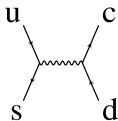


Correlations in Double Parton Scattering

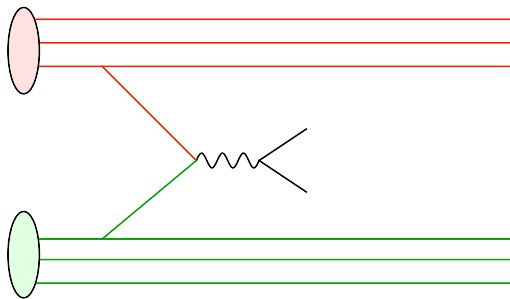
Wouter Waalewijn
UCSD



Pheno 2012

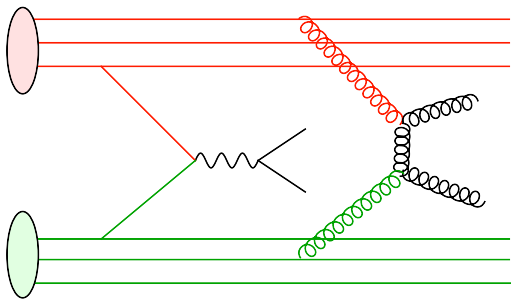
In collaboration with Aneesh Manohar
arXiv: 1202.3794, 1202.5034

What is Double Parton Scattering?



- ▶ Single parton scattering (SPS): one partonic collision for colliding protons

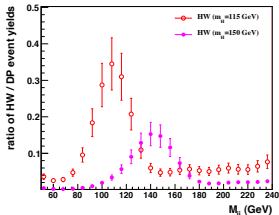
What is Double Parton Scattering?



- ▶ Single parton scattering (SPS): one partonic collision for colliding protons
- ▶ Double parton scattering (DPS): two partonic collisions (**not pile-up!**)
- ▶ DPS is $\Lambda_{\text{QCD}}^2/Q^2$ suppressed, only important when signal is small

Examples

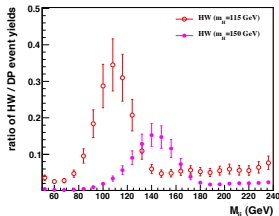
- ▶ Light Higgs searches, $pp \rightarrow WH \rightarrow \ell\nu b\bar{b}$: DPS background important [Del Fabbro, Treleani; Hussein; Berger, Jackson, Shaughnessy; Bandurin, Golovanov, Skachkov]



[Bandurin, Golovanov, Skachkov (1011.2186)]

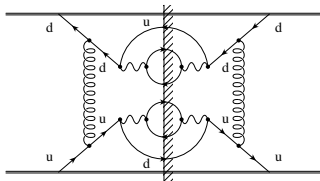
Examples

- ▶ Light Higgs searches, $pp \rightarrow WH \rightarrow \ell\nu b\bar{b}$: DPS background important [Del Fabbro, Treleani; Hussein; Berger, Jackson, Shaughnessy; Bandurin, Golovanov, Skachkov]



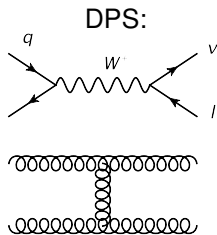
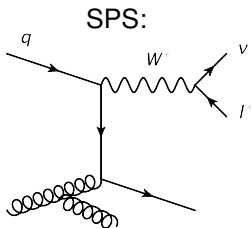
[Bandurin, Golovanov, Skachkov (1011.2186)]

- ▶ Same-sign leptons, $pp \rightarrow W^+W^+$: SPS requires additional jets [Kulesza, Stirling, Gaunt, Kom; Cattaruzza, Del Fabbro, Treleani; Meina]

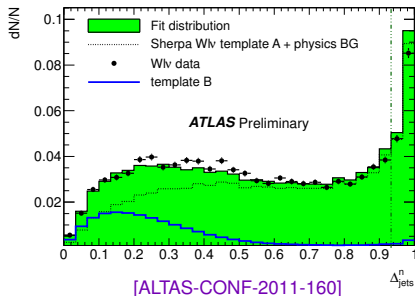


Examples

- $pp \rightarrow W + 2 \text{ jets}$: DPS jets are back-to-back in transverse plane

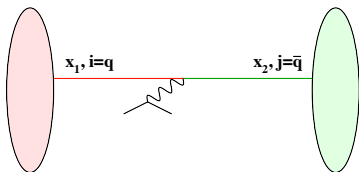


Double Parton Scattering at the LHC



- ▶ $pp \rightarrow W + 2 \text{ jets}$ with $p_T^{\text{jet}} > 20 \text{ GeV}$
- ▶ Use $\Delta_{\text{jets}}^n = |\vec{p}_T^{\text{jet},1} + \vec{p}_T^{\text{jet},2}| / (|\vec{p}_T^{\text{jet},1}| + |\vec{p}_T^{\text{jet},2}|)$
- ▶ DPS observed in 33 pb^{-1} of data (no pile-up)
- ▶ Fit to shape of SPS/DPS from Monte Carlo
→ extract fraction of DPS events $f_{\text{DPS}} = 16\%$

Calculating Cross Sections (Until Recently)

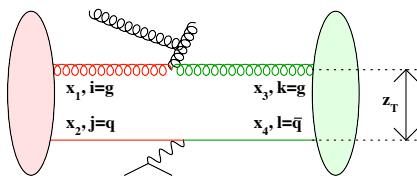


SPS:

$$d\sigma = \sum_{i,j} f_i(x_1) f_j(x_2) \hat{\sigma}_{ij}(x_1 x_2 s)$$

- ▶ PDF $f_i(x)$ is probability in x for finding parton of type $i = g, u, \bar{u}, d, \dots$
- ▶ $\hat{\sigma}_{ij}$ is partonic cross section for $ij \rightarrow$ final state

Calculating Cross Sections (Until Recently)



SPS:

$$d\sigma = \sum_{i,j} f_i(\mathbf{x}_1) f_j(\mathbf{x}_2) \hat{\sigma}_{ij}(\mathbf{x}_1 \mathbf{x}_2 s)$$

- ▶ PDF $f_i(\mathbf{x})$ is probability in \mathbf{x} for finding parton of type $i = g, u, \bar{u}, d, \dots$
- ▶ $\hat{\sigma}_{ij}$ is partonic cross section for $ij \rightarrow$ final state

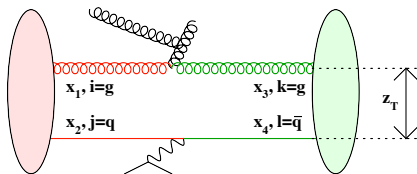
DPS:

$$d\sigma = \int d^2 z_T \sum_{i,j,k,l} F_{ij}(\mathbf{x}_1, \mathbf{x}_2, z_T) F_{kl}(\mathbf{x}_3, \mathbf{x}_4, z_T) \hat{\sigma}_{ik}(\mathbf{x}_1 \mathbf{x}_3 s) \hat{\sigma}_{jl}(\mathbf{x}_2 \mathbf{x}_4 s)$$

[Paver, Treleani]

- ▶ Double PDF $F_{ij}(\mathbf{x}_1, \mathbf{x}_2, z_T)$ depends on transverse separation z_T

Effective Cross Section

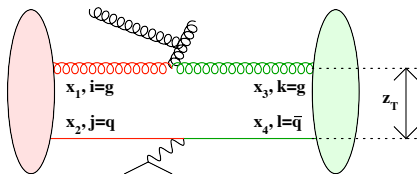


$$d\sigma = \int d^2 z_T \sum_{i,j,k,l} F_{ij}(x_1, x_2, z_T) F_{kl}(x_3, x_4, z_T) \hat{\sigma}_{ik}(x_1 x_3 s) \hat{\sigma}_{jl}(x_2 x_4 s)$$

Commonly-used assumptions:

- ▶ No correlation between x_i and z_T : $F_i(x_1, x_2, z_T) = F_i(x_1, x_2) F(z_T)$
- ▶ Uncorrelated partons: $F_{ij}(x_1, x_2) = f_i(x_1) f_j(x_2)$
Not valid at large x , since $x_1 + x_2 \leq 1$

Effective Cross Section



$$d\sigma = \int d^2 z_T \sum_{i,j,k,l} F_{ij}(x_1, x_2, z_T) F_{kl}(x_3, x_4, z_T) \hat{\sigma}_{ik}(x_1 x_3 s) \hat{\sigma}_{jl}(x_2 x_4 s)$$

Commonly-used assumptions:

- ▶ No correlation between x_i and z_T : $F_i(x_1, x_2, z_T) = F_i(x_1, x_2) F(z_T)$
- ▶ Uncorrelated partons: $F_{ij}(x_1, x_2) = f_i(x_1) f_j(x_2)$
Not valid at large x , since $x_1 + x_2 \leq 1$

Effective cross section:

$$\sigma \sim \sigma_1 \sigma_2 \int d^2 z_T F(z_T)^2 = \frac{\sigma_1 \sigma_2}{\sigma_{\text{eff}}}$$

- ▶ $\sigma_{\text{eff}} \sim 1/\Lambda_{\text{QCD}}^2 \sim \text{mB}$.
- ▶ Experimentally: $\sigma_{\text{eff}} \sim 5 - 15 \text{ mB}$

Full QCD analysis

Our work:

- ▶ QCD factorization of the cross section (**without assumptions**)
- ▶ Field-theoretic definition of double PDFs → **derive** RGE and properties
- ▶ Addresses obstacles:
 - ▶ Rapidity divergences complicate factorization
 - ▶ Overlap between single and double PDFs

Full QCD analysis

Our work:

- ▶ QCD factorization of the cross section (without assumptions)
- ▶ Field-theoretic definition of double PDFs → derive RGE and properties
- ▶ Addresses obstacles:
 - ▶ Rapidity divergences complicate factorization
 - ▶ Overlap between single and double PDFs

Factorization for double Drell-Yan [Manohar, WW]

$$\begin{aligned}
 \frac{d\sigma^{\text{DPS}}}{dq_1^2 dY_1 dq_2^2 dY_2} &= \left(\frac{4\pi\alpha^2 Q_q^2}{3N_c s} \right)^2 \frac{1}{q_1^2 q_2^2} \int d^2 z_T \left\{ \left[F_{qq}^1 F_{\bar{q}\bar{q}}^1 + F_{q\bar{q}}^1 F_{\bar{q}q}^1 + F_{\Delta q\Delta q}^1 F_{\Delta\bar{q}\Delta\bar{q}}^1 + F_{\Delta q\Delta\bar{q}}^1 F_{\Delta\bar{q}\Delta q}^1 \right] \right. \\
 &+ \frac{2N_c}{C_F} \left[(F_{qq}^T F_{\bar{q}\bar{q}}^T + F_{\Delta q\Delta q}^T F_{\Delta\bar{q}\Delta\bar{q}}^T) + (F_{q\bar{q}}^T F_{\bar{q}q}^T + F_{\Delta q\Delta\bar{q}}^T F_{\Delta\bar{q}\Delta q}^T) \right] S^{TT} \\
 &+ \frac{1}{2} \left[(I_{\bar{q}q}^1 + I_{\Delta\bar{q}\Delta q}^1)(I_{q\bar{q}}^1 + I_{\Delta q\Delta\bar{q}}^1) + I_{\delta\bar{q}\delta q}^1 I_{\delta q\delta\bar{q}}^1 \right] S_I^{11} \\
 &+ \frac{N_c}{2} \left[(I_{\bar{q}q}^T + I_{\Delta\bar{q}\Delta q}^T)(I_{q\bar{q}}^T + I_{\Delta q\Delta\bar{q}}^T) + I_{\delta\bar{q}\delta q}^T I_{\delta q\delta\bar{q}}^T + (1 \leftrightarrow T) \right] S_I^{T1} \\
 &\left. + \frac{N_c}{C_F} \left[(I_{\bar{q}q}^T + I_{\Delta\bar{q}\Delta q}^T)(I_{q\bar{q}}^T + I_{\Delta q\Delta\bar{q}}^T) + I_{\delta\bar{q}\delta q}^T I_{\delta q\delta\bar{q}}^T \right] S_I^{TT} + (q \leftrightarrow \bar{q}) \right\}
 \end{aligned}$$

- ▶ New effects: spin correlations, color correlations, interference in fermion number, soft radiation

Full QCD analysis

Our work:

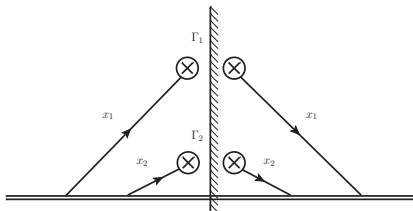
- ▶ QCD factorization of the cross section (without assumptions)
- ▶ Field-theoretic definition of double PDFs → derive RGE and properties
- ▶ Addresses obstacles:
 - ▶ Rapidity divergences complicate factorization
 - ▶ Overlap between single and double PDFs

Factorization for double Drell-Yan [Manohar, WW]

$$\frac{d\sigma^{\text{DPS}}}{dq_1^2 dY_1 dq_2^2 dY_2} = \left(\frac{4\pi\alpha^2 Q_q^2}{3N_c s}\right)^2 \frac{1}{q_1^2 q_2^2} \int d^2 z_T \left\{ \left[F_{qq}^1 F_{\bar{q}\bar{q}}^1 + F_{q\bar{q}}^1 F_{\bar{q}q}^1 + F_{\Delta q \Delta q}^1 F_{\Delta \bar{q} \Delta \bar{q}}^1 + F_{\Delta q \Delta \bar{q}}^1 F_{\Delta \bar{q} \Delta q}^1 \right] \right. \\ \left. + \frac{2N_c}{C_F} \left[(F_{qq}^T F_{\bar{q}\bar{q}}^T + F_{\Delta q \Delta q}^T F_{\Delta \bar{q} \Delta \bar{q}}^T) + (F_{q\bar{q}}^T F_{\bar{q}q}^T + F_{\Delta q \Delta \bar{q}}^T F_{\Delta \bar{q} \Delta q}^T) \right] S^{TT} \right. \\ \left. + \frac{1}{2} \left[(I_{\bar{q}q}^1 + I_{\Delta \bar{q} \Delta q}^1)(I_{q\bar{q}}^1 + I_{\Delta q \Delta \bar{q}}^1) + I_{\delta \bar{q} \delta q}^1 I_{\delta q \delta \bar{q}}^1 \right] S_I^{11} \right. \\ \left. + \frac{N_c}{2} \left[(I_{\bar{q}q}^T + I_{\Delta \bar{q} \Delta q}^T)(I_{q\bar{q}}^T + I_{\Delta q \Delta \bar{q}}^T) + I_{\delta \bar{q} \delta q}^T I_{\delta q \delta \bar{q}}^T + (1 \leftrightarrow T) \right] S_I^{T1} \right. \\ \left. + \frac{N_c}{C_F} \left[(I_{\bar{q}q}^T + I_{\Delta \bar{q} \Delta q}^T)(I_{q\bar{q}}^T + I_{\Delta q \Delta \bar{q}}^T) + I_{\delta \bar{q} \delta q}^T I_{\delta q \delta \bar{q}}^T \right] S_I^{TT} + (q \leftrightarrow \bar{q}) \right\}$$

- ▶ New effects: spin correlations, color correlations, interference in fermion number, soft radiation
- ▶ We find: color correlations and interferences are Sudakov suppressed!

Double PDF Definition

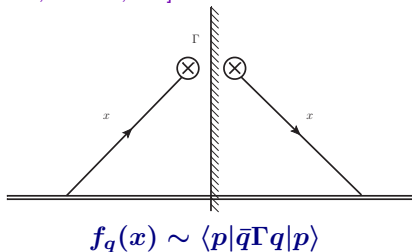


$$\begin{aligned}
 F_{qq}^1\left(\frac{q_1^-}{p_1^-}, \frac{q_2^-}{p_1^-}, z_T\right) &= -4\pi p_1^- \int \frac{dz_1^+}{4\pi} \frac{dz_2^+}{4\pi} \frac{dz_3^+}{4\pi} e^{-iq_1^- z_1^+/2} e^{-iq_2^- z_2^+/2} e^{iq_1^- z_3^+/2} \\
 &\quad \times \langle p_1 | \bar{T} \left\{ \left[\bar{\psi}(z_1^+, 0, z_T) \frac{\vec{\eta}}{2} \right]_a \left[\bar{\psi}(z_2^+, 0, 0_T) \frac{\vec{\eta}}{2} \right]_b \right\} \\
 &\quad T \left\{ \psi_a(z_3^+, 0, z_T) \psi_b(0) \right\} | p_1 \rangle
 \end{aligned}$$

- ▶ Like a PDF at z_T and 0_T
- ▶ Wilson lines ensure gauge invariance (not shown)

Spin Correlations for Single PDF

[Mekhfi; Diehl, Ostermeier, Schäfer; Manohar, WW]

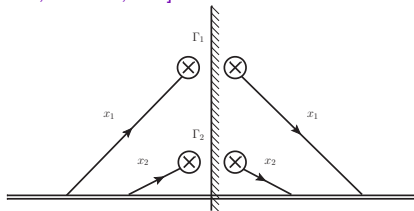


► Single PDF spin structures:

$\Gamma = \frac{1}{2} \vec{n}$	$q(x)$	unpolarized
$\Gamma = \frac{1}{2} \vec{n} \gamma_5$	$\Delta q(x)$	longitudinally polarized
$\Gamma = \frac{i}{2} \vec{n}_\nu \sigma^{\mu\nu} \gamma_5$	$\delta q(x)$	transversely polarized

Spin Correlations for Double PDF

[Mekhfi; Diehl, Ostermeier, Schäfer; Manohar, WW]



$$F_{qq}(x_1, x_2, z_T) \sim \langle p | \bar{q}_1 \Gamma_1 q_1 \bar{q}_2 \Gamma_2 q_2 | p \rangle$$

- ▶ Single PDF spin structures:

$$\Gamma = \frac{1}{2} \bar{\eta} \quad q(x) \quad \text{unpolarized}$$

$$\Gamma = \frac{1}{2} \bar{\eta} \gamma_5 \quad \Delta q(x) \quad \text{longitudinally polarized}$$

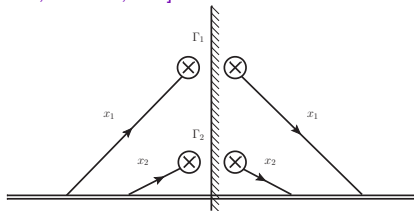
$$\Gamma = \frac{i}{2} \bar{n}_\nu \sigma^{\mu\nu} \gamma_5 \quad \delta q(x) \quad \text{transversely polarized}$$

- ▶ **Unpolarized** double PDF has spin correlations:

$$\Gamma_1 \otimes \Gamma_2 = \frac{1}{2} \bar{\eta} \otimes \frac{1}{2} \bar{\eta}, \quad \frac{1}{2} \bar{\eta} \gamma_5 \otimes \frac{1}{2} \bar{\eta} \gamma_5, \quad \frac{1}{2} \bar{\eta} \otimes \frac{i}{2} \bar{n}^\mu \sigma_{\mu\rho} \gamma_5 z_T^\rho, \dots$$

Spin Correlations for Double PDF

[Mekhfi; Diehl, Ostermeier, Schäfer; Manohar, WW]



$$F_{qq}(x_1, x_2, z_T) \sim \langle p | \bar{q}_1 \Gamma_1 q_1 \bar{q}_2 \Gamma_2 q_2 | p \rangle$$

- ▶ Single PDF spin structures:

$$\Gamma = \frac{1}{2} \not{n} \quad q(x) \quad \text{unpolarized}$$

$$\Gamma = \frac{1}{2} \not{n} \gamma_5 \quad \Delta q(x) \quad \text{longitudinally polarized}$$

$$\Gamma = \frac{i}{2} \bar{n}_\nu \sigma^{\mu\nu} \gamma_5 \quad \delta q(x) \quad \text{transversely polarized}$$

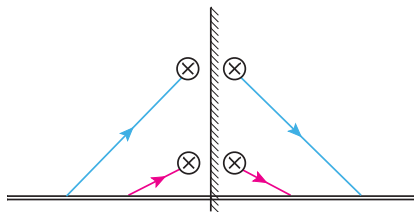
- ▶ **Unpolarized** double PDF has spin correlations:

$$\Gamma_1 \otimes \Gamma_2 = \frac{1}{2} \not{n} \otimes \frac{1}{2} \not{n}, \quad \frac{1}{2} \not{n} \gamma_5 \otimes \frac{1}{2} \not{n} \gamma_5, \quad \frac{1}{2} \not{n} \otimes \frac{i}{2} \bar{n}^\mu \sigma_{\mu\rho} \gamma_5 z_T^\rho, \dots$$

- ▶ Interpretation:

$$F_{\Delta q \Delta q} \sim \langle p | (a_{1R}^\dagger a_{1R} - a_{1L}^\dagger a_{1L}) (a_{2R}^\dagger a_{2R} - a_{2L}^\dagger a_{2L}) | p \rangle$$

Color Correlations

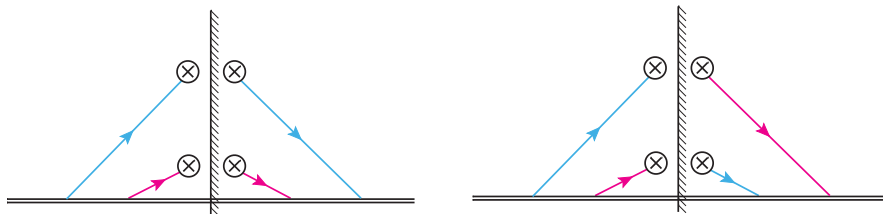


$$F_{q\bar{q}}(x_1, x_2, z_T) \sim \langle p | \bar{q}_1 \Gamma_1 q_1 \bar{q}_2 \Gamma_2 q_2 | p \rangle$$

- ▶ Double PDF color structures:

$$\Gamma_1 \otimes \Gamma_2 = \mathbf{1} \otimes \mathbf{1}, \quad T^A \otimes T^A$$

Color Correlations

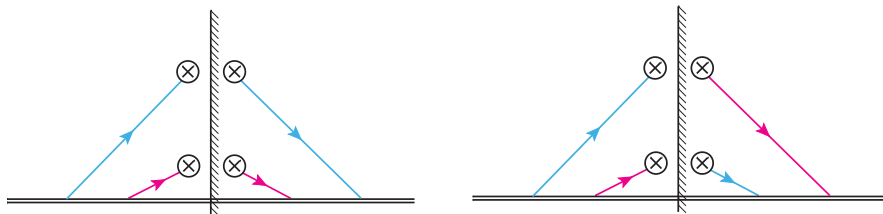


$$F_{q\bar{q}}(x_1, x_2, z_T) \sim \langle p | \bar{q}_1 \Gamma_1 q_1 \bar{q}_2 \Gamma_2 q_2 | p \rangle$$

- ▶ Double PDF color structures:

$$\Gamma_1 \otimes \Gamma_2 = 1 \otimes 1, \quad T^A \otimes T^A$$

Color Correlations



$$F_{q\bar{q}}(x_1, x_2, z_T) \sim \langle p | \bar{q}_1 \Gamma_1 q_1 \bar{q}_2 \Gamma_2 q_2 | p \rangle$$

- ▶ Double PDF color structures:

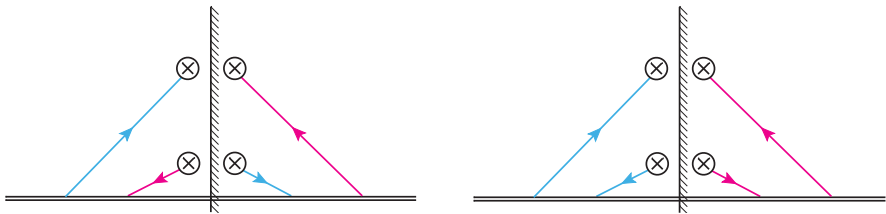
$$\Gamma_1 \otimes \Gamma_2 = \mathbf{1} \otimes \mathbf{1}, \quad T^A \otimes T^A$$

- ▶ $T^A \otimes T^A$ measures color correlation

$$F_{q\bar{q}}^{(1)} = F_{q\bar{q}}^1 + \frac{N^2 - 1}{2N} F_{q\bar{q}}^T, \quad F_{q\bar{q}}^{(8)} = F_{q\bar{q}}^1 - \frac{1}{2N} F_{q\bar{q}}^T$$

$$F_{qq}^{(6)} = F_{qq}^1 + \frac{N - 1}{2N} F_{qq}^T, \quad F_{qq}^{(\bar{3})} = F_{qq}^1 - \frac{N + 1}{2N} F_{qq}^T$$

Interference Effects

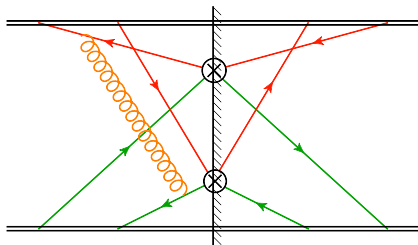


- ▶ Interference in fermion number is possible

$$I_{q\bar{q}}(x_1, x_2, z_T) \sim \langle p | \bar{q}_2 \Gamma_1 q_1 \bar{q}_2 \Gamma_2 q_1 | p \rangle$$

- ▶ The interference double PDF does not have to be real: $I_{q\bar{q}}^* = I_{\bar{q}q}$

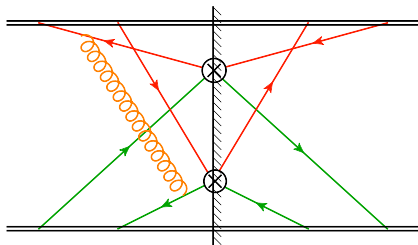
Soft Gluon Emissions



$$\sigma \sim \hat{\sigma} \hat{\sigma} \int d^2 z_T [F_{qq}^1(x_1, x_2, z_T) F_{\bar{q}\bar{q}}^1(x_3, x_4, z_T) S^{11}(z_T) + F_{qq}^T(x_1, x_2, z_T) F_{\bar{q}\bar{q}}^T(x_3, x_4, z_T) S^{TT}(z_T) + \dots]$$

- ▶ Soft radiation resolves large $z_T \sim 1/\Lambda_{\text{QCD}}$ separation

Soft Gluon Emissions



$$\sigma \sim \hat{\sigma} \int d^2 z_T [F_{qq}^1(x_1, x_2, z_T) F_{\bar{q}\bar{q}}^1(x_3, x_4, z_T) S^{11}(z_T) + F_{qq}^T(x_1, x_2, z_T) F_{\bar{q}\bar{q}}^T(x_3, x_4, z_T) S^{TT}(z_T) + \dots]$$

- ▶ Soft radiation resolves large $z_T \sim 1/\Lambda_{\text{QCD}}$ separation
- ▶ Soft gluon emissions summed in eikonal Wilson lines S_n and $S_{\bar{n}}$

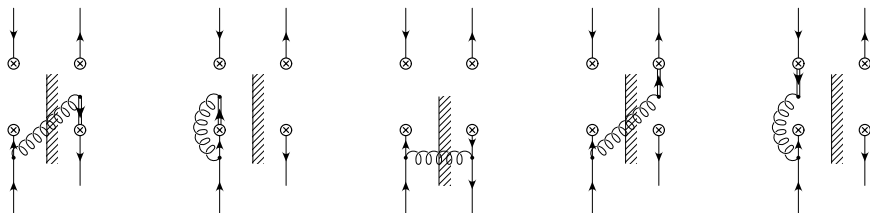
$$S^{11}(z_T) = \langle 0 | \text{tr}[S_{\bar{n}}^\dagger S_n S_n^\dagger S_{\bar{n}}](0) \text{tr}[S_{\bar{n}}^\dagger S_n S_n^\dagger S_{\bar{n}}](z_T) | 0 \rangle = \text{tr}[1]^2 = N_c^2$$

$$S^{TT}(z_T) = \langle 0 | \text{tr}[T^A S_{\bar{n}}^\dagger S_n T^B S_n^\dagger S_{\bar{n}}](0) \text{tr}[T^A S_{\bar{n}}^\dagger S_n T^B S_n^\dagger S_{\bar{n}}](z_T) | 0 \rangle$$

[Manohar, WW]

Double PDF Evolution

[Manohar, WW]

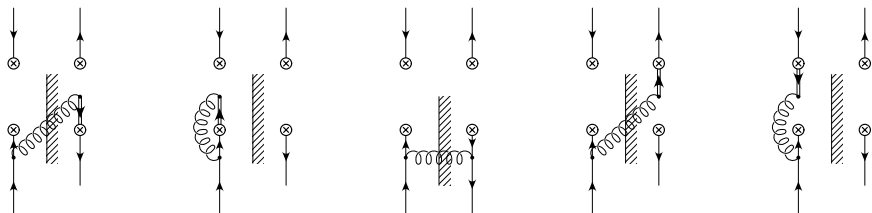


$$\gamma_{\mu}^{F^1} = \frac{\alpha_s C_F}{\pi} P_{qq}(x_1) \delta(1-x_2)$$

- ▶ $+(x_1 \leftrightarrow x_2)$ and mixing contributions not included
- ▶ F_{qq}^1 has the usual **splitting function** evolution

Double PDF Evolution

[Manohar, WW]

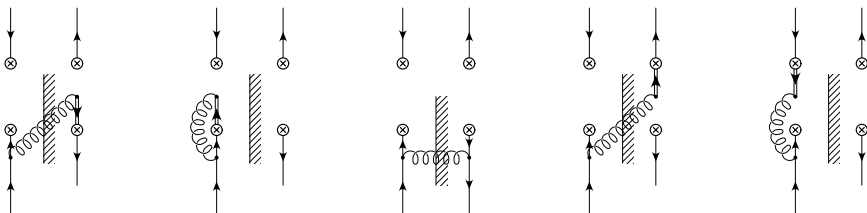


$$\gamma_{\mu}^{F^T} + \frac{1}{2}\gamma_{\mu}^{S^T} = \frac{\alpha_s}{\pi} \left[\left(C_F - \frac{1}{2}C_A \right) P_{qq}(x_1) + C_A \left(\frac{3}{4} - \ln \frac{p_1^-}{\mu} \right) \delta(1-x_1) \right] \delta(1-x_2)$$

- ▶ $(x_1 \leftrightarrow x_2)$ and mixing contributions not included
- ▶ Rapidity divergences require extra regulator, use [Chiu, Jain, Neill, Rothstein]
- ▶ F_{qq}^T has color factor $C_F - \frac{1}{2}C_A = -1/6 \rightarrow$ “reverse” evolution

Double PDF Evolution

[Manohar, WW]



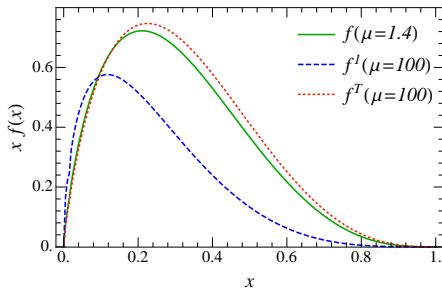
$$\gamma_{\mu}^{F^T} + \frac{1}{2}\gamma_{\mu}^{S^T} = \frac{\alpha_s}{\pi} \left[\left(C_F - \frac{1}{2}C_A \right) P_{qq}(x_1) + C_A \left(\frac{3}{4} - \ln \frac{p_1^-}{\mu} \right) \delta(1-x_1) \right] \delta(1-x_2)$$

- ▶ $(x_1 \leftrightarrow x_2)$ and mixing contributions not included
- ▶ Rapidity divergences require extra regulator, use [Chiu, Jain, Neill, Rothstein]
- ▶ F_{qq}^T has color factor $C_F - \frac{1}{2}C_A = -1/6 \rightarrow$ “reverse” evolution
- ▶ **Remaining terms** affect overall normalization \rightarrow Sudakov suppression:

$$U_{\mu}(\Lambda, Q) = \exp \left[- \frac{\alpha_s C_A}{\pi} \left(\frac{1}{2} \ln^2 \frac{Q^2}{\Lambda^2} - \frac{3}{2} \ln \frac{Q^2}{\Lambda^2} \right) \right]$$

[Agrees with Mekhfi, Artru]

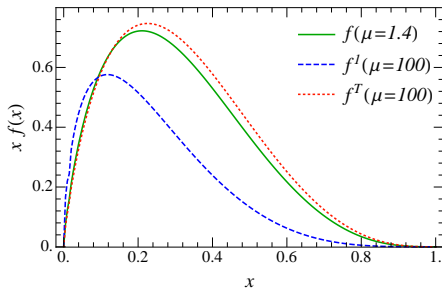
Numerical Results for Color Correlations



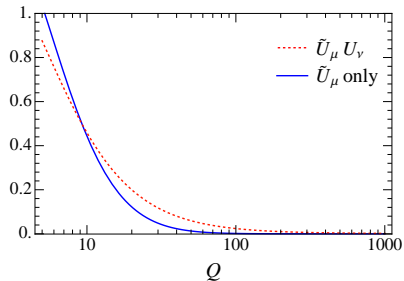
[Using HOPPET (Salam, Rojo)]

- ▶ The reverse evolution in x is shown for a sample PDF

Numerical Results for Color Correlations

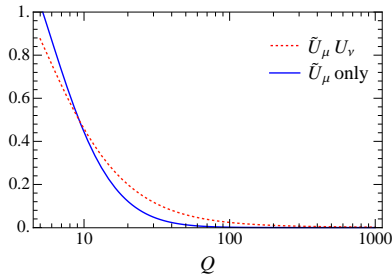
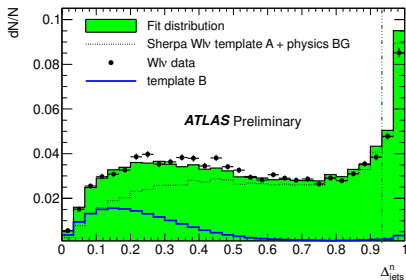


[Using HOPPET (Salam, Rojo)]



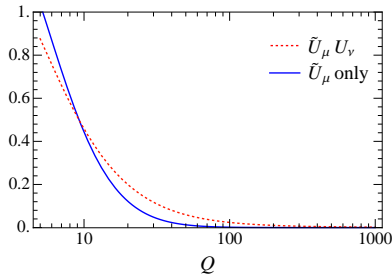
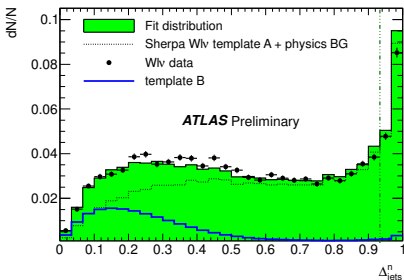
- ▶ The reverse evolution in x is shown for a sample PDF
- ▶ Sudakov double logarithmic suppression \tilde{U}_μ and Rapidity single logarithmic enhancement U_ν
- ▶ Mixing with single PDFs generates color correlations and is not included

Conclusions



- ▶ Double parton scattering has been observed
- ▶ Higher twist but important for e.g. light Higgs
- ▶ Approximate description $\sigma \sim \sigma_1 \sigma_2 / \sigma_{\text{eff}}$
- ▶ Our QCD analysis:
 - ▶ Full factorization analysis of cross section
 - ▶ Field-theoretic definition of double PDF \rightarrow derive evolution
 - ▶ Color correlations and interferences are Sudakov suppressed

Conclusions



- ▶ Double parton scattering has been observed
- ▶ Higher twist but important for e.g. light Higgs
- ▶ Approximate description $\sigma \sim \sigma_1 \sigma_2 / \sigma_{\text{eff}}$
- ▶ Our QCD analysis:
 - ▶ Full factorization analysis of cross section
 - ▶ Field-theoretic definition of double PDF \rightarrow derive evolution
 - ▶ Color correlations and interferences are Sudakov suppressed

Thank You!