

FBPS Generators for NLO MC's

In collaboration with:

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(arXiv::1106.5045 [hep-ph])

- Ciaran Williams and John Campbell
(arXiv:1204.4424 [hep-ph])

- Forward Branching Phase Space generators:
 - The idea
 - The implementation
 - The application
 - The future

The idea:

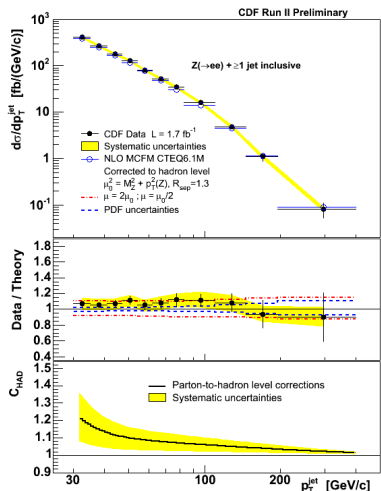
- Standard NLO predictions follow a simple prescription:
 - Generate m weighted born+virtual events.
 - Generate $n \gg m$ weighted bremsstrahlung events.
 - Combine the events in a histogram to make physical prediction and compare to data.

These parton level MC's have been used very successfully in the last 20+ years, but have drawbacks:

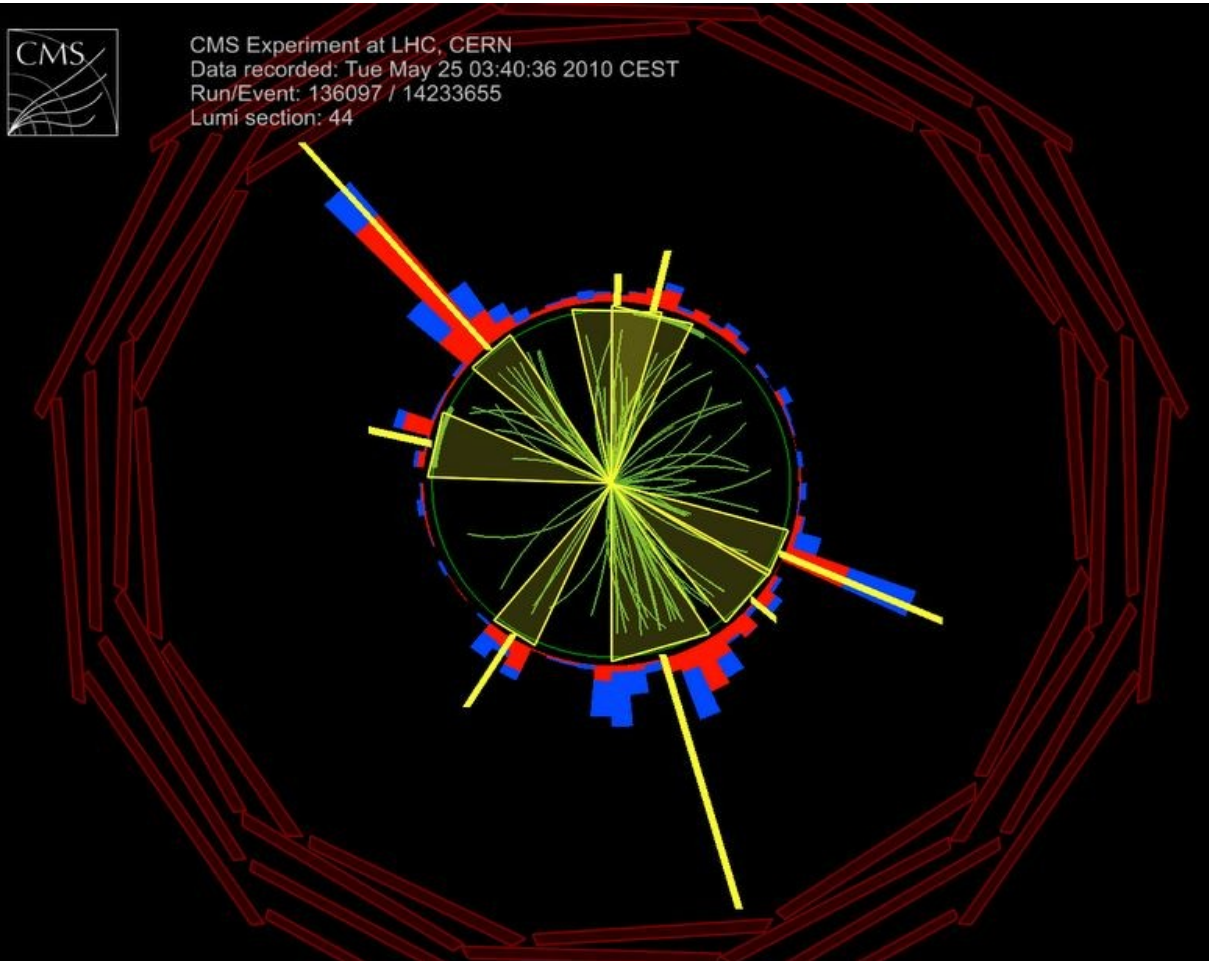
- No event-by-event physical interpretation
- Cancellation between virtual and real through MC integration
- High multiplicity final stats give complicated phase spaces

Result:

- Need lots of computer power to get results, especially as we go to higher and higher multiplicity.
- While it is straightforward to generate high multiplicity LO and NLO matrix elements, the MC phase space integration is a limiting factor in making NLO predictions.



How to do better:



A **PP** \rightarrow **8 jets** event at CMS:

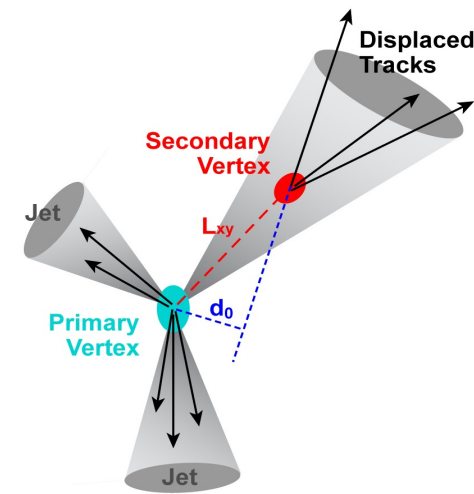
- The jet algorithm assigns each hadron to either one of the jets *or* it is unclustered (assigned to the “beam”-jet).
- Many hadronic configurations gives the identical exclusive jet final state.

- Experimental collisions produce well defined final states, organized in exclusive objects (but still hadron inclusive).
- This exclusive jet final state has a well defined scattering probability.
- Can we define this within the context of pQCD?
- If we can, it will result in a powerful event generator.

- Event complicated in terms of hadrons.
- Event simple in term of objects

What changes:

For a given exclusive jet event we calculate the scattering probability:



- Just one Born and one Virtual event contributes.
- Generate limited set of bremsstrahlung events:
 - These events do *not* change the observed jet final state.
 - We obtain a positive definite probability for the NLO jet event.
 - Unweighted NLO!
- In words: we integrate out all physics (partonic configurations) below the jet resolution scale.

How does this help:

- Generation of bremsstrahlung 100% correlated with the single virtual event (fixed jets).
- NLO phase space is now a simple 2-parameter (+ 1 trivial parameter). Implementable on GPU or SCC chips for massive speed-ups.
- Generator:
 - Generate unweighted LO events.
 - For each LO, calculate the virtual event and add bremsstrahlung to get NLO K-factor.
 - Unweight NLO with high efficiency.
- Non-binned predictions, e.g. predict at LO and NLO the differential transverse momentum prediction at 50 GeV, 75 GeV,.... and make a smooth interpolation.



Implementation:

Issues:

- At LO jets are massless (one parton per jet).
- At LO jets are balanced (no unclustered partons).
- Beyond LO jets can be massive and unbalanced.



After applying a jet algorithm to an hadronic event, we need to map the jet events onto the LO phase space before we can proceed:

- By using the unknowable longitudinal boost we can define massless jets.
- By using the unclustered hadrons we can define a balanced frame. (As we use more event information we can expect better agreement with data.)

Born Phase Space (jet mass):

- The longitudinal boost is undetermined.
- We can use this to scale the jets massless in the following manner:
 - Rescale energy and longitudinal component with the same factor such that the jet becomes massless
 - This leaves the transverse momentum, the rapidity and azimuthal angle of the jet invariant.
- In other words, we fix the jet's transverse momentum, rapidity and azimuthal angles while the jet mass is integrated out (as it is a jet property depending on internal partons).

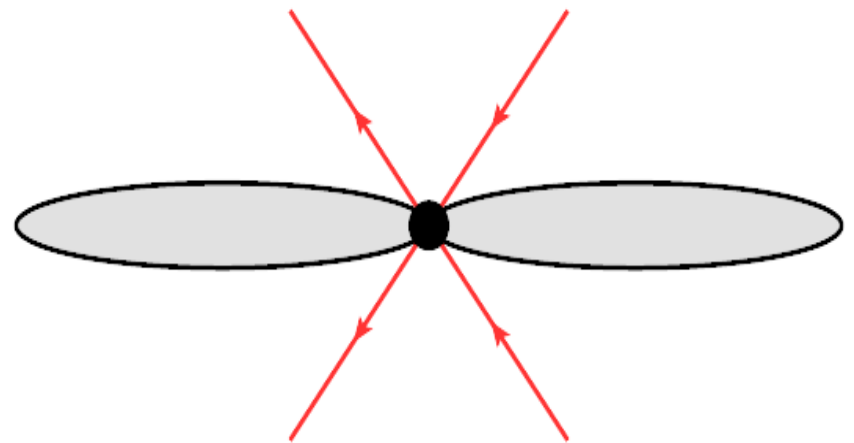
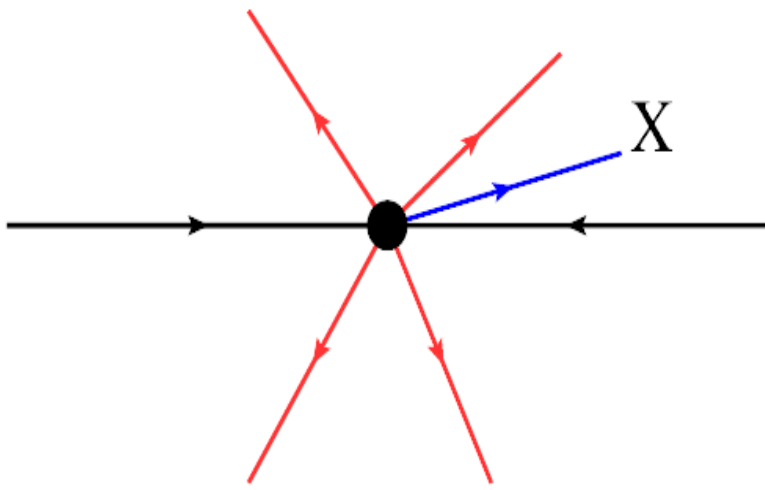
Born Phase Space (event balance):

- An event will have the reconstructed objects and a vector \mathbf{X} , which is the sum of all unclustered hadron momenta (at LO $\mathbf{X}=0$).
- We apply a transverse boost to the frame where the transverse momentum of \mathbf{X} is zero.
- We only consider observables invariant under longitudinal boost, so all possible transverse boosts are equivalent (leave the observables invariant).
- Define the observables in this Born frame. Invariant masses are the same in LAB and Born frame, other observables such as transverse momentum are not.
- Complication: cuts are defined in the LAB frame.

Born Phase Space (born frame):

This is a very natural frame:

- Instead of having the unphysical constraint of the incoming parton along the proton direction (requires infinite resolution), we have the physical constraint of having the beam jet (cluster of hadrons not associated with the jet) along the proton direction.
- The bremsstrahlung events “wobble” the incoming parton along the proton direction. This can be seen as a consequence of earlier bremsstrahlung.



Forward Branching PS Generators:

- We can now simply generate bremsstrahlung events directly in the Born-frame.
- That is, we split (forward branch) an incoming or outgoing parton without changing the Born-frame observables.
- As an example, the initial state branching (generates the unclustered parton phase space):

$$d\Phi(p_a + p_b \rightarrow Q + p_r) = d\Phi(\hat{p}_a + \hat{p}_b \rightarrow Q) \times d\Phi_{\text{FBPS}}(p_a, p_b, p_r) \times \theta_{\text{veto}}$$

$$d\Phi_{\text{FBPS}}(p_a, p_b, p_r) = \frac{1}{(2\pi)^3} \left(\frac{\hat{s}_{ab}}{s_{ab}} \right) dt_{ar} dt_{rb} d\phi \quad \theta_{\text{veto}}(p_r) = \theta \left[p_T^{\text{lab}}(p_r) < p_T^{\text{min}}(\text{jet}) \right]$$

- **Some complication:**
 - Cuts are in the LAB frame.
 - The jet algorithm is in the LAB frame.

LAB Cuts and Jet Algorithms:

- These issues are easily resolved by rewriting:

$$p_T^{lab,i} = \sqrt{\frac{s_{ai}s_{ib}}{s_{ab}}}, \quad \eta^{lab,i} = \frac{1}{2} \log \left(\frac{x_a^2 s_{ib}}{s_{ab} s_{ai}} \right)$$

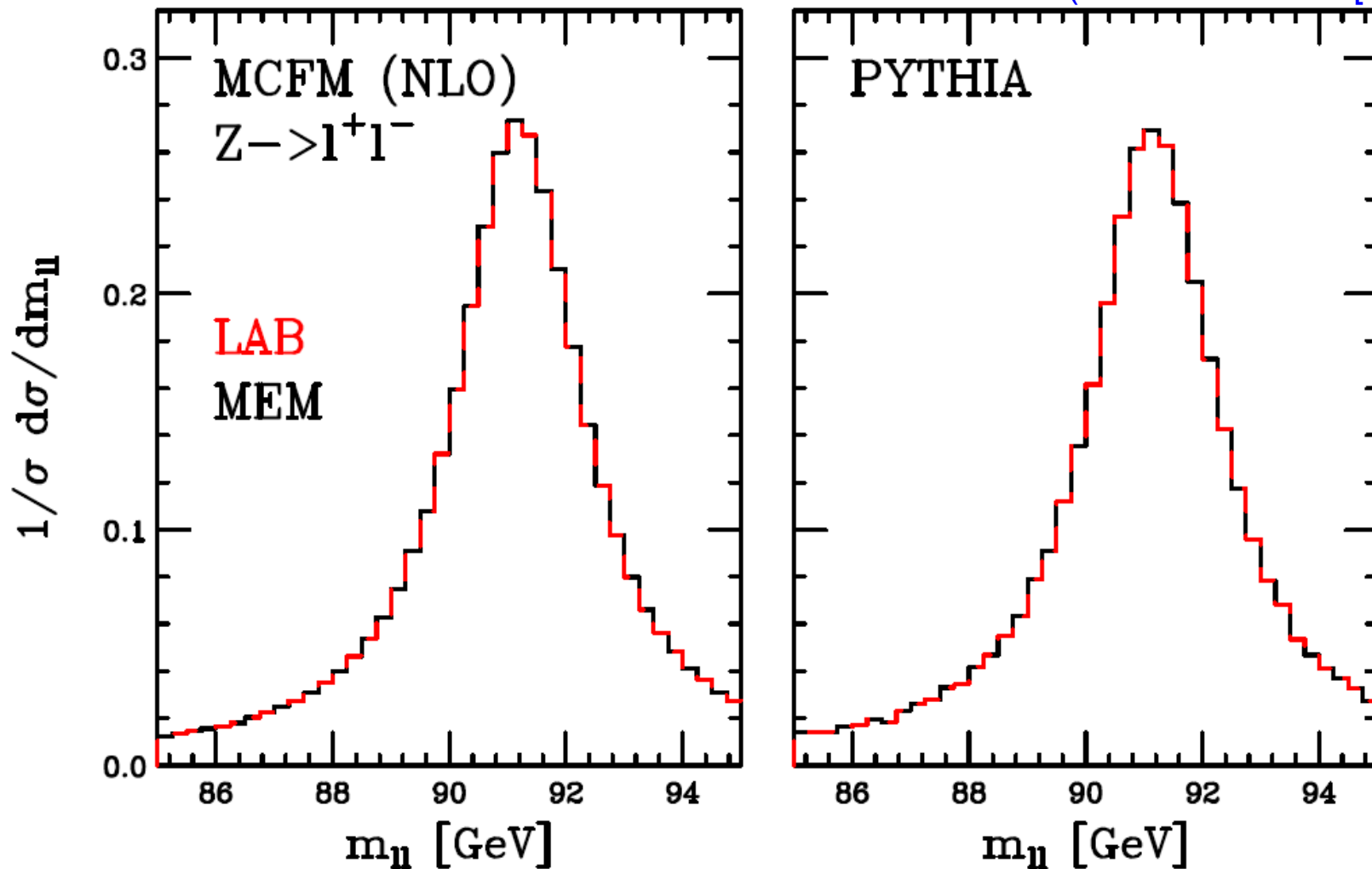
$$\Delta\eta_{ij}^{lab} = \frac{1}{2} \log \left(\frac{s_{bi} s_{aj}}{s_{ai} s_{bj}} \right), \quad \Delta\phi_{ij}^{lab} = \cos^{-1} \left(\cosh(\Delta\eta_{ij}^{lab}) - \frac{s_{ij}}{2p_T^{lab,i} p_T^{lab,j}} \right)$$

- That is, we can calculate the relevant LAB frame observables directly in any frame.
- The cuts and jet algorithm are dynamical in the Born frame.

Example: DY Production at LHC

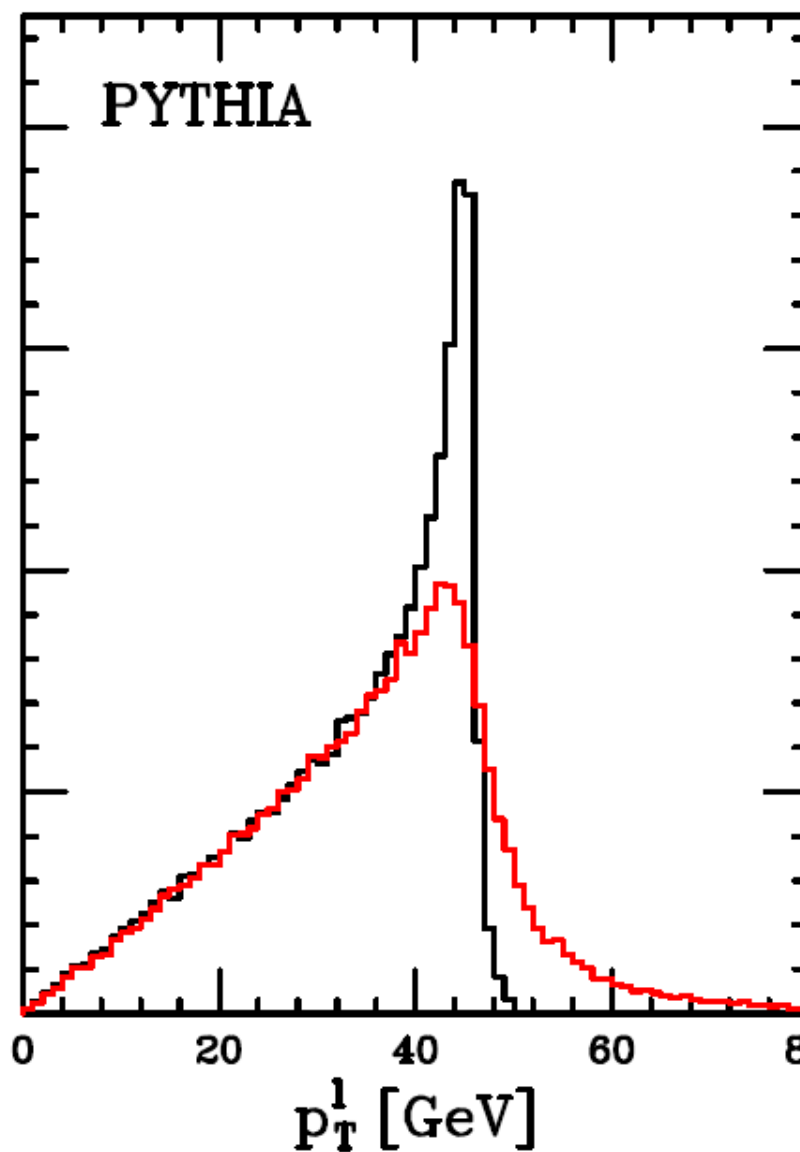
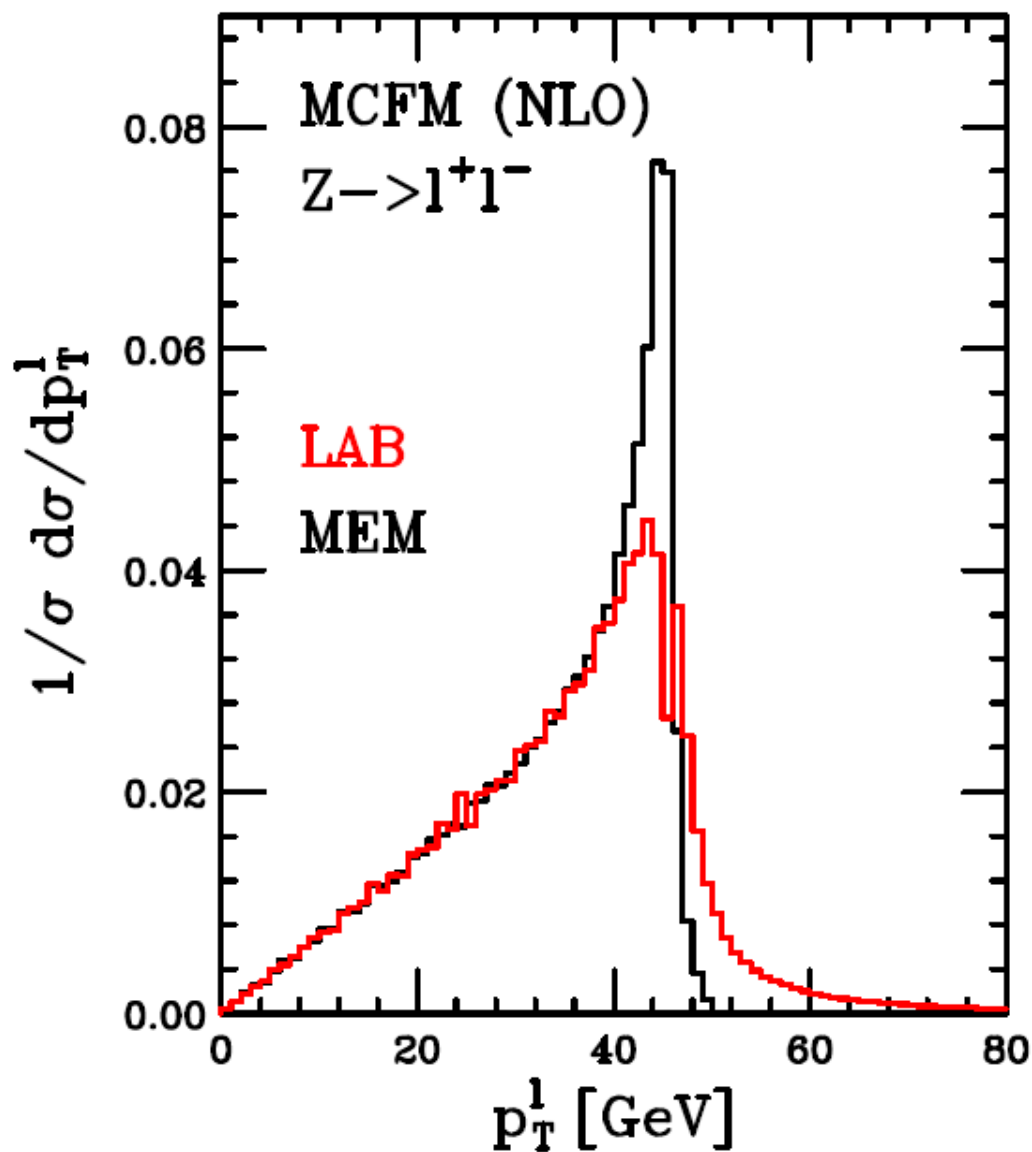
- The di-lepton invariant mass is Lorentz invariant. Same in any frame (MEM = Born-frame)

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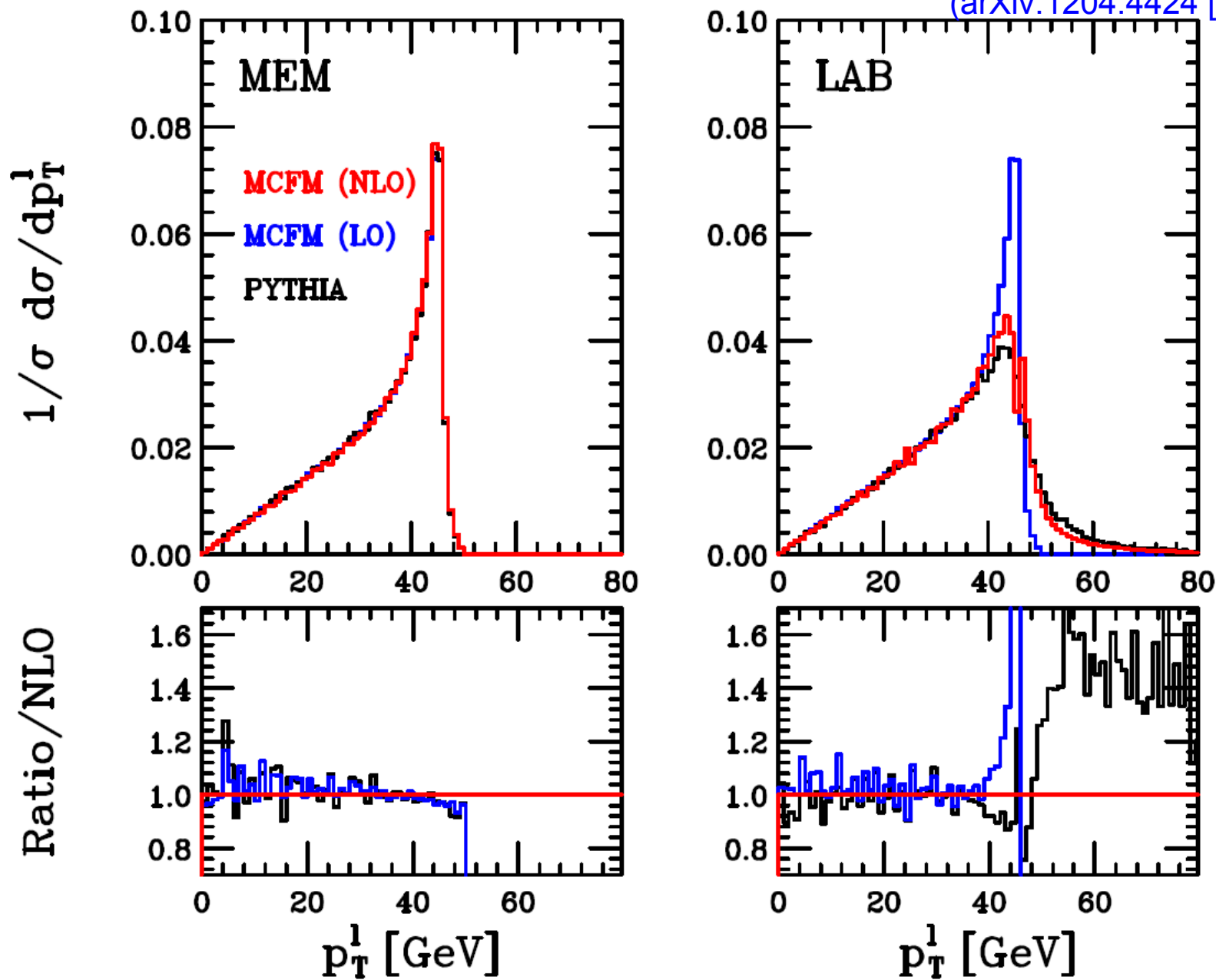
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Example: DY Production at LHC

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Example: Jet Production

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arXiv:1106.5045 [hep-ph]

Scale choice: average di-jet mass

Gluons only...

| jets | r -factor | -- ++ ... + | | --- + ... ++ | | - + - + ... - + | |
|------|---------------|---------------|-----------------|-----------------------|-----------------|--------------------------|-----------------|
| | | $ m^{(0)} ^2$ | k -factor | $ m^{(0)} ^2$ | k -factor | $ m^{(0)} ^2$ | k -factor |
| 2 | 172 ± 1 | 1.72216 | 1.15 ± 0.05 | 1.6×10^{-31} | --- | 0.00552438 | 1.09 ± 0.05 |
| 3 | 243 ± 2 | 120.638 | 1.13 ± 0.08 | 0.043632 | 1.18 ± 0.08 | 5.98249 | 1.10 ± 0.08 |
| 4 | 392 ± 3 | 125.234 | 1.30 ± 0.13 | 0.282847 | 1.17 ± 0.13 | 0.0498892 | 1.18 ± 0.13 |
| 5 | 366 ± 4 | 5941.55 | 0.94 ± 0.17 | 849.054 | 0.87 ± 0.17 | 31.5083 | 0.80 ± 0.17 |
| 6 | 529 ± 5 | 1202.54 | 1.15 ± 0.24 | 69.0066 | 1.06 ± 0.24 | 0.469815 | 0.82 ± 0.24 |
| 8 | 650 ± 7 | 26732.0 | 1.41 ± 0.34 | 1364.49 | 1.32 ± 0.34 | 1.41604 | 1.15 ± 0.34 |
| 10 | 844 ± 11 | 6575.23 | 1.49 ± 0.49 | 579.066 | 1.26 ± 0.49 | 6.09232×10^{-6} | 0.97 ± 0.49 |
| 15 | 1264 ± 20 | 4690.02 | 1.39 ± 0.95 | 671.554 | 1.28 ± 0.95 | 4.37178×10^{-7} | 1.24 ± 0.95 |

Table 1: The LO ordered amplitude squared $|m^{(0)}(J_a, J_b, J_1, \dots, J_n)|^2$ and its corresponding $r(s_{\min})$ and ordered k -factor as defined in Eq. (33) for an exclusive n -jet event. The explicit jet momenta for the different jet multiplicities are given in Appendix B. The slicing scale s_{\min} is set to $10^{-4} \times S$ and the Monte Carlo integration over the

The Future:

- We developed a new phase space integration method for higher order corrections.
- It calculates higher order corrections on an event-by-event basis.
- Defining observables in the Born frame gives a better pQCD expansion (removes large logs).
- The method is ready for serious applications.
- The first one is the MEM@NLO (see next talk).
- The second application is a unweighted NLO multi-jet generator: **$PP \rightarrow n \text{ jets}$** (in progress).