Chiral Symmetry in Holographic QCD

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Outline of Talk

1. Motivation.
2. The Chiral Lagrangian
3. Chiral Symmetry in Holographic QCD
4. Conclusions.

Motivation:

History:

- Son and Stephanov (2000): Chiral Lagrangian with μ_I – pions condense; phase transition is second order (@ $T = 0$).
  $\rightarrow$ based on symmetries.
- Kim, Kim and Lee (2007): No pion condensation in holographic QCD with μ_I.
  – But we expect the chiral Lagrangian is the low energy theory of holographic QCD.
- D.A. and Erlich (2010): Found pion condensation in holographic QCD, but with a first order phase transition.

Chiral Lagrangian

Symmetry considerations:

- The pattern of symmetry breaking: $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$.
- The pions, $\Sigma = \exp[2i\pi/f_\pi^2]$, transform as $\Sigma \to L\Sigma R^\dagger$.
- M_q is the source for explicit symmetry breaking.
- Full chiral $\mathrm{SU}(2)$ symmetry if $M_q \to L M_q R^\dagger$.

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The chiral Lagrangian (at E^2 in the energy expansion):

$$\mathscr{L}_{\text{eff}} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(\partial_\nu \Sigma \partial^\nu \Sigma^\dagger\right) + \frac{f_\pi^2 B_0}{2}\mathrm{Tr}\left(M_q \Sigma^\dagger + \Sigma M_q^\dagger\right).$$

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From this we find the GOR relation:

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2}\langle \bar{q} q \rangle = \frac{2m_q}{f_\pi^2}\sigma.$$

AdS/CFT

Recipe for model building:

AdS	$\leftrightarrow$	CFT
Fields	$\leftrightarrow$	Operators
Gauge fields in bulk	$\leftrightarrow$	Global Symmetry
KK modes in bulk	$\leftrightarrow$	States of CFT

Holographic QCD

We start with $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ gauge theory with bifundamental field $X(x,z)$:

$$S = \int d^5x \sqrt{g}\, \mathrm{Tr}\left\{ |DX|^2 + 3\,|X|^2 - \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right) \right\},$$

where $X(x,z)$ is dual to $q_L \overline{q}_R$.

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A slice of AdS space:

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right), \qquad \epsilon \le z \le z_m,$$

where ϵ plays the role of UV cutoff and z_m cuts off the geometry – modeling confinement.

Holographic QCD

Near boundary behavior of $X(x,z)$:
– Non-normalizable part $\sim m_q(x)z$.
– Normalizable part $\sim \langle q_L \overline{q}_R(x)\rangle z^3$.

Pattern of chiral symmetry breaking:
– The source m_q provides explicit breaking
– And the vev $\sigma \equiv \langle q\overline{q}\rangle = 2\langle q_L \overline{q}_R\rangle$ the spontaneous breaking.

$$\tilde{X}_0 = \frac{1}{2}\left(\tilde{m}_q z + \sigma z^3\right).$$

Normalization such that $\tilde{m}_q$ sources σ in the boundary theory.

Enter the Pions

We add the pion fluctuations

$$X(x,z) = (\tilde{X}_0 + \tilde{S}(x,z))e^{i2\tilde{\pi}^a(x,z)T^a},$$

where $\tilde{S}$ are the scalars and $\tilde{\pi}^a$ are the pions.

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KK decomposition: $\tilde{\pi}^a(x,z) = \sum \pi^a_n(x)\psi_n(z)$, and similar for $\tilde{S}(x,z)$.
We integrate out heavy physics, assuming

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Then

$$X(x,z) \to \tilde{X}_0 e^{i2\pi^a(x)\psi(z)T^a}.$$

Enter the Pions

At quadratic order in the pions

- Pick some natural boundary conditions.
- Solve the Sturm-Liouville problem for the lightest pion.

We can derive (GOR)

$$m_\pi^2 f_\pi^2 = 2\tilde{m}_q \sigma.$$

$\rightarrow$ We expect the 4D effective theory to be the chiral Lagrangian.

An Important Piont

Looking at the z-derivative term of the 5D Lagrangian,

$$\mathcal{L} \supset \partial_z X \partial_z X^\dagger.$$

If $X \to \exp\left[2i\pi^a(x)\psi(z)T^a\right]$,

$$\partial_z X \partial_z X^\dagger \to (\partial_z \psi(z))^2 \pi^a(x)\pi^a(x)$$

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$\rightarrow$ Motivated to try $X = \frac{\tilde{m}_q z}{2} + (\frac{\sigma z^3}{2} + S)e^{2i\pi}$.

Field Redefined $X(x,z)$

With $X = \frac{\tilde{m}_q z}{2} + \left(\frac{\sigma z^3}{2}\right) e^{2i\pi}$, the action has the form:

$$I = \int d^5x \, \mathrm{Tr}\left\{ \frac{\sigma^2 z^3}{4} (\partial_\mu U \partial^\mu U^\dagger - \partial_z U \partial_z U^\dagger) \right\} - \int d^4x \, \mathrm{Tr}\left\{ \frac{\tilde{m}_q \sigma}{4} (U + U^\dagger)\Big|_{z_m} \right\},$$

where $U = e^{2i\pi}$.

- Boundary term looks like the chiral Lagrangian mass term.

But it would imply a GOR-like relation

$$m_\pi^2 f_\pi^2 = -\tilde{m}_q \sigma.$$

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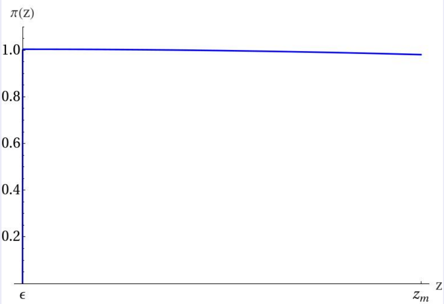
General considerations (symmetry, m_q as source) $\Rightarrow \tilde{m}_q = -2m_q$.

Different Background X_0 ($\tilde{m}_q = -2m_q$)

It turns out that $\psi(z) \approx 1$ over the entire interval:

$\psi(z)$



Different Background X_0 ($\tilde{m}_q = -2m_q$)

Changing to $X = -m_q z + (\frac{\sigma z^3}{2})e^{2i\pi}$, evaluating the action on the linearized EOMs, and performing the z-integrals we get

$$I = \int d^5x \, \mathrm{Tr}\left\{\frac{\sigma^2 z^3}{4}(\partial_\mu U \partial^\mu U^\dagger - \partial_z U \partial_z U^\dagger)\right\} + \int d^4x \, \mathrm{Tr}\left\{\frac{m_q \sigma}{2}(U + U^\dagger)\Big|_{z_m}\right\}.$$

$$\Downarrow$$

$$I = \int d^4x \, \mathrm{Tr}\left\{\frac{f_\pi^2}{4}\partial_\mu \Sigma \partial^\mu \Sigma^\dagger + \frac{m_\pi^2 f_\pi^2}{4}(\Sigma + \Sigma^\dagger)\right\}.$$

But why expand about different backgrounds?

An Example

Consider the Lagrangian:

$$\mathscr{L} = \frac{1}{2}\partial_N\phi_1\partial^N\phi_1 + \frac{1}{2}\partial_N\phi_2\partial^N\phi_2 - \frac{1}{2}M^2(\phi_1^2 + \phi_2^2),$$

with boundary conditions in the compact direction

$$\delta\phi_1 = 0, \quad \text{and} \qquad \partial_z\phi_2 = 0.$$

Transforming the fields $\phi_\pm = \frac{1}{\sqrt{2}}(\phi_1 \pm \phi_2) \Rightarrow$

$$\delta\phi_+ = -\delta\phi_-, \quad \text{and} \qquad \partial_z\phi_+ = \partial_z\phi_-.$$

What We've Found

Compare the two forms for $X(x,z)$:

$$X = \left(\frac{1}{2}(\tilde{m}_q z + \sigma z^3) + \tilde{S}\right) e^{2i\tilde{\pi}} \quad \text{v.s.} \quad X = \frac{\tilde{m}_q z}{2} + (\frac{\sigma z^3}{2} + S)e^{2i\pi}.$$

- Go from $(S,\pi) \to \left(\tilde{S}(S,\pi), \tilde{\pi}(S,\pi)\right)$.
- Unusual boundary conditions – nonlinear, mixing fields. - E.g. $\delta S = 0 \Rightarrow \delta\tilde{S} = -\delta f(\tilde{S},\tilde{\pi})$. (Not conventional Sturm-Liouville.)
- With these BCs, we need to change the background for both $\mathscr{L}(\tilde{S},\tilde{\pi})$ and $\mathscr{L}(S,\pi)$; $\tilde{m}_q \to -2m_q$.
- Expect to maintain the pattern of chiral symmetry breaking using either form for X.

To Conclude

Things to take away:

- We have a model which maintains the pattern of chiral symmetry.
- The chosen field representation admits a good Sturm-Liouville problem.
- The mathematical question of finding representations that allow good Sturm-Liouville systems – i.e, simple linear boundary conditions.