Alternative subtraction scheme using Nagy-Soper dipoles

Tania Robens

.

based on

C.H.Chung, TR (arXiv:1001.2704)

C.H.Chung, M. Krämer, TR (arXiv:1012.4948 (JHEP), arXiv:1105.5327)

C.H.Chung, PhD thesis (06/11)

C.H.Chung, T. Robens (work in progress)

IKTP, TU Dresden

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1. NLO calculations - pole structure and treatments
   - Singularity structure of NLO calculations
   - Subtraction schemes
2. Nagy Soper subtraction scheme
   - General setup
3. Nagy Soper subtraction scheme
   - Applications
4. Summary and Outlook

Introduction (1)

- era of LHC: QCD-governed processes, **large NLO contributions**
- want fully differential distributions: need **stable + fast NLO Monte Carlo Event generators**
- full description ⇒ **combination w parton showers** necessary

A lot of progress on all above points in recent years...

(cf eg P.Nason, B.Webber, "Next-to-Leading-Order Event Generators", arXiv:1202.1251)

Introduction (2) (and pre-summary)

Motivation for new NLO subtraction scheme

- number of phase space mappings in the scheme greatly reduced

$\Rightarrow$ especially important for large number of external particles

- dipoles: derived from splitting functions of "Parton shower with quantum interference" (Nagy, Soper,2007)

$\Rightarrow$ once shower and scheme are implemented: facilitates matching with NLO calculations

- here: **concentrate on subtraction scheme**

NLO corrections: general structure

Masterformula

for m particles in the final state

$$\sigma_{\text{NLO,tot}} = \sigma_{\text{LO}} + \sigma_{\text{NLO}},$$

$$\sigma_{\text{LO}} = \int d\Gamma_m \, |\mathcal{M}^{(m)}_{\text{Born}}|^2(s) \qquad \text{leading order contribution}$$

$$\sigma_{\text{NLO}} = \sigma_{\text{real}} + \sigma_{\text{virt}},$$

$$\sigma_{\text{real}} = \int d\Gamma_{m+1} \, |\mathcal{M}^{(m+1)}|^2 \qquad \text{real emission}$$

$$\sigma_{\text{virt}} = \int d\Gamma_m \, 2\, Re(\mathcal{M}^{(m)}_{\text{Born}} \, (\mathcal{M}^{(m)}_{\text{virt}})^*) \qquad \text{virtual contribution}$$

with $d\Gamma$: phase space integral, $\mathcal{M}$ matrix elements
(here: flux factors etc implicit)

Infrared divergencies in NLO corrections

- source of infrared divergence: integration over phase space of emitted massless particles in real and virtual contribution (poles cancel in $\sigma_{\text{real}} + \sigma_{\text{virt}}$)
- appear in matrix elements as terms $\frac{1}{p_i p_j} = \frac{1}{E_i E_j \,(1-\cos\theta_{ij})}$
  $E_j \to 0$: soft divergence, $\cos\theta_{ij} \to 1$: collinear divergence
- poles arise from **integration** of phase space of p_j
- eg in QCD $\tilde{p}_i \to p_i + p_j$ (omitted color factors etc)

$$q \to q\, g :\propto \frac{1}{\varepsilon^2} + \frac{3}{2\,\varepsilon},\; g \to q\,\bar{q} :\propto -\frac{1}{3\varepsilon}$$

- important: **this behaviour is the same for all processes**

Dipole subtraction: general idea

- know that pole structure always the same
- matrix element level: in the singular limits,

$$|\mathcal{M}^{(m+1)}|^2 \longrightarrow D_{ij}(p_i, p_j)\, |\mathcal{M}^{(m)}|^2, \quad D_{ij} \sim \frac{1}{p_i p_j} \qquad (1)$$

- D_{ij}: **dipoles**, contain complete singularity structure
- also means that

$$\int d\Gamma_{m+1} \left(|\mathcal{M}^{(m+1)}|^2 - \sum_{ij} D_{ij} |\mathcal{M}^{(m)}|^2 \right) = \text{finite}$$

- **general idea of dipole subtraction:** make use of (1), shift singular parts from $m+1$ to m particle phase space

  $\Rightarrow$ **need to have a good (analytical) parametrization of the singularity structure**

Dipole subtraction for total cross sections

Master formula

$$\sigma = \sigma^{LO} + \sigma^{NLO}$$

$$\sigma^{NLO} = \int_{m+1} d\sigma^{R} + \int_{m} d\sigma^{V} + \int d\sigma^{C}$$

$$= \int_{m+1} \left(d\sigma^{R} - d\sigma^{A}\right) + \int_{m} \left(d\sigma^{\tilde{A}} + d\sigma^{V} + d\sigma^{C}\right),$$

$\Rightarrow$ effectively added "0"; both integrals finite

$$\sigma_{m}^{\text{NLO}}(s) = \int_{m} \Big\{ |\widetilde{\mathcal{M}}_{\text{virt}}(s;\varepsilon)|^2 + \mathbf{I}(\varepsilon)|\mathcal{M}_{\text{Born}}(s)|^2 + \int_0^1 dx\, \left(\mathbf{K}(x) + \mathbf{P}(x;\mu_F)\right) |\mathcal{M}_{\text{Born}}(x,s)|^2 \Big\}$$

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Dipole subtraction for total cross sections

Master formula

$$\begin{aligned} \sigma &= \sigma^{LO} + \sigma^{NLO} \\ \sigma^{NLO} &= \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int d\sigma^C \\ &= \int_{m+1} \left(d\sigma^R - d\sigma^A\right) + \int_m \left(d\sigma^{\tilde{A}} + d\sigma^V + d\sigma^C\right), \end{aligned}$$

$\Rightarrow$ effectively added "0"; both integrals finite

$$\begin{aligned} \sigma_m^{\mathrm{NLO}}(s) = \int_m \Big\{ & |\widetilde{\mathcal{M}}_{\mathrm{virt}}(s;\varepsilon)|^2 + \mathbf{I}(\varepsilon)|\mathcal{M}_{\mathrm{Born}}(s)|^2 \\ & + \int_0^1 dx\, \left(\mathbf{K}(x) + \mathbf{P}(x;\mu_F)\right) |\mathcal{M}_{\mathrm{Born}}(x,s)|^2 \Big\} \end{aligned}$$

Ingredient for subtraction schemes: momentum mapping

- previous slide: add and subtract "0" in terms of

$$\int d\Gamma_m \widetilde{F}_{\text{sing}} |\mathcal{M}^{(m)}_{\text{Born}}|^2 - \int d\Gamma_{m+1} F_{\text{sing}} |\mathcal{M}^{(m)}_{\text{Born}}|^2$$

- addition and subtraction takes place in different phase spaces

$$p_{\tilde{a}}^{(m)} = F\left(p_a^{(m+1)}, p_b^{(m+1)}, \dots\right)$$

This function is highly scheme dependent !!!

requirement: keep all particles onshell, total energy/ momentum conserved :

$$p_{\tilde{i}}^2 = p_i^2 = m_i^2, \quad \sum_m p_{\tilde{a}} \stackrel{!}{=} \sum_{m+1} p_a$$

(sum over outgoing particles only)

Nagy Soper subtraction scheme

- many different subtraction schemes are around (best known: Catani, Seymour, 1996)
- all schemes: poles have to be the same; finite parts can differ

Main motivation for new scheme

- proposal of improved parton shower: Nagy, Soper (arXiv:0706.0017, 0801.1917, 0805.0216)
- basic idea: can use the splitting functions of the new shower as dipole subtraction terms

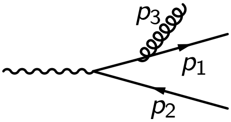
⇒ (cf Catani Seymour Showers in Sherpa (Schumann ea '07), Herwig++ (Plätzer ea '11), ... (Winter ea, Dinsdale ea '07, ...))

- introduce new mapping between m and $m+1$ phase spaces: spectator = whole remaining event

⇒ leads to a much smaller number of subtraction terms, $\sim N_{\text{fin}}^2/2$ (vs $\sim N_{\text{fin}}^3/2$ in Catani Seymour scheme) especially important for large number of external particles

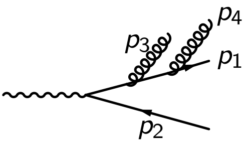
Shifting momenta: Example (1)

$$\gamma^* \longrightarrow q(p_1)\bar{q}(p_2)\,g(p_3)\text{ (@ NLO)}$$



part of Born contribution

real gluon emissions for this diagramm:

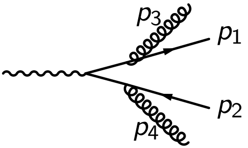


CS: 1 momentum shift/ spectator

p_2, p_3: 2 transformations

NS: 1 total transformation

Shifting momenta: Example (2)



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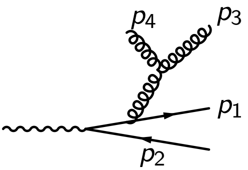

CS: 1 momentum shift/ spectator

p_1, p_2: 2 transformations

NS: 1 total transformation

$\Rightarrow$ from simple counting:

10 transformations using CS vs 5 using NS dipoles !!

(full process also includes $g \longrightarrow q\,\bar{q}$)

Final state $g \longrightarrow q\,\bar{q}$: Catani Seymour vs Nagy Soper (1)

- $g(\tilde{p}_i) \longrightarrow q(p_i) + \bar{q}(p_j)$,
  spectator: any other final state parton, p_k
- Dipole keeping angular correlations

$$\langle\mu|v_{ij}^2|\nu\rangle_{\text{NS, CS}} = -\underbrace{\frac{4\,\pi\,\alpha_s}{\hat{p}_i\hat{p}_j}}_{\text{sing}}\left[g^{\mu\nu} + 2\frac{k_\perp^\mu\,k_\perp^\nu}{\hat{p}_i\hat{p}_j}\right]$$

$$\textbf{NS: } k_\perp = p_i - (1-z)\gamma(y)\tilde{p}_i - \frac{z}{\gamma(y)}\,y\,\tilde{n}$$

$$\textbf{CS: } k_\perp = p_i - z\,\tilde{p}_i - y(1-z)\tilde{p}_k$$

- $\tilde{p}_i$, $\tilde{p}_k$: Born-type kinematics, mother parton/ spectator
- y: singular variable
- z: parametrization of angle between $(p_i,\,p_k)$ (CS), $(p_j,\tilde{n})$ (NS)

Final state $g \longrightarrow q\,\bar{q}$: Catani Seymour vs Nagy Soper (2)

- Dipole (in terms of integration variables):

$$D^{ij,k}_{\text{NS, CS}} \propto \underbrace{\frac{1}{y}}_{\text{sing}} \left[1 - \frac{z\,(1-z)}{1-\varepsilon}\right]$$

- NS definitions

$$y_{\text{NS}} = \frac{p_i\,p_j}{(p_i + p_j)Q - p_i\,p_j}, \quad z_{\text{NS}} = \frac{p_j\,\tilde{n}}{p_i\,\tilde{n} + p_j\,\tilde{n}}$$

$$\tilde{n} = \frac{1+y+\lambda}{2\lambda}\,Q - \frac{a}{\lambda}\left(p_i + p_j\right), \; \lambda = \sqrt{(1+y)^2 - 4\,a\,y}, \; a = \frac{Q^2}{(p_i+p_j)Q - p_i\,p_j}$$

- CS definitions:

$$y_{CS} = \frac{p_i\,p_j}{p_i\,p_j + p_i\,p_k + p_j\,p_k}, \quad z_{CS} = \frac{p_i\,p_k}{p_i\,p_k + p_j\,p_k}$$

Final state $g \longrightarrow q\,\bar{q}$: Catani Seymour vs Nagy Soper (3)

- CS matching (all other final state particles untouched)

$$\tilde{p}_i \;=\; p_i + p_j - \frac{y}{1-y} p_k,\;\; \tilde{p}_k^\mu \;=\; \frac{1}{1-y} p_k$$

- NS matching

$$\tilde{p}_i \;=\; \frac{1}{\lambda}(p_i + p_j) - \frac{1-\lambda+y}{2\,\lambda\,a}\,Q,\;\; \tilde{p}_k^\mu \;=\; \Lambda^\mu{}_\nu p_k^\nu \quad \textbf{all fs particles}$$

$$\Lambda^{\mu\nu} = g^{\mu\nu} - \frac{2\,(K+\widetilde{K})^\mu (K+\widetilde{K})^\nu}{(K+\widetilde{K})^2} + \frac{2\,K^\mu \widetilde{K}^\nu}{K^2},\; K=Q-p_i-p_j,\;\; \widetilde{K}=Q-\tilde{p}_i$$

- integration measure (identical, same pole structure)

$$[dp_j]_{\mathrm{CS}} \;=\; \frac{(2\,\tilde{p}_i\tilde{p}_k)^{1-\varepsilon}}{16\,\pi^2}\,\frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}}\,dz\,dy\,(1-y)^{1-2\,\varepsilon} y^{-\varepsilon}\,[z\,(1-z)]^{-\varepsilon},$$

$$[dp_j]_{\mathrm{NS}} \;=\; \frac{(2\,\tilde{p}_i\,Q)^{1-\varepsilon}}{16\,\pi^2}\,\frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}}\,dz\,dy\,\lambda^{1-2\,\varepsilon} y^{-\varepsilon}\,[z\,(1-z)]^{-\varepsilon}$$

Final state $g \longrightarrow q\,\bar{q}$: Catani Seymour vs Nagy Soper (4)

- result CS

$$\mu^{2\varepsilon}\int [dp_j]\, D^{ij,k} \;=\; \frac{\alpha_s}{2\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\,T_R\left(\frac{2\mu^2\pi}{\tilde{p}_i\tilde{p}_k}\right)^{\varepsilon}\left[-\frac{2}{3\,\varepsilon}-\frac{16}{9}\right]$$

- result NS

$$\mu^{2\varepsilon}\int [dp_j]\, D^{ij} \;=\; T_R\,\frac{\alpha_s}{2\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\left(\frac{2\,\pi\,\mu^2}{p_{\tilde{i}}Q}\right)^{\varepsilon}$$
$$\times\left[-\frac{2}{3\,\varepsilon}-\frac{16}{9}+\frac{2}{3}\left[(a-1)\ln(a-1)-a\ln a\right]\right],$$

- for $a = 1$, reduces completely to Catani Seymour result
- (reason: $a = 1$ implies only 2 particles in the final state, $\tilde{n} \longrightarrow p_k$, $\Rightarrow$ complete equivalence)
- **tradeoff:** all final state particles get additional momenta: **integrals more complicated, but fewer transformations necessary**

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Applications

Single W (✓)

Dijet production at lepton colliders (✓)

$p\bar{p} \longrightarrow H$ **and** $H \longrightarrow gg$ (✓)

Deep inelastic scattering (✓)

$e^{+}\,e^{-} \longrightarrow 3$ **jets**

[(✓): checked and published]

Deep inelastic scattering (subprocess of...)

- **considered process:**

$$e(p_{in})\, q(p_1) \longrightarrow e(p_{out})\, q(p_4)\ [g(p_3)]$$

- CS: spectator for final state gluon emission:

  initial state quark

- NS: spectator for final state gluon emission:

  final state lepton

- ("spectator" = spectator in momentum mapping)

⇒ first nontrivial check of NS scheme ⇐

DIS: Catani Seymour

Real emission subtraction terms

$$\mathbf{D}_{43,1} = \frac{4\pi\alpha_s}{p_3 p_4}\frac{1}{x_{43,1}} C_F \left[\frac{2}{1-\tilde{z}_4 + (1-x_{43,1})} - (1+\tilde{z}_4)\right] |\mathcal{M}|^2_{\text{Born}}(\tilde{p}_1, \tilde{p}_4)$$

$$\mathbf{D}_{13,4} = \frac{4\pi\alpha_s}{p_1 p_3}\frac{1}{x_{34,1}} C_F \left[\frac{2}{1- x_{34,1} + u_3} - (1+x_{34,1})\right] |\mathcal{M}|^2_{\text{Born}}(\tilde{p}_1, \tilde{p}_4)$$

$$\tilde{z}_4 = \frac{p_1 p_4}{(p_3+p_4)p_1},\quad x_{43,1} = x_{34,1} = \frac{p_i p_o}{p_1 p_4 + p_1 p_3},\quad u_3 = \frac{p_1 p_3}{(p_3+p_4)p_1}$$

Mapping

$$\tilde{\mathbf{p}}_1 = x_{43,1} p_1,\quad \tilde{\mathbf{p}}_4 = p_3 + p_4 - (1-x_{43,1})p_1$$

Integrated subtraction terms

$$\int_0^1 \mathbf{dx} |\mathcal{M}|^2_{2,\text{tot}} = \int_0^1 \frac{dx}{x}\left\{-\frac{9}{2}\frac{\alpha_s}{2\pi} C_F \delta(1-x) + K^{\text{eff}}_{\text{fin}}(x) + P^{\text{eff}}_{\text{fin}}(x;\mu_F^2)\right\} |\mathcal{M}|^2_{\text{Born}}(x p_1)$$

$$\mathbf{K}^{\text{eff}}(\mathbf{x}) = \frac{\alpha_s}{2\pi} C_F \left\{\left(\frac{1+x^2}{1-x}\ln\frac{1-x}{x}\right)_+ + \frac{1}{2}\delta(1-x) + (1-x) - \frac{3}{2}\frac{1}{(1-x)_+}\right\}$$

DIS: Nagy Soper - real emission terms

Initial state real emission subtraction

$$\mathbf{D}^{1,3} = \frac{4\,\pi\,\alpha_s}{x\,y\,\hat{p}_1\cdot\hat{p}_i}\;C_F\left(1-x-y+\frac{2\,\tilde{z}\,x}{v\,(1-x)+y}\right)|\mathcal{M}_{\text{Born}}(p)|^2$$

$$x=\frac{\hat{p}_o\cdot\hat{p}_4}{\hat{p}_i\cdot\hat{p}_1},\; y=\frac{\hat{p}_1\cdot\hat{p}_3}{\hat{p}_1\cdot\hat{p}_i},\; \tilde{z}=\frac{\hat{p}_1\cdot\hat{p}_4}{\hat{p}_4\cdot\hat{Q}},\; v=\frac{(\hat{p}_1\cdot\hat{p}_i)\,(\hat{p}_3\cdot\hat{p}_4)}{(\hat{p}_4\cdot\hat{Q})\,(\hat{p}_3\cdot\hat{Q})}.$$

Initial state: mapping

$$\mathbf{p_1} = x\,\hat{p}_1,\; \mathbf{p_i} = \hat{p}_i,\; \mathbf{p}^{\mu}_{\mathbf{o,4}} = \mathbf{\Lambda}^{\mu}{}_{\nu}(\mathbf{K},\hat{\mathbf{K}})\hat{p}^{\nu}_{o,4},\, K = x\,\hat{p}_1+\hat{p}_i,\; \hat{K} = \hat{p}_1+\hat{p}_i-\hat{p}_3\,.$$

Final state real emission subtraction

$$\mathbf{D}^{4,3} = \frac{4\pi\alpha_s\,C_F}{y(\hat{p}_i\cdot\hat{p}_1)}\left[\frac{y}{1-y}F_{\text{eik}}+z+2\frac{(1-v)(1-z(1-y))}{v\,[1-z(1-y)]+y\,[(1-y)\tilde{a}+1]}\right]|\mathcal{M}_{\text{Born}}(p)|^2$$

$$y=\frac{\hat{p}_3\cdot\hat{p}_4}{\hat{p}_1\cdot\hat{p}_i},\; z=\frac{\hat{p}_3\cdot\hat{p}_o}{\hat{p}_3\cdot\hat{p}_o+\hat{p}_4\cdot\hat{p}_o},\; v=\frac{\hat{p}_1\cdot\hat{p}_3}{\hat{p}_1\cdot\hat{p}_3+\hat{p}_1\cdot\hat{p}_4},\; F_{\text{eik}} = 2\,\frac{(\hat{p}_3\cdot\hat{p}_o)\,(\hat{p}_4\cdot\hat{p}_o)}{(\hat{p}_3\cdot\hat{Q})^2}.$$

Final state: mapping

$$\mathbf{p_i} = \hat{p}_i,\; \mathbf{p_1} = \hat{p}_1,\; \mathbf{p_4} = \frac{1}{1-y}\left[\hat{p}_3+\hat{p}_4-y\,(\hat{p}_1+\hat{p}_i)\right],\; \mathbf{p_o} = \frac{\hat{p}_o}{1-y}.$$

DIS: Nagy Soper - subtraction in the virtual contribution

$$\int_0^1 \mathbf{dx}\,|\mathcal{M}|_2^2 = \int_0^1 dx \left\{ \frac{\alpha_s}{2\,\pi}\, C_F\, \delta(1-x) \left[-9 + \frac{1}{3}\pi^2 - \frac{1}{2}\mathrm{Li}_2[(1-\tilde{z}_0)^2] \right.\right.$$

$$\left. + 2\,\ln\,2\,\ln\tilde{z}_0 + 3\,\ln\tilde{z}_0 + 3\,\mathrm{Li}_2(1-\tilde{z}_0) + \mathbf{I}_{\mathrm{fin}}^{\mathrm{tot},0}(\tilde{\mathbf{z}}_0) + \mathbf{I}_{\mathrm{fin}}^{1}(\tilde{\mathbf{a}}) \right]$$

$$\left. + K_{\mathrm{fin}}^{\mathrm{tot}}(x;\tilde{z}) + P_{\mathrm{fin}}^{\mathrm{tot}}(x;\mu_F^2) \right\} |\mathcal{M}|_{\mathrm{Born}}^2(x\,p_1),$$

$$\mathbf{K}_{\mathrm{fin}}^{\mathrm{tot}}(\mathbf{x};\tilde{\mathbf{z}}) = \frac{\alpha_s}{2\,\pi}\, C_F \left\{ \frac{1}{x} \left[2\,(1-x)\,\ln\,(1-x) - \left(\frac{1+x^2}{1-x}\right)_+ \ln\,x \right.\right.$$

$$\left.\left. + 4\,x \left(\frac{\ln(1-x)}{1-x}\right)_+ \right] + \mathbf{I}_{\mathrm{fin}}^{1}(\tilde{\mathbf{z}},\mathbf{x}) \right\},$$

⇒ **contains integrals which need to be evaluated numerically** ⇐

DIS: Nagy Soper - integrals to be evaluated numerically

$\Rightarrow$ **Integrals contain nontrivial functions depending on m and $m+1$ four-momenta** $\Leftarrow$

$$\mathbf{I}_{\text{fin}}^{\text{tot},0}(\tilde{\mathbf{z}}_0) = 2\int_0^1 \frac{dy}{y}\left\{\frac{\tilde{z}_0}{\sqrt{4\,y^2\,(1-\tilde{z}_0)+\tilde{z}_0^2}}\right.$$

$$\left.\times\ \ln\left[\frac{2\,z\,\sqrt{4\,y^2\,(1-\tilde{z}_0)+\tilde{z}_0^2}\,(1-y)}{\left(2\,y+\tilde{z}_0-2\,y\,\tilde{z}_0+\sqrt{4\,y^2\,(1-\tilde{z}_0)+\tilde{z}_0^2}\right)^2}\right]+\ln 2\right\}.$$

$$\mathbf{I}_{\text{fin}}^{\mathbf{1}}(\tilde{\mathbf{a}}) = 2\int_0^1 \frac{du}{u}\int_0^1 \frac{dx}{x}$$

$$\times\left[\frac{\mathbf{x}\,(\mathbf{1}-\mathbf{x}+\mathbf{u}\,\mathbf{x}\,[(\mathbf{1}-\mathbf{u}\,\mathbf{x})\,\tilde{\mathbf{a}}+\mathbf{2}])}{\mathbf{k}(\mathbf{u},\mathbf{x},\tilde{\mathbf{a}})}-\frac{1}{\sqrt{1+4\,\tilde{a}_0\,u^2\,(1+\tilde{a}_0)}}\right].$$

$$\mathbf{I}_{\text{fin}}^{\mathbf{1}}(\tilde{\mathbf{z}},\mathbf{x}) = \frac{2}{(1-x)_+}\,\frac{1}{\pi}\int_0^1\frac{dy'}{y'}\left[\int_0^1\frac{dv}{\sqrt{v\,(1-v)}}\,\frac{\tilde{\mathbf{z}}}{\mathbf{N}(\mathbf{x},\mathbf{y}',\tilde{\mathbf{z}},\mathbf{v})}-1\right],$$

DIS: Nagy Soper - variables in integrals to be evaluated numerically

for some integrals, $m+1$ variables have to be reconstructed

in initial state subtraction terms

$$\mathbf{N} = \frac{\hat{p}_3 \cdot \hat{p}_4}{\hat{p}_4 \cdot \hat{Q}} \frac{1}{1-x} + y', \tilde{z} = \frac{1}{x} \frac{p_1 \cdot \hat{p}_4}{\hat{p}_4 \cdot \hat{Q}}$$

$$\hat{\mathbf{p}}_3 = \underbrace{\frac{(1-\mathbf{x})(1-\mathbf{y}')}{\mathbf{x}}}_{\alpha} \mathbf{p}_1 + \underbrace{(1-\mathbf{x})\,\mathbf{y}'}_{\beta} \mathbf{p}_i - \mathbf{k}_\perp, \hat{\mathbf{p}}_4^\mu = \Lambda^\mu{}_\nu(\hat{\mathbf{K}}, \mathbf{K})\hat{\mathbf{p}}_4^\nu$$

$$k_\perp^2 = -2\,\alpha\,\beta\,p_1 \cdot p_i, \; k_\perp = -|k_\perp| \begin{pmatrix} 0 \\ 1-2\,v \\ 2\sqrt{v\,(1-v)} \\ 0 \end{pmatrix},$$

in final state subtraction terms

$$\mathbf{k}^2(\mathbf{x}, \mathbf{u}, \tilde{\mathbf{a}}) = \left[(1+ux-x)(z-z') + ux\left((1-ux)\,\tilde{a}+1\right)\right]^2 + 4\,u\,x\,z'\,(1-z)\,(1+u\,x-x)\,\left((1-ux)\,\tilde{a}+1\right)$$

$$\tilde{a} = \frac{p_1 \cdot p_o}{p_1 \cdot (p_i - (1-y)p_o)}$$

DIS: Nagy Soper - variables in integrals to be evaluated numerically

for some integrals, $m+1$ variables have to be reconstructed

in initial state subtraction terms

$$\mathbf{N} = \frac{\hat{p}_3 \cdot \hat{p}_4}{\hat{p}_4 \cdot \hat{Q}} \frac{1}{1-x} + y', \tilde{z} = \frac{1}{x} \frac{p_1 \cdot \hat{p}_4}{\hat{p}_4 \cdot \hat{Q}}$$

$$\hat{\mathbf{p}}_3 = \underbrace{\frac{(1-\mathbf{x})(1-\mathbf{y}')}{\mathbf{x}}}_{\alpha} \mathbf{p}_1 + \underbrace{(1-\mathbf{x})\,\mathbf{y}'}_{\beta} \mathbf{p}_i - \mathbf{k}_\perp, \hat{\mathbf{p}}_4^\mu = \Lambda^\mu{}_\nu(\hat{\mathbf{K}}, \mathbf{K}) \hat{\mathbf{p}}_4^\nu$$

$$k_\perp^2 = -2\,\alpha\,\beta\,p_1 \cdot p_i,\; k_\perp = -|k_\perp| \begin{pmatrix} 0 \\ 1-2\,v \\ 2\sqrt{v\,(1-v)} \\ 0 \end{pmatrix},$$

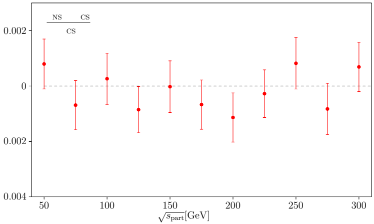
in final state subtraction terms

$$\mathbf{k}^2(\mathbf{x}, \mathbf{u}, \tilde{\mathbf{a}}) = \left[(1+ux-x)(z-z') + ux\,((1-ux)\,\tilde{a}+1)\right]^2 + 4\,u\,x\,z'\,(1-z)\,(1+u\,x-x)\,((1-ux)\,\tilde{a}+1)$$

$$\tilde{a} = \frac{p_1 \cdot p_o}{p_1 \cdot (p_i - (1-y)p_o)}$$

DIS: Numerical results

consistency check: get the same result



relative difference between CS and NS: $\frac{\sigma_{CS}-\sigma_{NS}}{\sigma_{CS}}$

agree on the sub-permill level ✓

$e^+ e^- \longrightarrow$ 3 jets: status report (1)

- consider process
$$e^+ e^- \longrightarrow q\,\bar{q}\,g$$
at NLO
- real emission contributions:
$$e^+ e^- \longrightarrow q\,\bar{q}\,q\,\bar{q},\; q\bar{q}\,g\,g$$
- number of necessary mappings (in total):
$$(8+10)_{CS} \text{ vs } (4+5)_{NS}$$
- 3 different color structures: $C_A\,C_F^2,\; C_A\,C_F\,n_f\,T_R,\; C_A^2\,C_F$
- singular parts: $C_A\,C_F\,n_f\,T_R$: $q\bar{q}\,q\,\bar{q}$ only,
$C_A^2\,C_F,\; C_A\,C_F^2$: $q\bar{q}\,gg$ only
- result known for a long time: Ellis ea 1980
(also Kuijf 1991, Giele ea 1992)

$e^+ e^- \longrightarrow$ 3 jets: status report (2)

- singularity structure in integrated dipoles for all color configurations: done ✓
- $q\bar{q}q\bar{q}$ real emission terms and all finite contributions: done ✓
- $q\bar{q}gg$ real emission terms and all finite contributions: done(✓)

- current work: improve numerics, especially for $g \rightarrow g\,g$ splittings

- infrared safe observable: C-distribution (Ellis ea 1980),

$$C^{(n)} = 3\left\{1 - \sum_{i,j=1,\, i<j}^{n} \frac{s_{ij}^2}{(2\,p_i \cdot Q)\,(2\,p_j \cdot Q)}\right\}$$

$e^+ e^- \longrightarrow$ 3 jets: status report (2)

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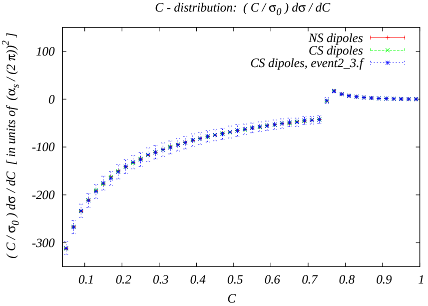
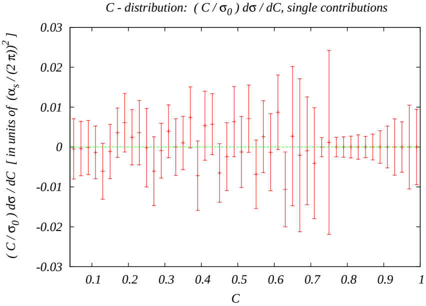
$e^+ e^- \longrightarrow$ 3 jets ($n_f T_R$ component): results



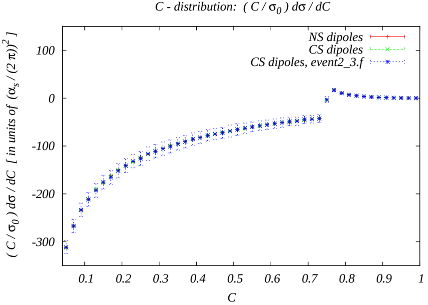
Comparison between NS and CS implementation as well as event2_3.f from M. Seymour



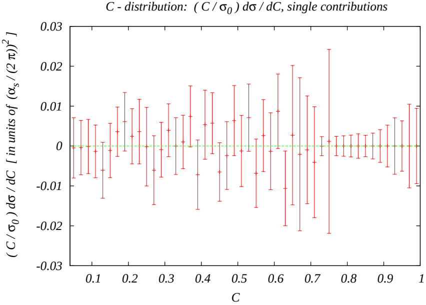
Relative difference between NS and CS implementation

- results agree on the (sub)-percent level, compatibel with 0 ✓ (integration not completely optimized yet)
- remark: for $C > 0.75$, only real emission contributes $\Rightarrow$ difference exactly 0 (when using the same setup)

$e^+ e^- \longrightarrow$ 3 jets ($n_f T_R$ component): results



Comparison between NS and CS implementation as well as event2_3.f from M. Seymour



Relative difference between NS and CS implementation

- results agree on the (sub)-percent level, compatibel with 0 ✓ (integration not completely optimized yet)
- remark: for $C > 0.75$, only real emission contributes $\Rightarrow$ difference exactly 0 (when using the same setup)

Difference 2: Combining showers and NLO (1)

Very very short...

- double counting: hard real emissions are described in both shower and "real emission" matrix element
- want:hard: matrix element, soft: shower (always talk about 1 jet)
- can be achieved by adding and subtracting a counterterm

$$-\int_{m+1} d\sigma^{\mathrm{PS}}|_{m+1} + \int_{m+1} d\sigma^{\mathrm{PS}}|_{m}$$

details eg in hep-ph/0204244: "Matching NLO QCD computations and parton shower simulations" (Frixione, Webber), MC@NLO

Difference 2: Combining showers and NLO (2)

- important: have new terms in $m+1$ phase space

$$\int_{m+1} \left(d\sigma^R \underbrace{-d\sigma^A + d\sigma^{PS}|_m}_{=0} - d\sigma^{PS}|_{m+1} \right)$$

- same splitting functions: second and third term cancel !! left with

$$\int_{m+1} \left(d\sigma^R - d\sigma^{PS}|_{m+1} \right)$$

$\Rightarrow$ improves numerical efficiency

- more details on this, also for MC@NLO vs Powheg: S. Hoeche ea, "A critical appraisal of NLO+PS matching methods", arXiv:1111.1220

Status quo (instead of Summary)

- goal: establish NS dipole formalism
- all integrals are done ✓
- all singularity structures checked ✓
- all finite term cross checked ✓

⇒ Massless scheme validated ⇐

"Test" processes
(w extensive documentation)

- single W at hadron colliders
- Dijet production at lepton colliders
- $p\bar{p} \longrightarrow H$ and $H \longrightarrow g\,g$
- DIS
- $e\,e \longrightarrow$ 3 jets

Outlook

Outlook

- make generically available for application in new higher order calculations
- **implementation in current NLO tool(s)**
- interpolation of finite integrals á la Somogyi, Szor, Trocsanyi [1]
- extension to massive scheme
- combination with parton shower

! Thanks for listening !

[1] (cf Les Houches NLO working group summary report, arXiv:1203.6803)

Appendix

Maximal number of transformations

Maximal number of **momentum mappings** using
Catani Seymour or Nagy Soper scheme
counting: consider gluon-splittings only
(maximal number of mappings)

emitter, spectator	**CS**, (ij)	**CS**, k	**NS**, (ij)
fin,fin	$\binom{N'}{2}$	$(N'-2)$	$\binom{N'}{2}$
fin,ini	$\binom{N'}{2}$	2	$-$
ini,fin	$2\,N'$	$(N'-1)$	$2\,N'$
ini,ini	$2N'$	1	$-$
total $(\sum_{\text{comb's}}(ij)\times(k))$	$N'^2(N'+3)/2 = (N+1)^2(N+4)/2$		$N'(N'+3)/2 = (N+1)(N+4)/2$
$\sim$	$N^3/2$		$N^2/2$

(N' number of real emission, N number of Born type final state particles)

Dipole subtraction: Real master formula

Real Masterformula $(s = (p_a + p_b)^2)$

$$\sigma(s) = \int_m d\Phi^{(m)}(s) \frac{1}{n_c(a) n_c(b)} |\mathcal{M}^{(m)}|^2(s) F_J^{(m)}$$

$$+ \int d\Phi^{(m+1)}(s) \left\{ \frac{1}{n_c(a) n_c(b)} |\mathcal{M}^{(m+1)}|^2(s)) F_J^{(m+1)} - \sum_{\text{dipoles}} (\mathcal{D} \cdot F_J^{(m)}) \right\}$$

$$+ \int d\Phi^{(m)}(s) \left\{ \frac{1}{n_c(a) n_c(b)} |\mathcal{M}^{(m)}|^2_{1\,\text{loop}}(p_a, p_b) + \mathbf{I}(\varepsilon) |\mathcal{M}^{(m)}|^2(s) \right\}_{\varepsilon=0} F_J^{(m)}$$

$$+ \left\{ \int dx_a\, dx_b \delta(x - x_a)\, \delta(x_b - 1) \int d\Phi^{(m)}(x_a p_a, x_b p_b) |\mathcal{M}^{(m)}|^2 (x_a p_a, x_b p_b) \right.$$

$$\left. \times \left(\mathbf{K}^{a,a'}(x) + \mathbf{P}^{a,a'}(x_a p_a, x_b p_b, x; \mu_F^2) \right) \right\} + (a \leftrightarrow b)$$

where all colour/ phase space factors have been accounted for

Second ingredient: Parametrization of integration variables

- again: remember you have

$$F_{\text{sing}} \propto D_{ij},\ \widetilde{F}_{sing} = \int d\Gamma_1\, D_{ij},\ d\Gamma_1 \propto d^4 p_j\, \delta(p_j^2)$$

$$\Longrightarrow \widetilde{F}_{sing} \propto \int d^4 p_j\, \delta(p_j^2)\, D_{ij}$$

- 3 free variables (in D dimensions: $D-1$)

  !! need to be written in terms of m particle variables !!
- now all ingredients:
  total energy momentum conservation, onshellness of external particles, need for integration variables

Integrated Dipoles in more details: I, K, P (1)

$m+1$ **phase space: in principle easy**

$$\int d\Gamma_{m+1}\,\left(|\mathcal{M}_{\text{real}}|^2 - \sum D\right),\quad \text{finite}$$

m **particle phase space: more complicated**

need integration variables (emission from p_1):

$$x\,=\,1-\frac{p_4(p_1+p_2)}{p_1 p_2}\ \text{softness},\ \tilde{v}\,=\,\frac{p_1 p_4}{p_1 p_2}\ \text{collinearity}$$

Integrated Dipoles in more details: I, K, P (2)

- in principle, obtain $\int d\Gamma_1 \, D = \int_0^1 dx \left(\mathbf{I}(\varepsilon) + \tilde{\mathbf{K}}(x,\varepsilon) \right)$
- $\mathbf{I}(\varepsilon) \propto \delta(1-x)$: corresponds to loop part
- $\tilde{\mathbf{K}}(x,\varepsilon)$ contains finite parts as well as **collinear singularities**
- latter need to be cancelled by adding **collinear counterterm**

$$\frac{1}{\epsilon} \left(\frac{4\pi\mu^2}{\mu_F^2} \right)^{\epsilon} P^{qq}(x)$$

  depends on factorization scale μ_F ($P^{qq}(x)$ splitting function)
- PDFs come in again: term already accounted for by folding w PDF, needs to be subtracted
- for $q\,g \rightarrow W\,q$ like processes, only singularity which appears

$q \longrightarrow q\,g$ for initial state quarks: Catani Seymour (1)

- $q(\tilde{p}_1) \longrightarrow q(p_1) + g(p_4)$, q enters hard interaction
- Dipole:

$$D^{14,2} = -\frac{8\,\pi\,\mu^2\,\alpha_s\,C_F}{s+t+u}\left(\frac{2\,s\,(s+t+u)}{t\,(t+u)} + (1-\varepsilon)\,\frac{t+u}{t}\right)$$

- matching ($\tilde{p}_2 = p_2$)

$$\tilde{p}_1 = x\,p_1, \quad x = 1 - \frac{p_4\,(p_1+p_2)}{(p_1\,p_2)}$$

$$\tilde{p}_k^{\mu} = \Lambda^{\mu}{}_{\nu}\,p_k^{\nu}, \quad (k\text{: final state particles})$$

$$\Lambda^{\mu\nu} = -g^{\mu\nu} - \frac{2\,(K+\widetilde{K})^{\mu}(K+\widetilde{K})^{\nu}}{(K+\widetilde{K})^2} + \frac{2\,K^{\mu}\widetilde{K}^{\nu}}{K^2}$$

$$K = p_1 + p_2 - p_4, \quad \widetilde{K} = \tilde{p}_1 + p_2$$

$q \longrightarrow q\,g$ for initial state quarks: Catani Seymour (2)

- integration variables:

$$v \;=\; \frac{p_1 p_4}{p_1 p_2},\; x \;=\; 1 - \frac{p_4\,(p_1 + p_2)}{(p_1 p_2)}$$

- in p_1, p_2 cm system: $E_4 \longrightarrow 0 \;\Rightarrow\; x \longrightarrow 1$ (softness)
  $\cos\theta_{14} \longrightarrow 1 \;\Rightarrow\; v \longrightarrow 0$ (collinearity)
- Dipole in terms of integration variables

$$D^{14,2} \;=\; -\frac{8\,\pi\,\alpha_s\,C_F}{v\,x\,s}\left(\frac{1+x^2}{1-x} - \varepsilon(1-x)\right)$$

- integration measure

$$[dp_j] \;=\; \frac{(2\,p_1 p_2)^{1-\varepsilon}}{16\,\pi^2}\,\frac{d\Omega_{d-3}}{(2\,\pi)^{1-\varepsilon}}\,dv\,dx\,(1-x)^{-2\,\varepsilon}\left[\frac{v}{1-x}\left(1-\frac{v}{1-x}\right)\right]^{-\varepsilon}$$

where $v \leq 1 - x$ and all integrals between 0 and 1

$q \longrightarrow q\,g$ for initial state quarks: Catani Seymour (3)

- result

$$\mu^{2\varepsilon}\int [dp_j]\, D^{14,2} = \frac{\alpha_s}{2\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\,C_F\,\left(\frac{2\mu^2\pi}{p_1 p_2}\right)^{\varepsilon}$$

$$\times\int_0^1 dx\,\left(\mathbf{I}(\varepsilon)\delta(1-x) + \tilde{\mathbf{K}}(x,\varepsilon)\,\underbrace{-\frac{1}{\varepsilon}P^{qq}(x)}_{\text{killed by coll CT}}\right)$$

with

$$\mathbf{I}(\epsilon) = \frac{1}{\epsilon^2}+\frac{3}{2\epsilon}-\frac{\pi^2}{6}$$

$$\mathbf{K}(x) = (1-x)-2\,(1+x)\ln(1-x)+\left(\frac{4}{1-x}\ln(1-x)\right)_+$$

$$P^{qq}(x) = \left(\frac{1+x^2}{1-x}\right)_+ \quad \text{regularized splitting function}$$

$q \longrightarrow q\,g$ for initial state quarks: Nagy Soper (1)

- $q(\tilde{p}_1) \longrightarrow q(p_1) + g(p_4)$, q enters hard interaction
- Dipole:

$$D^{14,2} = -\frac{8\,\pi\,\mu^2\,\alpha_s\,C_F}{s+t+u}\left(\frac{2\,s\,u\,(s+t+u)}{t\,(t^2+u^2)} + (1-\varepsilon)\,\frac{u}{t}\right)$$

as CS, same pole structure as CS

- matching, integration variables, integration measure: as Catani Seymour($v \leftrightarrow y$)
- Dipole in terms of integration variables

$$D^{14,2} = -\frac{8\pi\alpha_s\,C_F}{x\,s} \times \left(\frac{1-x-y}{y}(1-\varepsilon) + \frac{2\,x}{y\,(1-x)} - \frac{2\,x\,[2\,y-(1-x)]}{(1-x)[y^2+(1-x-y)^2]}\right)$$

$q \longrightarrow q\,g$ for initial state quarks: Nagy Soper (2)

- result

$$\mu^{2\varepsilon}\int [dp_j]\, D^{14,2} = \frac{\alpha_s}{2\,\pi}\,\frac{1}{\Gamma(1-\varepsilon)}\,C_F\,\left(\frac{2\,\mu^2\pi}{p_1 p_2}\right)^{\varepsilon}$$

$$\times \int_0^1 dx\,\left(\mathbf{I}(\varepsilon)\delta(1-x) + \tilde{\mathbf{K}}(x,\varepsilon)\;\underbrace{-\frac{1}{\varepsilon}P^{qq}(x)}_{\text{killed by coll CT}}\right)$$

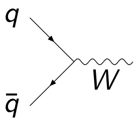
with

$$\mathbf{K}(x) =$$

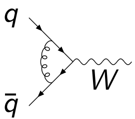
$$(1-x) - 2\,(1+x)\ln(1-x) + \left(\frac{4}{1-x}\ln(1-x)\right)_+ -(1-x)$$

- equivalence of dipoles schemes checked analytically

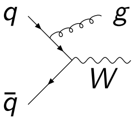
Single W production (slide by C. Chung)

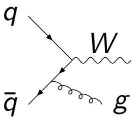


Tree level: $q\bar{q} \rightarrow W$

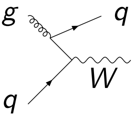


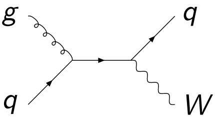
Virtual corrections: $q\bar{q} \rightarrow W$





Real corrections: $q\bar{q} \rightarrow Wg$





$gq \rightarrow Wq$ (+ 2 more diagrams)

$$\frac{1}{4}\frac{1}{9}|\mathcal{M}_B|^2 = \frac{g^2}{12}|V_{qq'}|^2 M_W^2, \quad \frac{1}{4}\frac{1}{9}\sum|\mathcal{M}_R|^2 = \frac{8g^2\pi\alpha_s}{9}|V_{qq'}|^2\frac{\hat{t}^2+\hat{u}^2+2M_W^2\hat{s}}{\hat{t}\hat{u}}$$

$$|\mathcal{M}_V|^2 = |\mathcal{M}_B|^2 \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\left\{-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon)\right\}$$

Subtraction terms: Nagy Soper vs Catani Seymour

NS, CS-NS, CS= NS+CS-NS

- **2 particle phase space (real emission)**

$$\mathcal{D}^{14,2} + \mathcal{D}^{24,1} = \underbrace{\frac{1}{4}\frac{1}{9}\sum|\mathcal{M}_{\text{real}}|^2}_{\text{singular}} + \underbrace{\frac{16}{9}g^2\,\alpha_s\,\pi}_{\text{finite}}$$

- **1 particle phase space (virtual contribution)**

$$\mathbf{I}(\epsilon)|\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi}\frac{1}{\Gamma(1-\epsilon)}\left(-8+\frac{2}{3}\pi^2\right)|\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}}_v|^2}_{\text{singular (+finite)}}$$

$$\begin{aligned}\mathbf{K}^a(xp_a) = \; & \frac{\alpha_s}{2\pi}C_F\frac{1}{\Gamma(1-\epsilon)}\Big[-(1-x)\ln x + 2(1-x)\ln(1-x) \\ & +4x\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{2x\ln x}{(1-x)_+} - \left(\frac{1+x^2}{1-x}\right)_+ \ln\left(\frac{4\pi\mu^2}{2xp_a\cdot p_b}\right) \\ & +(1-x)\Big]\end{aligned}$$

compare to Nagy Soper :

pole structure the same, finite terms differ ✓

Subtraction terms: Nagy Soper vs Catani Seymour

NS, CS-NS, CS= NS+CS-NS

- 2 **particle phase space (real emission)**

$$\mathcal{D}^{14,2}+\mathcal{D}^{24,1} = \underbrace{\frac{1}{4}\frac{1}{9}\sum|\mathcal{M}_{\text{real}}|^2}_{\text{singular}} + \underbrace{\frac{16}{9}g^2\,\alpha_s\,\pi}_{\text{finite}}$$

- 1 **particle phase space (virtual contribution)**

$$\mathbf{I}(\epsilon)|\mathcal{M}_b|^2 = \underbrace{\frac{2\alpha_s}{3\pi}\frac{1}{\Gamma(1-\epsilon)}(-8+\frac{2}{3}\pi^2)|\mathcal{M}_b|^2}_{\text{finite}} - \underbrace{|\widetilde{\mathcal{M}_v}|^2}_{\text{singular (+finite)}}$$

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Subtraction terms: Nagy Soper vs Catani Seymour

NS, CS-NS, CS= NS+CS-NS

- **2 particle phase space (real emission)**

$$\mathcal{D}^{14,2} + \mathcal{D}^{24,1} = \underbrace{\frac{1}{4}\frac{1}{9}\sum |\mathcal{M}_{\text{real}}|^2}_{\text{singular}} + \underbrace{\frac{16}{9} g^2 \alpha_s \pi}_{\text{finite}}$$

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$$\mathbf{K}^a(xp_a) = \frac{\alpha_s}{2\pi} C_F \frac{1}{\Gamma(1-\epsilon)} \Big[-(1-x)\ln x + 2(1-x)\ln(1-x) + 4x\left(\frac{\ln(1-x)}{1-x}\right)_+ - \frac{2x\ln x}{(1-x)_+} - \left(\frac{1+x^2}{1-x}\right)_+ \ln\left(\frac{4\pi\mu^2}{2xp_a\cdot p_b}\right) + (1-x)\Big]$$

compare to Nagy Soper :

pole structure the same, finite terms differ ✓

Subtraction terms: Nagy Soper vs Catani Seymour

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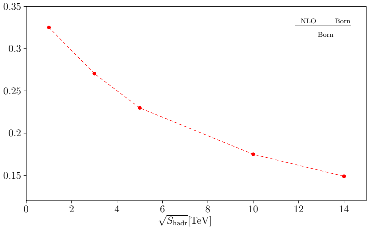
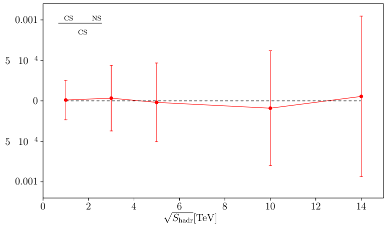
compare to Nagy Soper :
pole structure the same, finite terms differ ✓

Numerical results for single W (slide by C. Chung)

input: $M_W = 80.35$ GeV, PDF $\Rightarrow$ cteq6m, $\alpha_s(M_W) = 0.120299$



$\frac{\sigma_{NLO}-\sigma_{LO}}{\sigma_{LO}}$ as a function of $\sqrt{S_{\rm hadr}}$
corrections up to 30%

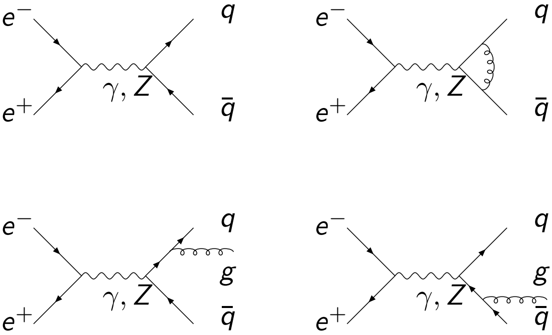


relative difference between CS and NS: $\frac{\sigma_{CS}-\sigma_{NS}}{\sigma_{CS}}$
agree on the sub-permill level ✓

difference between schemes:
subtraction in m particle phase space, $\mathbf{K}(x)$ terms

pole structure the same, finite terms shifted around ✓

Applications: $e^+e^- \longrightarrow 2\,\text{jets}\ (1)$ (slide by C.Chung)



Tree level diagram:
$e^+e^- \rightarrow q(p_1) + \bar{q}(p_2)$

Virtual corrections:
$e^+e^- \rightarrow q(p_1) + \bar{q}(p_2)$

Real corrections:
$e^+e^- \rightarrow$
$q(p_1) + \bar{q}(p_2) + g(p_3)$

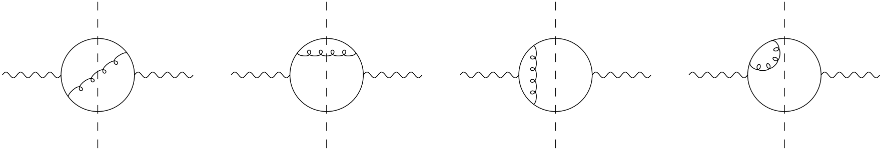
The matrix element for NLO real emission (three particle ps):

$$|\,\mathcal{M}_3(p_1,p_2,p_3)\,|^2 = \ C_F \frac{8\pi\alpha_s}{Q^2} \frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}\,|\,\mathcal{M}_2\,|^2,\ \ x_i = \frac{2p_i\cdot Q}{Q^2}$$

($\mathcal{M}_2,\ \mathcal{M}_3$ averaged over angles)

soft/ collinear singularities from $x_i \rightarrow 1$

Applications: $e^+e^- \longrightarrow 2\,\text{jets}$ (2) (slide by C. Chung)



2 dipole contributions $\mathcal{D}_1$ and $\mathcal{D}_2$ (in 3 particle ps):

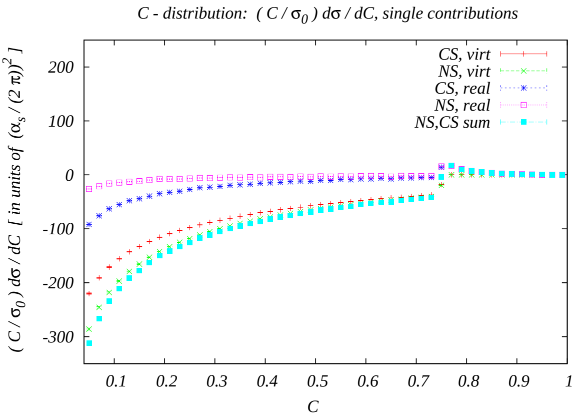
$$\mathcal{D}_1 = v_{qqg}^2 - v_{\text{soft}}^2 = \left(v_{qqg}^2 - v_{\text{eik}}^2\right) + \left(v_{\text{eik}}^2 - v_{\text{soft}}^2\right)$$
$$= \frac{4}{\hat{Q}^2}\left\{\left(\frac{1}{x_2}\right)\left[2\left(\frac{x_1}{2-x_1-x_2} - \frac{1-x_2}{(2-x_1-x_2)^2}\right) + \frac{1-x_1}{1-x_2}\right]\right.$$
$$\left. +2\left(\frac{x_1+x_2-1}{1-x_2}\right)\frac{x_1}{(1-x_1)x_1+(1-x_2)x_2}\right\}$$

Integration over dipole

$$2\left(\frac{4\pi\alpha_s}{2}\right)\mu^{2\epsilon}C_F\int d\zeta_p\mathcal{D}_1 = \frac{\alpha_s}{2\pi}C_F\frac{1}{\Gamma(1-\epsilon)}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-2+\frac{\pi^2}{3}\right)$$

$$\sigma^{NLO} = \sigma^{NLO\,\{2\}} + \sigma^{NLO\,\{3\}} = \frac{3}{4}\frac{\alpha_s}{\pi}C_F\sigma^{LO} \quad (\checkmark)$$

$e\, e \longrightarrow q\, \bar{q}\, q\, \bar{q}$: single components



real and virtual contributions from CS and NS dipoles respectively as well as sum