The Focus Point for Non-Zero *A*

David Sanford

Work with J.L. Feng
UC Irvine

Pheno 2012 - Tuesday May 8, 2012

Status of Supersymmetry

- LHC places stringent limits on the masses of colored superpartners (5 fb^{-1})

$$m_{\tilde{g}} \gtrsim 900 \text{ GeV}$$
$$m_{\{\tilde{u},\tilde{d},\tilde{c},\tilde{s}\}} \gtrsim 1.2 - 1.4 \text{ TeV}$$

- Heavy superpartners imply significant fine-tuning of the weak scale

What possibilities remain for natural weak-scale SUSY?

- Weaker heavy flavor bounds allow light $\tilde{t}$, reducing quadratic corrections
- Heavier sfermions with dynamically-generated naturalness

In Defense of Heavy Scalars

- Heavy scalars are consistent with the data!
  - Look for RPV, squashed spectra, dark matter detection, electroweak production, etc.
- Heavy stops are needed to produce a light Higgs mass of 125 GeV in the absence of large mixing
- Heavy scalars are more consistent with flavor and CP constraints

Focus point scenario provides for "dynamically natural" heavy scalars

Matchev and Feng (2000)

- Renormalization suppresses shifts in electroweak potential due to large scalar masses
- Previously realized in $A_0 = 0$ limit, requiring $m_{\tilde{t}} \sim 10$ TeV and $\sim 0.05\%$ fine-tuning for $m_h = 125$ GeV

Focus Point SUSY: Natural Heavy Scalars

- In the limit of moderate to large tan β, observed values for electroweak symmetry breaking require balancing the values of μ^2 and $m_{H_u}^2$

$$m_Z^2 \approx -2\mu^2 - 2m_{H_u}^2(m_W)$$

- If $m_{H_u}^2$ is small at the high scale fine-tuning will be low
- Fine-tuning will also be low if $m_{H_u}^2$ is large but runs to a small value due to RGEs
- Focus point provides a set of boundary conditions to produce this running
- For $A_0 = 0$, the CMSSM unified mass condition produces the proper running $\rightarrow$ "focus point region" of CMSSM

Wish to generalize to $A_0 \neq 0$

Renormalization Group and the Focus Point

- For moderate $\tan\beta$, the dominant RG behavior of $m_{H_u}^2$ and stop masses is proportional to y_t

$$\frac{d}{d\ln Q}\begin{bmatrix} m_{H_u}^2 \\ m_{\bar{U}_3}^2 \\ m_{Q_3}^2 \\ A_t^2 \end{bmatrix} = \frac{y_t^2}{8\pi}\begin{bmatrix} 3 & 3 & 3 & 3 \\ 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 12 \end{bmatrix}\begin{bmatrix} m_{H_u}^2 \\ m_{\bar{U}_3}^2 \\ m_{Q_3}^2 \\ A_t^2 \end{bmatrix}$$

- Decomposition into eigenvalues produces approximate RG trajectory

$$\begin{bmatrix} m_{H_u}^2(Q) \\ m_{\bar{U}_3}^2(Q) \\ m_{Q_3}^2(Q) \\ A_t^2(Q) \end{bmatrix} = \kappa_{12}\begin{bmatrix} 3 \\ 2 \\ 1 \\ 6 \end{bmatrix} e^{12I(Q)} + \kappa_6 \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix} e^{6I(Q)} + \kappa_0 \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} + \kappa_0' \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

$$I(Q) = \int_{\ln Q_0}^{\ln Q} \frac{y_t^2(Q')}{8\pi^2}\, d\ln Q'$$

Renormalization Group and the Focus Point

Specialize to CMSSM extension with modified scalar mass boundary condition

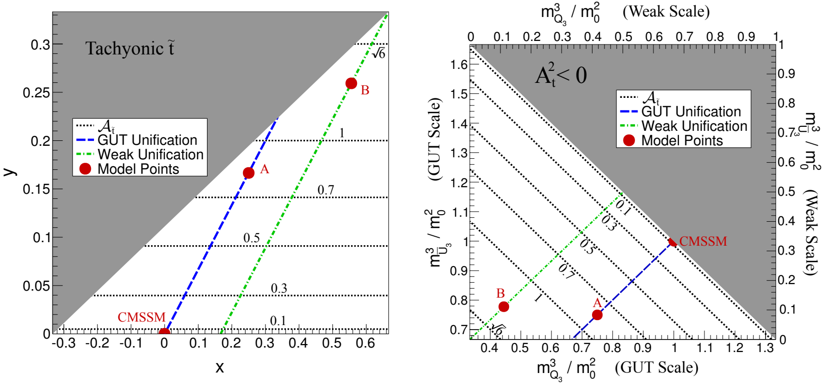
- Overall mass scale $m^2_{H_u}(m_{\mathrm{GUT}}) = m_0^2$
- Fine-tuning is reduced for $m^2_{H_u}(m_W) = 0$
- $e^{6I(m_W)} \simeq \frac{1}{3}$ for masses generated at $m_{\mathrm{GUT}} \simeq 10^{16}$ GeV

$$\begin{aligned}\left[m^2_{H_u}, m^2_{\bar{U}_3}, m^2_{Q_3}, A_t^2\right](m_{\mathrm{GUT}}) &= m_0^2\,[1, 1-x, 1+x-3y, 9y] \\ \left[m^2_{H_u}, m^2_{\bar{U}_3}, m^2_{Q_3}, A_t^2\right](m_W) &= m_0^2\left[1, \frac{2}{3}-x, \frac{1}{3}+x-3y, y\right]\end{aligned}$$

- Requring $\left\{m^2_{\bar{U}_3}, m^2_{Q_3}, A_t^2\right\}(m_W) \geq 0$ implies

$$0 \leq y \leq \frac{1}{3}, \qquad -\frac{1}{3}+3y \leq x \leq \frac{2}{3}$$

Parameter Space of the Focus Point



$$\mathcal{A}_{\tilde{t}} = \frac{|A_t(m_W)|}{\sqrt{\frac{1}{2}\left(m^2_{Q_3}(m_W) + m^2_{\bar{U}_3}(m_W)\right)}} = \frac{\sqrt{y}}{\sqrt{\frac{1}{2} - \frac{3}{2}y}}$$

Maximal contribution to Higgs mass at $\mathcal{A}_{\tilde{t}} \simeq \sqrt{6}$

Fine-Tuning Measure

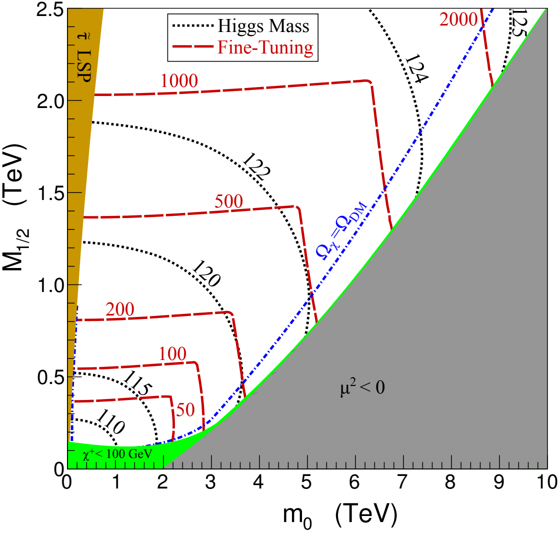
Fine-tuning is inherently subjective and fine-tuning measures fragile

- Highly model-dependent
- Focus is dependent on motivations
- "Unreasonable" level of fine-tuning varies from person to person
- Nevertheless, need numerical measure...

$$c_a \equiv \left| \frac{\partial \ln m_Z^2}{\partial \ln a^2} \right|$$

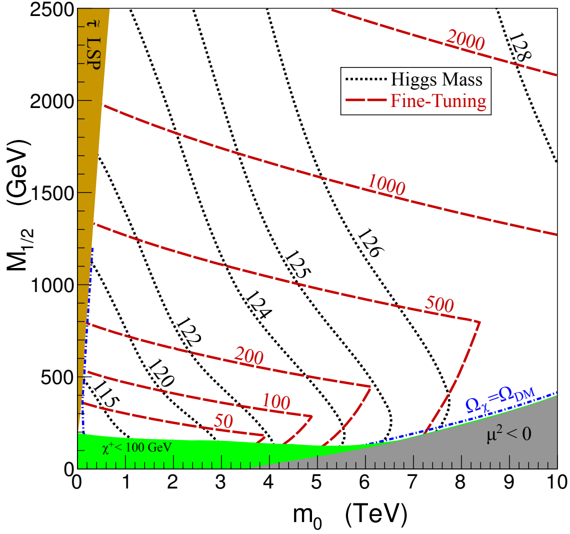
$$a \subset \{m_0, M_{1/2}, \mu, \sqrt{B}\}$$

Results: CMSSM ($x = 0$, $y = 0$)



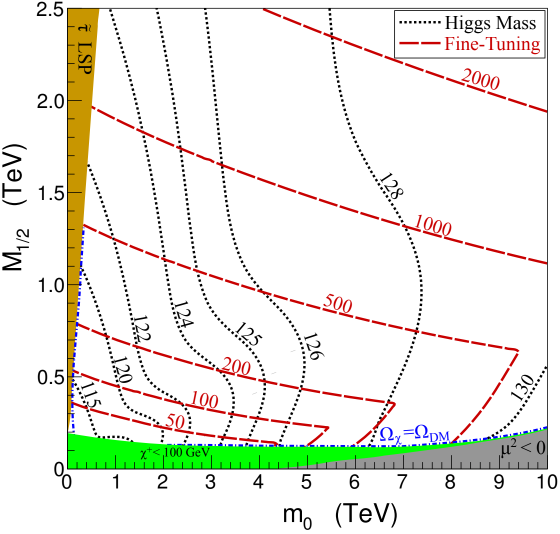
$c \sim 2000$ for $m_h = 125$ GeV

Results: Model A ($x = 1/4$, $y = 1/6$)



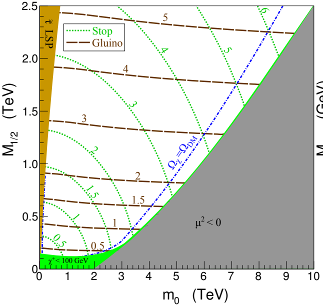
$c \sim 300$ for $m_h = 125$ GeV

Results: Model B ($x = 5/9$, $y = 7/27$)



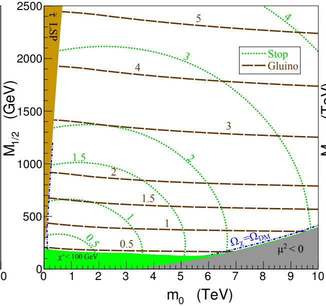
$c \sim 50$ for $m_h = 125$ GeV

$M_{\tilde{g}}$ and $m_{\tilde{t}_1}$ for $m_h = 125$ GeV



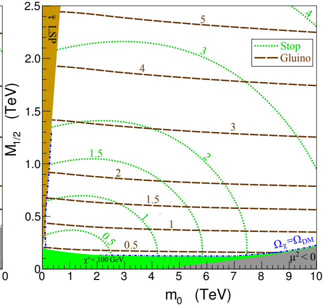
CMSSM

$m_{\tilde{t}_1} \sim 6$ TeV



Model A

$m_{\tilde{t}_1} \sim 2$ TeV



Model B

$m_{\tilde{t}_1} \sim 1$ TeV

Dark Matter Considerations

- $\Omega_\chi = \Omega_{\rm DM}$ curve found for $M_{1/2} \sim 150 - 200$ GeV

Ruled out by gluino searches: $m_{\tilde{g}} \gtrsim 900$ GeV

Significant overdensity for larger $M_{1/2}$ in accordance with gluino bound

- R-parity violation avoids constraint
- Can introduce gluino/axino dark matter
- Can produce consistency in MSSM wtih non-unified gaugino masses

At gluino bound, $c \simeq 300$ (Model A), 110 (Model B) for $m_h = 125$ GeV

Conclusion

- Heavy scalars can still be natural with appropriate boundary conditions!
- *A*-terms can be included while retaining naturalness to reduce fine-tuning for $m_h = 125$ GeV
- Can achieve $\mathcal{O}(.3 - .4\%)$ fine-tuning for models with GUT unification of stop masses and $\mathcal{O}(1\%)$ for more complicated boundary conditions
- Best results have lighter stops (but still ≥ 1 TeV), but require additional input for consistent dark matter

Backup Slides

Backup Slide: Deflection of the Focus Point

Why does the $\mu^2 < 0$ region move?

- Only considered part of $m^2_{H_u}$ RGE

$$\frac{dm^2_{H_u}}{d\ln Q} = \frac{1}{16\pi^2}\Big[2y_t^2\left(m^2_{H_u} + m^2_{Q_3} + m^2_{\bar{U}_3} + A_t^2\right) - 6g_2^2M_2^2 - \frac{6}{5}g_1^2M_1^2 + \frac{3}{5}g_1^2 S\Big]$$

$$S = m^2_{H_u} - m^2_{H_d} + \mathrm{tr}[m^2_Q - m^2_L - 2m^2_{\bar{U}} + m^2_{\bar{D}} + m^2_{\bar{E}}]$$

- Second line generally deflects $m^2_{H_u}$ to positive values
- Significant corrections to $m^2_{H_u}$ from deflection of A_t

$$\frac{dA_t}{d\ln Q} = \frac{1}{16\pi^2}\left[12y_t^2A_t + 2y_b^2A_b + \frac{32}{3}g_3^2M_3 + 6g_2^2M_2 + \frac{26}{15}g_1^2M_1\right]$$

- Negative contribution to $m^2_{H_u}$ for A_t, M_3 with opposite signs