The Matrix Element Method at NLO

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Outline

- MEM: a brief Introduction
- MEM at LO
- MEM at NLO
- Example : DY
- Example : $H \longrightarrow ZZ$

The Matrix Element Method (MEM)

- All measurements of SM parameters and searches for new physics rely on matrix elements at some level.

- The Matrix element contains the maximal amount of theoretical information available (for the hard scattering process).

- The goal of the MEM is to perform a measurement using the matrix element to create a probability distribution function.

$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_{\Omega}^{LO}} \int dx_a dx_b \, d\mathbf{y} \sum_{ij} \frac{f_i(x_a) f_j(x_b)}{x_a x_b s} \, \mathcal{B}_{\Omega}^{ij}(p_a, p_b, \mathbf{y}) \, W(\mathbf{x}, \mathbf{y}) \ .$$

- Then this can be used to build a Likelihood for the model (Omega) under investigation.

$$\mathcal{L}(\mathbf{x}|\Omega) = f(N) \prod_{i=1,N} \mathcal{P}(\mathbf{x_i}|\Omega).$$

Pros and cons of the method.

- Clean separation between theory and experimental inputs
- Utilizes full ME.
- Many potential applications.
- Ripe for parallelisation

- Computationally expensive
- Need for simplifications:
  - Transfer function form
  - LO ME elements

This talk

$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_\Omega^{LO}} \int dx_a dx_b \, d\mathbf{y} \sum_{ij} \frac{f_i(x_a) f_j(x_b)}{x_a x_b s} \, \mathcal{B}_\Omega^{ij}(p_a, p_b, \mathbf{y}) \, W(\mathbf{x}, \mathbf{y}) \ .$$

Theory input

Experimental input

Theoretical MEM tools.

- Experimentalists have multiple in-house MEM codes (for top mass etc.) using various LO MEs.

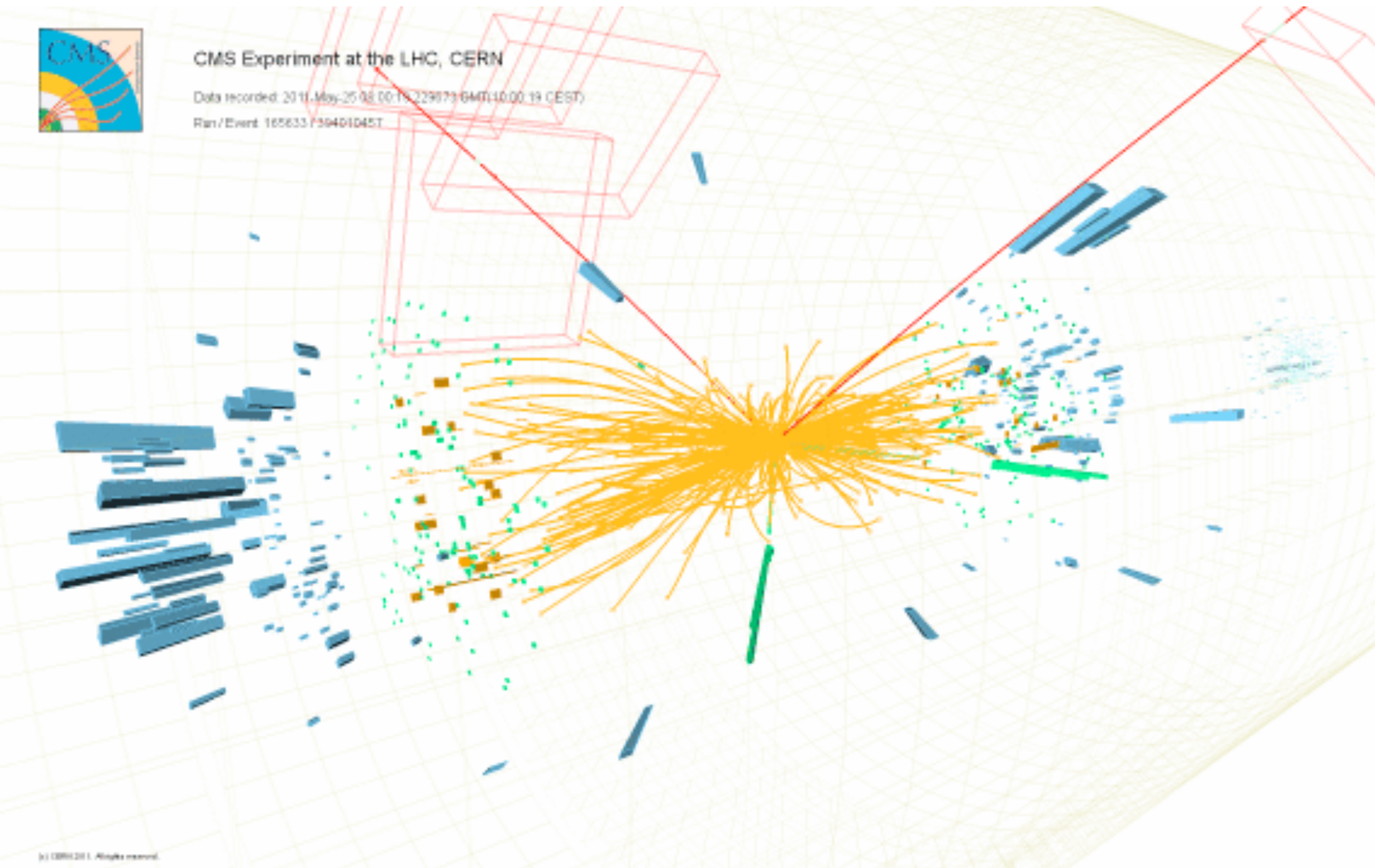
- A nice implementation of the MEM for general BSM scenarios has been provided at LO in the Madgraph framework. (Artoisenet, Lemaitre, Maltoni, Mattelaer 1007.3300).

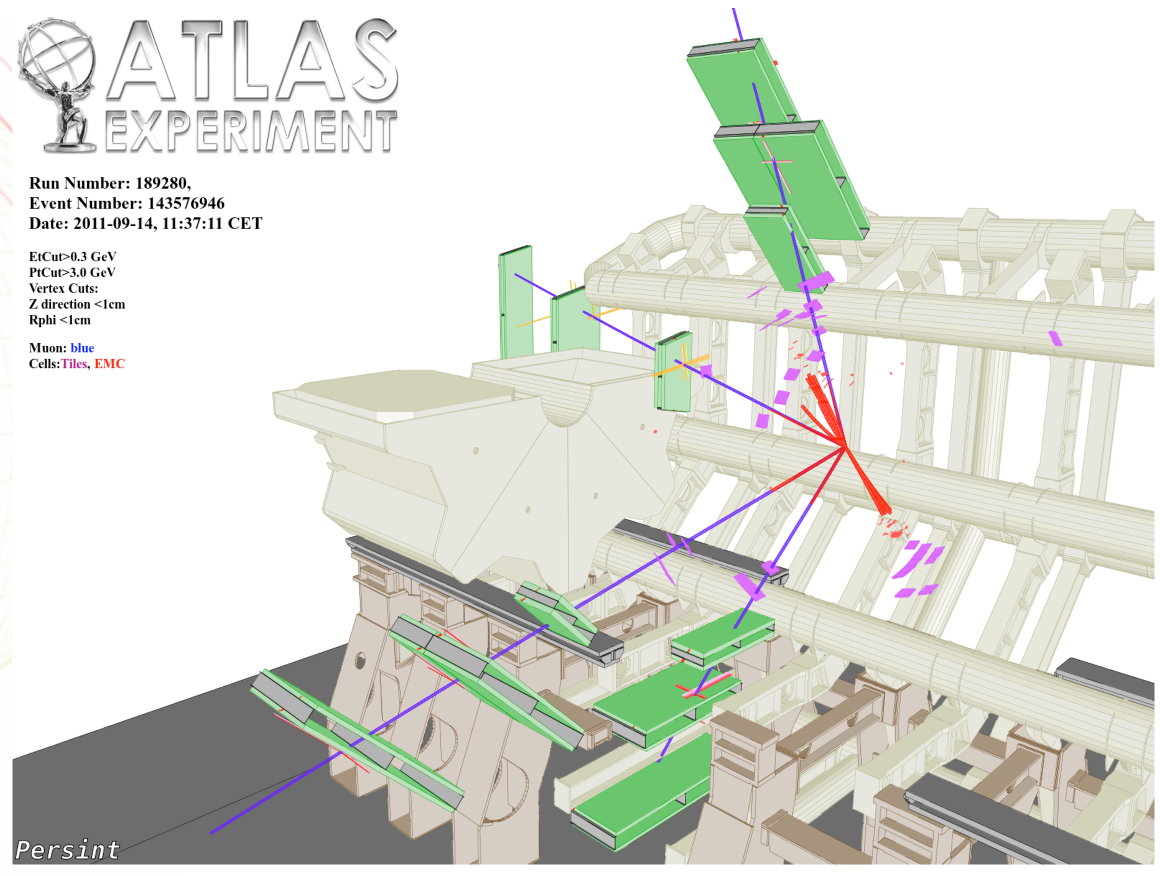
- This has also been extended to include some ISR modeling. (Alwall, Freitas, Mattelaer 1010.2263).

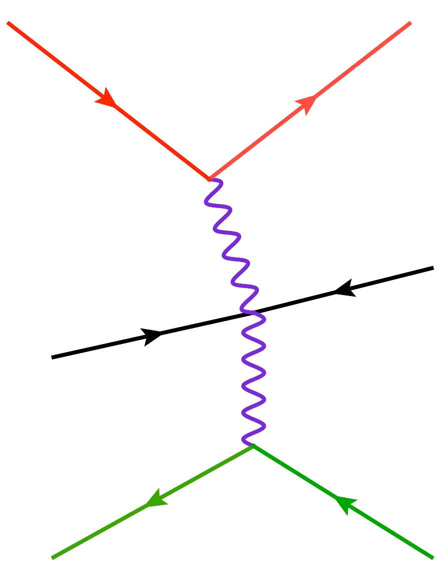
- Would be nice to have the situation where we can have NLO background + LO BSM signal

- Providing the NLO background is the goal of this work!

Experimental events versus fixed order weights.



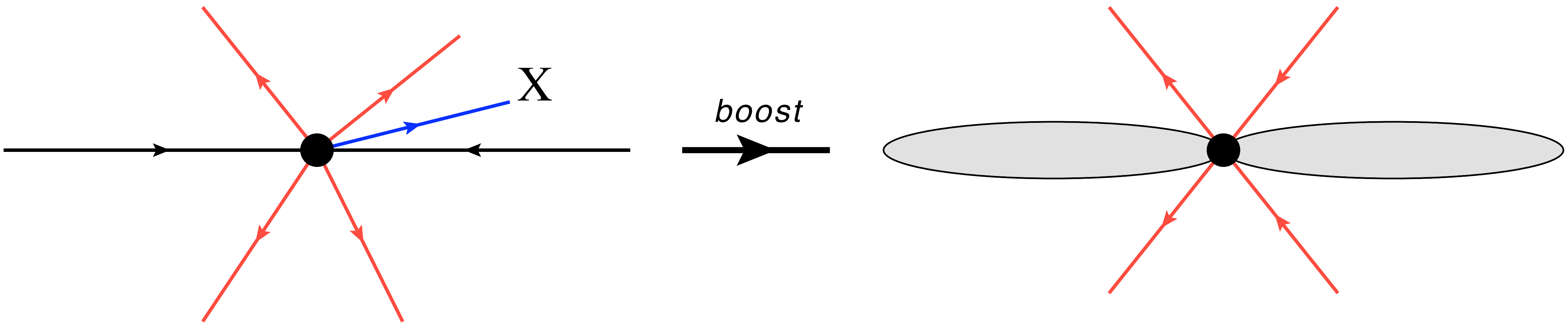




We want to weight an experimental event with a fixed order ME.

Experimental events contain more than the Born final state particles, we need to conserve momentum between the observed final state AND remain in the frame in which the PDFs are calculated (beams along the z-axis).

Mapping Data to Born



- Define the sum of all (Born) final state momenta as X.

$$X = -\sum_{i=1}^{n} \tilde{p}_i.$$

- Born phase space point (with beams along z-axis) require.

$$X^x \;=\; X^y \;=\; 0,$$

- In general this requirement is not satisfied in data!

Getting to the MEM frame

- What we can do is to perform a Lorentz transformation on the final state particles, (thus preserving all Lorentz invariant quantities).

$$p_i^\mu = \Lambda^\mu{}_\nu(X)\,\tilde{p}_i^\nu \qquad \text{with} \qquad \sum_{i=1}^{n} p_i^x = \sum_{i=1}^{n} p_i^y = 0\ .$$

- This transformation is not unique, what I do with the longitudinal component is a free choice.

- Recall that the longitudinal components specify the parton fractions,

$$x_a - x_b = \frac{2}{\sqrt{s}}\left(\sum_{i=1}^{n} p_i^z\right)\ , \qquad x_a + x_b = \frac{2}{\sqrt{s}}\left(\sum_{i=1}^{n} E_i\right)$$

- So in other words, our boosts do not fix xa and xb uniquely only the product.

$$\delta(x_a x_b s - Q^2)$$

More formally......

- One can start with the prediction for the total cross section,

$$\sigma_{\Omega}^{LO} = (2\pi)^{4-3n} \int dx_a \, dx_b \, dQ^2 \, \delta(x_a x_b s - Q^2) \times \prod_{m=1}^{n} \left(\frac{d^3\mathbf{p}_m}{2E_m} \right) \frac{f_i(x_a) f_j(x_b)}{x_a x_b s} \mathcal{B}_{\Omega}^{ij} \, \delta^{(4)} \left(Q - \sum_{m=1}^{n} p_m \right)$$

- Ideally we want to factorize this into production and decay, the data then specifies the decay products, we didnt observe the initial state...

$$\sigma_{\Omega}^{LO} = \int dx_a \, dx_b \, d\mathbf{x} \, \delta(x_a x_b s - Q^2) \frac{f_i(x_a) f_j(x_b)}{x_a x_b s} \mathcal{B}_{\Omega}^{ij}(p_a, p_b, \mathbf{x}) \; .$$

- A fully reconstructed (i.e. no MET) fixes a unique dx which we can use to build our MEM pdf.

The LO MEM

- In the ideal setup (i.e. perfect detector so no transfer functions and a fully reconstructed final state) then the LO MEM takes on the following form,

$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_{\Omega}^{LO}} \mathcal{L}_{ij}(s_{ab}, x_l, x_u) \mathcal{B}_{\Omega}^{ij}(p_a, p_b, \mathbf{x}) \ .$$

- where the only boost-dependent term is process independent and grouped ino the first term (L)

$$\mathcal{L}_{ij}(s_{ab}, x_l, x_u) = \int dx_a dx_b \, \frac{f_i(x_a) f_j(x_b)}{x_a x_b s} \, \delta(x_a x_b s - s_{ab})$$

- We define the LO event by event weight as,

$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_{\Omega}^{LO}} \, B_{\Omega}(\mathbf{x}).$$

The complication of cuts!

- Cross sections and events are defined in the lab frame, we want to perform our calculation in the MEM frame => Need a map for fiducial cuts.

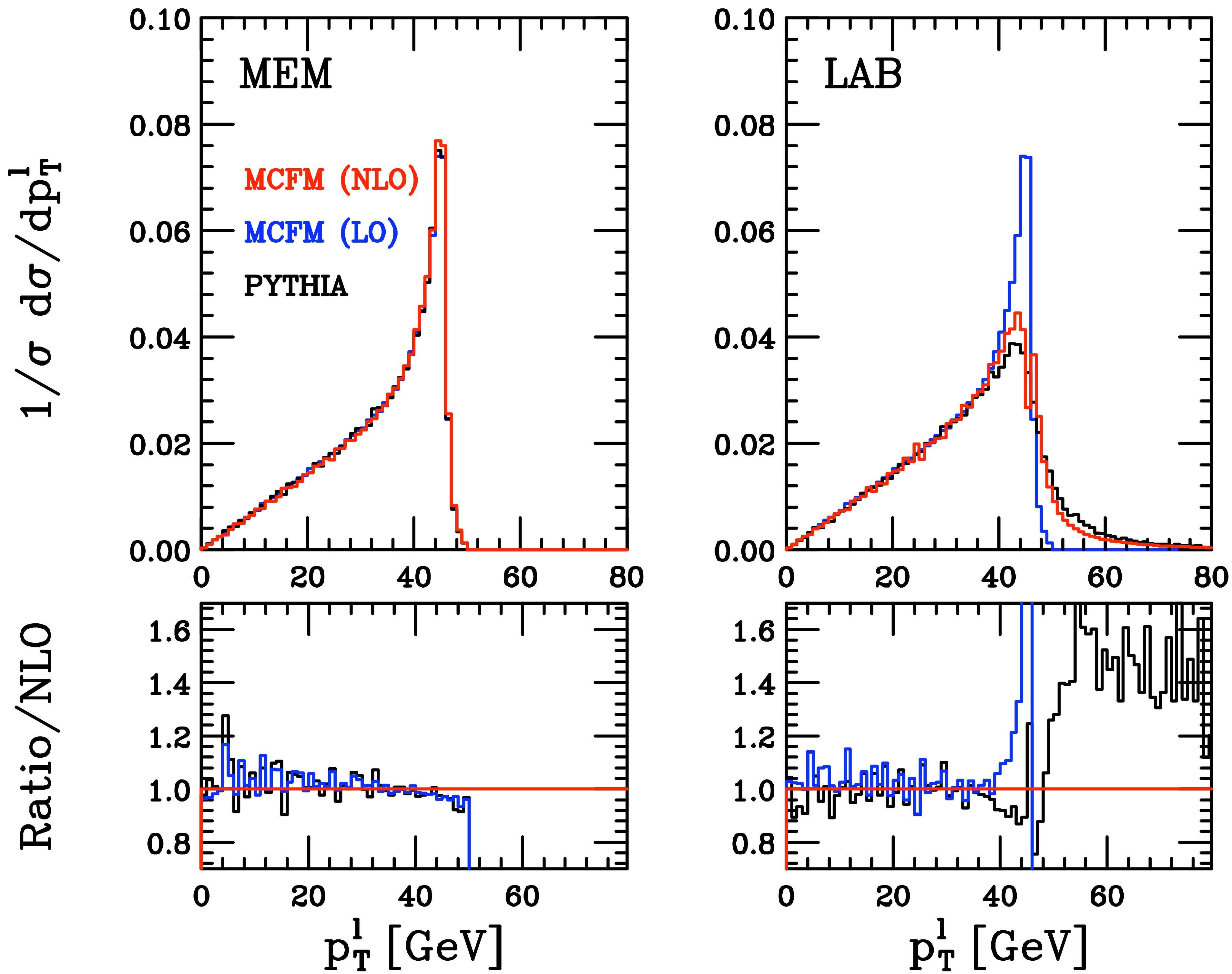
- This is defined by

$$p_T^{lab,i} = \sqrt{\frac{s_{ai}s_{ib}}{s_{ab}}}\,, \quad \eta^{lab,i} = \frac{1}{2}\log\left(\frac{x_a^2 s}{s_{ab}}\frac{s_{ib}}{s_{ai}}\right).$$

- Note pT is defined in terms of invariants, rapidity is boost dependent. In fact cuts on rapidity actually fix the upper and lower bounds on the boost integration,

$$\mathcal{L}_{ij}(s_{ab}, x_l, x_u) = \int_{x_l}^{x_u} dx_a \, \frac{f_i(x_a) f_j(s_{ab}/(s x_a))}{s x_a s_{ab}}$$

Insane in the MEM frame.



- We look at lab frame pT,

$$p_T^{lab,i} = \sqrt{\frac{s_{ai}s_{ib}}{s_{ab}}}$$

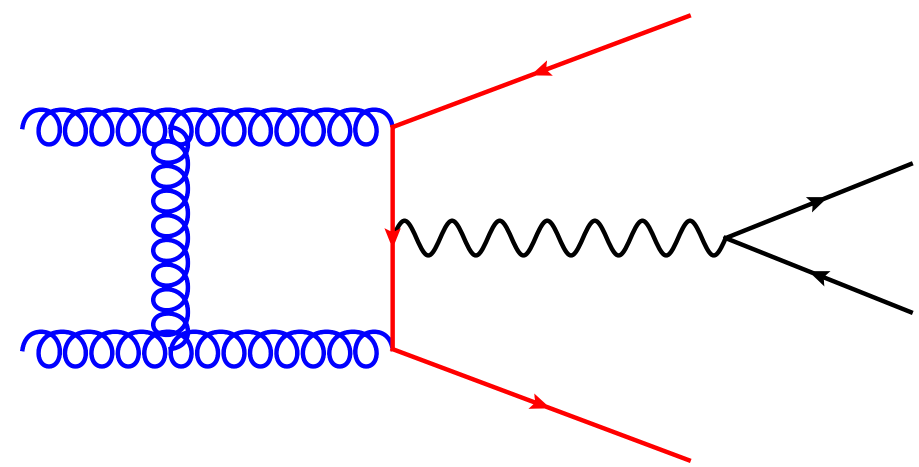
- and MEM frame pT

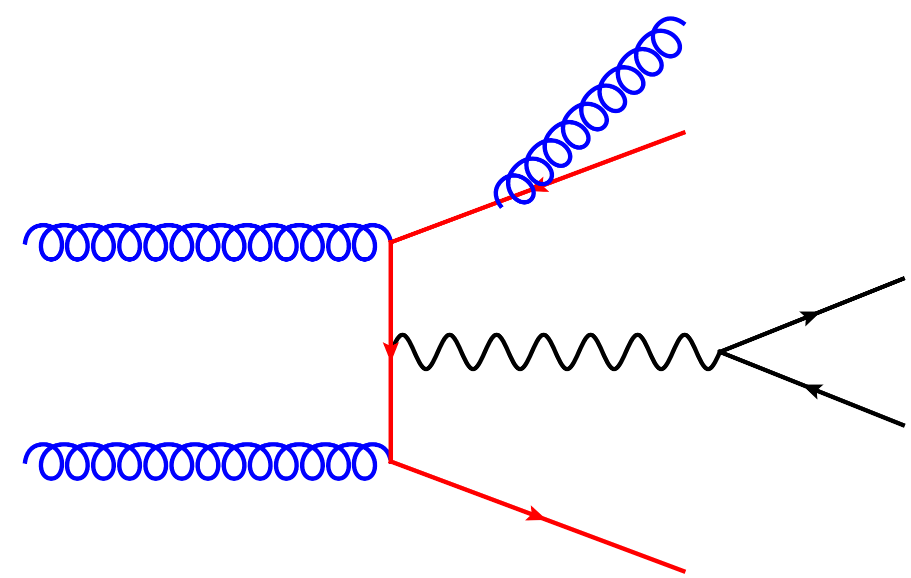
$$p_T^{MEM,i} = \sqrt{(p_i^x)^2 + (p_i^y)^2}$$

- Using MCFM and Pythia
- Clearly the MEM has some very nice features!

NLO parton level

- At NLO in perturbation theory one has to deal with divergences
- Virtual diagrams contain UV and IR divergences which typically manifest themselves as poles an analytic.
- Real diagrams contain an emission of an additional parton. Although 4 dimensional they develop singular regions in phase space when the extra parton is unresolved.





$$\sigma_{NLO} = \int_m d\sigma_{LO} + \int_m d\sigma_V + \int_{m+1} d\sigma_R$$

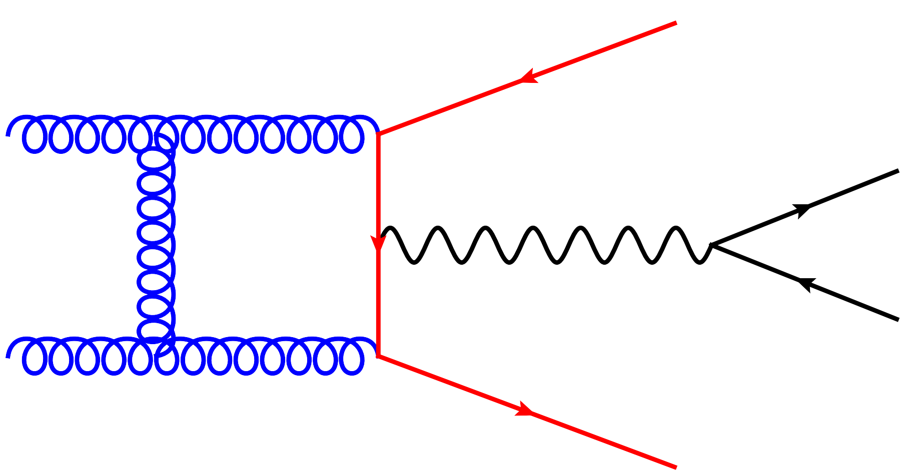
MEM at NLO

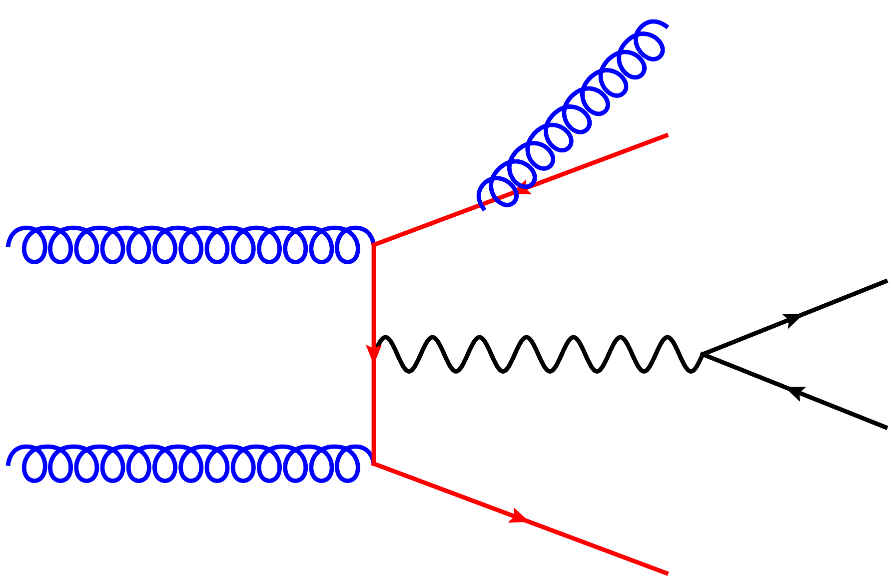
- Naively one might expect the MEM to be impossible at NLO.

- This is because NLO calculations include two sorts of contributions which live in different phase spaces.

- The virtual (loop) diagrams can easily be incorporated into the method, since they share the same phase space as the Born.

- The issue lies in the generation of the real phase space, which contain one extra parton. We need to define a map for these to a Born topology.





MEM at NLO

- Our goal is to define the NLO cross section in terms of a single identified Born final state.

$$\frac{d\,\sigma_{\Omega}^{NLO}(\mathbf{x})}{d\mathbf{x}} = R_{\Omega}(\mathbf{x}) + V_{\Omega}(\mathbf{x}) \ .$$

- Where R and V represent the real and virtual pieces.

- The above can be used to define unique NLO weights for an exclusive event.

- V can be defined similarly to the Born since they share a phase space.

- The real pieces are more tricky, one way to accomplish our goal is to use a forward branching phase space generator. 1106.5045 (Giele, Stavenga, Winter). and Walter's talk.

The FBPS Generator (c.f. Walter's talk)

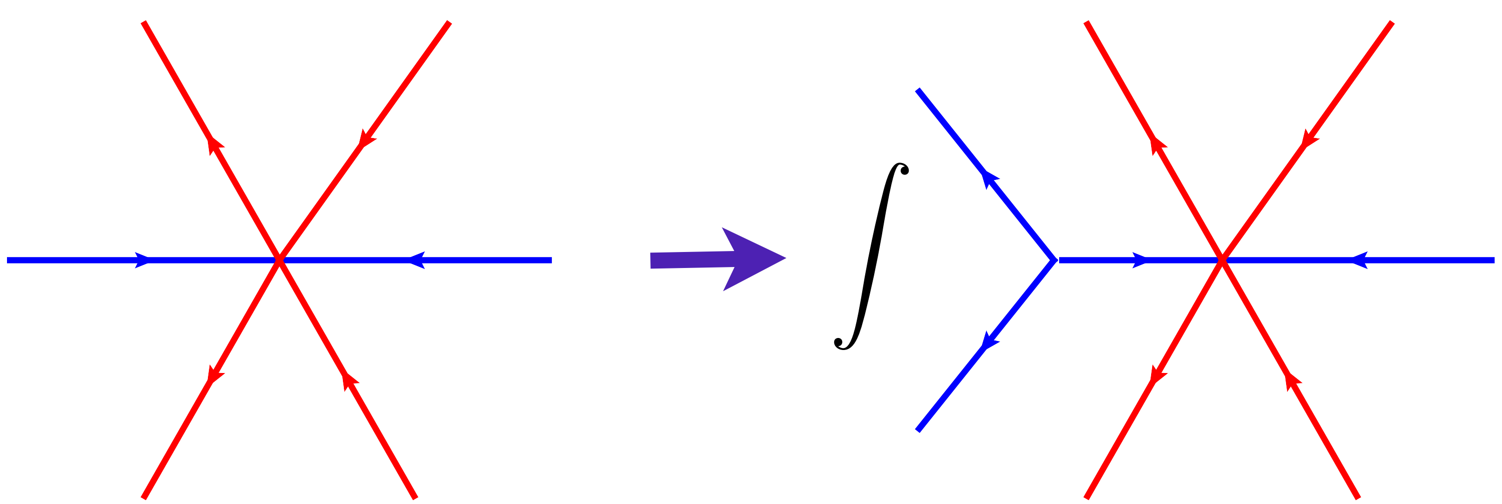
- Mathematically we need to factorize the real phase space into the following,

$$d\Phi(p_a + p_b \to Q + p_r) = d\,\Phi(\widehat{p}_a + \widehat{p}_b \to Q) \times d\,\Phi_{\text{FBPS}}(p_a, p_b, p_r) \times \theta_{\text{veto}}$$

- Then Q is identified with the observed final state, from this we derive the form of the FBPS integration

$$d\,\Phi_{\text{FBPS}}(p_a, p_b, p_r) = \frac{1}{(2\pi)^3}\left(\frac{\widehat{s}_{ab}}{s_{ab}}\right) d\,t_{ar} d\,t_{rb} d\,\phi\ ,$$

- We then explicitly integrate out these quantities for each event.



The MEM at NLO.

- We now have everything we need to define the MEM at NLO,

$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_{\Omega}^{NLO}}\Big(V_{\Omega}(\mathbf{x}) + R_{\Omega}(\mathbf{x})\Big)$$

- Note that the real and virtual are both defined for the observed Born topology x. Recall (from Walter's talk)...

$$V_{\Omega}(\mathbf{x}) = \mathcal{L}_{ij}(s_{ab}, x_l, x_u)\Big(\mathcal{B}_{\Omega}^{ij}(p_a, p_b, \mathbf{x}) + \mathcal{V}_{\Omega}^{ij}(p_a, p_b, \mathbf{x})\Big)$$

$$+ \sum_{m=0}^{2} \int dz \Big(\mathcal{D}_m(z, \mathbf{x}) \otimes \mathcal{L}_m(z, s_{ab}, x_l, x_u)\Big)_{ij} \mathcal{B}_{\Omega}^{ij}(p_a, p_b, \mathbf{x}).$$

$$R_{\Omega}(\mathbf{x}) = \int d\,\Phi_{\mathrm{FBPS}}(p_a, p_b, p_r)\Big(\mathcal{L}_{ij}(s_{ab}, x_l, x_u)\mathcal{R}_{\Omega}^{ij}(p_a, p_b, \mathbf{x}, p_r)$$

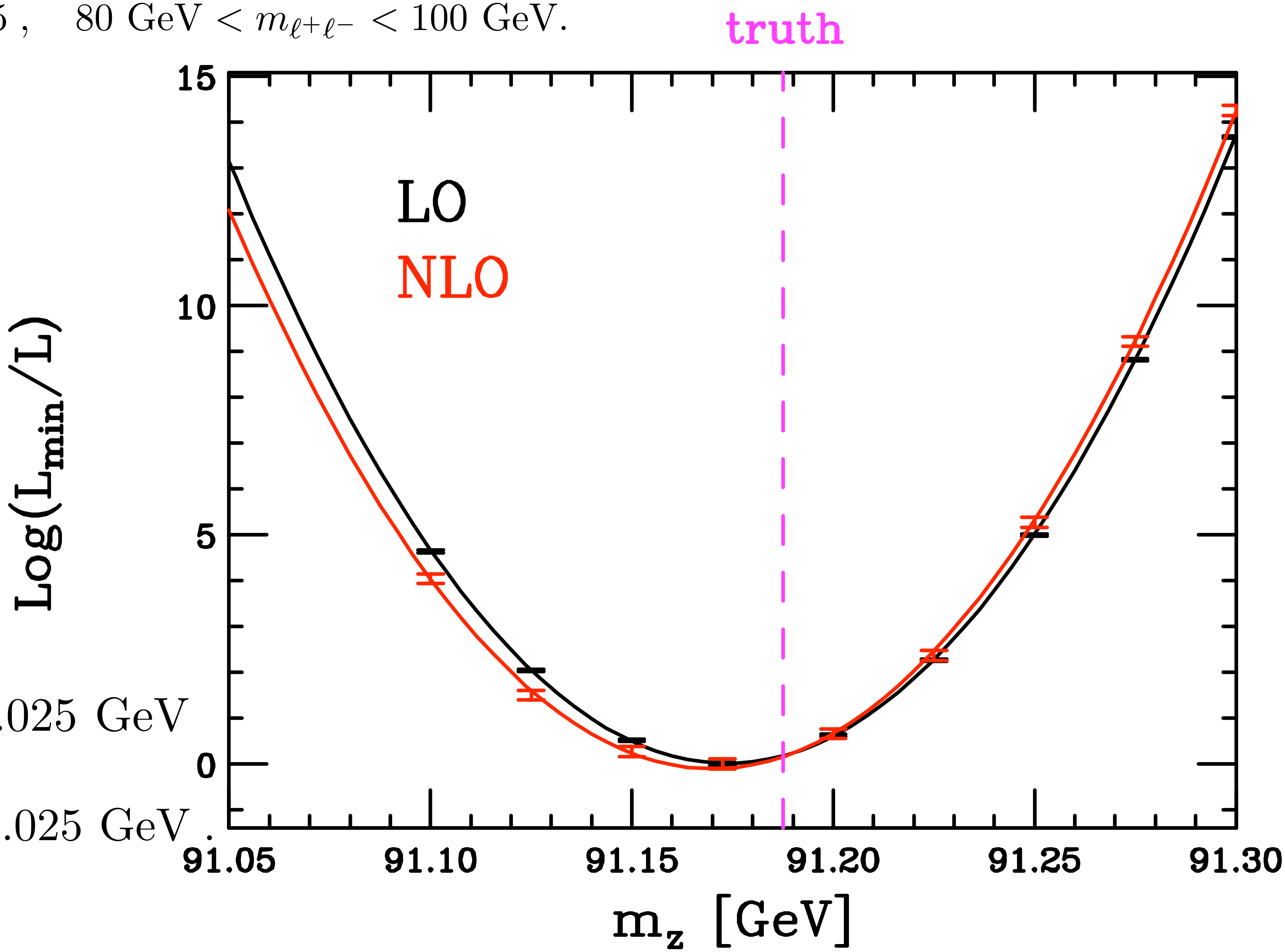
$$- \sum_{m} \mathcal{L}_{ij}(s_{ab}, x_l^m, x_u^m) D^m(p_a, p_b, p_r)\mathcal{B}_{\Omega}^{ij}(\hat{p}_a, \hat{p}_b, \mathbf{x})\Big).$$

Example : measuring the mass of the Z boson

$p_T^\ell > 15 \text{ GeV}\,, \quad |\eta_\ell| < 2.5\,, \quad 80 \text{ GeV} < m_{\ell^+\ell^-} < 100 \text{ GeV}.$

Generate 5000 events with Pythia and try to measure the Z mass.

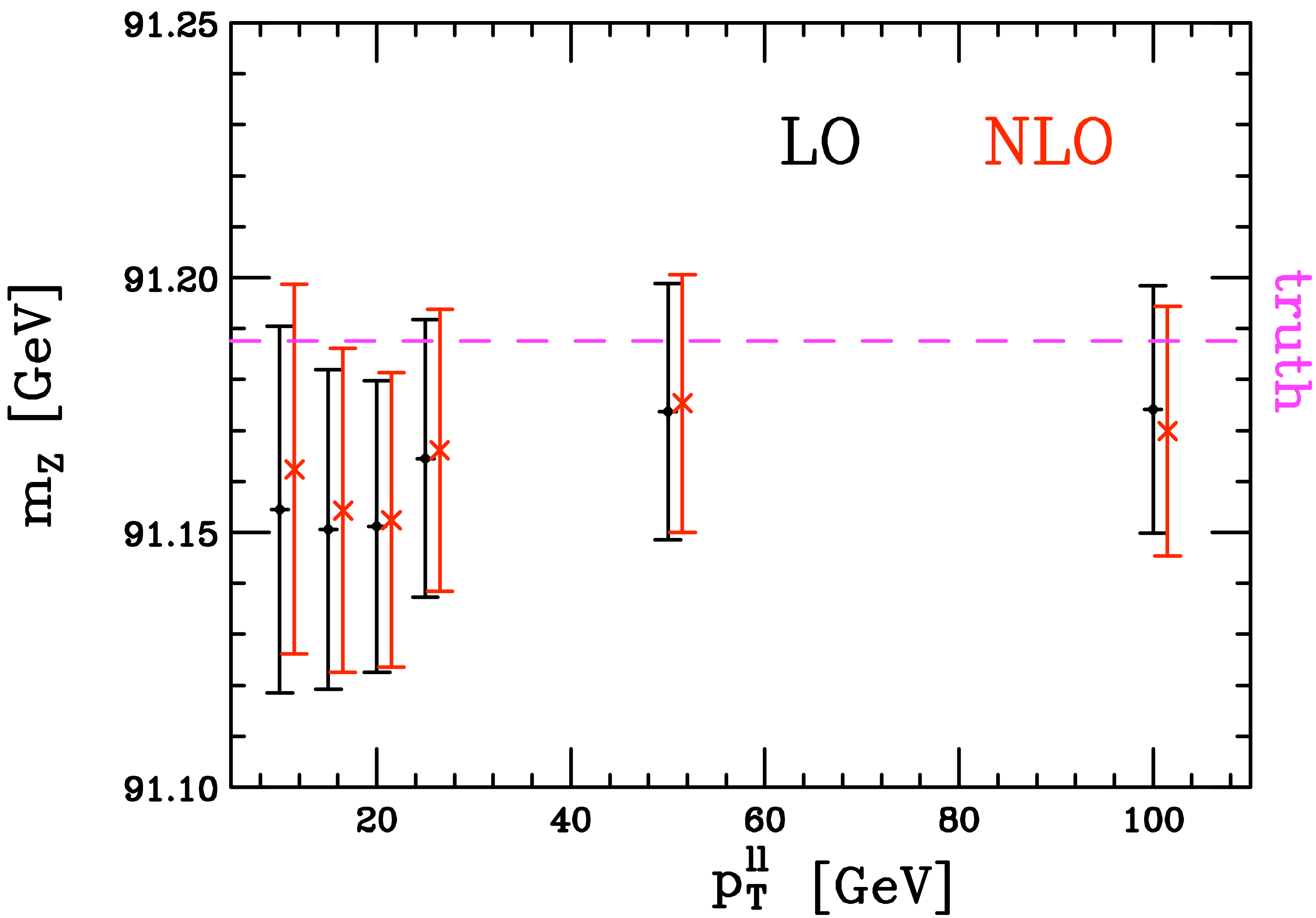
We know NLO corrections are very small (saw almost identical MEM frame kinematics).



$$\text{LO:} \quad m_Z = 91.170 \pm 0.025 \text{ GeV}$$

$$\text{NLO:} \quad m_Z = 91.174 \pm 0.025 \text{ GeV}\,.$$

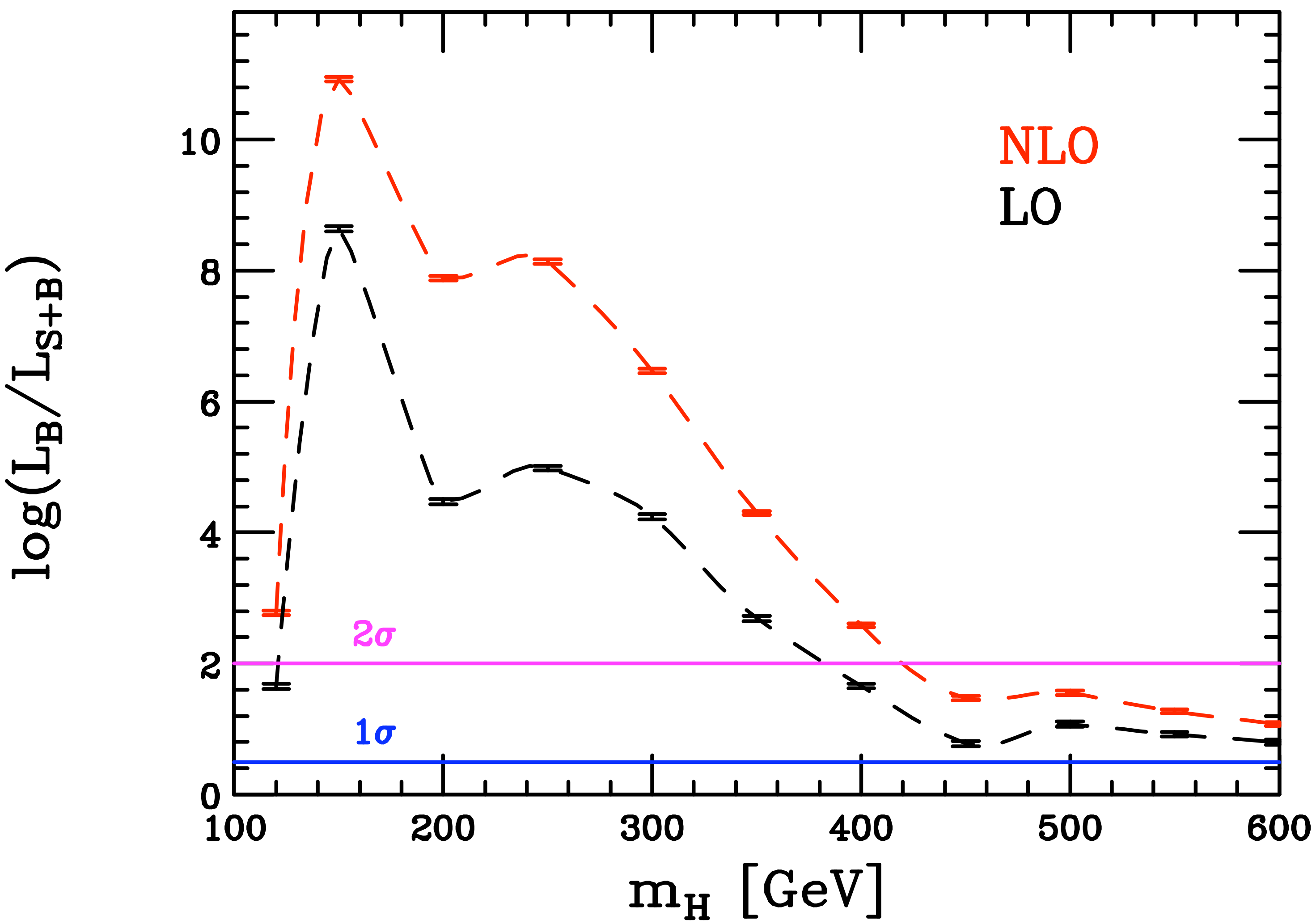
What about the extra radiation??



Might worry that the boost does a bad job when there is a large amount of showered radiation.

It appears that for measurements of EWK masses at least, the inclusive approach is valid.

Example: Setting Limits on the Higgs

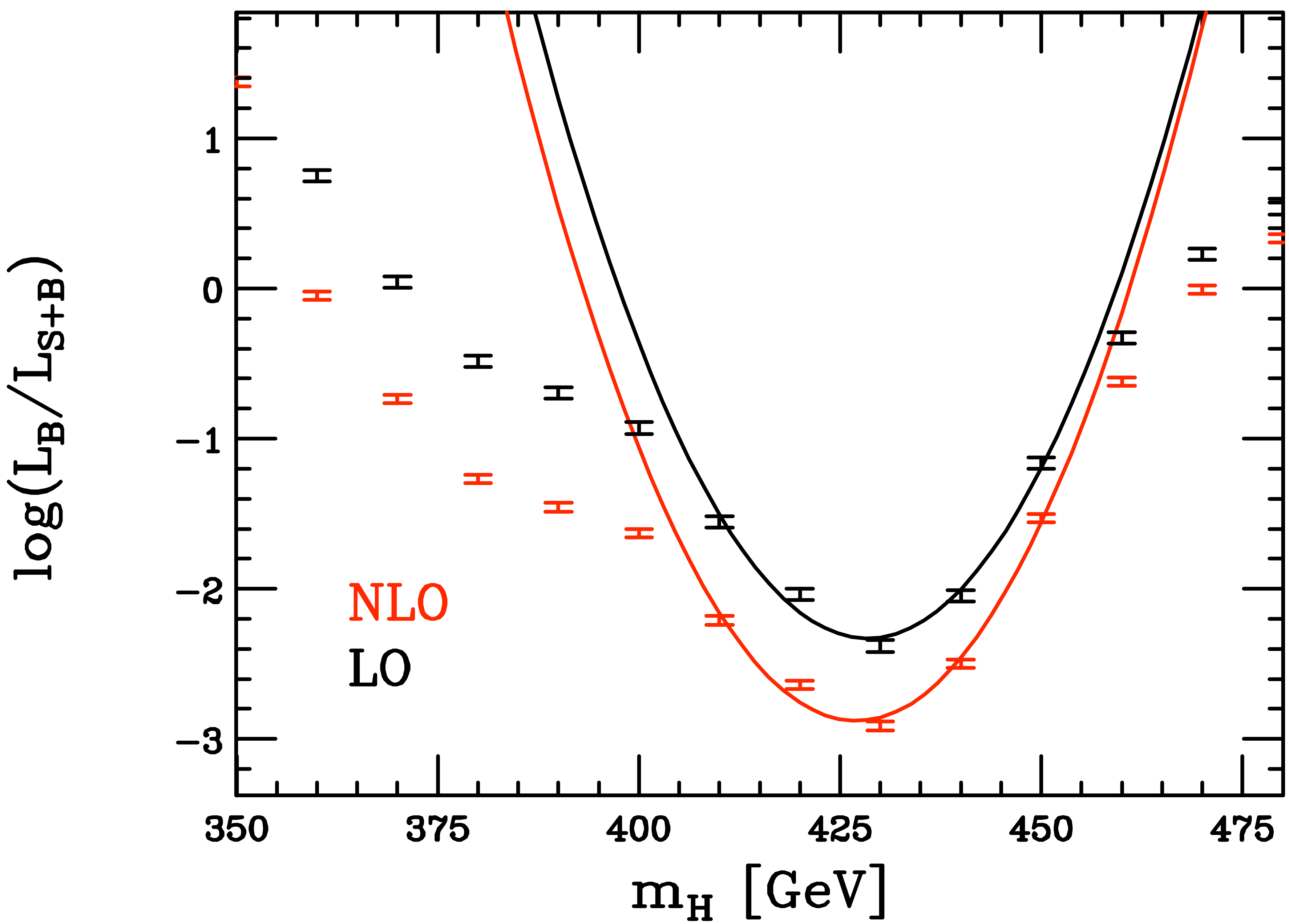


- A more interesting example is to use the MEM to constrain the Higgs
- We generate ~ 250 ZZ->4l unweighted NLO events.
- The background spectrum contains everything in MCFM (e.g. gg->ZZ, singly resonant Z..).

$$\text{LO MEM exclusion:} \qquad 120 \text{ GeV} < m_H < 380 \text{ GeV} ,$$

$$\text{NLO MEM exclusion:} \qquad 100 \text{ GeV} < m_H < 430 \text{ GeV} .$$

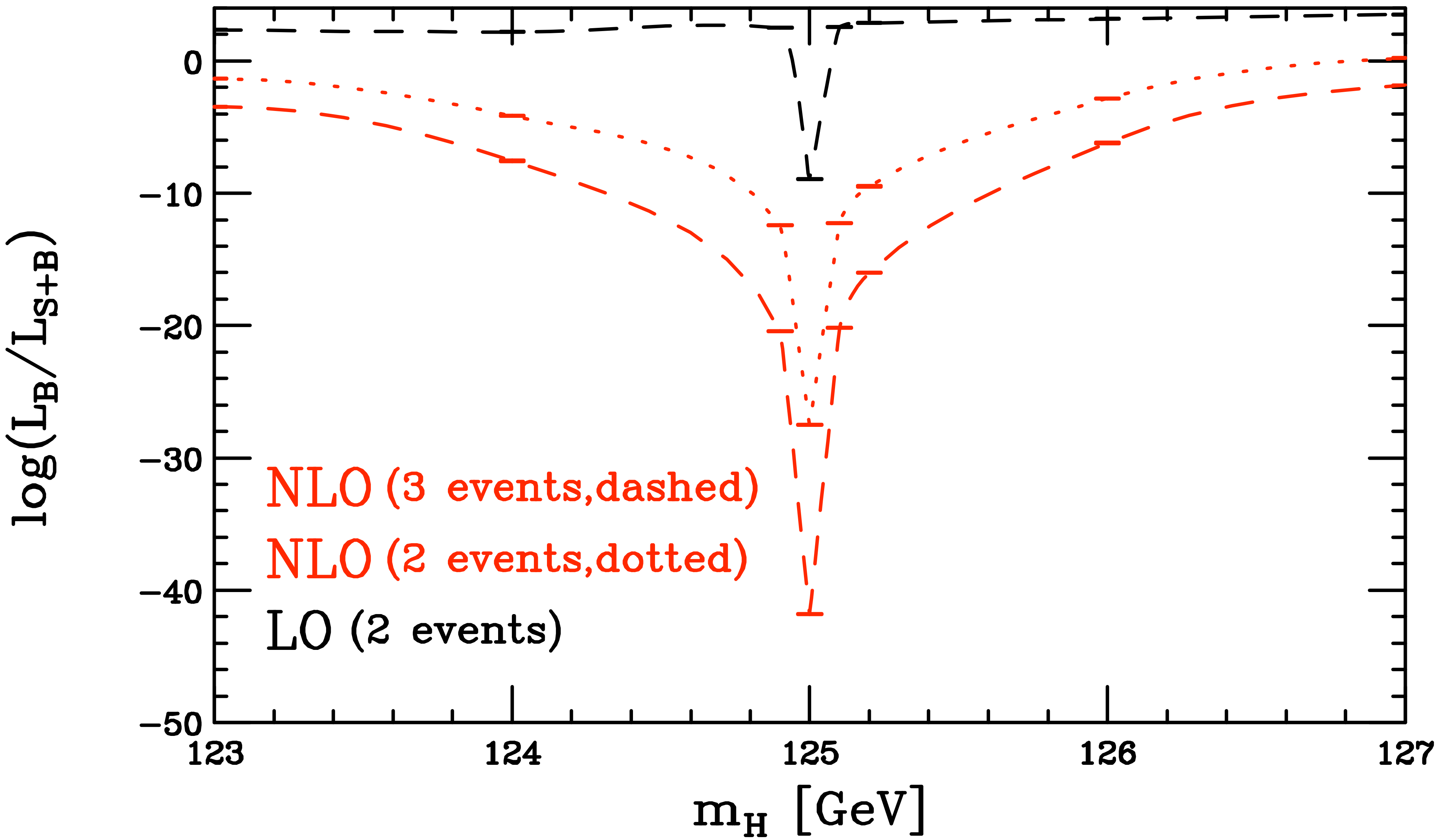
Example: Heavy Higgs.



- Inject a signal at 425 using the same background sample as the previous slide.
- Can measure the Higgs mass to within a few % with a handful of events!

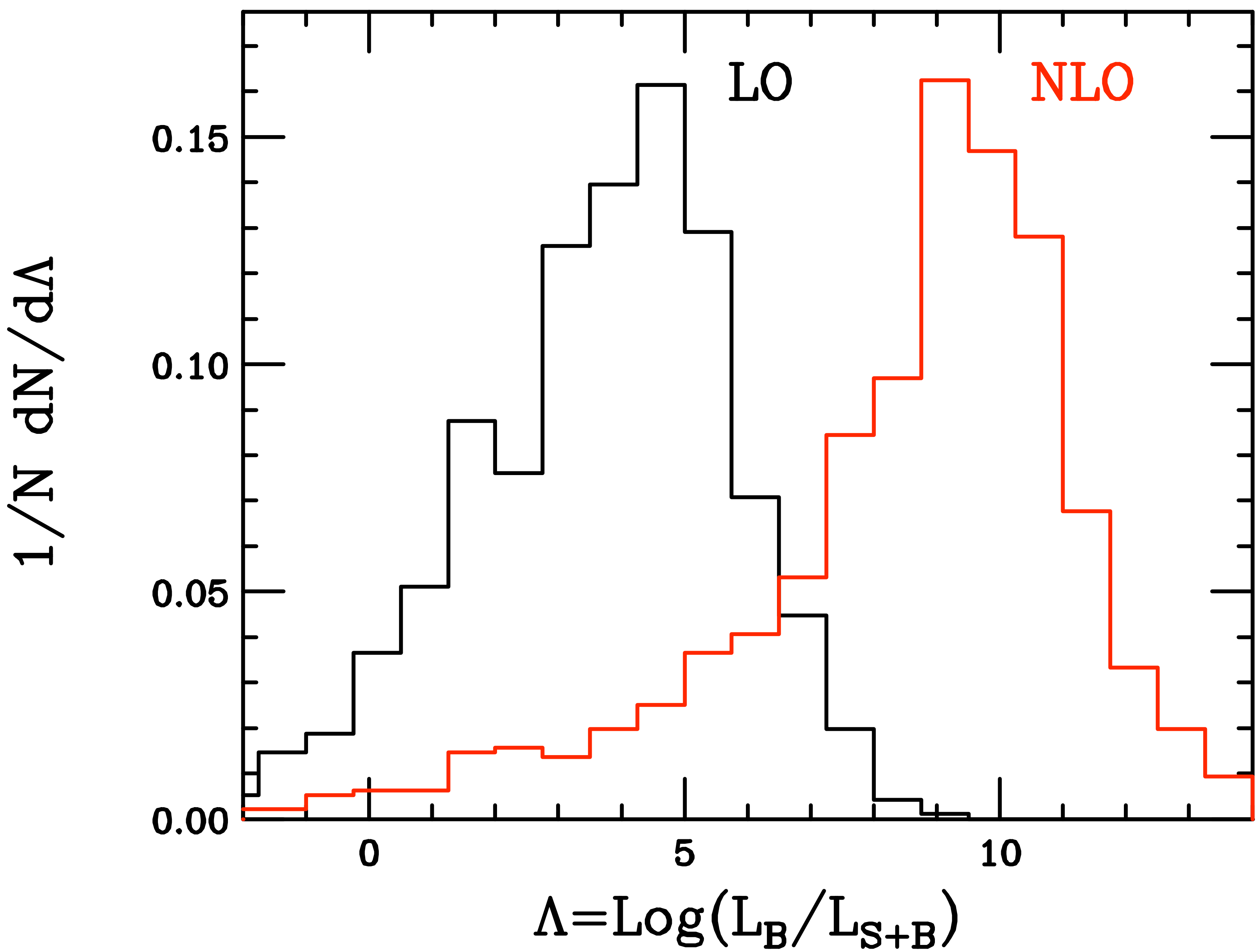
$$m_H^{\text{fit}}(\text{LO}) = 428 \pm 14 \text{ GeV} , \qquad m_H^{\text{fit}}(\text{NLO}) = 427 \pm 14 \text{ GeV} .$$

Example: Higgs mass measurements.



The MEM can also be used for a light Higgs, however since the width is smaller than the lepton resolution, transfer function modeling becomes necessary.

Systematic study of the NLO/LO differences.



It is interesting to compare LO and NLO over a wider range of pseudo experiments to observe if the differences are systematic.

In our setup NLO consistently sets better limits for a Higgs of 200 GeV. (Not surprising since we used NLO data!)

Exclusion	% LO	% NLO
$> 1\sigma$	91.1	98.2
$> 2\sigma$	77.3	96.1
$> 3\sigma$	38.1	90.1
$> 4\sigma$	0.521	67.3
$> 5\sigma$	0.00	3.75

Conclusions.

- We have illustrated how the MEM can be theoretically well defined at all orders, presented simple examples at NLO of H->4l and Z->ll

- In order to define a fixed order weight for an experimental event one must boost to a frame in which the final state is balanced.

- Since a given boost is not unique, we must integrate over all equivalent boosts, the Matrix Element doesn’t care but the PDFs do.

- Our approach does not change the experimental input (transfer functions).

Future study

We are keen to extend the method to other measurements, in particular....

- Measurement of the top mass at the LHC and Tevatron (flagship application of the MEM).
- Higgs in other channels, associated production, two photons etc. Confirming SM properties, BR, spin etc. WW....
- Measurement of/Limits on triple anomalous gauge couplings.
- ....

We gladly welcome experimental input! Beta code of NLOME is available, first release expected in May/June. BIG Thank you to experimentalists who have helped so far!