

BR($\bar{B} \rightarrow X_s \gamma$) in 2HDMs to NNLO

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Outline

1 Introduction

2 BR($\bar{B} \rightarrow X_s \gamma$) in the SM

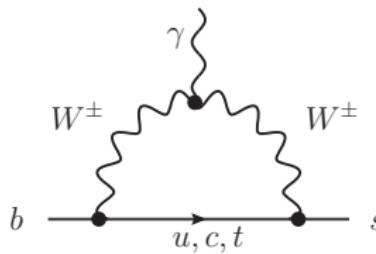
3 BR($\bar{B} \rightarrow X_s \gamma$) in 2HDM

- Two Higgs doublet models
 - Calculation of C_7 and C_8 to 3L
 - 2HDM Type II
 - Lower bound of M_{H^\pm}

4 Conclusion

Introduction $\bar{B} \rightarrow X_s \gamma$

$$\Gamma(\bar{B} \rightarrow X_s \gamma) |_{E_\gamma > E_0} = \Gamma(b \rightarrow X_s^p \gamma) |_{E_\gamma > E_0} + \left(\begin{array}{l} \text{non-perturbative contributions} \\ \sim \pm 5\% \text{ [Benzke et al. 2010]} \end{array} \right)$$



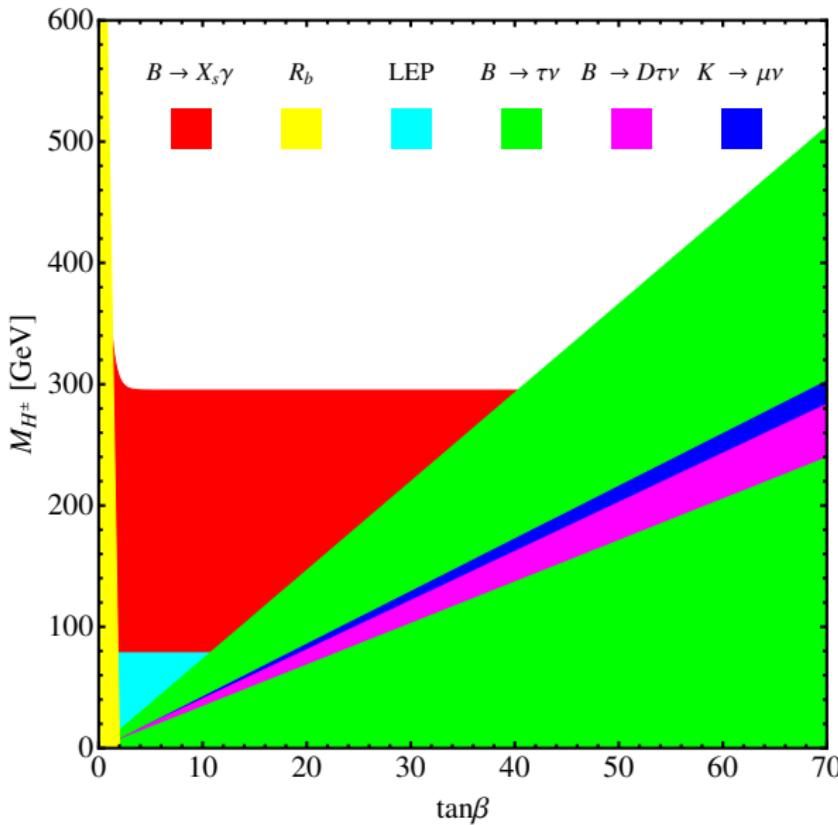
Exp. world average:

$$\text{BR}(\bar{B} \rightarrow X_s \gamma) |_{E_\gamma > 1.6 \text{ GeV}} = \begin{cases} (3.55 \pm 0.24 \pm 0.09) \cdot 10^{-4} & [\text{HFAG 2010}] \\ (3.50 \pm 0.17) \cdot 10^{-4} & [\text{Artuso et al. 2009}] \end{cases}$$

SM NNLO prediction:

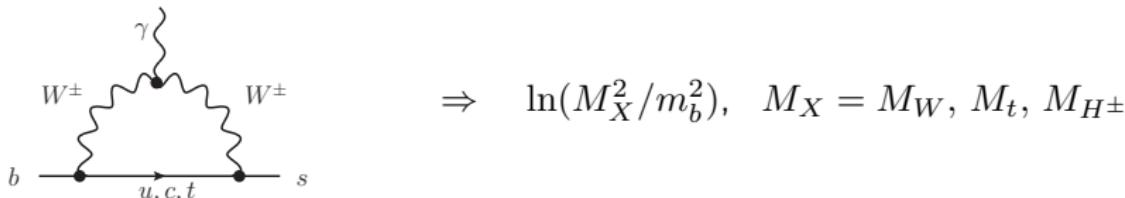
$$\text{BR}(\bar{B} \rightarrow X_s \gamma) |_{E_\gamma > 1.6 \text{ GeV}} = \begin{cases} (3.15 \pm 0.23) \cdot 10^{-4} & [\text{Misiak et al. 2006}] \\ (3.28 \pm 0.25) \cdot 10^{-4} & [\text{Gambino, Giordano 2008}] \end{cases}$$

Bounds on M_{H^+} in 2HDM Type II



Ulrich Haisch
arXiv:0805.2141

Effective theory



Resummation of large logarithms \Rightarrow effective theory approach
integrating out heavy particles: W^\pm, t, H^\pm

$$\mathcal{L} = \mathcal{L}_{\text{QCD} \times \text{QED}}(u, d, s, c, b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^8 C_i Q_i$$

- Q_i dimension 5 and 6 operators
 - $C_i(\mu)$ Wilson coefficients

Electroweak-scale physics in Wilson coefficients C_i

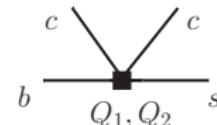
Effective theory

current-current operators:

$$Q_1 = (\bar{s}_L \gamma_\mu T^a c_L) (\bar{c}_L \gamma^\mu T^a b_L)$$

$$Q_2 = (\bar{s}_L \gamma_\mu c_L) (\bar{c}_L \gamma^\mu b_L)$$

[Chetyrkin, Misiak, Muenz 1998]



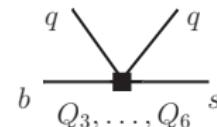
QCD penguin operators:

$$Q_3 = (\bar{s}_L \gamma_\mu b_L) \sum (\bar{q} \gamma^\mu q)$$

$$Q_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum (\bar{q} \gamma^\mu T^a q)$$

$$Q_5 = (\bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \sum (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q)$$

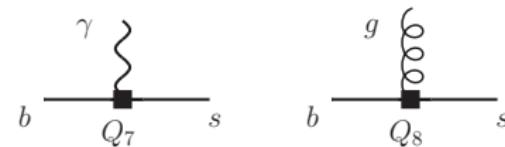
$$Q_6 = (\bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \sum_i (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q)$$



dipole operators:

$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

$$Q_8 = \frac{g}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G_{\mu\nu}^a$$



Calculations in the effective theory

Calculation:

① Matching:

Calculation of $C_i(\mu_0)$ at the matching scale
 $\mu_0 \sim M_W, M_t, \dots$

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② Mixing (RGE):

Q_i mix under renormalization
Calculation of anomalous dimension matrix of Q_i

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ji} C_j(\mu) \quad \Rightarrow \quad C_i(\mu_B) = U_{ij}(\mu_B, \mu_0) C_j(\mu_0)$$

$\mu_B \sim m_b$, $\eta = \alpha_s(\mu_0)/\alpha_s(\mu_B)$
anomalous dimensions to 3L and 4L

[Gorbahn, Haisch, Misiak 2005]
[Czakon, Haisch, Misiak 2007]

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③ On-shell matrix elements:

Calculation of the on-shell matrix elements in the effective theory

$\text{BR}(\overline{B} \rightarrow X_s \gamma)$ in the SM

$$\Gamma(b \rightarrow X_s^p \gamma) |_{E_\gamma > E_0} = \frac{G_F^2 m_b^5 \alpha_{em}}{32\pi^4} |V_{ts}^\star V_{tb}|^2 \sum_{i,j=1}^8 C_i(\mu_B) C_j(\mu_B) G_{ij}(E_0, \mu_B)$$

Matrix elements G_{ij} :

LO: $G_{ij} = \delta_{i7}\delta_{j7} + (\text{small tree-level})$

NLO: G_{ij} complete [Ali, Buras, Czarnecki, Greub, Hurth, Misiak, Pott, Urban, Wyler 1991-2002]

NNLO:

- G_{77} fully known [Blokland, Czarnecki, Misiak, Slusarczyk, Tkachov 2005]
 [Melnikov, Mitov 2005]
 [Asatrian et al 2006-2007]
 - G_{78} fully known [Asatrian et al. 2010]
 - G_{ij} for $i, j \in (1, 2, 7, 8)$ partly known (BLM approx. and more)
 [Bieri, Greub, Steinhauser 2003]
 [Ligeti, Luke, Manohar, Wise 1999]
 [Asatrian, Boughezal, Czakon, Ewerth, Ferroglia, Gabrielyan, Greub, Haisch, Misiak, Poradzinski, Schutzmeier 2007-2011]
 - beyond BLM: $m_c \gg m_b/2$ limit, interpolation to physical value m_c
 \Rightarrow uncertainty of 3% to BR [Misiak, Steinhauser 2006]

Electroweak contributions NLO:

[Gambino, Haisch 2001]

Non-perturbative power corrections:

- [Bigi et al. 1992]
- [Falk, Luke, Savage 1993]
- [Voloshin 1996]
- [Buchalla, Isidori, Rey 1997]
- [Bauer 1997]
- [Gambino, Ewerth, Nandi 2009]
- [Benzke et al. 2010]

Normalization:

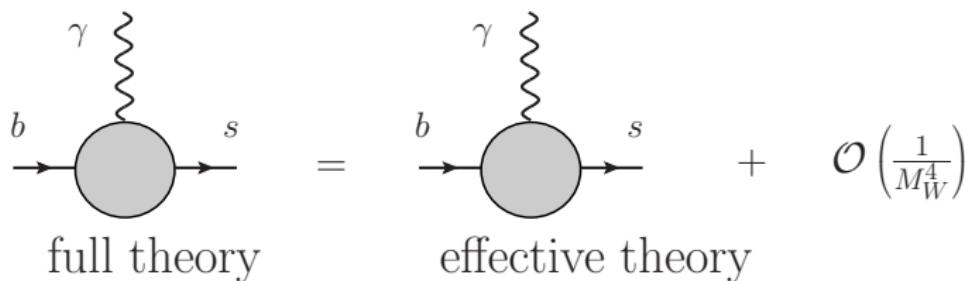
$$\text{BR}(\overline{B} \rightarrow X_s \gamma)|_{E_\gamma > E_0} = \frac{\text{BR}(\overline{B} \rightarrow X_c e \bar{\nu})_{\text{exp}}}{C_{\text{fit}}} \left(\frac{\Gamma(\overline{B} \rightarrow X_s \gamma)|_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2 \Gamma(\overline{B} \rightarrow X_u e \bar{\nu})} \right)_{\text{th}}$$

semileptonic phase space ratio

$$C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma(\bar{B} \rightarrow X_c e \bar{\nu})}{\Gamma(\bar{B} \rightarrow X_u e \bar{\nu})}$$

Matching

Matching:



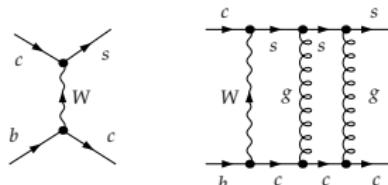
Off-shell matching:

- off-shell 1LPI amputated Green functions
 - Taylor expansion to second order in external momenta \Rightarrow tadpole diagrams
 - loop diagrams vanish in effective theory (massless tadpoles)

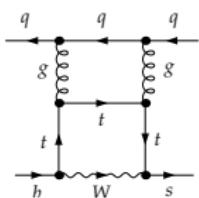
Wilson coefficients in the SM

2 loop Wilson coefficients:

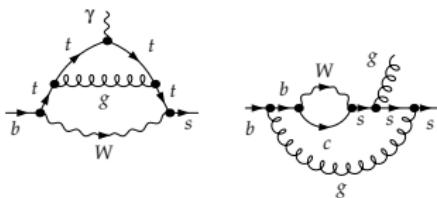
[Bobeth, Misiak, Urban 2000]



$\Rightarrow C_1^c, C_2^c$



$$\Rightarrow C_3^t, \dots, C_6^t$$



$\Rightarrow C_7^t, C_8^c$

subtleties: evanescent operators, non-physical operators off-shell

Wilson coefficients in the SM

3 loop Wilson coefficients:

[Misiak, Steinhäuser 2004]



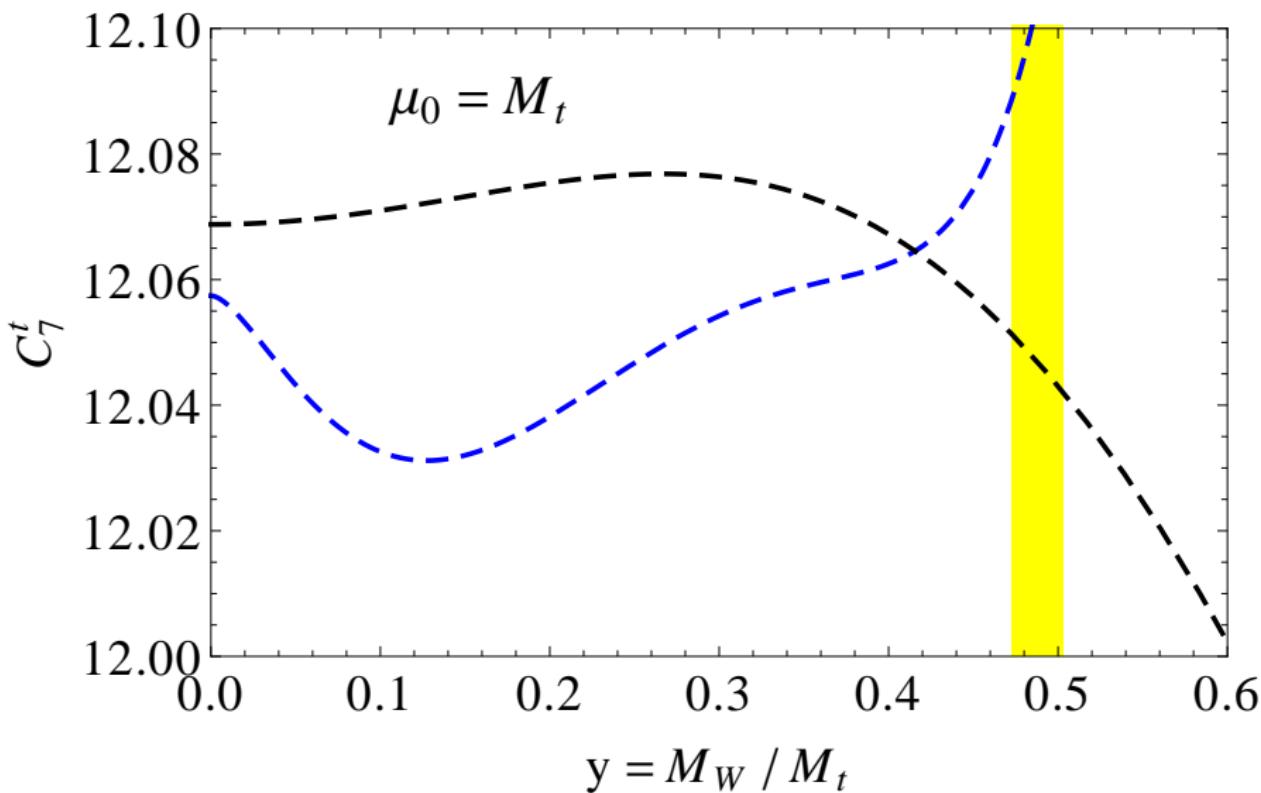
- ① $M_t \gg M_W$: asymptotic expansion $(M_W/M_t)^8$
 - ② $M_t \approx M_W$: ordinary Taylor expansion $(M_W^2 - M_t^2)^8$

improved calculation:

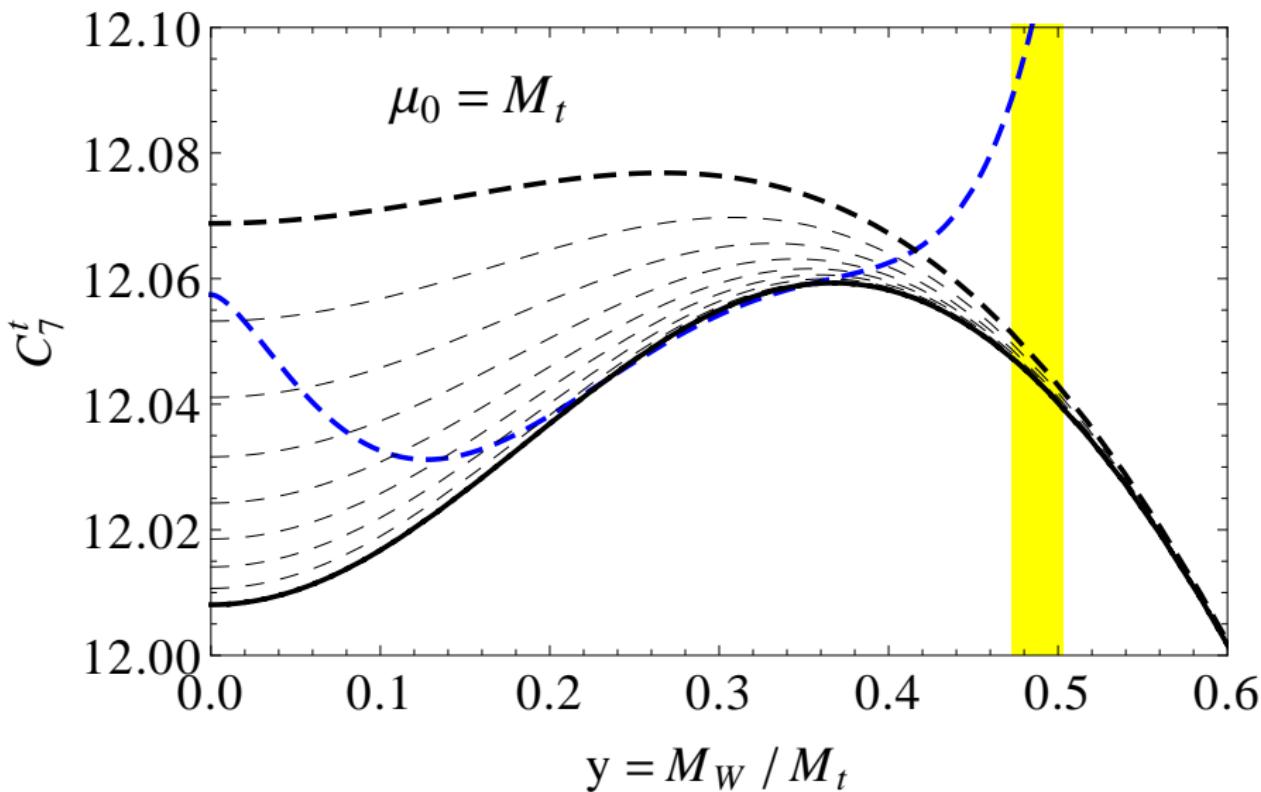
[TH, Misiak, Steinhauser]

$M_t \approx M_W$: ordinary Taylor expansion $(M_W^2 - M_t^2)^{16}$

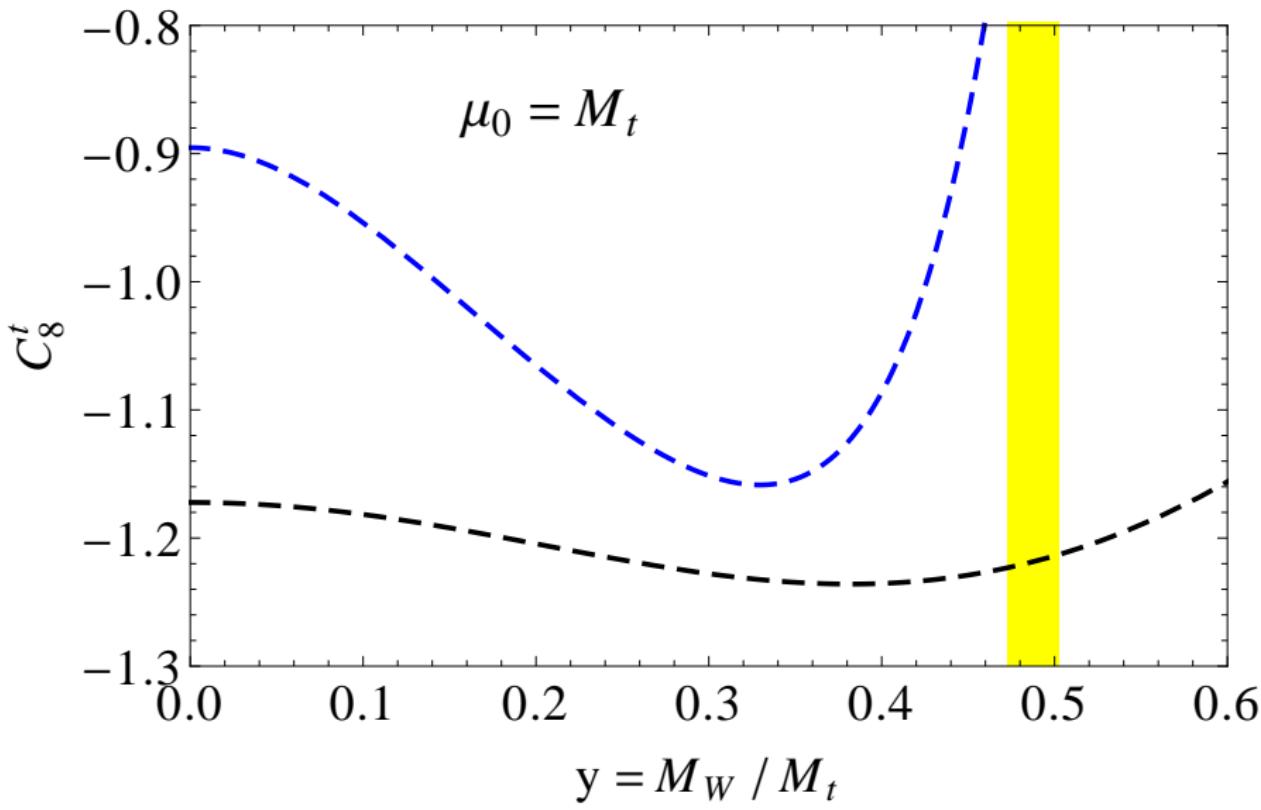
$C_7(\mu_0 = M_t)$ 3L in the SM, 2004



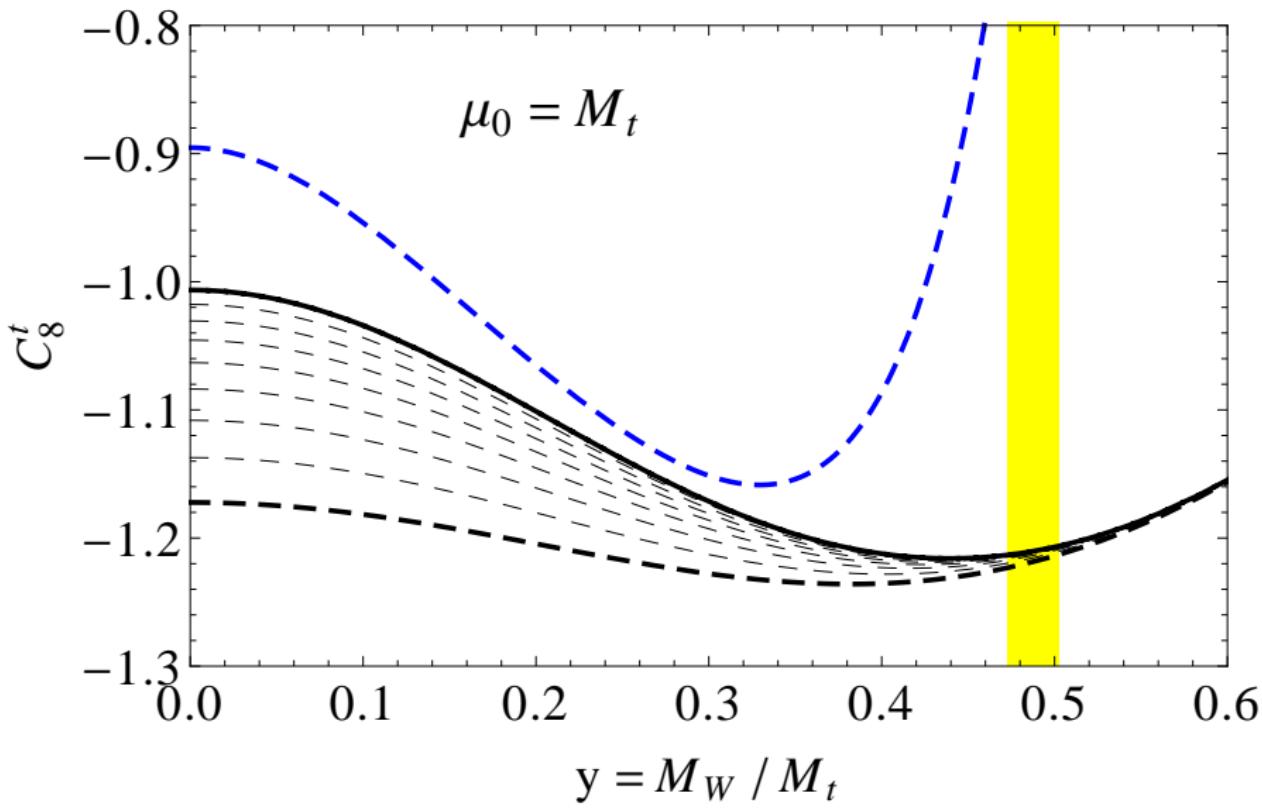
$C_7(\mu_0 = M_t)$ 3L in the SM, 2012



$C_8(\mu_0 = M_t)$ 3L in the SM, 2004



$C_8(\mu_0 = M_t)$ 3L in the SM, 2012



Two Higgs doublet models

- physical basis: h , H , A and H^\pm
 - interaction between charged Higgs H^\pm and quarks:

$$\mathcal{L} = (2\sqrt{2}G_F)^{1/2} \sum_{i,j=1}^3 \bar{u}_i (A_u m_{u_i} V_{ij} P_L - A_d m_{d_j} V_{ij} P_R) d_j H^+ + h.c.$$

2HDM Type I:

$$A_u = A_d = \frac{1}{\tan \beta}$$

2HDM Type II (e.g. MSSM):

$$A_u = -\frac{1}{A_d} = \frac{1}{\tan \beta}$$

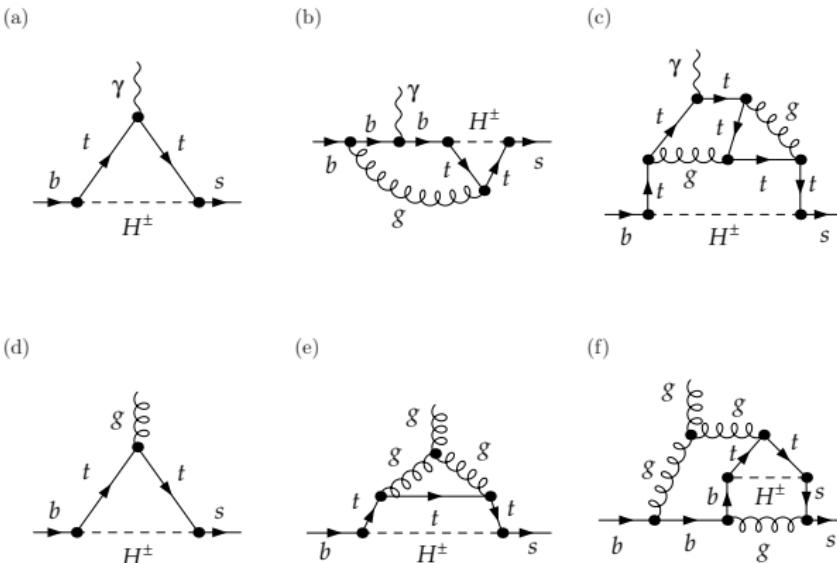
with $\tan \beta = \frac{\langle \phi_2^0 \rangle}{\langle \phi_1^0 \rangle}$

Wilson coefficients: $C_i = A_d A_u^* \dots + A_u A_u^* \dots$

Calculation of C_7 and C_8 to 3L

C_7 : amputated 1PI Green function $b \rightarrow s\gamma$

C_8 : amputated 1PI Green function $b \rightarrow s g$



number of diagrams C_7 (C_8) 3L: 350 (500)

Calculation of C_7 and C_8 to 3L

NLO:

Wilson coefficients to NLO in 2HDM [Ciafaloni, Romanino and Strumia 1997]

[Ciuchini, Degrassi, Gambino and Giudice 1997]

[Borzumati and Greub 1998]

$C_3^{(2)}, \dots, C_6^{(2)}$: in MSSM

[Bobeth, Buras, Ewerth 2005]

NNLO:

[TH, Misiak, Steinhauser]

Three-loop vacuum integrals with two different mass scales: M_H and M_t

Three different mass hierarchies:

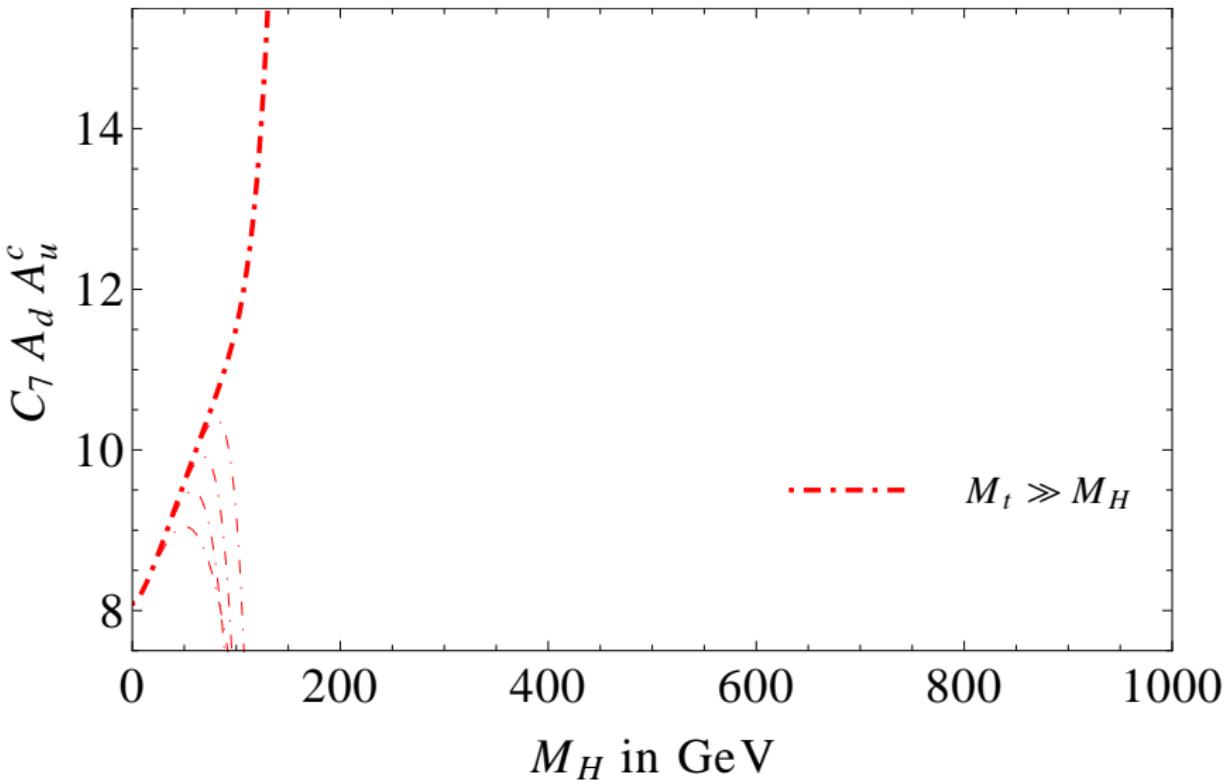
- ① $M_H \gg M_t$: asymptotic expansion $(M_t/M_H)^{10}$
 - ② $M_H \approx M_t$: ordinary Taylor expansion $(M_H^2 - M_t^2)^{16}$
 - ③ $M_H \ll M_t$: asymptotic expansion $(M_H/M_t)^{10}$

Asymptotic expansion: Q2E/EXP

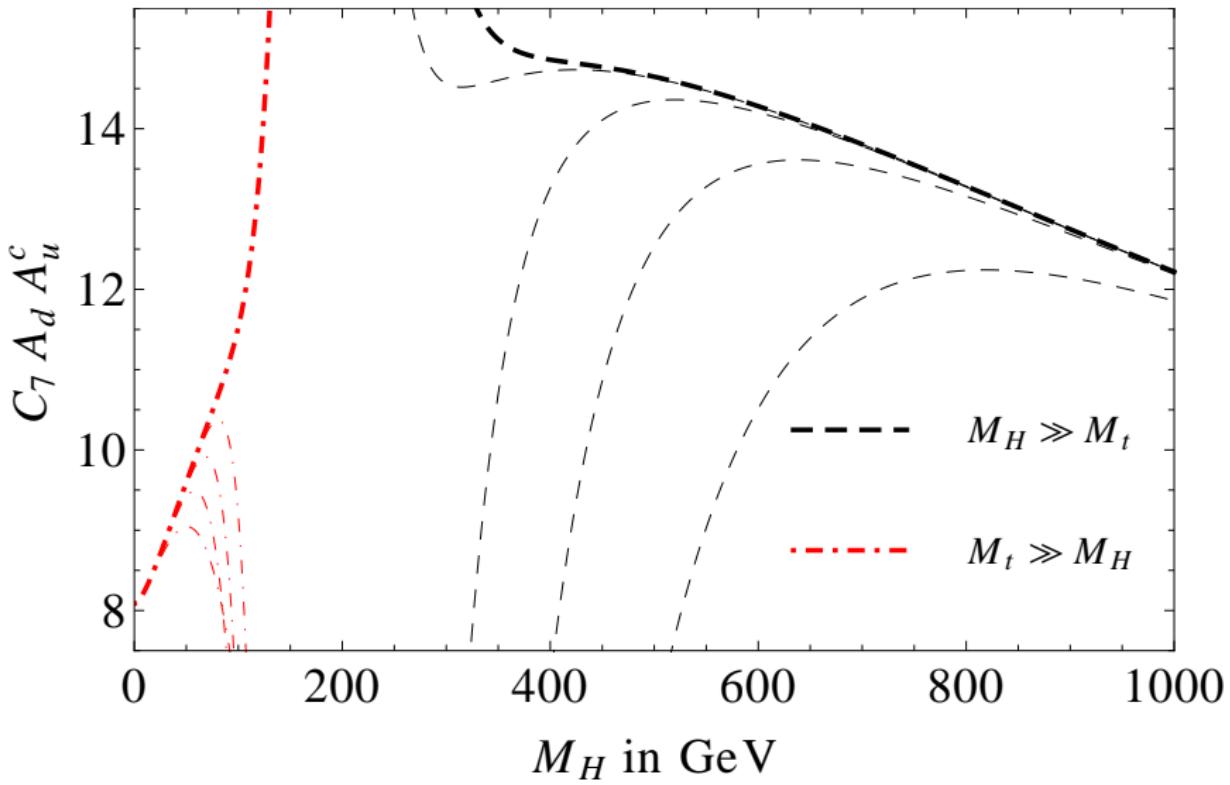
[Harlander, Seidensticker, Steinhauser 1998 and Seidensticker 1999]

Three-loop massive tadpoles: MATAD [Steinhauser 2001]

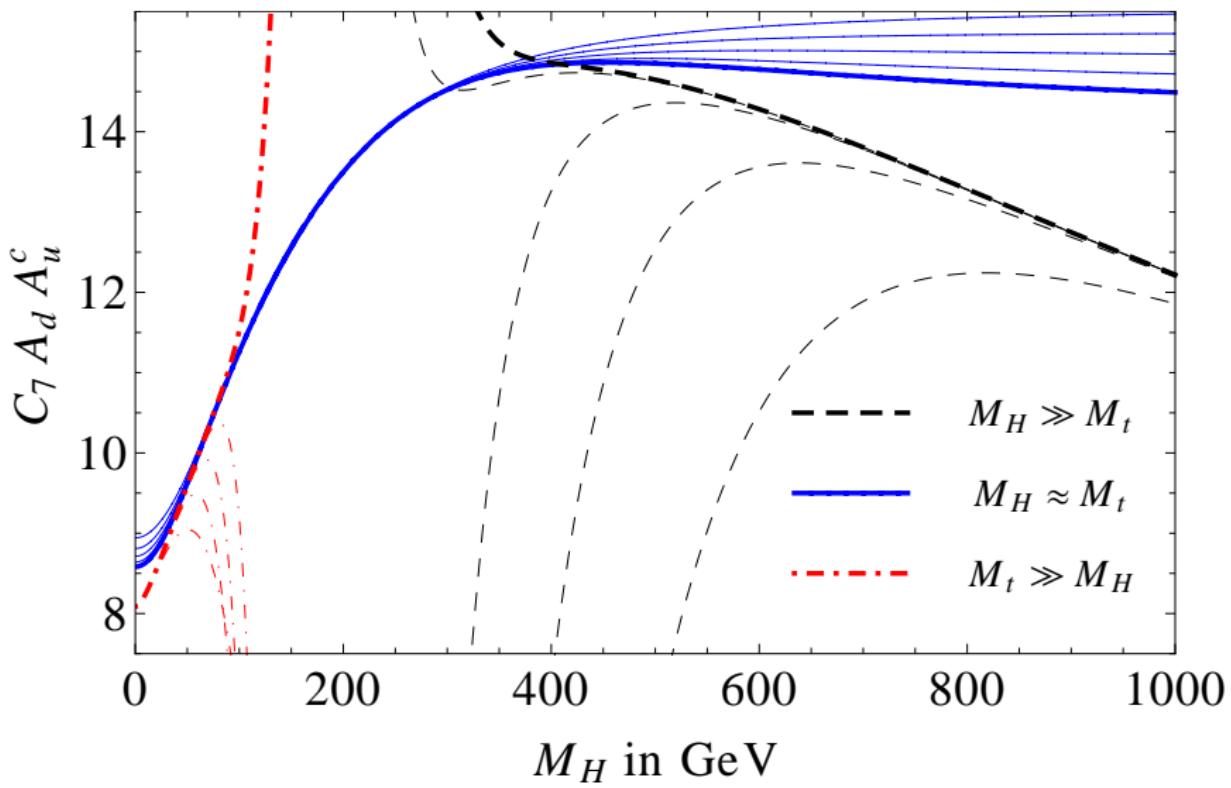
Results for C_7 : $A_d A_u^*$



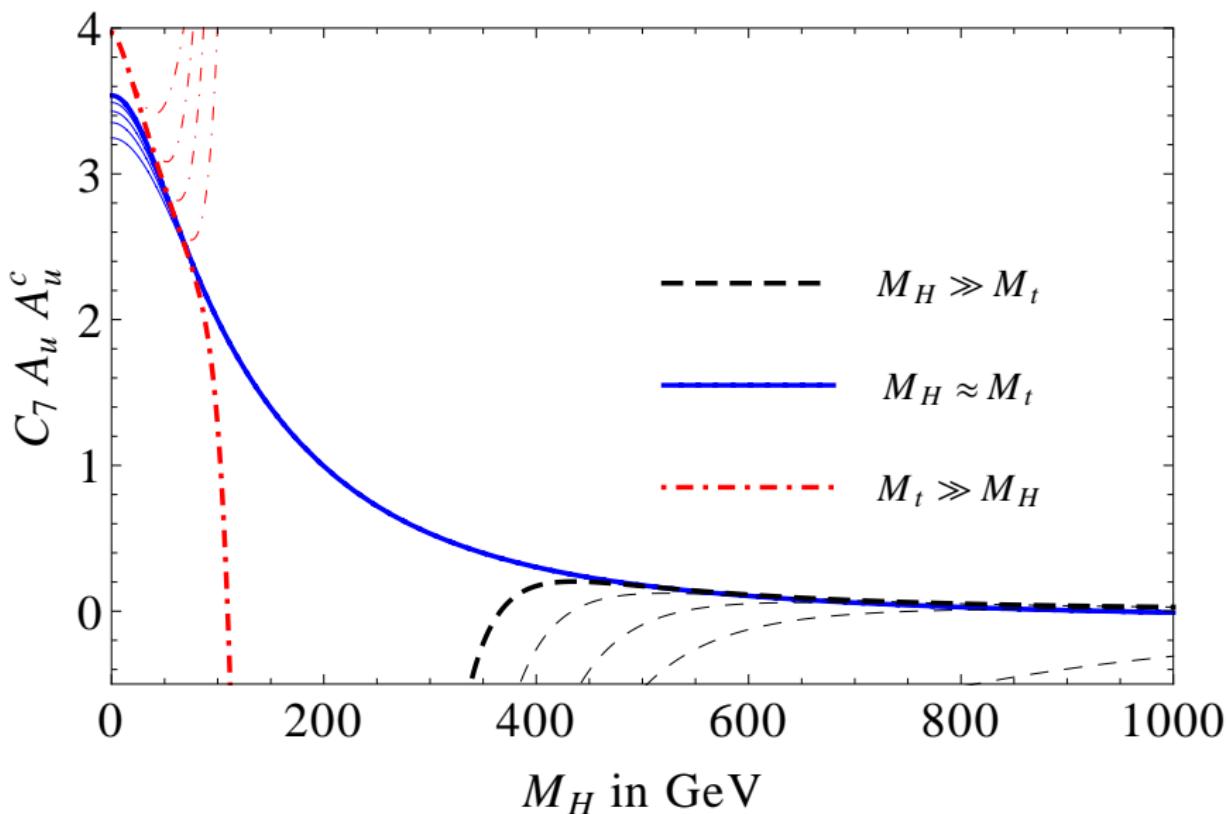
Results for C_7 : $A_d A_u^*$



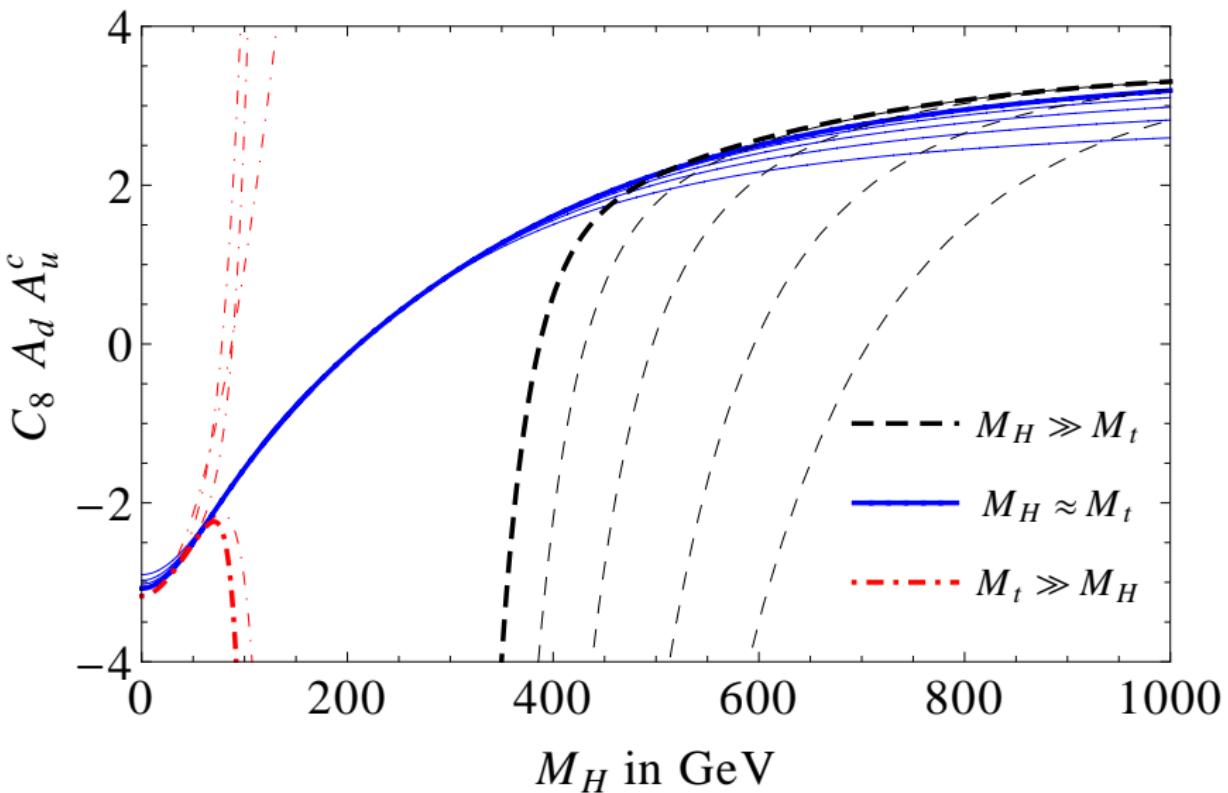
Results for C_7 : $A_d A_u^*$

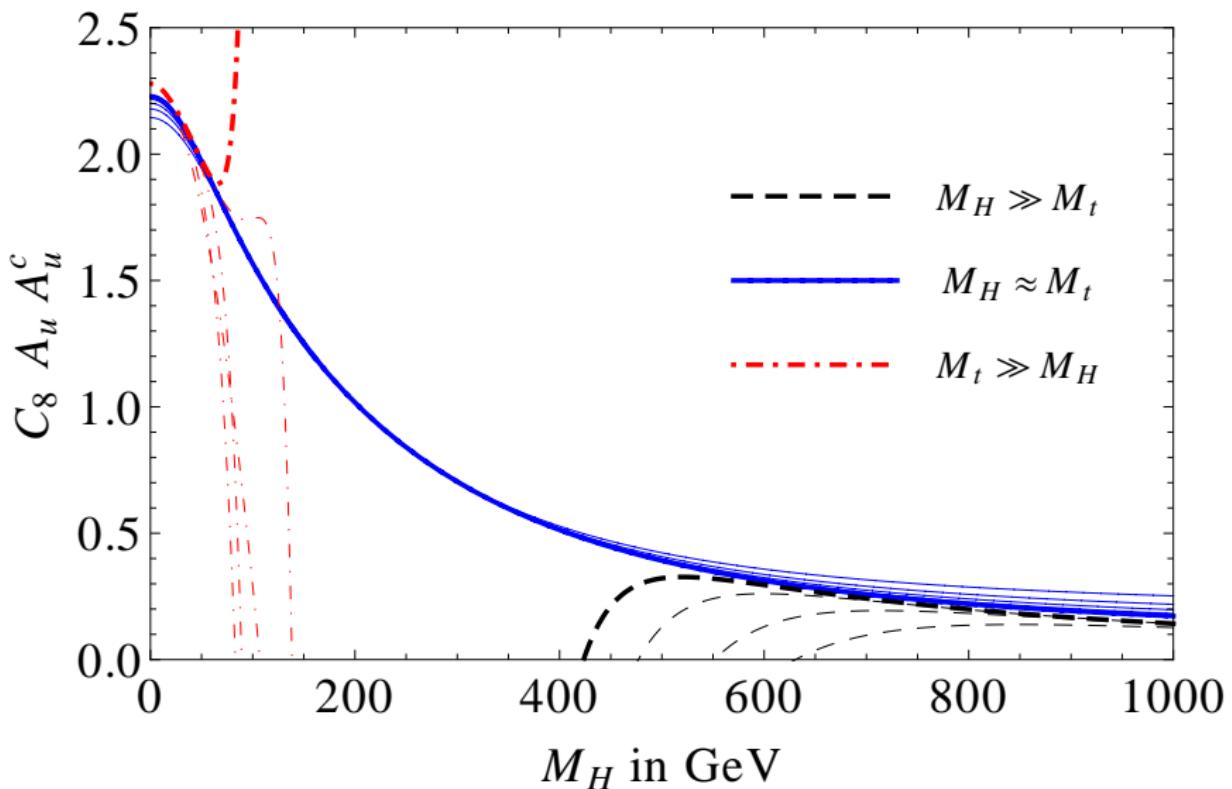


Results for C_7 : $A_u A_u^*$



Results for C_8 : $A_d A_u^*$



Results for $C_8: A_u A_u^*$ 

Branching ratio in 2HDMs

Calculation of branching ratio to NNLO

[Misiak, Steinhauser 2006]

Parameters:

$$\text{BR}(B \rightarrow X_c e \bar{\nu})_{\text{exp}} = 0.1061 \pm 0.0017$$

$$C = 0.580 \pm 0.016$$

$$|V_{ts}^* V_{cb} / V_{cb}|^2 = 0.9676 \pm 0.0033$$

$$\alpha_{\text{em}}(0) = 1/137.036$$

$$M_Z = 91.1876 \text{ GeV}$$

$$\alpha_s(M_Z) = 0.1189 \pm 0.0020$$

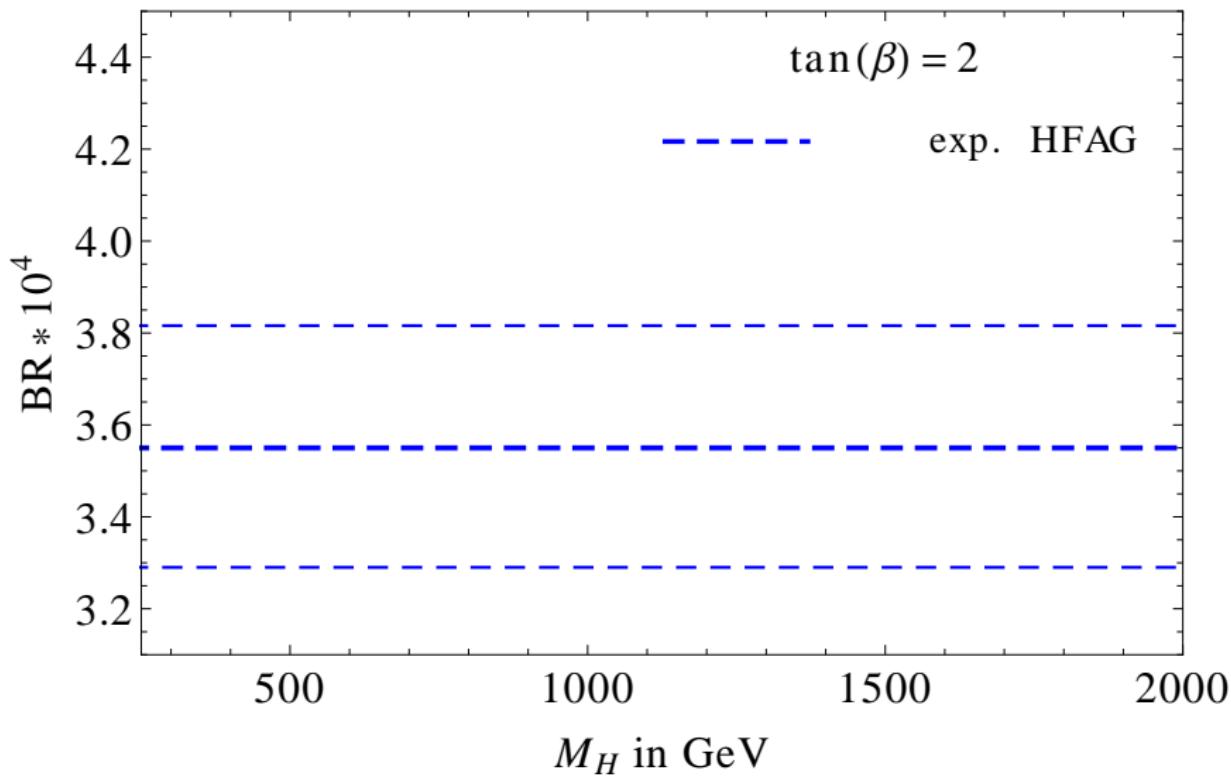
⋮

Renormalization scales:

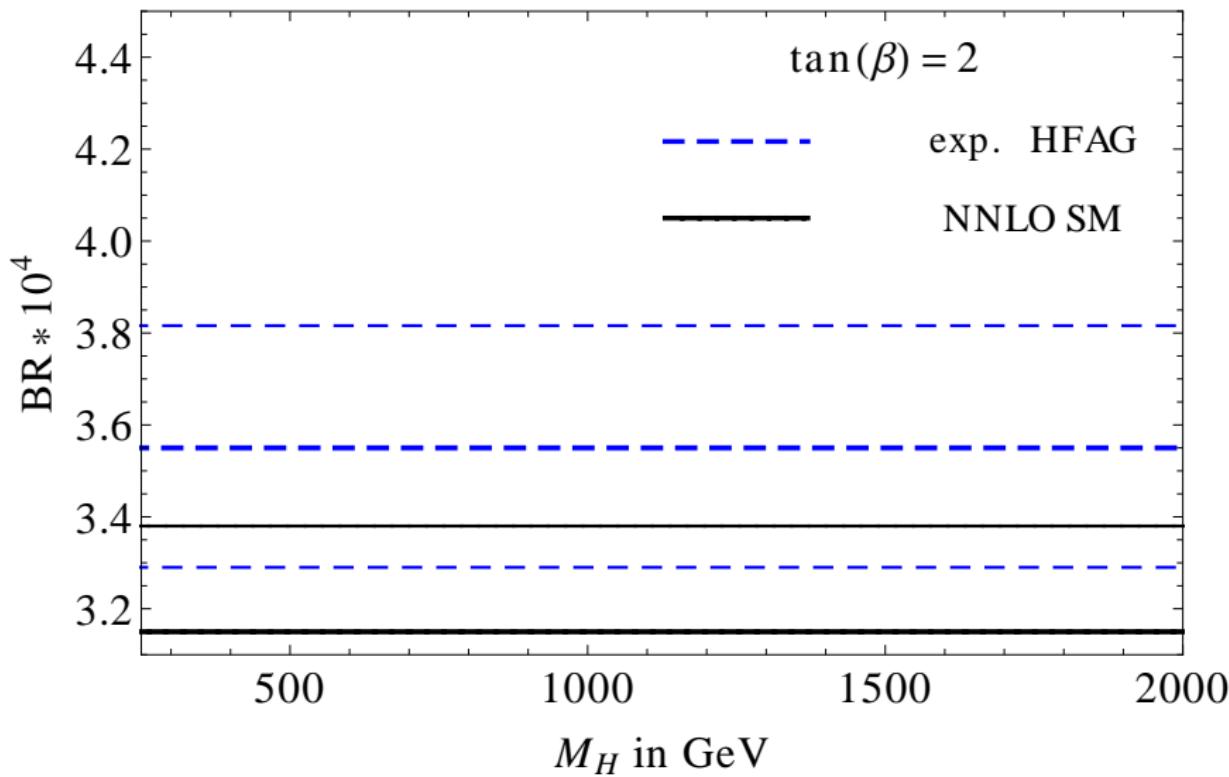
$$\mu_0 = 160 \text{ GeV}, \mu_B = 2.5 \text{ GeV} \text{ and } \mu_C = 1.5 \text{ GeV}$$

Partial NNLO in 2HDM type II without $C_{7,8}^{(2)\text{eff, 2HDM}}(\mu_0)$ [Misiak et al. 2006]

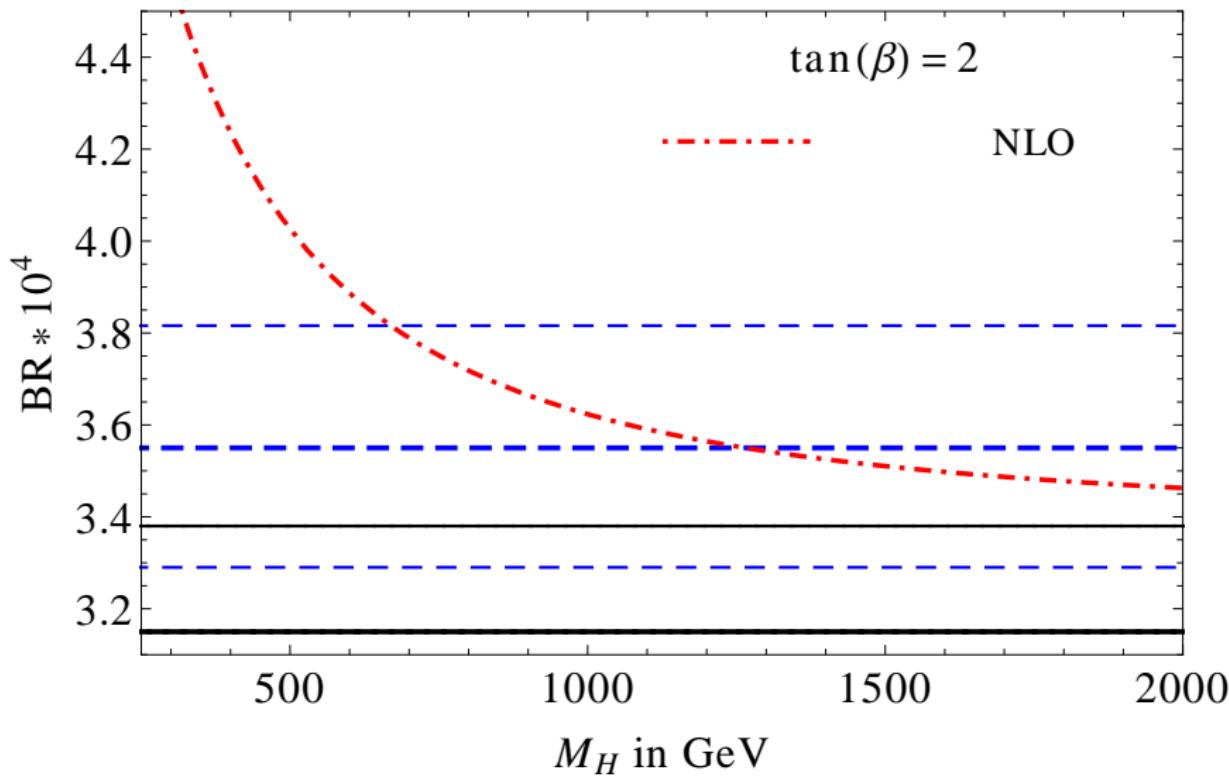
Branching ratio in 2HDM type II



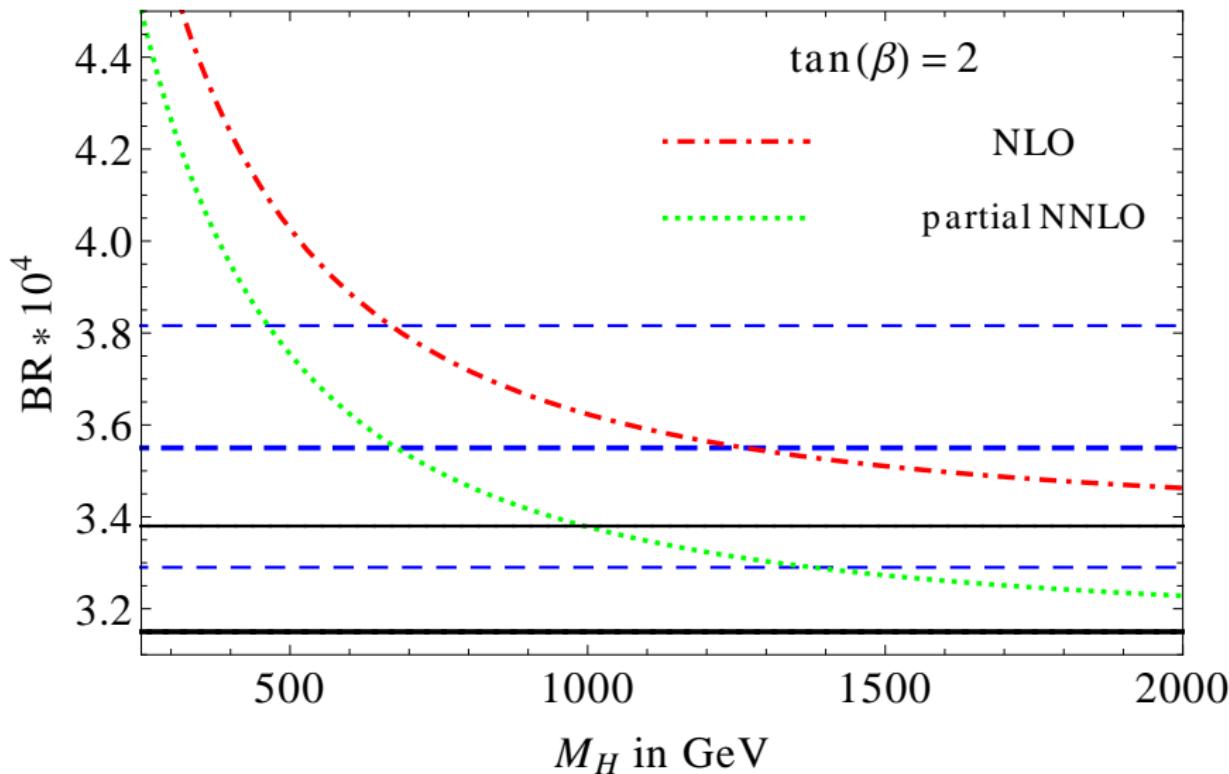
Branching ratio in 2HDM type II



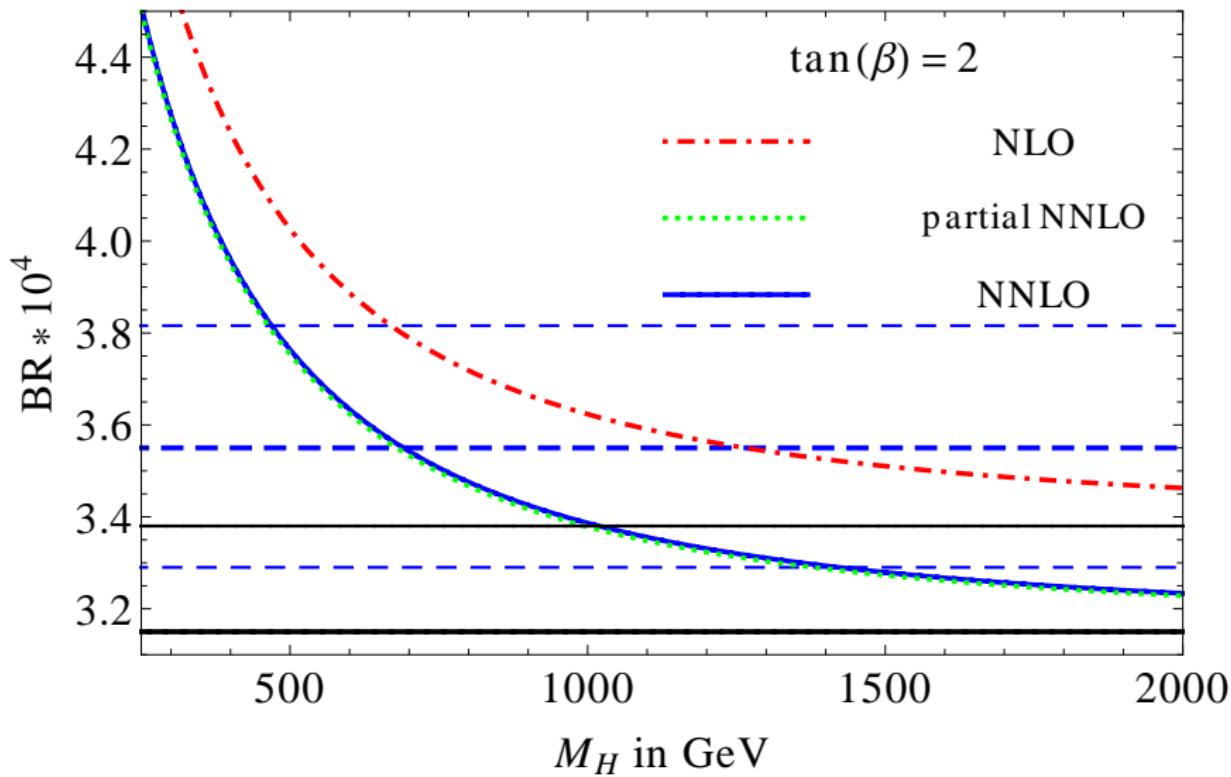
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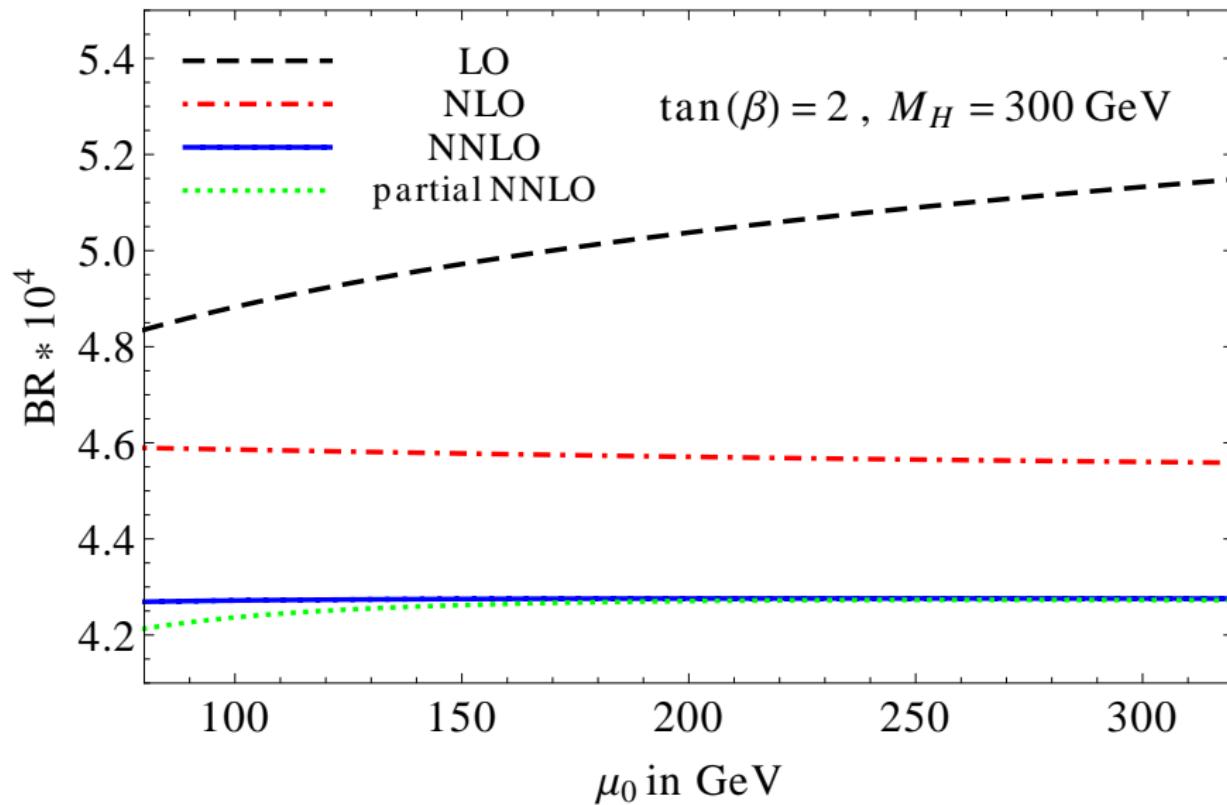
Branching ratio in 2HDM type II



Branching ratio in 2HDM type II



Matching scale 2HDM type II



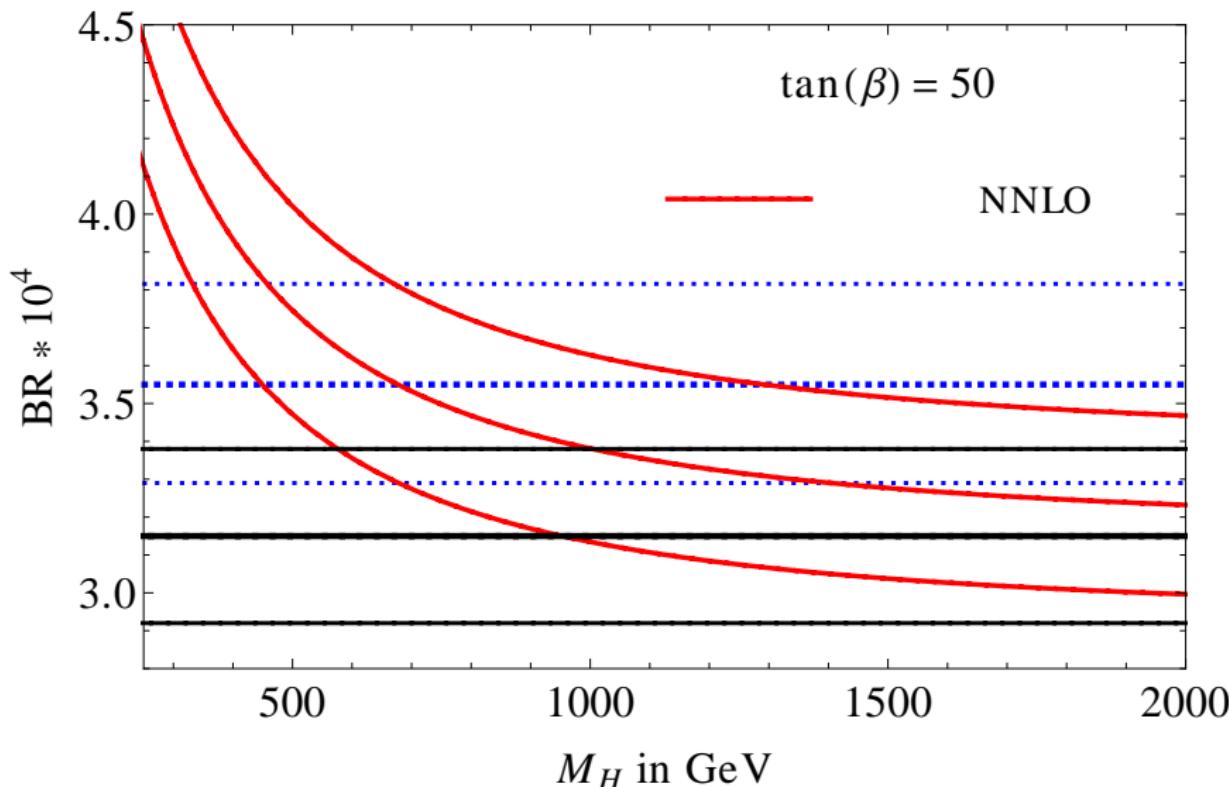
Uncertainty band 2HDM type II

$\tan(\beta) = 50$ and $M_{H^\pm} = 300$ GeV

- ➊ m_c interpolation: 3% [Misiak, Steinhauser 2006]
- ➋ scale uncertainty: 3.9%
 - $\mu_0 = 80$ GeV, 160 GeV, 320 GeV $\Rightarrow 0.06\%$
 - $\mu_B = 1.25$ GeV, 2.5 GeV, 5 GeV $\Rightarrow 3.3\%$
 - $\mu_C = 1.224$ GeV, 1.5 GeV, 4.68 GeV $\Rightarrow 2.0\%$
- ➌ parameter uncertainty: 2.5%
 - $\text{BR}(B \rightarrow X_c e \bar{\nu})_{\text{exp}}$: 1.6%
 - semileptonic phase space ratio C and $m_c(m_c)$: 1.1% correlated [Hoang, Manohar 2006]
 - $\alpha_s(M_Z)$: 1.1%
 - ...
- ➍ non-perturbative uncertainty: 5%

uncertainties added in quadrature $\Rightarrow 7.4\%$

Uncertainty band 2HDM type II



Lower bound of M_{H^\pm} 2HDM type II

$$\tan(\beta) = 50$$

practically constant to $\tan(\beta) \simeq 2$

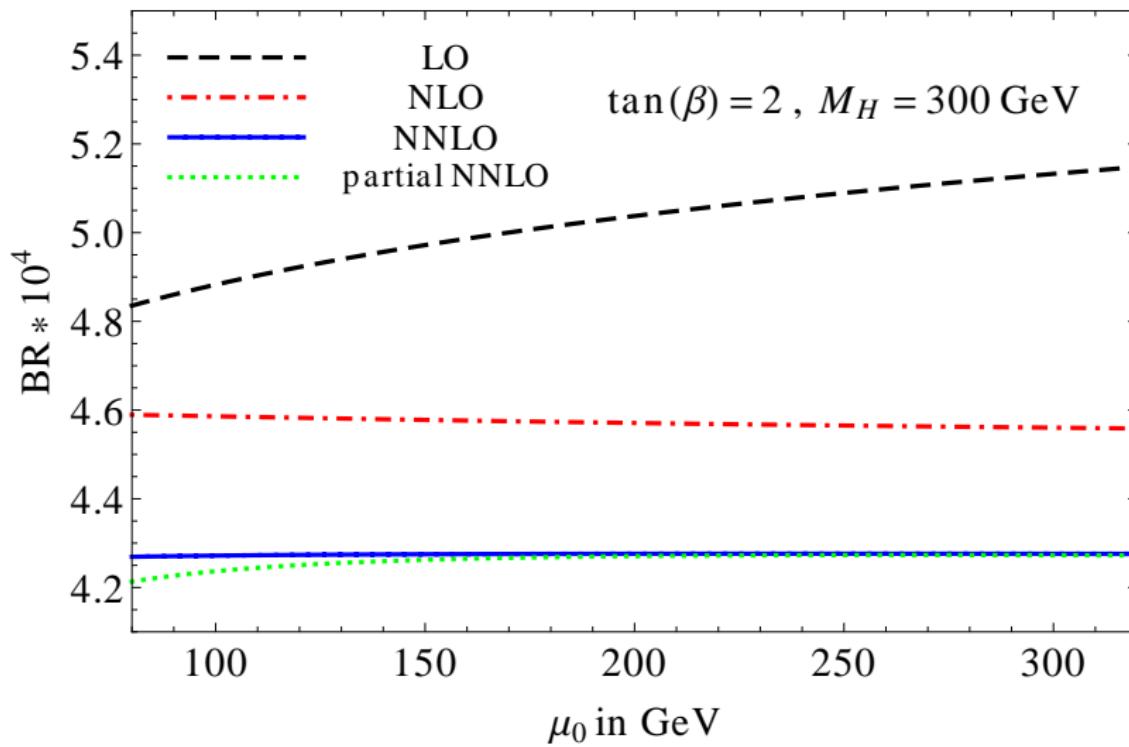
for $\tan(\beta) < 2$ BR and lower bound increase

- $M_{H^+} \geq 300 \text{ GeV}$ with 95% CL
with $C_{7,8}^{(2)\text{eff, 2HDM}}(\mu_0)$ $\Rightarrow M_{H^+} \geq 303 \text{ GeV}$

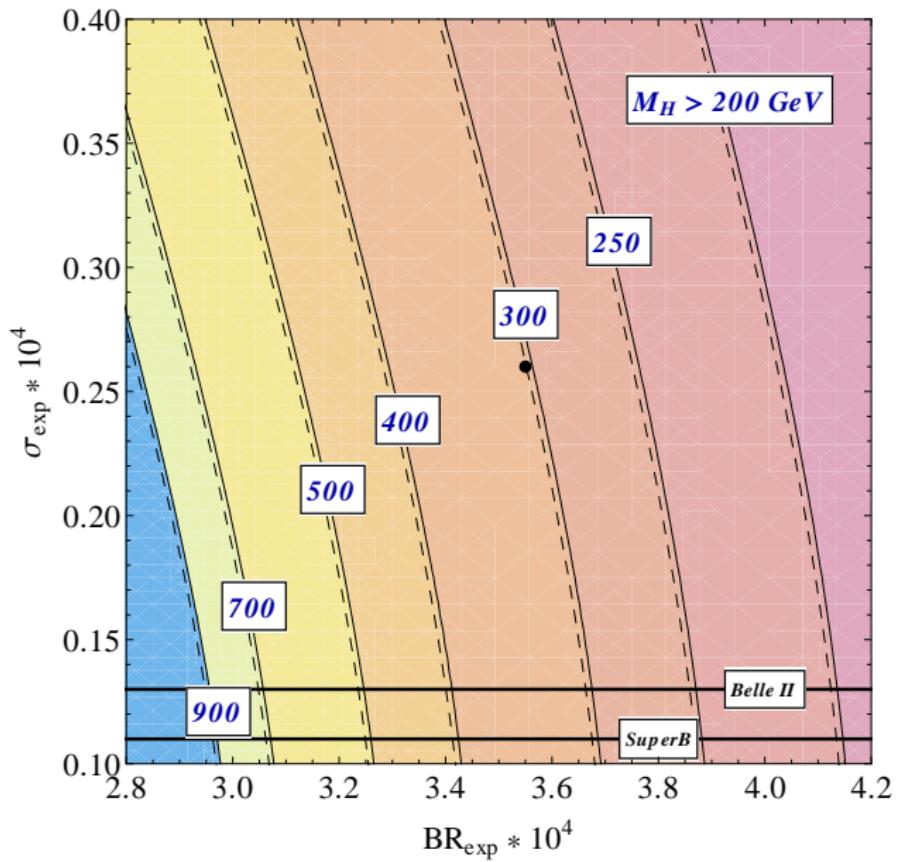
- $M_{H^+} \geq 230 \text{ GeV}$ with 99% CL
with $C_{7,8}^{(2)\text{eff, 2HDM}}(\mu_0)$ $\Rightarrow M_{H^+} \geq 233 \text{ GeV}$

small shift of 3 GeV due to $C_{7,8}^{(2)\text{eff, 2HDM}}(\mu_0)$

Matching scale 2HDM type II



lower bound for different scales $\sim 20 \text{ GeV}$

Lower bound of M_{H^\pm} 2HDM type II

• [HFAG 2010]

$Belle II$ and $SuperB$

[Meadows et al. 2011]

Conclusion

- C_7 and C_8 to three-loop order in 2HDMs
- consistent NNLO estimation in 2HDMs
- reduction of matching scale dependence $\sim 1\%$ to $\sim 0.1\%$
- lower bound in 2HDM type II:

$$M_{H^+} \geq 300 \text{ GeV with } 95\% \text{ CL}$$

Branching ratio in 2HDM type I

