Rapidity Renormalization Group

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Introduction

- Many observables in QCD develop poorly behaved perturbation series in certain limits of phase space.
- Observables then often dominated by radiation collinear to a light-cone direction, or soft radiation.
- When the collinear and soft radiation virtuality is small but of the same order, special difficulties arise.

(Not Quite) Back to Back jets

$e^+e^- \to 2j$ with the event shape Jet Broadening.

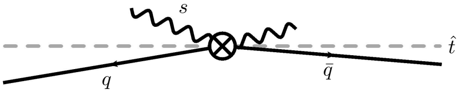

Figure: Jet Broadening Collinear Displacement From Thrust Axis

(Not Quite) Back to Back jets

- $e = \sum_i \frac{|\vec{k}_{ti}|}{Q}$ jet broadening event shape.
- Thrust axis $\hat{\mathbf{t}}$ of the event defines directions:

$$n = (1, \hat{\mathbf{t}}) \qquad \bar{n} = (1, -\hat{\mathbf{t}}) \qquad \vec{p}_t . \hat{\mathbf{t}} = 0$$

$$p = (\bar{n}.p, n.p, \vec{p}_t)$$

- Demand $e \ll 1$ for dijets.
- Relevant on-shell modes must have $\vec{p}_t \sim Qe$.
- Softs $\sim Q(e, e, e)$ and collinears $\sim Q(1, e^2, e)$ or $\sim Q(e^2, 1, e)$

Perturbative Expansion

$$L = \log(e)$$

$$
\begin{aligned}
\frac{d\sigma}{de} = \sigma_0 (1 + \quad & \alpha_s [c_{12} L^2 && + c_{11} L && + c_{10}] && && \text{(LO)} \\
+ \quad & \alpha_s^2 [c_{24} L^4 && + c_{23} L^3 && + c_{22} L^2 && + c_{21} L && + c_{20}] \quad \text{(NLO)} \\
+ \quad & \alpha_s^3 [c_{36} L^6 && + c_{35} L^5 && + c_{34} L^4 && + c_{33} L^3 && + \ldots \\
& \quad LL && \quad NLL && \quad NLL' && \quad NNLL &&
\end{aligned}
$$

In dijet limit, perturbation theory is poorly behaved.

Factorization with SCET

$$\mathcal{L}_{SCET} = [\mathcal{L}_{QCD}]_n + [\mathcal{L}_{QCD}]_{\bar{n}} + [\mathcal{L}_{QCD}]_{soft} + p.c.$$

- Use formalism of Soft-Collinear Effective Theory to organize factorization.
- On-shell external states that can give contribution to the observable defines modes of theory.
- Effective theory built on each mode having its own (QCD) Lagrangian. No interactions linking modes at leading power.

(Bauer, Fleming, Luke 2000; Bauer, et al. 2001; Bauer, Pirjol, Stewart 2002; ...)

Factorization Theorem

$$\frac{d\sigma}{de} = N \int de_n de_{\bar{n}} de_s \delta(e - e_n - e_{\bar{n}} - e_s)$$
$$\int \frac{d^2\vec{p}_{t1}}{(2\pi)^2} \frac{d^2\vec{p}_{t2}}{(2\pi)^2} J_n(e_n, \vec{p}_{t1}) J_{\bar{n}}(e_{\bar{n}}, \vec{p}_{t2}) S(e_s, \vec{p}_{1t}, \vec{p}_{2t})$$

Where the jet and soft functions are defined as:

$$J_{\bar{n}}(e_{\bar{n}}, \vec{p}_{t2}) = \frac{(2\pi)^3}{N_c} \mathrm{tr}\langle 0 | \bar{\chi}_{\bar{n}} \delta(n \cdot \hat{P} - Q) \delta(e_{\bar{n}} - \hat{e}_{\bar{n}}) \delta(\hat{P}_\perp - p_{2\perp}) \frac{\not{n}}{2} \chi_{\bar{n}} | 0 \rangle$$
$$J_n(e_n, \vec{p}_{t1}) = \frac{(2\pi)^3}{N_c} \mathrm{tr}\langle 0 | \frac{\bar{\not{n}}}{2} \chi_n \delta(\bar{n} \cdot \hat{P} - Q) \delta(e_n - \hat{e}_{\hat{n}}) \delta(\hat{P}_\perp - p_{1\perp}) \bar{\chi}_n | 0 \rangle$$
$$S(e_s, \vec{p}_{1t}, \vec{p}_{2t}) = \frac{1}{N_c} \mathrm{tr}\langle 0 | S_{\bar{n}} S_n^\dagger \delta(\mathbb{P}_{n\perp} + p_{1\perp}) \delta(\bar{\mathbb{P}}_{n\perp} + p_{2\perp}) \delta(e_s - \hat{e}_s) S_n S_{\bar{n}}^\dagger | 0 \rangle$$

Where $\mathbb{P}_{n\perp}$ and $\bar{\mathbb{P}}_{n\perp}$ are operators picking out the transverse momenta being contributed in each hemisphere.

Naive Dim-Reg Calculation

- Bare jet function:

$$J_n(e_n, 0) = \frac{\alpha_s C_f}{\pi} \left(\frac{\mu^2}{Q^2 e_n^2} \right)^{\epsilon} (e_n)^{-1} \int_0^1 dz \frac{1 + (1 - z)^2}{z}$$

$$z = \frac{\overline{n}.l}{Q} \quad l \text{ momentum of gluon crossing cut}$$

- Integral ill-defined at $z = 0$, the soft region.
- Divergence multiplies non-zero e_n terms that virtuals cannot cancel.

Problem of Scales

Factorization in dim-reg: $d\sigma = H(\mu)\, J_n(\mu) \otimes J_{\bar{n}}(\mu) \otimes S(\mu)$

- The μ parameter in Dim-Reg is sensitive only to the invariant mass of the sector.
- Hard function contains μ-logarithms of hard scale Q^2.
- Low scale functions contains μ-logarithms of low scale eQ. e sets low scale invariant mass.

Problem of Scales

$$d\sigma = H(\mu)\, J_n(\mu) \otimes J_{\bar{n}}(\mu) \otimes S(\mu)$$

- From fixed order cross-section, there are large double logs to be resummed.
- Double logs appear in factorized form in each sector, now dependent on factorization scale μ
- μ variation of the hard function must cancel in low scale matrix elements. But hard function will have double logs: $H(\mu) = 1 + a\, \text{Log}^2\left(\frac{Q^2}{\mu^2}\right) + ...$
- Leading μ variation is $a\text{Log}\left(\frac{Q^2}{\mu^2}\right)$.
- Must be able to generate such log in low scale matrix elements. But dim-reg factorizes Q^2 scale from the these matrix elements!

Strategy

- Further factorization must be performed. Low-scale modes are differentiated only in rapidity, modes must be factorized accordingly.
- Factorization always introduces new divergences: now in light-cone integrations.

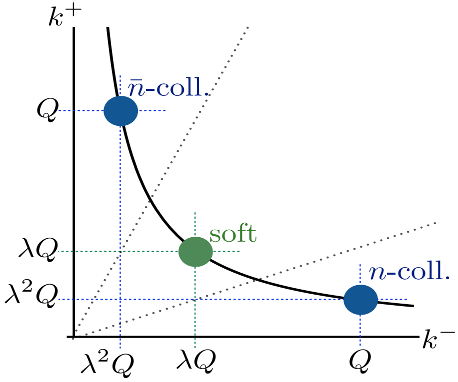

Figure: Rapidity Factorization of low scale Modes

Strategy

- Introduce a rapidity regulator to control divergences.
- After appropriate subtractions (zero- or soft-bin), renormalize these divergences.
- Left over renormalization parameter allows resummation of logarithms in controlled fashion.

New regulators

To regulate Rapidity divergences, one may use any regulator up to certain requirements:

- Maintains gauge invariance.
- Preserves eikonal identities.
- Cleanly separates Invariant Mass and Rapidity Logarithms/Divergences.
- Enters each function in universal fashion.
- All jet functions appropriately soft-subtracted to disentagle overlaps.

New regulators

Example regulators where such features can be achieved:

- δ-regulator (Chui et al.), eikonal propagators receive small mass: $\frac{1}{\bar{n}.k} \to \frac{1}{\bar{n}.k+\delta_{\bar{n}}}$
- Titling Wilson lines off the light-cone.
- η-regulator (Chui et al.)

η regulator

- Regulates in a minimal fashion: Operates on gauge invariant CWEB structures defined by non-abelian exponentiation.
- CWEB minimally divergent: only one overall rapidity divergence.
- Within a CWEB structure regulate the total k_3 momentum flowing between the n and $\bar{n}$ directions, by multipling integrand by $\nu^{-\eta}|k_3|^{\eta}$, expanded according to the power counting of each sector.
- At one loop, this reduces to regulating the k_3 momentum flowing onto the wilson line insertion graph.

Jet Function Redux: ν Logs

Now with η regulator in place, we look at the laplace transformed jet function:

$$J_n(\tau,0) = -\frac{\alpha_s C_f}{\pi}\left(\frac{\mu^2\tau^2}{Q^2}\right)^{\epsilon}\Gamma(-2\epsilon)\int_0^1 dz(z+2\left(\frac{\nu}{Q}\right)^{\eta}\frac{1-z}{z^{1+\eta}})$$

$$= \frac{\alpha_s C_f}{\pi}\left(\frac{\mu^2\tau^2}{Q^2}\right)^{\epsilon}\Gamma(-2\epsilon)\left(\frac{2}{\eta}\right) - \frac{\alpha_s C_f}{\pi}\frac{3}{4\epsilon} - \frac{\alpha_s C_f}{2\pi\epsilon}\mathrm{Log}\left(\frac{\nu^2}{Q^2}\right)$$

$$-\frac{\alpha_s C_f}{2\pi}\mathrm{Log}\left(\frac{\nu^2}{Q^2}\right)\mathrm{Log}\left(\frac{\mu^2\tau^2}{Q^2}\right) - \frac{\alpha_s C_f}{\pi}\frac{3}{4}\mathrm{Log}\left(\frac{\mu^2\tau^2}{Q^2}\right)$$

Structure of η divergences

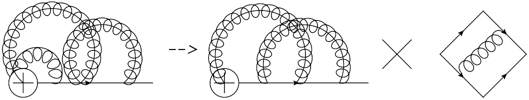

Figure: Factorization of η Divergences

- Combining jet and soft sectors, η divergences and ν dependence cancels.
- Within a sector η divergences exponentiate.
  - Pattern of exponentiation follows non-abelian exponentiation theorems for eikonal processes.
  - Treat removal of η divergences with multiplicative renormalization.

ν Renormalization

Now there are renormalization factors $Z_n, Z_{\bar{n}}, Z_s$ such

$$J_n^{\mathrm{B}}(\tau, b_1) J_{\bar{n}}^{\mathrm{B}}(\tau, b_2) S^{\mathrm{B}}(\tau, b_1, b_2) = \Big(Z_n(\nu,\mu) J_n^{\mathrm{R}}(\tau, b_1, \nu, \mu)\Big)\Big(Z_{\bar{n}}(\nu,\mu) J_{\bar{n}}^{\mathrm{R}}(\tau, b_2, \nu, \mu)\Big) \Big(Z_s(\nu,\mu) S^{\mathrm{R}}(\tau, b_1, b_2, \nu, \mu)\Big)$$

Where

$$Z_n(\nu,\mu) Z_{\bar{n}}(\nu,\mu) Z_s(\nu,\mu) = Z_H^{-1}(\mu)$$

$$Z_n(\nu,\mu) = 1 + \frac{\alpha_s C_f}{\pi}\left(\frac{\mu^2\tau^2}{Q^2}\right)^{\epsilon}\Gamma(-2\epsilon)\left(\frac{2}{\eta}\right) - \frac{\alpha_s C_f}{\pi}\frac{3}{4\epsilon} - \frac{\alpha_s C_f}{2\pi\epsilon}\mathrm{Log}\left(\frac{\nu^2}{Q^2}\right)$$

ν RG

One can calculate the RG equations as:

$$\nu \frac{d}{d\nu} F^{\mathrm{R}}(\nu,\mu) = \gamma_F^{\nu} F^{\mathrm{R}}(\nu,\mu)$$
$$\mu \frac{d}{d\mu} F^{\mathrm{R}}(\nu,\mu) = \gamma_F^{\mu} F^{\mathrm{R}}(\nu,\mu)$$

For the case of the jet function at NLO:

$$\gamma_J^{\nu} = \frac{\alpha_s C_f}{\pi} \mathrm{Log}\left(\frac{\mu^2 \tau^2}{Q^2}\right), \qquad \gamma_J^{\mu} = \frac{\alpha_s C_f}{\pi} \mathrm{Log}\left(\frac{\nu^2}{Q^2}\right) + \frac{3\alpha_s C_f}{2\pi}$$

NB: Running in ν and μ commute.

ν RG

nu parameter acts as an effective cutoff in rapidity fluctuations in each sector. Using evolution equations, one moves fluctuations into and out of a sector along invariant mass hyperbola.

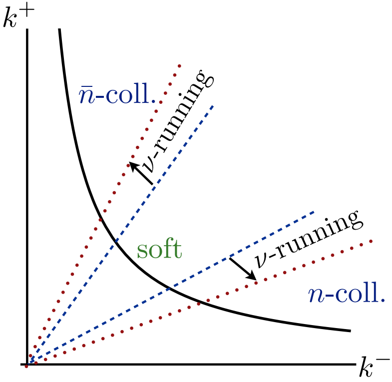

Figure: RRG along an invariant mass hyperbola.

The Strategy of Running

- μ Run hard function down to scale eQ
- ν Run soft function up to scale Q

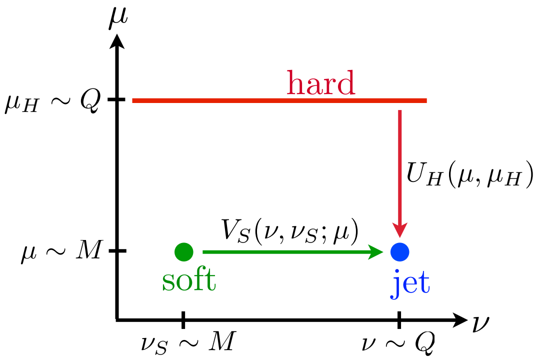

Figure: Running Strategy

Structure of Resummed Cross-Section

$$
\begin{aligned}
\frac{d\sigma}{de} &= H\, U \otimes J_n \otimes J_{\bar{n}} \otimes S \\
U &= Exp\Big[\Gamma(\alpha_S)L^2 - 2\Gamma(\alpha_S)L\tilde{L} + ...\Big] \\
L &= \mathrm{Log}\frac{Q^2}{\mu^2} \\
\tilde{L} &= \mathrm{Log}\frac{e^2 Q^2}{\mu^2} \\
\mu &\sim eQ
\end{aligned}
$$

Leading Log variation cancels in the exponent (problem of scales).
All logarithms minimized in all sectors.

Results at NLL

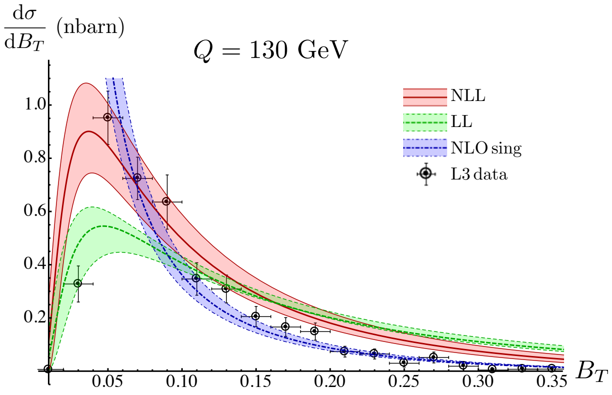

Figure: Jet Broadening Differential Cross-Section

NLL error bars include geometric mean of ν and μ variation.

Why RG technique?

The key feature of the renormalization group technique is that it allows a clean definition of the matrix elements, and how to exponentiate the large logarithms from the matrix elements.

- Factorize at any common scale ν.
- Evolve each matrix element to natural scale.
- Can precise quantify ν dependence of cross-section: cross-section ν independent at all orders in perturbation theory. There is always subleading ν dependence at given resummation order.

Applications

Beyond jet broadening, many applications. Broadly called as "Soft Recoil Sensitive Observables" (SRSO's), and provides a universal formalism for all such processes:

- Transverse Momentum PDF's
- Transverse Momentum Fragmentation functions
- End-point Singularities in exclusive B-decays
- Double Parton Distribution Functions (Manohar&Waalewijn 2012)
- Jet Algorithms (Cheung& Freedman 2012)

Comparison to CSS

Factorization in Rapidities (Collins&Soper 1982) for TM fragmentation functions.

- CS equation: $Q^- \frac{d}{dQ^-} F = (G + K)F$.
- G corresponds to hard double logs, K to infra-red double logs.
- Inadequate factorization: hard double logs should only reside in hard function.
- Hard matching coefficient ambiguous as it depends on rapidity regularization parameter.
- Power corrections problematic.

Comparison to CSS

- ν equation: $\nu \frac{d}{d\nu} F = K\, F$.
- G term gone: factorization is complete.
- hard matching coefficient depends on only on μ renormalization scale: factorized in invariant mass and well-defined independent of rapidity regulator.
- Power Corrections straight forward to implement in EFT framework.

Conclusion

- Introduced formalism applicable to observables where soft and collinear radiation with same invariant mass dominate, and fixed order cross-section contains large double logarithmic series (SRSO's).
- Allows for consistent resummation of all large logarithms, gives universal low scale matrix elements.
- Formalism applicable in wide variety of observables (not process dependent).
- Organized in SCET framework for conceptual ease of use.