Dark energy from a renormalization group flow

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Particle Physics

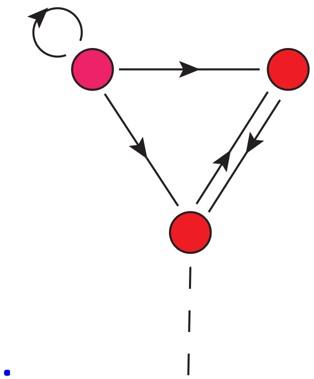
- Well tested Standard Model
- Many free parameters
- High energy completion?
- New particles?

Cosmology

- Well tested Standard Model
- Many free parameters
- Early universe theory?
- Dark matter/dark energy?

Standard Model of Particle Physics and Cosmology?

Quiver gauge theories



- Theories with many gauge group factors
- Representation of fermions and scalars can be
  - adjoint
  - bifundamental
- Some of them may be easily realized in string theory in terms of D-branes at orbifold/Calabi-Yau singularities
- Have been suggested as a means of embedding the standard model in string theory

e.g. Verlinde, Wijnholt

- Fields in fundamental representation ⟶ some gauge group factors may be non-dynamical ⟶ left-over sectors with no gauge interactions with standard model
- In presence of gravity coupling still exists
  - potential observable effects
  - potential (un)desirable effects
- quivers associated to orbifolds may be a typical presence

The orbifold of a field theory

Consider a QFT with matrix-valued fields and some global symmetry

e.g. $\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(\partial^\mu\Phi\partial_\mu\Phi) + \frac{1}{4}\mathrm{Tr}(\Phi^4)$ $\quad \Phi \to O^T\Phi O$ with $O \in SO(N)$

Orbifold field theory = truncate the field content to the fields invariant under some discrete global symmetry group

e.g. $\mathbf{Z}_2 \in SO(2N)$ $\quad \gamma = \begin{pmatrix} \mathbf{1}_N & 0 \\ 0 & -\mathbf{1}_N \end{pmatrix}$

$$\begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} = \Phi = \gamma^T\Phi\gamma = \begin{pmatrix} \Phi_{11} & -\Phi_{12} \\ -\Phi_{21} & \Phi_{22} \end{pmatrix} \to \Phi = \begin{pmatrix} \Phi_{11} & 0 \\ 0 & \Phi_{22} \end{pmatrix}$$

- same Lagrangian, restricted fields
- more interesting in the presence of several symmetries, including local
- planar perturbation theory is inherited from the parent theory: finite

Bershadsky, Johansen; Bershadsky, Kakushadze, Vafa

- However, renormalizability requires addition at tree level of dim=4 2-trace op's, e.g. $|\mathrm{Tr}[g\Phi^A\Phi_A]|^2$; renormalize $\longrightarrow \beta$ -function

Zarembo, Tseytlin; Adams, Silverstein; Dymarsky, Klebanov, Roiban

In the absence of supersymmetry this β-function is nonzero and positive

$\longrightarrow$ conformal invariance weakly broken

Dymarsky, Klebanov, Roiban

Immediate consequence: Landau pole for the 2-trace coupling f_n

$$\frac{dg_{YM}}{d\ln\Lambda} = \beta_{g_{YM}}(\lambda, f_k) = 0 \qquad \frac{df_n}{d\ln\Lambda} = \beta_n(\lambda, f_k) > 0$$

$$\beta_n = \lambda^2 a_O + 2\lambda\gamma_O f_n + v_O f_n^2 \qquad f(M) = -\frac{\gamma_O\lambda}{v_O} + \frac{b\,\lambda}{v_O}\tan\left(\frac{b\,\lambda}{v_O}\ln\frac{M}{\mu_r}\right)$$

$$\text{IR:} \quad M = \mu e^{-\pi v_0/(b\lambda)} \longrightarrow f_k \to -\infty$$
$$\text{UV:} \quad M = \mu e^{+\pi v_0/(b\lambda)} \longrightarrow f_k \to +\infty$$

When instability sets in, dynamics is driven by the double-trace operator with fastest running; the other interactions contribute solely to render the theory finite at the planar level and renormalizable at O(1/N).

Instability cures itself through condensation of the two factors of the 2-trace operators

(unpublished) Dymarsky, Franco, Klebanov, Roiban

Horowitz, Orgera, Polchinski

Supported by comparison with the string theory dual

Condensation is captured by a single scalar field model:

$$\mathcal{L} = -N\mathrm{Tr}[\partial^\mu \bar{\Phi} \partial_\mu \Phi] + f(M)|\mathrm{Tr}[g\Phi\bar{\Phi}]|^2$$

i.e. the structure of the action for the condensate is the same but the coefficients are slightly different in the complete theory.

For simplicity focus on this model and incorporate the necessary changes.

Condensation of the single-trace factors is standard:

-- linearize and integrate out matrix degrees of freedom

$$\mathcal{L} = -N\mathrm{Tr}[\partial^\mu \bar{\Phi} \partial_\mu \Phi] + \frac{1}{f(M)}\bar{\varphi}\varphi - \varphi\mathrm{Tr}[g^{-1}\Phi\bar{\Phi}] - \bar{\varphi}\mathrm{Tr}[g\Phi\bar{\Phi}]$$

⟶ effective action for φ; justified *a posteriori* by $\langle\varphi\rangle \neq 0$

In complete theory: consistent picture of the theory around condensate

Curved Space

Want to preserve scale invariance

- in flat space only weakly broken by running of f

→ add conformal coupling ξ : required by renormalization

$$L = \sqrt{-g}\left(-N g^{\mu\nu} Tr[\partial_\mu \bar{\Phi} \partial_\nu \Phi] - \xi\, R\, Tr[\bar{\Phi}\Phi] - f|Tr[\gamma\Phi\bar{\Phi}]|^2\right)$$

For FRW geometry and in conformal time:

$$L = -N Tr[\partial_\mu \bar{\hat{\Phi}} \partial^\mu \hat{\Phi}] + \frac{1}{f}\bar{\hat{\varphi}}\hat{\varphi} - \hat{\varphi} Tr[\gamma^{-1}\hat{\Phi}\bar{\hat{\Phi}}] - \bar{\hat{\varphi}} Tr[\gamma\hat{\Phi}\bar{\hat{\Phi}}]$$

with:

$$\hat{\Phi} = a(\eta)\,\Phi \qquad \hat{\varphi} = a(\eta)^2\,\varphi$$

→ L quadratic → Coleman-Weinberg effective action:

$$K_B = -\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{1}{N}\hat{\varphi}\,\gamma^{-1} + \frac{1}{N}\bar{\hat{\varphi}}\gamma \quad , \qquad S_{\text{eff}} = i \ln\det K_B$$

$$S_{eff} = V_{eff} + \delta L_K$$

Effective action:

$$K_B = -\eta^{\mu\nu}\partial_\mu\partial_\nu + \frac{1}{N}\hat{\varphi}\,\gamma^{-1} + \frac{1}{N}\bar{\hat{\varphi}}\gamma \quad , \qquad S_{\rm eff} = i \ln\det K_B$$

$$S_{eff} = V_{eff} + \delta L_K$$

Coleman-Weinberg potential:

$$V_{\rm eff} = \frac{1}{(4\pi)^2} Tr\left[\mathcal{M}^2\left(\ln\frac{\mathcal{M}^2}{\Lambda^4} - 1\right)\right] \qquad \mathcal{M} = \frac{1}{N}\left(\hat{\varphi}\,\gamma^{-1} + \bar{\hat{\varphi}}\gamma\right)$$

$$\delta L_K = \frac{1}{2}Tr\left[\frac{1}{p^2+\mathcal{M}}\mathcal{M}(-q)\frac{1}{(p+q)^2+\mathcal{M}}\mathcal{M}(q)\right] - \frac{1}{2}Tr\left[\frac{1}{(p^2+\mathcal{M})^2}\right]Tr[\mathcal{M}(-q)\mathcal{M}(q)]$$

An example: the $\mathbb{Z}_2$ orbifold

$$V^{\mathbb{Z}_2} = \frac{c_0}{f(M)}\hat{\varphi}_0^2 - \frac{\hat{\varphi}^2}{f(M)} + V_{\rm eff}^{\mathbb{Z}_2} \qquad V_{\rm eff}^{\mathbb{Z}_2} = +\frac{8\,\hat{\phi}^2}{(4\pi)^2}\left(\ln\frac{4\hat{\phi}^2}{N^2\,M^4} - 1\right)$$

$$\delta L_K = -\frac{N}{6(4\pi)^2}\,\hat{\varphi}^{-1}\,\partial^\mu\hat{\varphi}\partial_\mu\hat{\varphi} + \ldots$$

Restore FRW metric + comoving frame:

$$L_{\rm eff} = \frac{1}{2G} R + \delta L - V$$

$M^2 \to a(\eta)^2 M^2$ (scale: covariant interpretation → square taken with inverse metric)

$$V^{\mathbb{Z}_2} = a(\eta)^4 \left[\frac{c_0}{f(M)} \varphi_0^2 - \frac{\varphi^2}{f(M)} + \frac{8}{(4\pi)^2} \varphi^2 \left(\ln \frac{4\varphi^2}{N^2 M^4} - 1 \right) \right] = \sqrt{-g}\, V(\varphi)$$

$$\delta L_K = - \frac{N}{6(4\pi)^2} \sqrt{-g}\, \frac{1}{a(\eta)^4 \varphi} g^{\mu\nu} \partial_\mu (a(\eta)^2 \varphi) \partial_\nu (a(\eta)^2 \varphi) + \ldots$$

$$= - \frac{N}{6(4\pi)^2} \sqrt{-g} \left[\frac{1}{\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \frac{2}{3} \varphi\, R \right] + \ldots$$

Field redefinition:

$$\varphi = \frac{3}{4N} (4\pi)^2 \zeta^2$$

$$V(\zeta) = \frac{c_0}{f(M)} \varphi_0^2 - \frac{9(4\pi)^4}{16\, N^2\, f(M)} \zeta^4 + \frac{9(4\pi)^2}{2\, N^2} \zeta^4 \left(\ln \frac{9(4\pi)^4 \zeta^4}{4\, N^4\, M^4} - 1 \right)$$

$$\delta L = - \frac{1}{2} \left[g^{\mu\nu} \partial_\mu \zeta \partial_\nu \zeta + \frac{1}{6} \zeta^2\, R \right] + \ldots$$

Cosmology

$$L_{\text{eff}} = \frac{1}{2G}R + \delta L - V$$

$$V(\zeta) = \frac{c_0}{f(M)}\varphi_0^2 - \frac{9(4\pi)^4}{16\,N^2\,f(M)}\zeta^4 + \frac{9(4\pi)^2}{2\,N^2}\zeta^4\left(\ln\frac{9(4\pi)^4\zeta^4}{4\,N^4\,M^4} - 1\right)$$

$$\delta L = -\frac{1}{2}\left[g^{\mu\nu}\partial_\mu\zeta\partial_\nu\zeta + \frac{1}{6}\zeta^2\,R\right] + \ldots$$

- Model has features of:
  - modified gravity: $G \to G/\left(1 - \frac{G}{6}\zeta^2\right)$
  - quintessence: $V(\zeta)$
  - K-essence : δL

EOM for ζ and Einstein eq. in isotropic, homogenous (FRW) universe:

$$\ddot{\zeta} + 3H\dot{\zeta} + \frac{2G}{3}\zeta V + \left(1 - \frac{G}{6}\zeta^2\right)\frac{\partial V}{\partial \zeta} + \frac{G}{6}\zeta\left(-\sqrt{2x}|\zeta|\frac{n_0}{a(t)^2} - \tilde{\rho}^m + 3\tilde{p}^m\right) = 0$$

$$3H^2 = G(\rho^\zeta + \rho^m)$$

$$\dot{\rho}^m = -\dot{\rho}^\zeta - 3\frac{\dot{a}}{a}\left(\rho^\zeta + \rho^m + p^\zeta + p^m\right)$$

$$\rho^m = \frac{\tilde{\rho}^m}{(1 - \frac{G}{6}\zeta^2)}$$ effective matter energy density

$$\rho^\zeta = \frac{1}{1 - \frac{G}{6}\zeta^2}\left(V + \frac{1}{2}\dot{\zeta}^2 + H\zeta\dot{\zeta} + \sqrt{2x}|\zeta|\frac{n_0}{a(t)^2}\right)$$ ζ energy density

$$p^\zeta = \frac{1}{1 - \frac{G}{6}\zeta^2}\left(-V + \frac{1}{6}\dot{\zeta}^2 - \frac{1}{3}\zeta\left(\ddot{\zeta} + 2H\dot{\zeta}\right)\right)$$ ζ pressure

n_0: finite density of Φ quanta

→ negative `spatial` curvature in Friedman equation
does not affect curvature of spatial slices: unconstrained by obs.

A Special Solution

Expect far future time-dependent solution asymptotes to ct. field cfg.

$$n_0 = 0, \ \rho_m = 0 \Rightarrow \zeta = \zeta_0 = \text{constant}$$

$$\frac{2G}{3}\zeta_0 V(\zeta_0) + \left(1 - \frac{G}{6}\zeta_0^2\right)\frac{\partial V}{\partial \zeta}\Big|_{\zeta=\zeta_0} = 0$$

$$\frac{G\,V(\zeta_0)}{3\left(1 - \frac{G}{6}\zeta_0^2\right)} = H^2$$

In absence of conformal coupling: ζ_0 given by position of min. of V

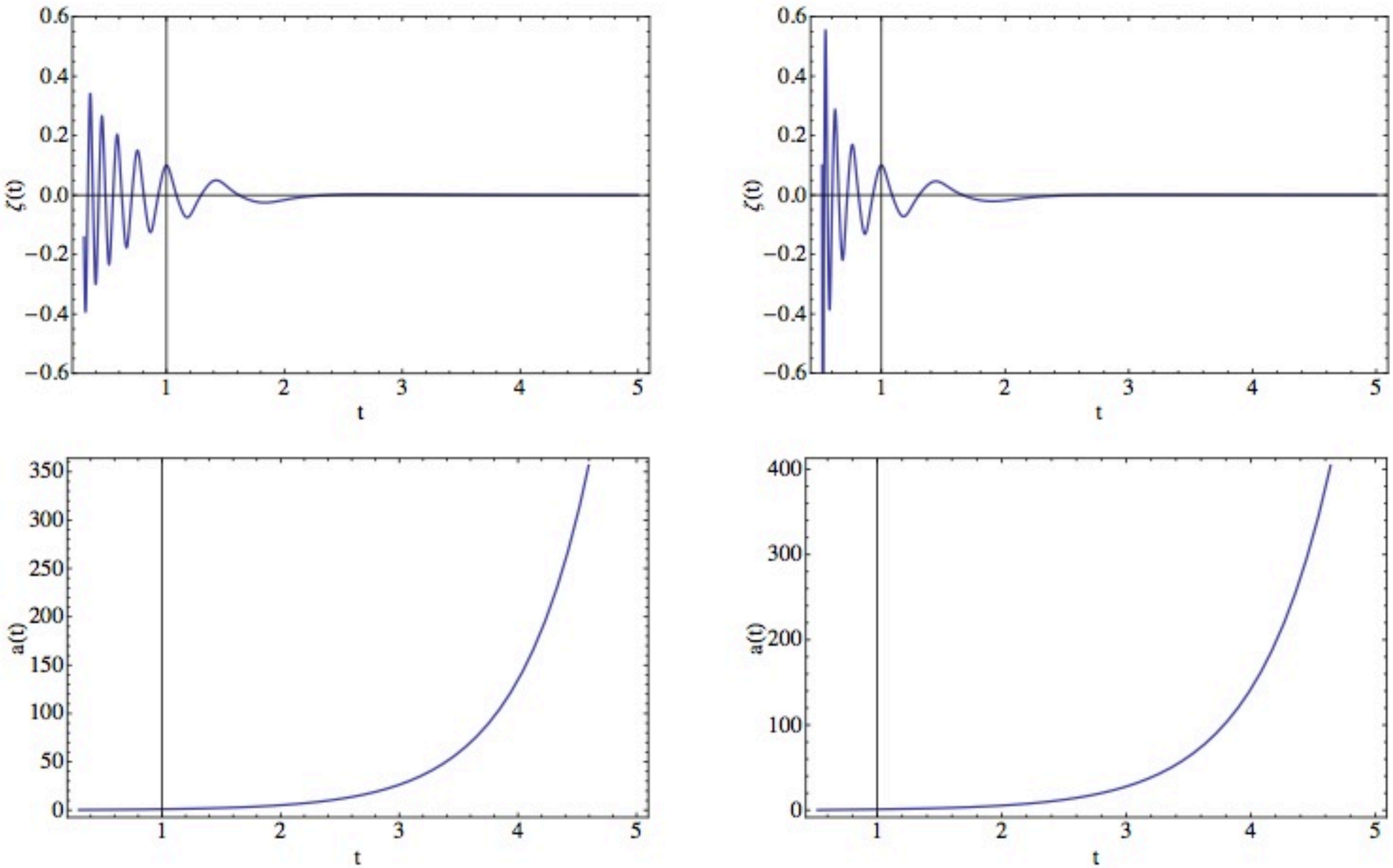
→ standard scale factor in presence of cosmological constant and effective Newton constant

$$\bar{\zeta}_0 = \pm \frac{N\,M}{2\pi\sqrt{6}}\, e^{\pi^2/2f(M)}$$

- Effective cosmological constant exponentially smaller than scale determining dynamics

$$\Lambda_{\text{eff}} = \frac{V(\zeta_0)}{\left(1 - \frac{G}{6}\zeta_0^2\right)} \propto N^4 M^4 e^{-\frac{2\pi^2}{|f(M)|}} \left(1 + \sum_{n\geq 1} e_n G^n \bar{\zeta}_0^n\right)$$

General Solution



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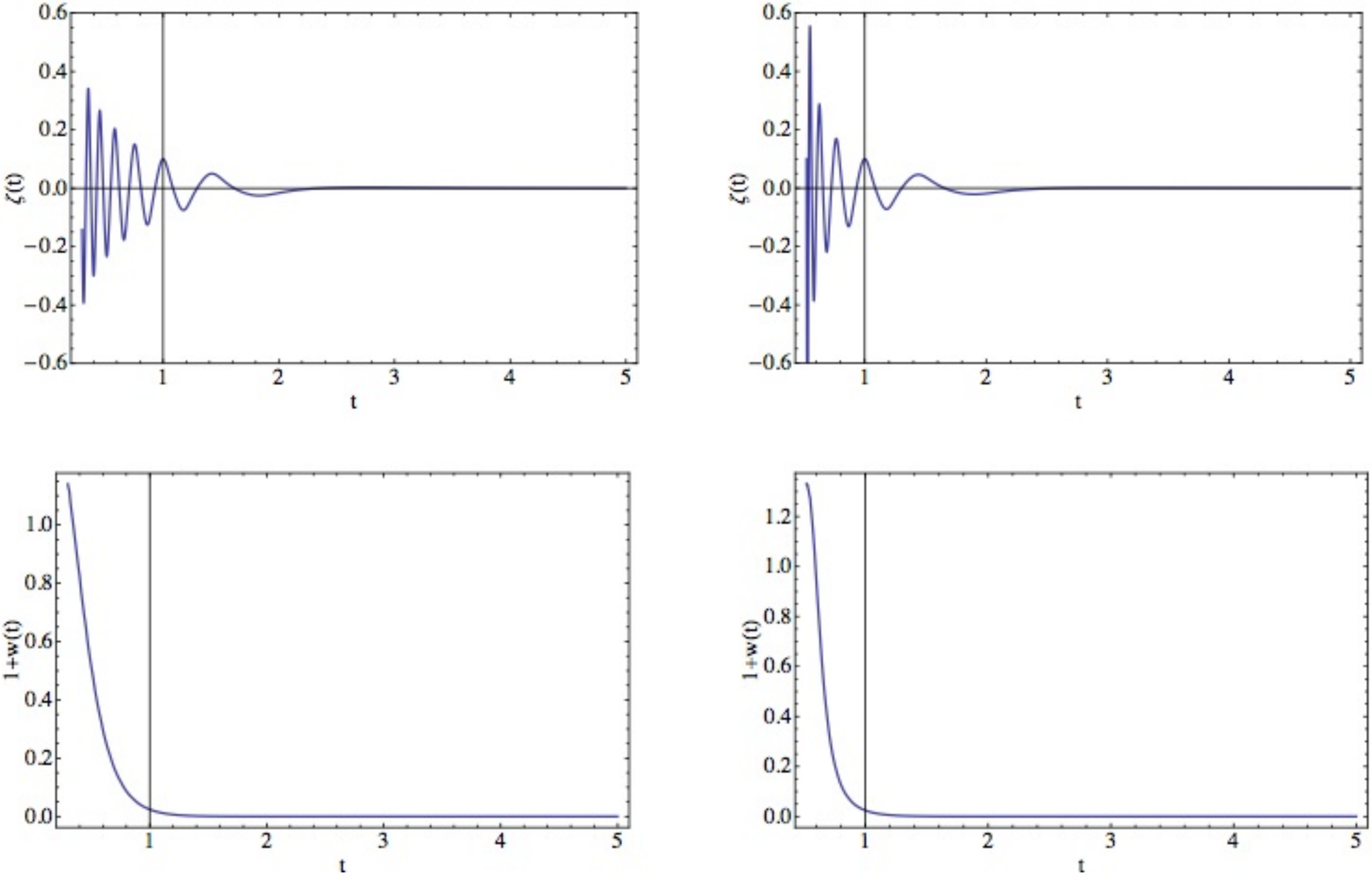

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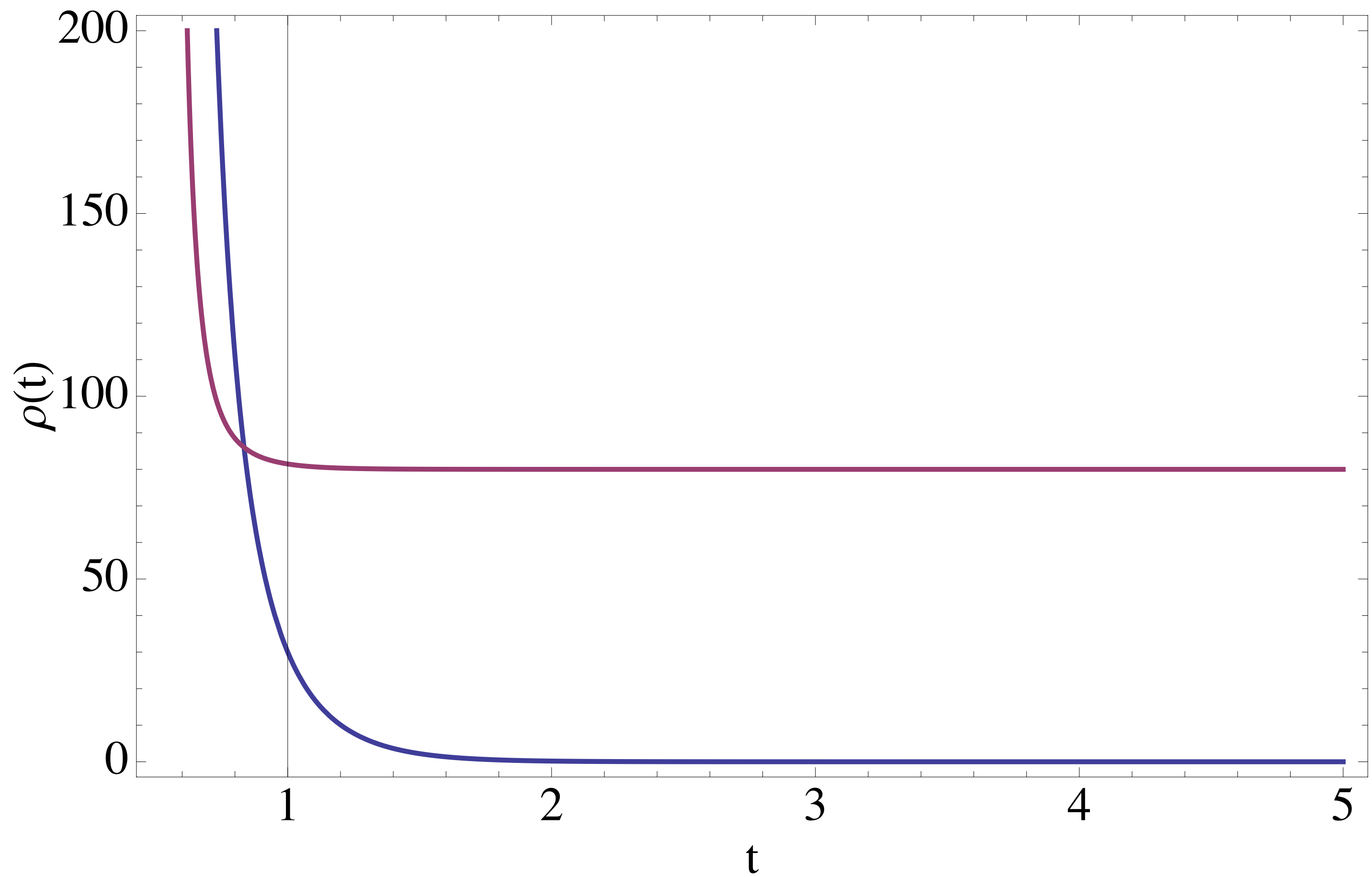

Outlook

- Model motivated by potential embedding of Standard Model in String Theory via Quiver Gauge Theories

- Cosmology: features of
    - modified gravity
    - K essence
    - quintessence

- Effective cosmological constant exponentially smaller than scale determining dynamics

- Finite density of Φ : negative spatial curvature

- Finite temperature

Extra Slides

$\mathcal{N}$=4 super-Yang-Mills theory and its orbifolds

Fields: $A_\mu, \ \Psi^i, \ \Phi^{ij}$

$SU(|\Gamma|N)$ gauge sym. $\Phi^{ij} \mapsto g^{-1}\Phi^{ij}g, \ \text{etc}$

$SO(6) \simeq SU(4) \ R$ sym. $\Psi^i \mapsto \gamma^i{}_j \Psi^j, \ \text{etc}$

Some integer

$$\mathcal{L} \sim -\frac{1}{2}\mathrm{Tr}(F^2) - \frac{1}{2}\mathrm{Tr}(D_\mu \Phi^A D^\mu \Phi_A) + \frac{1}{2}\mathrm{Tr}([\Phi^A, \Phi^B][\Phi_A, \Phi_B])$$

$$\Phi^{ij} = (\rho_A)^{ij}\Phi^A \qquad\qquad + \bar{\Psi}_i \not{D} \Psi^i + \bar{\Psi}_i (\rho_A)^i{}_j [\Phi^A, \Psi^j]$$

Discrete group with $|\Gamma|$ elem's: $\Gamma_\gamma \in SU(4)$ and $\Gamma_g \in SU(|\Gamma|N) \ \ \Gamma_\gamma = \Gamma_g$

- Truncate to fields invariant under simultaneous action of Γ_γ and Γ_g

$$A_\mu = g^{-1}A_\mu g \ , \ \Psi^i = \gamma^i{}_k \, g^{-1}\Psi^k g \ , \ \Phi^{ij} = \gamma^i{}_k \gamma^j{}_l \, g^{-1}\Phi^{kl} g$$

- planar perturbation theory is the same as that of $\mathcal{N}$=4 sYM theory

$\longrightarrow$ planar UV-finite; $\beta_{g_{YM}} = 0$ Bershadsky, Johansen; Bershadsky, Kakushadze, Vafa

- However, renormalizability requires addition at tree level of dim=4 2-trace op's, e.g. $|\mathrm{Tr}[g\Phi^A\Phi_A]|^2$; renormalize $\longrightarrow$ β-function

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$$1 + w = \frac{1}{3}\frac{2\dot{\zeta}^2 - \zeta(\ddot{\zeta} - H\dot{\zeta}) + 12\pi\sqrt{\frac{3}{2N}}|\zeta|\frac{n_0}{a(t)^2}}{V + \frac{1}{2}\dot{\zeta}^2 + H\zeta\dot{\zeta} + 12\pi\sqrt{\frac{3}{2N}}|\zeta|\frac{n_0}{a(t)^2}}$$