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NNLO and N^3LO DIS structure functions in ACOT scheme

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Outline

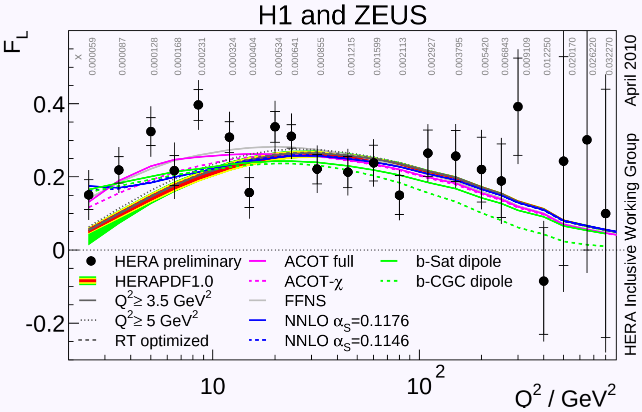
- Motivations
- ACOT scheme
  - basic features
  - extension beyond NLO
- Results for F_2 and F_L structure functions
- Summary

Motivations

- Increasing precision of experimental data → precision of theoretical calculations → heavy quark schemes beyond NLO
- DIS is one of the most important processes: → precision HERA data + precision calculations → precise PDFs → accurate predictions for LHC

Motivations

Higher order corrections and heavy quark schemes are particularly important for F_L in DIS.



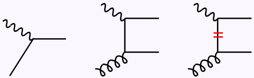
massless quarks → F_L vanishes at LO → first unsuppressed contribution at NLO → NNLO & N^{3}LO corrections more important than for F_2

ACOT scheme

ACOT scheme provides a mechanism to incorporate the heavy quark mass into the calculation of heavy quark production. It is based on heavy quarks factorization theorem of Collins.

The key ingredient of ACOT scheme is subtraction term (SUB), at NLO we have

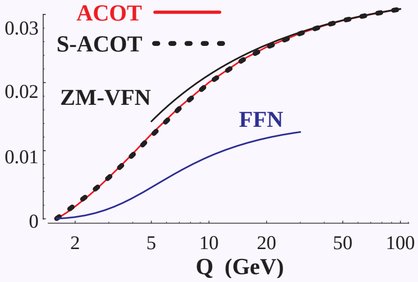
$$\sigma_{TOT} = \sigma_{LO} + \{\sigma_{NLO} - \sigma_{SUB}\}$$

for gluon-initiated processes: 

$$\sigma_{SUB} = f_g \otimes \tilde{P}_{g\to Q} \otimes \sigma_{QV\to Q}$$

$\tilde{P}_{g\to Q}$ is the $\overline{MS}$ splitting times the logarithm $\ln(\mu^2/m_Q^2)$

Mass limits of ACOT scheme

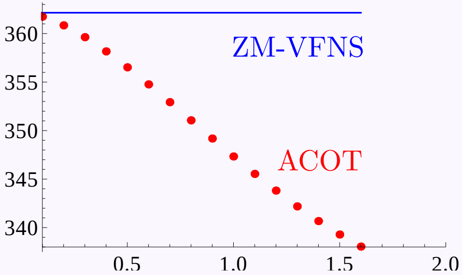


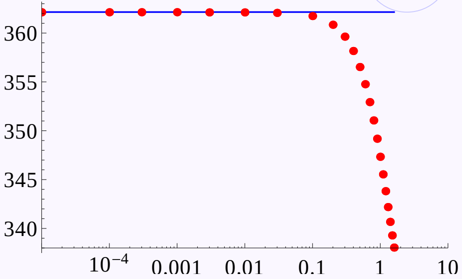
F_2^c at $x = 0.1$ for NLO DIS heavy quark production for different schemes:

- ACOT, S-ACOT,
- Fixed-Flavor-Number-Scheme (FFNS),
- Zero-Mass Variable-Flavor-Number-Scheme (ZM-VFNS)

- $\mu \lesssim m_Q$: ACOT $\rightarrow$ FFNS
  heavy quark is treated as extrinsic to hadron, $f_Q(x, \mu) = 0$
- $\mu \gtrsim m_Q$: ACOT $\rightarrow$ $\overline{MS}$ ZM-VFNS (exactly–**without** any finite renormalizations) quark mass m serves purely as a regulator – no dynamical role

Mass limits of ACOT scheme





F_2^c (scaled by 10^4) vs. quark mass m[GeV] at $x = 0.1$ and $Q = 10$GeV for NLO DIS heavy quark production for different schemes:

- logarithmic plot shows explicitly the result holds precisely in the $m \to 0$ limit and **no finite renormalization are needed**!
- ACOT is a **minimal massive extension** of $\overline{MS}$ ZM-VFNS.

ACOT scheme beyond NLO

ACOT is a factorization scheme valid to all orders of pQCD $\rightarrow$ we can use it beyond NLO

Problem: we need massive Wilson coefficients $\rightarrow$ they haven't been calculated.

The massless Wilson coefficients for NC F_2 and F_L structure functions are known at NNLO and even N^3LO.

Question: can we use these results, and the knowledge that ACOT reduces to the massless $\overline{MS}$ (ZM-VFNS) for $m_Q \rightarrow 0$, to estimate mass effects at NNLO and N^3LO?

Mass dependence in ACOT scheme

There are two ways heavy quark mass enters calculation of a cross section in ACOT scheme

- **dynamically** – through the mass dependent Wilson coefficients
- **kinematically** – restricting available phase space

$$\sigma = f\left(\xi(x, m_{kin}), Q\right) \otimes \hat{\sigma}_{QV}(m_{dyn})$$

Without the knowledge of Wilson coefficients we cannot include **dynamic** mass effects but we can include effects of **kinematic** mass.

Kinematic mass dependence

- Mass enters the kinematics restricting the final state phase space, which is realized via a shift of Bjorken x variable.
- In general the shift depend on the process and can be found working out the full kinematics.

The following form of rescaling allows to encompass different possibilities

$$x \to x \left[1 + \left(\frac{nm}{Q} \right)^2 \right]$$

$n = 0$ represents massless result, $n = 1$ is the original Barnett rescaling, and $n = 2$ is the (preferred) χ-rescaling.

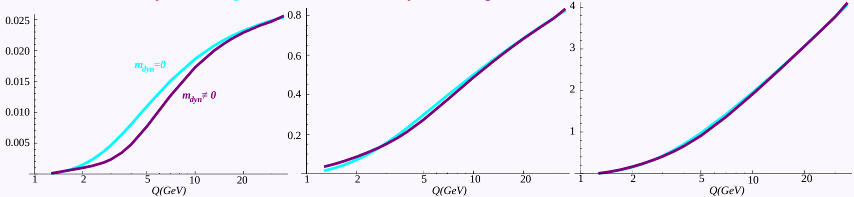
- It ensures that phase space is suppressed by appropriate factor $(nm/Q)^2$

Dynamic vs. kinematic mass dependence

Comparison of F_2^c for NLO ACOT

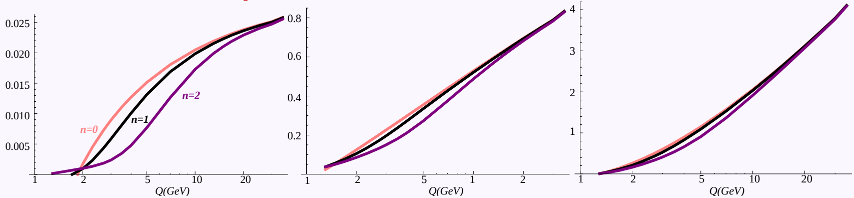
dynamic: $\hat{\sigma}_{QV}(m_{dyn} = 0)$ vs. $\hat{\sigma}_{QV}(m_{dyn} \neq 0)$ (for $n = 2$)

F_2^C



kinematic: (for $m_{dyn} = 0$)

F_2^C



$x = 10^{-1}$ $x = 10^{-3}$ $x = 10^{-5}$

ACOT scheme beyond NLO

Our strategy:

- use fully massive ACOT result to NLO,
- and add massless NNLO and N^{3}LO contributions with χ-rescaling.

$$\text{ACOT}[\mathcal{O}(\alpha_S^{0+1+2+3})] \;\simeq\; \text{ACOT}[\mathcal{O}(\alpha_S^{0+1})] + \text{ZM-VFNS}_{\chi(n)}[\mathcal{O}(\alpha_S^{2+3})]$$

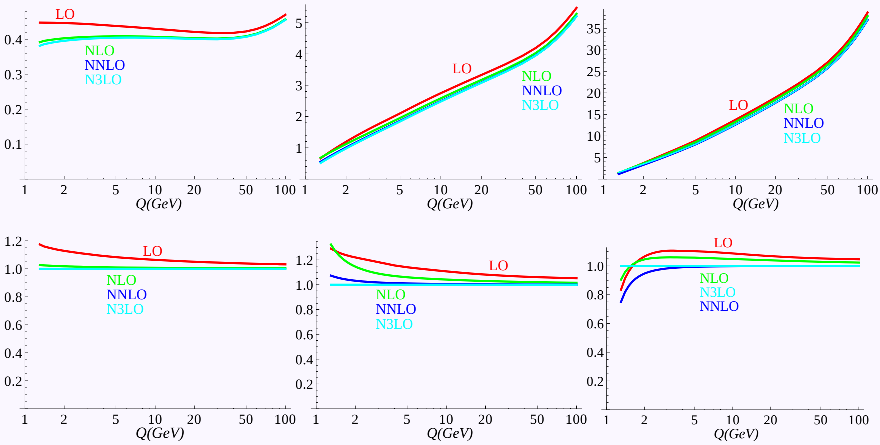
Based on NLO study using full ACOT result the above prescription provides a good approximation of the exact result. At worst, the error is of order $\alpha\,\alpha_S^2 \times [m^2/Q^2]$.

RESULTS

Results

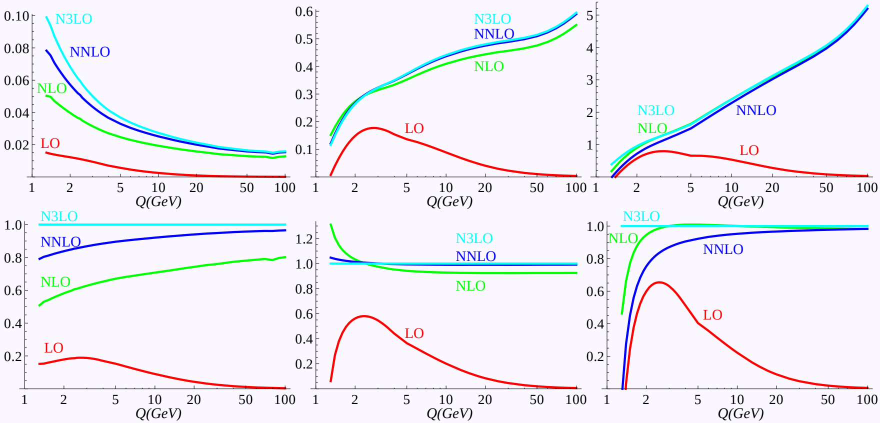
- evolution – QCDNUM in VFNS with NNLO DGLAP kernels;
- initial PDFs form Les Houches benchmark set (`hep-ph/0204316`);
- $m_c = 1.3$ GeV, $m_b = 4.5$ GeV, $\alpha_S(m_Z) = 0.118$

F_2 vs. Q at {LO, NLO, NNLO, N^{3}LO}



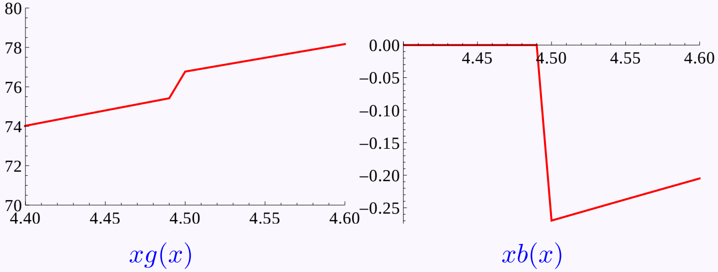
$x = 10^{-1}$ $x = 10^{-3}$ $x = 10^{-5}$

F_L vs. Q at {LO, NLO, NNLO, N^{3}LO}



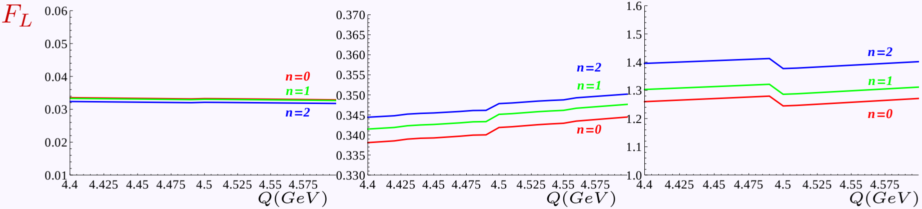
$x = 10^{-1}$ $x = 10^{-3}$ $x = 10^{-5}$

Discontinuities in PDFs $x = 10^{-5}$



- PDFs – $O(\alpha_s^2)$
- strong coupling – $O(\alpha_s^3)$

Discontinuities in F_L at NNLO ($m_b = 4.5$ GeV)



$x = 10^{-1}$ $x = 10^{-3}$ $x = 10^{-5}$

- PDFs – $O(\alpha_s^2)$
- strong coupling – $O(\alpha_s^3)$

- measurable quantities calculated at $O(\alpha_s^n)$

$$\sigma_{TOT}^{N_F=5} = \sigma_{TOT}^{N_F=4} + O(\alpha_s^{n+1})$$

- note change of sign

Discontinuities in measurable quantities

Matching condition:

$$f_a^{N_F+1} = A_{ab} \otimes f_b^{N_F}$$

for $\mu = m_b$:

$$\begin{aligned} f_b^5 &= \left\{0 + \frac{\alpha_s}{2\pi} P_{qg} \left(L + a_{qg}\right) + O(\alpha_s^2)\right\} \otimes f_g^4 \\ f_g^5 &= \left\{1 + \frac{\alpha_s}{2\pi} P_{gg} \left(L + a_{gg}\right) + O(\alpha_s^2)\right\} \otimes f_g^4 \end{aligned} \quad (1)$$

where $L = \ln(\mu^2/m_b^2)$

- for $\overline{MS}$ at NLO $a_{qg} = a_{gg} = 0$ $\rightarrow$ PDFs are continuous

Discontinuities in measurable quantities

What if $a \neq 0$ at NLO:

ACOT for $N_F = 5$ ($\mu > m_b$):

$$\begin{aligned}
\sigma_{LO} &= C^0 \otimes f_b^5 \simeq C^0 \otimes \left\{0 + \frac{\alpha_s}{2\pi} P_{qg}\left(L + a_{qg}\right)\right\} \otimes f_g^4 \\
\sigma_{NLO} &= C^1 \otimes f_g^5 \simeq C^1 \otimes \left\{1 + \frac{\alpha_s}{2\pi} P_{gg}\left(L + a_{gg}\right)\right\} \otimes f_g^4 \\
\sigma_{SUB} &= C^0 \otimes \widetilde{f}_{g\to q} \otimes f_g^5 \simeq C^0 \otimes \left\{\frac{\alpha_s}{2\pi} P_{qg}\left(L + a_{qg}\right)\right\} \\
&\otimes \left\{1 + \frac{\alpha_s}{2\pi} P_{gg}\left(L + a_{gg}\right)\right\} \otimes f_g^4
\end{aligned}$$

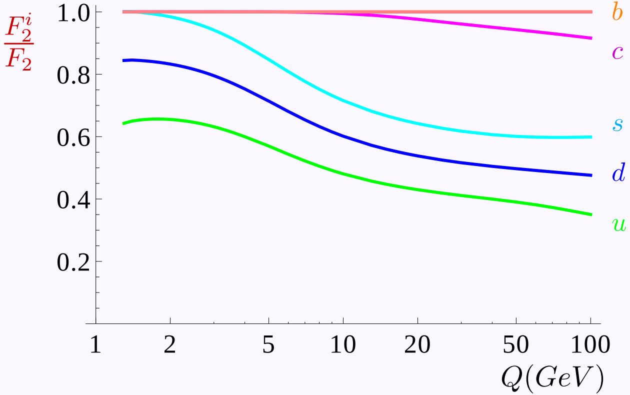
to $\mathcal{O}(\alpha_s^1)$ order:

$$\sigma_{TOT}^{N_F=5} = \sigma_{LO} + \sigma_{NLO} - \sigma_{SUB} = C^1 \otimes f_g^4 + O(\alpha_s^2)$$

ACOT for $N_F = 4$ ($\mu < m_b$):

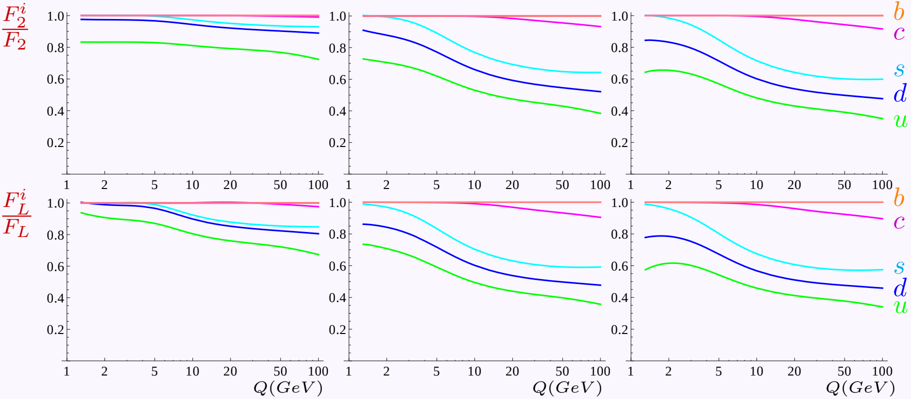
$$\sigma_{TOT}^{N_F=4} = \sigma_{NLO} = C^1 \otimes f_g^4 + O(\alpha_s^2)$$

Fractional flavor decomposition at N^{3}LO: $F^i_{2,L}/F_{2,L}$, $i = \{u, d, s, c, b\}$ $(n = 2)$



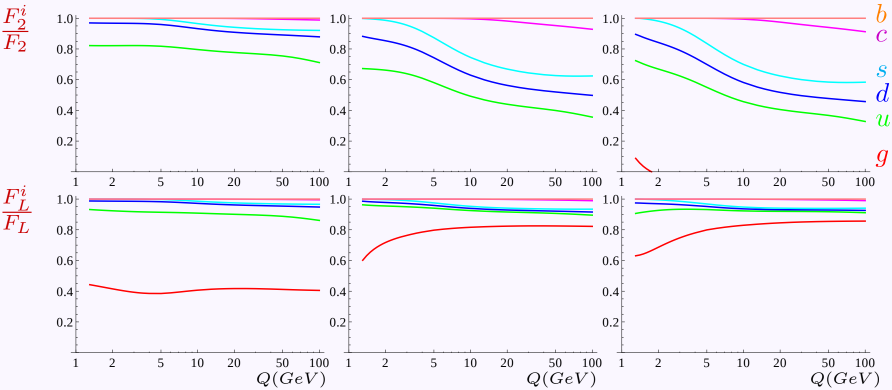
$x = 10^{-5}$

Fractional flavor decomposition at N^{3}LO: $F^i_{2,L}/F_{2,L}$, $i = \{u, d, s, c, b\}$ $(n = 2)$



$x = 10^{-1}$ $x = 10^{-3}$ $x = 10^{-5}$

Initial state flavor decomposition of F_2 and F_L (not an observable)

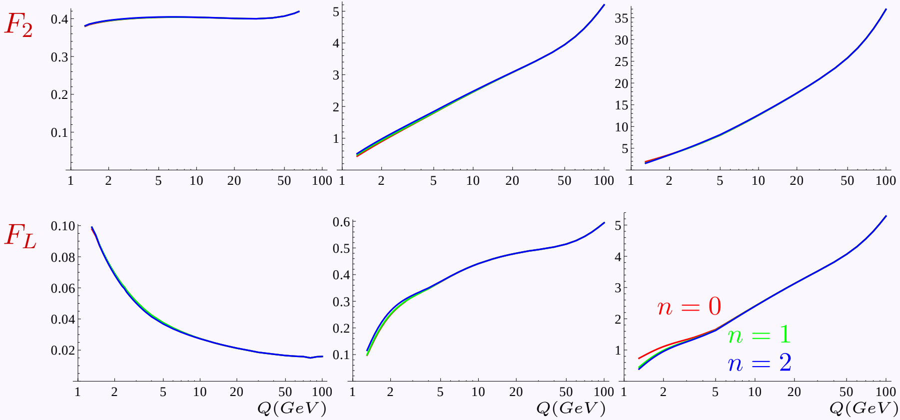


$x = 10^{-1}$ $x = 10^{-3}$ $x = 10^{-5}$

- sizable gluon contribution

n-scaling dependence for inclusive F_2, F_L at N^{3}LO

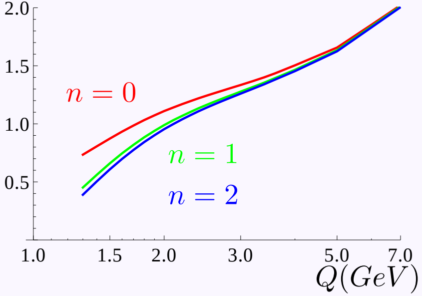
$$x \to x\left[1 + \left(\frac{nm}{Q}\right)^2\right]$$



$x = 10^{-1}$ $x = 10^{-3}$ $x = 10^{-5}$

n-scaling dependence for inclusive F_2, F_L at N^{3}LO

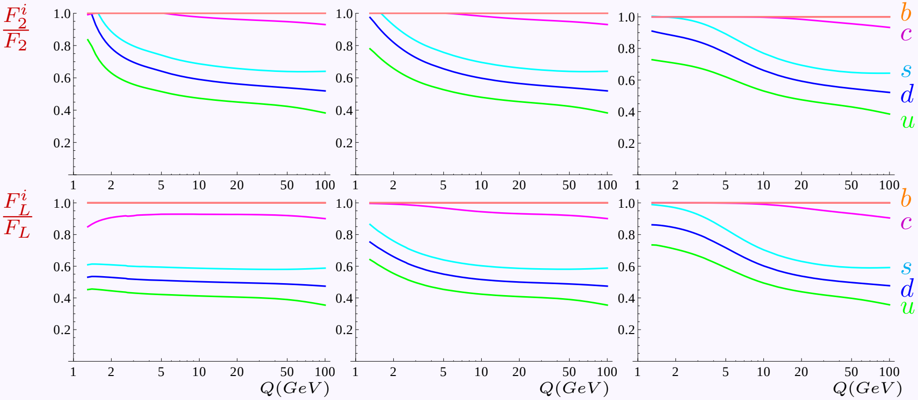
- Effect of n-scaling negligible except for small Q
  - heavy quarks are only fraction of total results
  - with growing Q we approach massless limit



$$x \to x\left[1 + \left(\frac{nm}{Q}\right)^2\right]$$

- $n = 1$ and $n = 2$ scalings are nearly identical
- but different that $n = 0$ (massless)
- we can use the difference between $n = 1$ and $n = 2$ as an uncertainty estimate

Fractional flavor $F_{2,L}$ decomposition for $x = 10^{-3}$ at N^{3}LO: $F^i_{2,L}/F_{2,L}$, $i = \{u, d, s, c, b\}$



$n = 0$ $n = 1$ $n = 2$

Summary

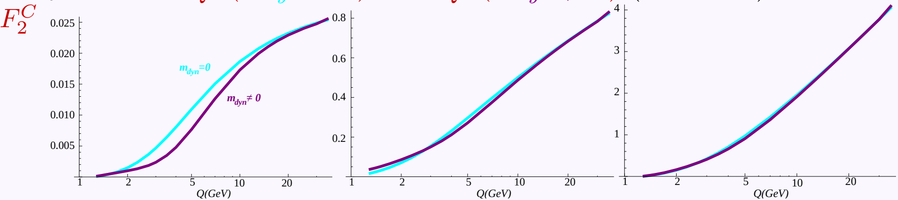
- We have computed the F_2 and F_L structure functions in the ACOT scheme at NNLO and N^{3}LO.
- The full mass dependence is computed to NLO, and the dominant mass effects for the higher orders are approximated using massless results supplemented with generalized rescaling prescription.
- This allows us to make detailed predictions throughout the kinematic range investigated by HERA.
- Together with the precise HERA data, these calculations facilitate accurate determination of PDFs.
- When the full massive Wilson coefficients will be calculated including them in our approach is straightforward.

BACKUP SLIDES

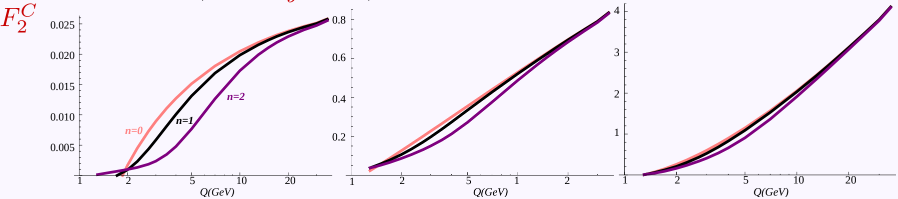
Dynamic vs. kinematic mass dependence

Comparison of F_2^c for NLO ACOT

dynamic: $\hat{\sigma}_{QV}(m_{dyn} = 0)$ vs. $\hat{\sigma}_{QV}(m_{dyn} \neq 0)$ (for $n = 2$)



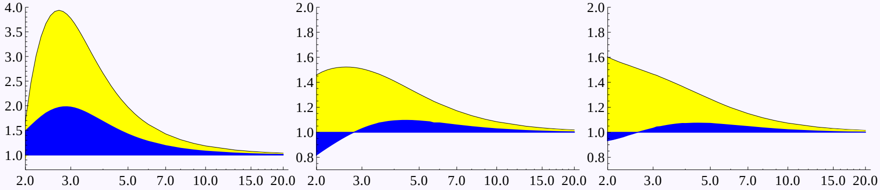
kinematic: (for $m_{dyn} = 0$)



$x = 10^{-1}$ $x = 10^{-3}$ $x = 10^{-5}$

Dynamic vs. kinematic mass dependence

$F_2^c/F_2^c(m_{dyn=0}, n=2)$ for NLO ACOT

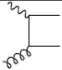


$x = 10^{-1}$ $x = 10^{-3}$ $x = 10^{-5}$

- yellow band – variation of the kinematic mass
- overlaid blue band – variation of the dynamic mass

Plots are scaled by the massless $n = 2$ result.

General rescaling

ξ	General	$m_1 = 0$	$m_1 = m_2 = m$	χ-scheme:
	$\eta\left[\frac{Q^2-m_1^2+m_2^2+\Delta[-Q^2,m_1^2,m_2^2]}{2Q^2}\right]$	$\eta\left[1+\frac{m_2^2}{Q^2}\right]$	$\eta\left[1+\frac{m^2}{Q^2}\right]$	$\eta\left[1+\frac{(2m)^2}{Q^2}\right]$
	$\eta\left[1+\left(\frac{m_1+m_2}{Q}\right)^2\right]$	$\eta\left[1+\frac{m_2^2}{Q^2}\right]$	$\eta\left[1+\frac{(2m)^2}{Q^2}\right]$	$\eta\left[1+\frac{(2m)^2}{Q^2}\right]$

$$\Delta[a,b,c] = \sqrt{a^2+b^2+c^2-2(ab+bc+ca)}$$

Flavor decomposition

Decomposition for final state (in case of charm):

$$F^c = \sum_{i=0}^{3} F^{i4} + \sum_{j=1}^{3} F^{4j} + F^{44}$$

for initial state (not observable):

$$F^i_{2,L} = \sum_{j=1}^{6} F^{ij}_{2,L}$$