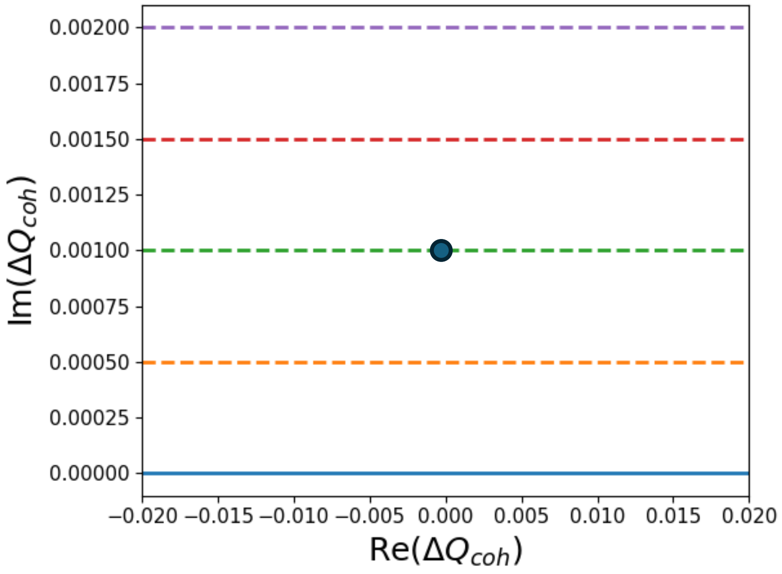
Anti-damper with monochromatic bunch

Set anti-damper growth rate to 1000 cells

$$\mathrm{Im}(q)_{AD} = 0.001$$

$$\mathrm{Re}(q)_{AD} = 0$$





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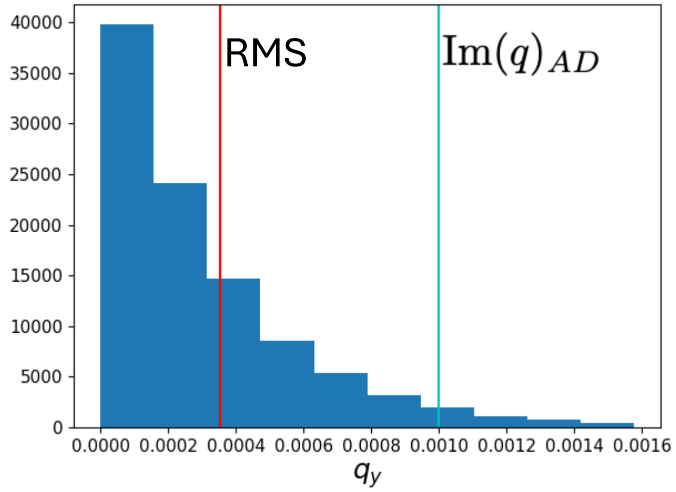
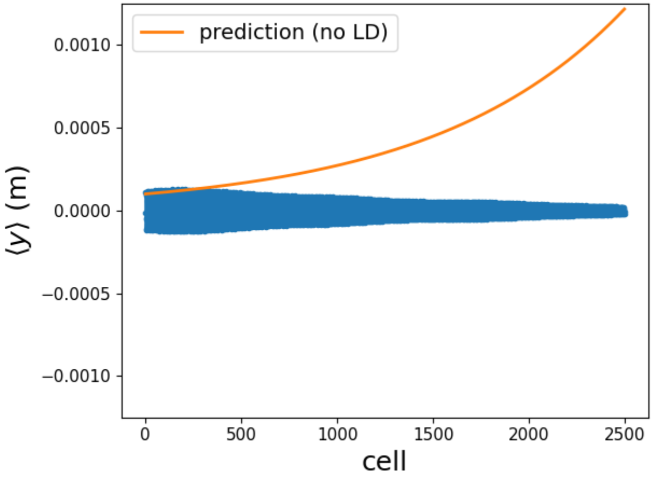
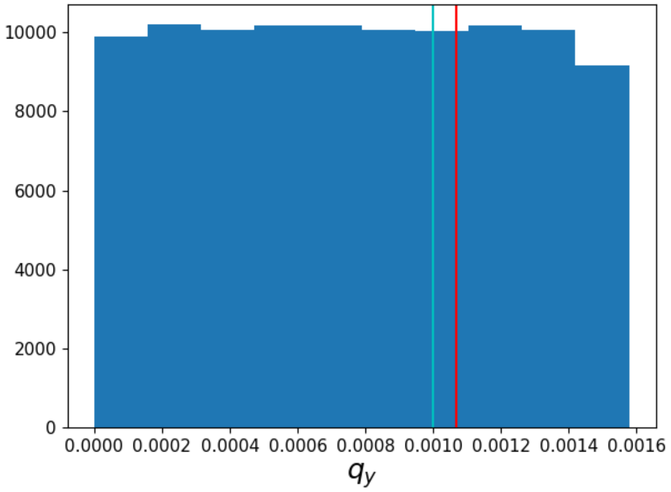
Effect of tune spread

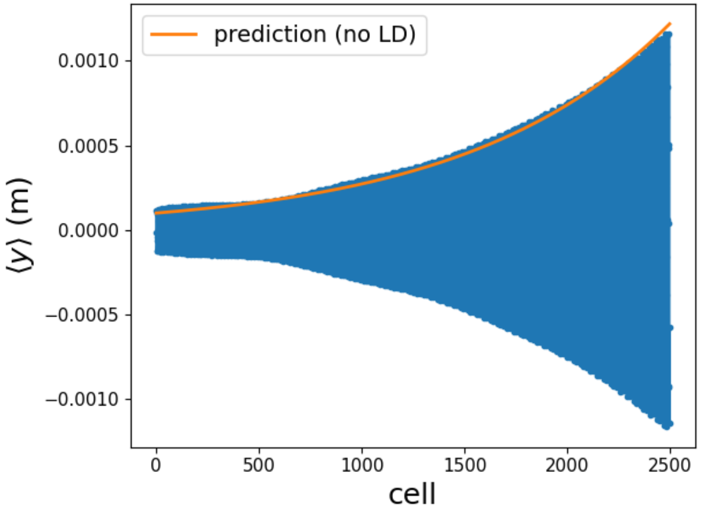
Gaussian distribution





Uniform distribution





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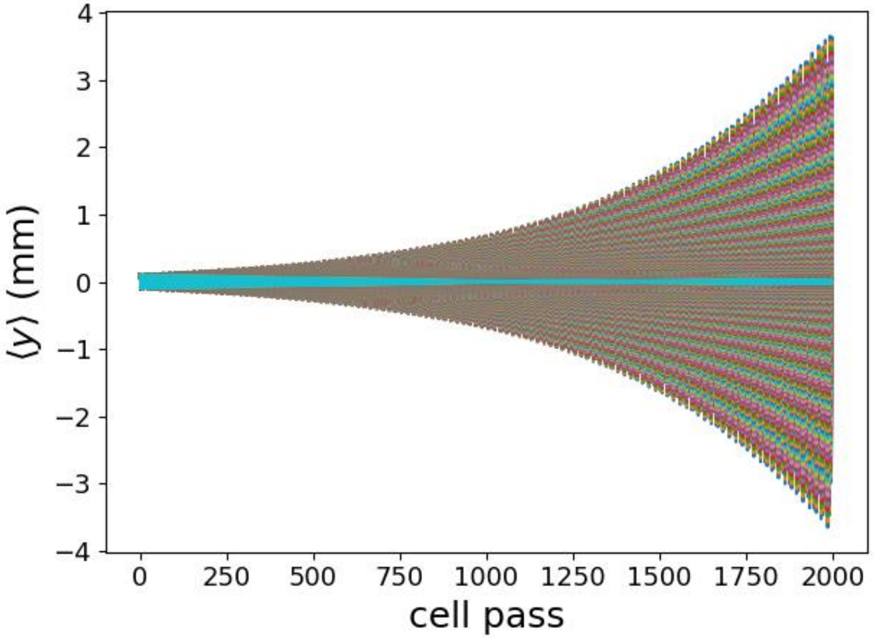
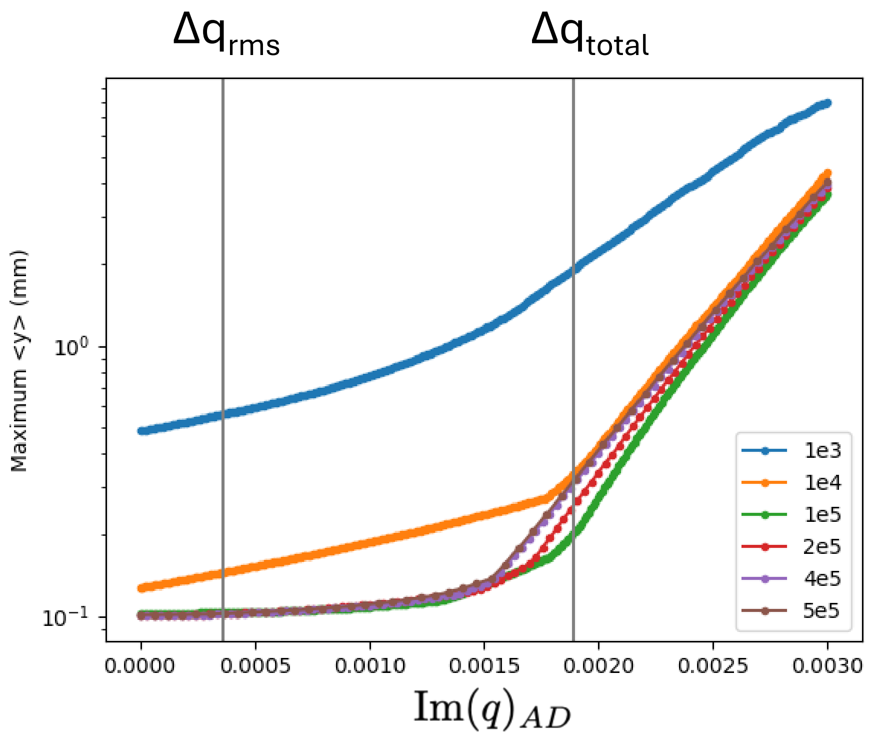
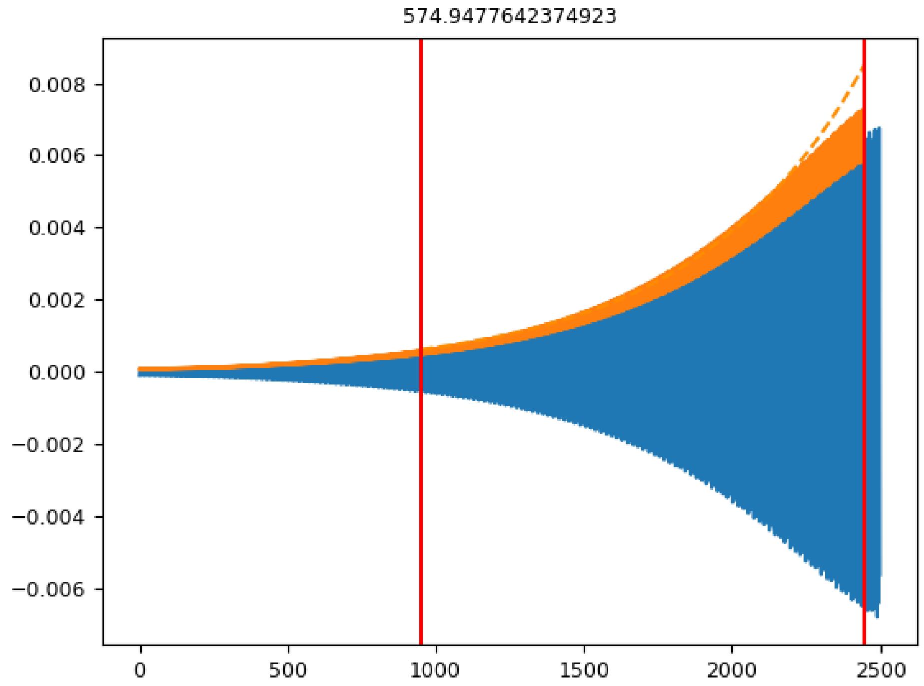
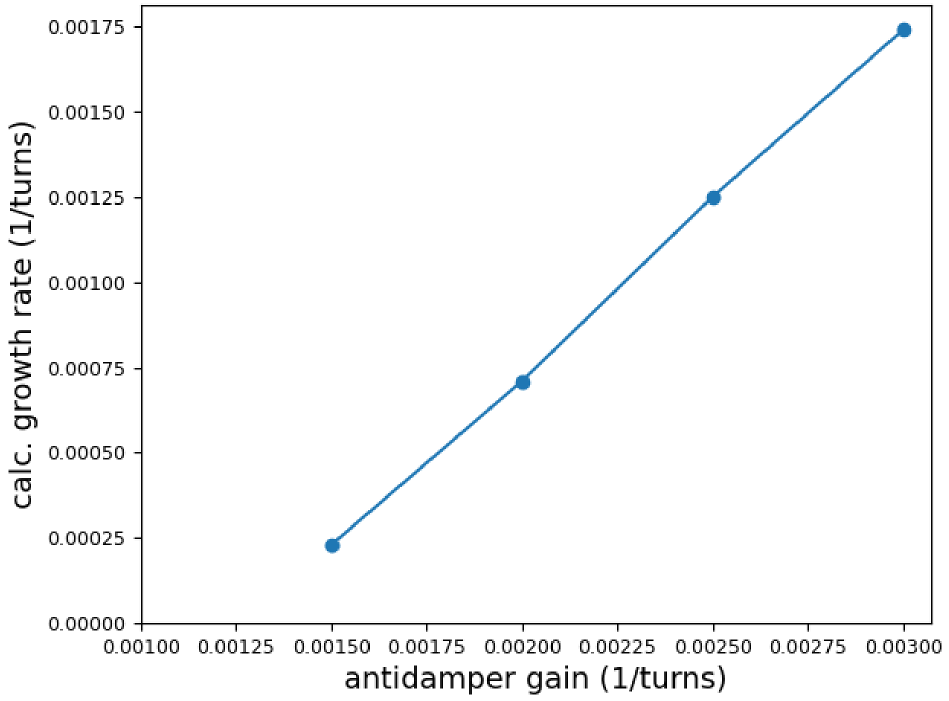
Max <y> vs anti-damper strength





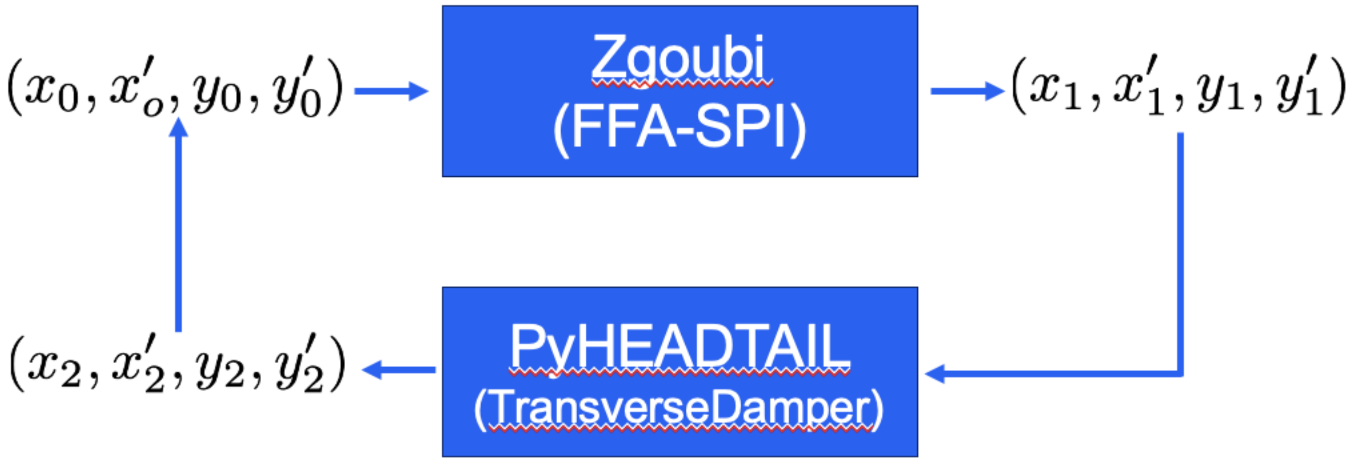
Calculated growth rates



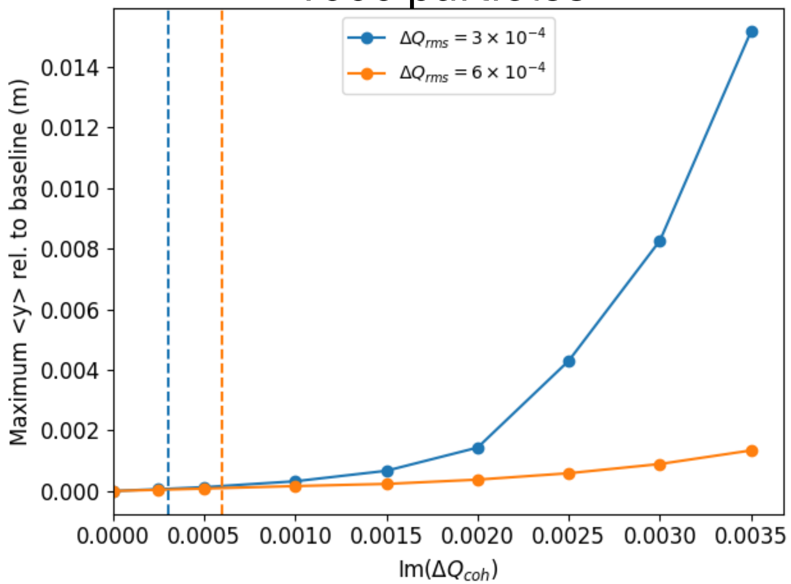


- FITX (A. Oeftiger) used to extract growth rate
- https://pypi.org/project/FITX/

Zgoubi with anti-damper step



1000 particles



- Track through cell using Zgoubi to include exact FFA scaling field.
- Just 1000 particles tracked in initial study, not enough to resolve stability limit.

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Future Work

- Extract high order maps from Zgoubi to speed up the computation.
- Simulate headtail instability - slow and fast.
- Include space charge (using PyHEADTAIL).
- Calculate impedance of vacuum vessel with rectangular cross-section (ImpedanceWake2D, CST]. Realistic values for the impedance of other elements (e.g. kicker magnet) should added.

Extra Slides

Dispersion relation

- J.S. Berg and F. Ruggiero derived analytic expressions for the stability limit for the case of 2D amplitude dependent tunes (CERN report SL-AP-96-71, 1996).
- The dispersion relation relates the coherent tune shift ΔQ_{coh} to Q-Q_x

$$1 = -\Delta Q_{coh} \int_0^\infty dJ_x \int_0^\infty dJ_y \frac{J_x \frac{\partial \rho(J_x, J_y)}{\partial J_x}}{Q - Q_x(J_x, J_y)}$$

- where $\rho(J_x, J_y)$ is the density distribution and, including just linear tune shift with amplitude,

$$Q_x(J_x, J_y) = Q_0 + aJ_x + bJ_y$$

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Beam transfer function (Gaussian case)

- Assuming a Gaussian distribution in 2D

$$\rho(J_x, J_y) = \frac{1}{\sigma^4} \exp(-\frac{J_x + J_y}{\sigma^2})$$

- Define normalized tune shifts with respect to the horizontal tune spread S = -aσ^2.

$$q_{coh} = \frac{\Delta Q_{coh}}{S} \qquad q = \frac{Q - Q_0}{S}$$

then the dispersion integral becomes

$$q_{coh} = \frac{i}{T(q)}$$

where the beam transfer function T(q) maps from complex q-plane to the complex q_{coh} plane (note c = b/a)

$$T(q) = i \int_0^\infty dJ_x \int_0^\infty dJ_y \frac{J_x \exp[-(J_x + J_y)]}{J_x + q + cJ_y}$$

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BTF solution

- The beam transfer function can be integrated analytically to obtain

$$T(q) = i\frac{1 - c - (q + c - cq)\exp[q]E_1(q) + c\exp[q/c]E_1(q/c)}{(1 - c)^2}$$

where $E_1(z)$ is the exponential integral function given by $E_1(z) = \int_z^\infty dt \exp[-t]/t$

- In the 1D case (c=0)

$$T(q) = i(1 - q\exp[q]E_1(q))$$

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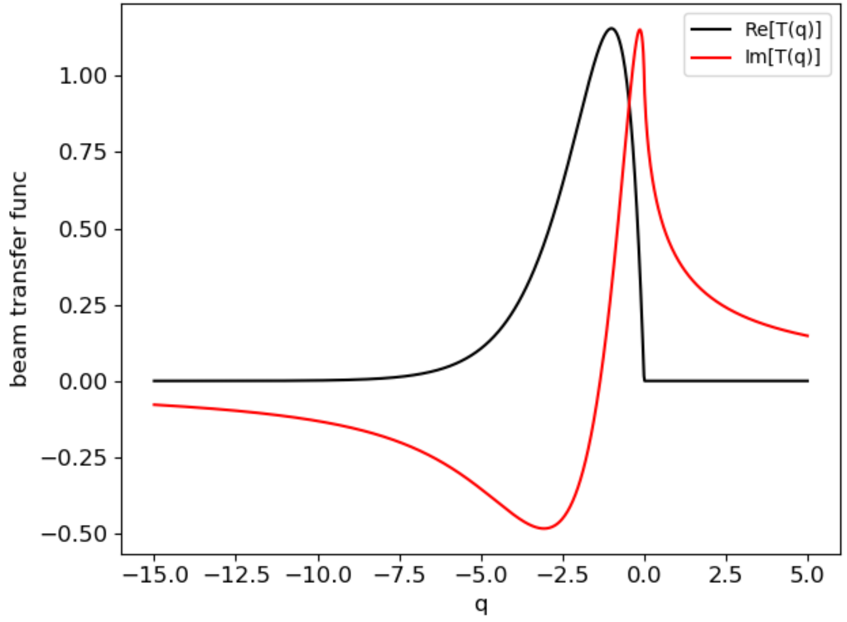
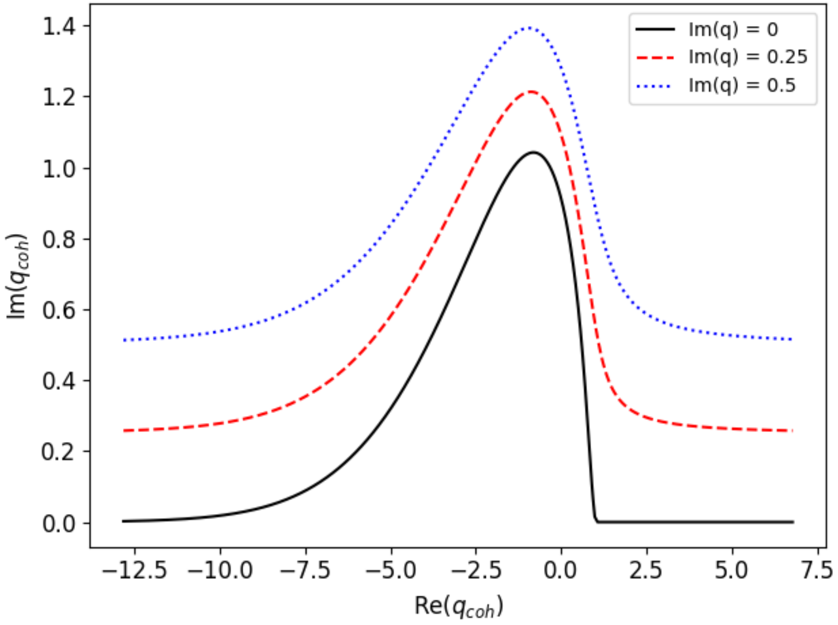
BTF and Stability Limit in 1D case



Stability Diagram



Stability is assured if imaginary and real coherent tune shifts from impedances stay within the stability limit.

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