Can the scale-invariant NMSSM Naturally give 125 GeV Higgs?

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PHENO Symposium, Pittsburgh, May 7, 2012

Outline

1. **Introduction: Naturalness for SUSY Higgs**
2. **Higgs in NMSSM: $m_h \approx 125\mathrm{GeV}$, GUT, LEP bound, tuning**
   - Pull-down Region
   - Push-up Region
3. **Conclusions**

What can we learn if $m_h \approx 125\mathrm{GeV}$?

Recent reports from ATLAS and CMS collaborations:
First hints for Higgs at LHC with $m_h \approx 125\mathrm{GeV}$

If there is a $m_h \approx 125\mathrm{GeV}$, further implication for underlying new physics?

- A pure SM Higgs?
  – Wide mass range can be accommodated (including $125\mathrm{GeV}$) by varying Higgs quartic coupling $\lambda = \frac{m_h}{v_{EW}}$;
  But well-known problem: **quadratic divergence of radiative correction to** $m_h \Rightarrow$ significant fine-tuning related to gauge hierarchy M_{pl}/M_{EW}...

- A SUSY Higgs? – *supersymmetry:* elegant solution to cancel quadratic divergence in δm_h^2
  But in MSSM m_h is "restricted" to be light at tree-level:

$$m_h^2 \approx m_Z^2 \cos^2 2\beta + \frac{3}{(4\Pi)^2}\frac{m_t^4}{v^2}\left[\ln\frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_t^2}\left(1 - \frac{X_t^2}{12 m_{\tilde{t}}^2}\right)\right]$$

  where $X_t = A_t - \mu\cos\beta$, $m_h^{\text{tree}} = m_Z \cos 2\beta \lesssim 90\mathrm{GeV}$,
  Get to $125\mathrm{GeV}$? **Large loop correction needed**:

  1. $X_t \sim 0, m_{\tilde{t}} \sim 5 - 10\mathrm{TeV}$
  2. Maximal $m_{\tilde{t}}$ mixing, large $X_t \approx \sqrt{6} m_{\tilde{t}} \gtrsim 1.5\mathrm{TeV}$

  – both reintroduce finetuning through $\delta m_{H_u}^2$
  (Draper, Meade, Reece and Shih *arxiv: 1112.3068*, Hall, Pinner and Ruderman *arxiv: 1112.2703*...)
  $\Rightarrow$ $125\mathrm{GeV}$ Higgs in MSSM betray Naturalness!

Does this mean $125\mathrm{GeV}$ *Higgs threatens natural SUSY "in general"?*

- No! That's only for MSSM $=$ **Minimal** supersymmetric SM. Non-minimal SUSY models, with extensions?...
- Theoretical appeal of SUSY: worth giving a harder try...

Interest of our work:

- ?Natural? in all aspects to get $125\mathrm{GeV}$ Higgs in a minimal, well-motivated extension of MSSM: scale-invariant NMSSM
  Natural: no EW tuning (m_Z, m_h), no tuning in model parameters

- Existing works for $125\mathrm{GeV}$ Higgs in scale-invariant NMSSM: pick benchmark points from numerical scan over all parameters
  Ellwanger *arXiv:1112.3548*, Kang, Li and Li *arxiv: 1201.5305*...
  – 'black-box'-like, hard to see hint for underlying UV physics, hidden new source of tuning...
- **Our goal:** More analytic, systematic approach, clearer view
  Separate discussions for "**pushup**" and "**pulldown**" regions, different cases: with small A-terms for singlet (favors gauge mediation) or moderate A-terms (favors gravity/anomaly mediation), preserve perturbativity up to GUT scale or accept lower Landau pole
  $\Rightarrow$ hint, guidelines for viable UV model

Review of Higgs sector in Scale Invariant NMSSM

Scale invariant NMSSM:
Simple extension of MSSM by adding a singlet chiral superfield S with coupling $\lambda S H_u H_d$ in superpotential

- Generates μ-term when $\langle S \rangle \neq 0$– a neat solution for μ-problem, esp. for gauge mediation
- Generates extra quartic coupling $\lambda^2 |H_u H_d|^2 \Rightarrow$ potential of raising m_h with moderate λ
- Scale invariant: no dimensionful terms in superpotential (Z_3 protected)– do not reintroduce μ-term type problem

$$
\begin{aligned}
W_{NMSSM} &= \lambda S H_u H_d + \frac{\kappa}{3} S^3 \\
V_{\mathrm{soft}} &= m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + \lambda A_\lambda H_u H_d S + \frac{1}{3} \kappa A_\kappa S^3
\end{aligned}
$$

Mass matrix for CP even scalar sector $\mathcal{M}_h^2$:

$$\begin{pmatrix} \lambda^2 v^2 \sin^2 2\beta + m_Z^2 \cos^2 2\beta & 2rv^2 \cot 2\beta & 2\lambda^2 sv - 2v^2 R \\ \cdot & -2v^2 r + \frac{2\lambda\kappa s^2 + 2A_\lambda s}{\sin 2\beta} & -2Rv \cot 2\beta \\ \cdot & \cdot & \kappa s(4\kappa s + A_\kappa) + \frac{v^2}{s} A_\lambda \lambda \sin 2\beta \end{pmatrix}$$

where $r \equiv \left(\frac{\lambda^2}{2} - \frac{M_Z^2}{v^2}\right) \sin^2 2\beta$, $R = \frac{1}{v}\lambda(\kappa s + A_\lambda) \sin 2\beta$

– Higgs related,

Mass matrix for CP odd scalar sector $\mathcal{M}_A^2$:

$$\begin{pmatrix} \frac{2\lambda s(A_\lambda + \kappa s)}{\sin 2\beta} & \lambda v(A_\lambda - 2\kappa s) \\ \cdot & \frac{\lambda v^2 (A_\lambda + 4\kappa s) \sin 2\beta}{2s} - 3\kappa A_\kappa s \end{pmatrix}$$

– Provide additional bound from Υ decay etc.

Higgs mass in NMSSM:

$$m_h^2(NMSSM) = (M_H^2)_{11} + \delta m^2_{\rm mix} + \delta m^2_{loop}$$

where $(M_H^2)_{11} = \lambda^2 v^2 \sin^2 2\beta + m_Z^2 \cos^2 2\beta$–maximized at low $\tan\beta$, $\delta m^2_{\rm mix}$ is shift due to mixing, δm^2_{loop} is loop correction dominated by stop. :

- **Pull-down** region: $\delta m^2_{\rm mix} < 0$, when $(M_H^2)_{11} < (M_H^2)_{33}$, i.e. heavier singlet sector
- **Push-up** region: $\delta m^2_{\rm mix} > 0$, when $(M_H^2)_{11} > (M_H^2)_{33}$, i.e. lighter singlet sector

Insist on "electroweak naturalness":
$\mu \lesssim 200\mathrm{GeV}$, $m_{\tilde t} \lesssim 500\mathrm{GeV} \Rightarrow$ Require $m_{\tilde t}$ at natural value, focus on (major) contribution coming from $(M_H^2)_{11} + \delta m^2_{\rm mix}$

Overview: $125\mathrm{GeV}$ vs. tuning NMSSM parameters

$m_h \approx 125\mathrm{GeV}$ + Other constraints/preference:

- LEP bound: chargino bound $\Rightarrow \mu \gtrsim 105\mathrm{GeV}$; scalar with mass $\ll 115\mathrm{GeV}$ has $g_{ZZs} \lesssim 0.1 g_{ZZh}(SM)$...
- Perturbativity up to GUT scale: $\lambda \lesssim 0.7$ for $\kappa \approx 0$

Combining these considerations:
specific tuning of model parameters required in NMSSM.

Quantification of fine-tuning – sensitivity measure:

- Electroweak naturalness: $\Delta_{EW} = \max \left| \frac{\partial \log m_h^2(m_Z^2)}{\partial \log X_i} \right|$, natural: $\Delta_{EW} \lesssim 5$ (Barbieri, Giudice 1987)
- New type of naturalness: $\Delta_{NMSSM} = \max \left| \frac{\partial \log \delta m^2_{\mathrm{mix}}}{\partial \log X_i} \right|$ where $\delta m^2_{\mathrm{mix}} \equiv m_h^2(\mathrm{tree}) - (M_H^2)_{11}$, natural: $\Delta_{NMSSM} \lesssim 5$

Our finding: $\Delta_{NMSSM} \gtrsim 5$ (with moderate A_λ, A_κ) or accept Landau pole below GUT scale...

Analytic analysis: clear view before numerics

Recall $\mathcal{M}_h^2$ in basis of (h, H, S) where $\langle h \rangle = v_{EW}, \langle H \rangle = 0$:

$$\begin{pmatrix} \lambda^2 v^2 \sin^2 2\beta + m_Z^2 \cos^2 2\beta & 2rv^2 \cot 2\beta & 2\lambda^2 sv - 2v^2 R \\ \cdot & -2v^2 r + \frac{2\lambda\kappa s^2 + 2A_\lambda s}{\sin 2\beta} & -2Rv \cot 2\beta \\ \cdot & \cdot & \kappa s(4\kappa s + A_\kappa) + \frac{v^2}{s} A_\lambda \lambda \sin 2\beta \end{pmatrix}$$

Good approximation to focus on (h, S) sub-matrix? Easier to do analytic analysis...

- **"Pull-down":** $(M_h^2)_{11} < (M_h^2)_{33}$. In case of small A_λ, A_κ, $\Rightarrow \lambda v < 2\kappa s$, $\theta_{12,23} \sim \frac{\lambda v}{2\kappa s} \frac{\cos 2\beta}{\sin^2 2\beta} \theta_{13} \ll \theta_{13}$, can safely decouple H for m_h consideration
- **"Push-up":** $(M_h^2)_{11} > (M_h^2)_{33}$. In case of small A_λ, A_κ, $\Rightarrow \lambda v > 2\kappa s$, $\theta_{12,23} \sim \frac{\cos 2\beta}{\sin^2 2\beta} \theta_{13}$, can safely decouple H when $\tan\beta \approx 1$

Pull-down Region

Simplified mass matrix:

$$\mathcal{M}^2 = \begin{pmatrix} \lambda^2 v^2 \sin^2 2\beta + m_Z^2 \cos^2 2\beta & 2\lambda^2 s v - (2\lambda\kappa s v + \lambda A_\lambda v)\sin 2\beta \\ \cdot & 4\kappa^2 s^2 + A_\kappa \kappa s + \frac{v^2}{2s} A_\lambda \lambda \sin 2\beta \end{pmatrix}$$

$\Rightarrow$ Lighter mass eigenstate as SM Higgs: $\rho \equiv \frac{\kappa}{\lambda}$

$$m_h^2(\mathrm{tree}) \approx m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta - \lambda^2 v^2 \frac{\left[\sin 2\beta\left(1 + \frac{A_\lambda}{2\kappa s}\right) - \frac{1}{\rho}\right]^2}{1 + \frac{A_\kappa}{4\kappa s} + \frac{A_\lambda \sin 2\beta\, v^2}{8\kappa\rho s^3}}$$

In limit of $A_\lambda, A_\kappa \to 0$ (as expected in GMSB models):

$$m_h^2(\mathrm{tree}) \approx m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta - \lambda^2 v^2 \left(\sin 2\beta - \frac{1}{\rho}\right)^2$$

- **Preserving GUT perturbativity:** $\lambda \lesssim 0.7, \tan\beta \gtrsim 1.5$, $m_h(\mathrm{tree})^{\mathrm{max}} \approx 118\mathrm{GeV} < 125\mathrm{GeV}$!!
- Need loop contribution: **Electroweak naturalness** $m_{\tilde{t}} \lesssim 500\mathrm{GeV} \Rightarrow m_h(\mathrm{tree})^{\mathrm{need}} \gtrsim 110\mathrm{GeV}$

Tuning Vs. GUT

Tight space between $m_h^2(\mathrm{tree})^{\mathrm{max}}$ and $m_h^2(\mathrm{tree})^{\mathrm{need}}$
$\Rightarrow \delta m_h^2(\mathrm{mix}) \to 0$, fine cancelation between model parameters in NMSSM

- $A_\lambda, A_\kappa \to 0$: $\delta m_h^2(\mathrm{mix}) = -\lambda^2 \mathrm{v}^2 \left(\sin 2\beta - \frac{1}{\rho}\right)^2$
  $\Rightarrow$ tune: $\lambda - \kappa \sin 2\beta \approx 0 \Rightarrow \kappa \sim 0.6$

  $\Delta_{NMSSM} \sim |\frac{\partial \log \delta m^2_{\mathrm{mix}}}{\partial \log \lambda}| > 5$, i.e. worse than 20% tuning

  However, $\lambda \approx 0.7, \kappa \approx 0.6 \Rightarrow$ perturbativity breaks well below GUT scale!
  $\Rightarrow$ **In the limit of small A_λ, A_κ (expected in GMSB), NMSSM can not simultaneously preserve GUT and accommodate $125\mathrm{GeV}$ Higgs, even with tuning!**

- At least $A_\lambda \neq 0$: requiring $\lambda \lesssim 0.7$ for GUT, still need tuning

$$\delta m_h^2(\mathrm{mix}) \to 0, \text{ now: } \delta m_h^2(\mathrm{mix}) = -\lambda^2 \mathrm{v}^2 \frac{\left[\sin 2\beta\left(1+\frac{\mathrm{A}_\lambda}{2\kappa \mathrm{s}}\right)-\frac{1}{\rho}\right]^2}{1+\frac{\mathrm{A}_\kappa}{4\kappa \mathrm{s}}+\frac{\mathrm{A}_\lambda \sin 2\beta\ \mathrm{v}^2}{8\kappa\rho \mathrm{s}^3}},$$

tune: $(\lambda - \kappa \sin 2\beta)s - A_\lambda \approx 0$

  - Allow $\kappa \ll \lambda$, **can preserve GUT perturbativity**
  - **Worse than** 20% **tuning persists:** $A_\lambda - 2\mu \approx 0$

Other option: Give up (conventional) GUT, accept lower Landau pole: $\lambda \gtrsim 0.7$, tuning typically alleviated

Numerical results for Pull-down region

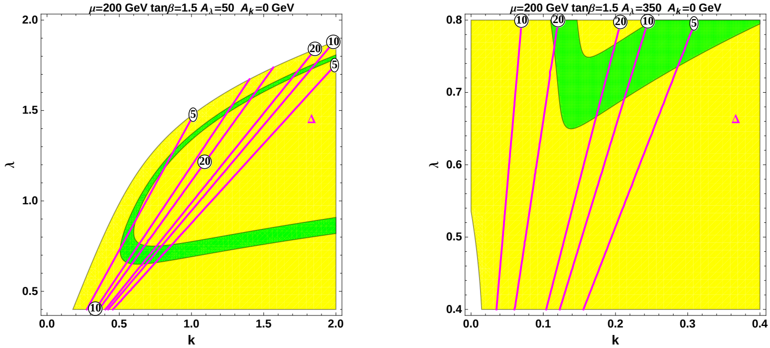

Figure: Allowed regions for "pull-down"scenario: yellow region allowed by vacuum condition and CP odd scalar bound, green band gives $110 - 125\mathrm{GeV}$ Higgs. Pink lines indicates tuning level Δ_{NMSSM}
Left: small A_λ, A_κ; **Right:** large A_λ

Push-up Region

Simplified mass matrix: taking $\tan\beta = 1$ to safely decouple H

$$\mathcal{M}^2 = \begin{pmatrix} \lambda^2 v^2 & 2\lambda^2 sv - (2\lambda\kappa sv + \lambda A_\lambda v) \\ \cdot & 4\kappa^2 s^2 + A_\kappa \kappa s + \frac{v^2}{2s} A_\lambda \lambda \end{pmatrix}$$

Can “push-up” do better than “pull-down”? naturally get $m_h \approx 125\mathrm{GeV}$ and preserve GUT?

- Hope: mixing now gives extra “increase” in m_h, may alleviate tension between $\lambda \lesssim 0.7$ and $m_h \approx 125\mathrm{GeV}$
- New constraint now: a singlet-like scalar $m_S < m_h$, LEP constraint on Z-coupling with scalar lighter than $115\mathrm{GeV} \Rightarrow$ limited “push-up” effect allowed from mixing

Simplified step-function for LEP bound on mixing as function of light scalar mass: (hep-ex/0602042)

$$\sin^2\theta \leq \begin{array}{ll} 0.01, & 0\mathrm{GeV} < \mathrm{m}_2 < 80\mathrm{GeV} \\ 0.1, & 80\mathrm{GeV} < \mathrm{m}_2 < 100\mathrm{GeV} \\ 0.4, & 100\mathrm{GeV} < \mathrm{m}_2 < 110\mathrm{GeV} \end{array}$$

In region where $\sin^2\theta \lesssim 0.1$, $m_2 < 100\mathrm{GeV}$,
$\tan 2\theta = 2M_{13}/(M_{11} - M_{33}) \ll 1$.
Expand the mass eigenvalues w.r.t.θ:

$$\begin{aligned} m_1^2 &= M_{11} + \theta^2(M_{11} - M_{33}) \\ m_2^2 &= M_{33} - \theta^2(M_{11} - M_{33}) \end{aligned}$$

$\Rightarrow$ **Bounds on matrix elements and degree of tuning defined by** $\Delta_{NMSSM} = \max |\frac{\partial \log \delta m^2_{\rm mix}}{\partial \log X_i}|$**:**

- Region-I: $\theta^2 = 0.1, 80 < m_2 < 100$, $m_1 = 110$
  $108^2 \lesssim M_{11} \lesssim 109^2$, $83 \lesssim M_{33} \lesssim 101^2$, $13^2 \lesssim M_{13} \lesssim 21^2 \Rightarrow$ $\sim 4\%$ tuning.
- Region-II: $\theta^2 = 0.01, 0 < m_2 < 80$, $m_1 = 110$
  $109.5^2 \lesssim M_{11} \lesssim 109.7^2$, $0 \lesssim M_{33} \lesssim 80^2$, $7.5^2 \lesssim M_{13} \lesssim 11^2$ $\Rightarrow < 1\%$ tuning.

– Worse tuning than "pull-down" case because of LEP bound!

Next: check implication on model parameters $\lambda, \kappa, A_\kappa, A_\lambda$

M_{11} lies in narrow region $108^2 - 110^2 \Rightarrow \lambda \approx 0.6$

- Case-I: $A_\kappa = 0, A_\lambda = 0$
  Bound on $M_{13} \Rightarrow \lambda \approx 0.6 \Rightarrow \kappa \approx \lambda \approx 0.6$
  $\Rightarrow M_{33} = 4\kappa^2 s^2 > M_{11}$! contradicts pushup condition $M_{11} > M_{33}$. **This region does not work in any way.**
- Case-II: $A_\kappa \neq 0, A_\lambda = 0$
  Still need tuning $\lambda \approx \kappa$, pushup condition and M_{33} bound can be accommodated by choosing proper value of $A_\kappa < 0$. **But not viable if we require perturbativity up to GUT scale because $\kappa \approx \lambda \approx 0.6$...**
- Case-III: $A_\kappa < 0$ and $A_\lambda > 0$
  3-parameter tuning $(\lambda - \kappa)s - A_\lambda/2 \approx 0$. Assuming $\kappa \ll \lambda$, tuning $A_\lambda - 2\mu \approx 0$, **this case viable, preserve perturbativity up to GUT scale with $< 5\%$ tuning.**

Numeric results for “push-up” scenario

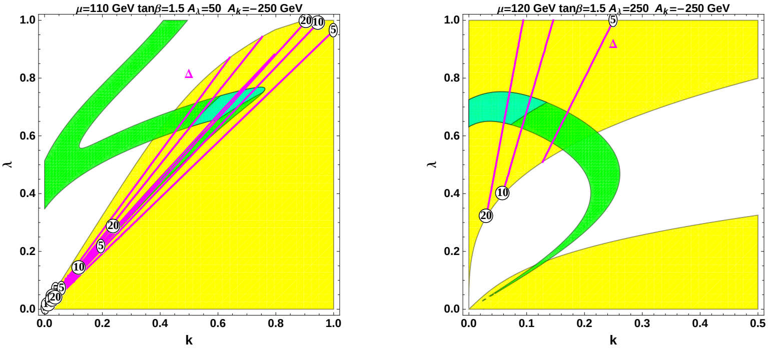

Figure: Allowed regions for “push-up”scenario: yellow region allowed by vacuum condition and CP odd scalar bound, green band gives $110 - 125\mathrm{GeV}$ Higgs, cyan region: also allowed by LEP. Pink lines indicates tuning level Δ_{NMSSM}. **Left:** small A_λ, $A_\kappa < 0$; **Right:** large $A_\lambda > 0$, $A_\kappa < 0$

Conclusions

We study the implication of a possible $125\mathrm{GeV}$ Higgs on scale-invariant NMSSM; analytic approach $\Rightarrow$ clearer picture:

- Preserving EW naturalness, $m_h \approx 125\mathrm{GeV}$ typically requires tuning/fine cancelation among model parameters in NMSSM to get $\delta m_h^2(\mathrm{mix}) \ll \mathrm{m_h^2}$
  Pressure of tuning from: GUT perturbativity or LEP bounds
- Sizeable A_λ, A_κ are necessary to get $m_h \approx 125\mathrm{GeV}$ and preserve perturbativity up to GUT scale, yet still in tuned way
- **The only region for good naturalness:** give up (conventional) GUT, allow low Landau pole

May provide guidelines for viable UV models that have NMSSM as EFT...

Backup Slides

Pull-down with no tuning

Other option: Give up GUT, accept lower Landau pole: $\lambda \gtrsim 0.7$, tuning typically alleviated (UV: Fat Higgs models...)
When $A_\lambda, A_\kappa \to 0$ rewrite as
$m_h^2(\mathrm{tree}) \approx \mathrm{m_Z^2} \cos^2 2\beta - \lambda^2 \mathrm{v}^2 \left(\frac{1}{\rho^2} - \frac{2}{\rho} \sin 2\beta\right)$ $(\rho \equiv \kappa/\lambda)$

- High Landau Pole $10\mathrm{TeV} \lesssim \Lambda < M_{\mathrm{GUT}}$: $\kappa < \lambda/(\sin 2\beta)$, may cause another type of tuning when taking $\rho \ll 1$, fine cancelation between two terms inside (...)
- Low Landau Pole $\Lambda \lesssim 10\mathrm{TeV}$: $\kappa > \lambda/(\sin 2\beta)$, natural in all aspects

Pushup Region-III: larger mixing region where $\tan 2\theta > 1$, now expand w.r.t $\epsilon = (M_{11} - M_{33})/(2M_{13}) < 1$:

$$m_1^2 = \frac{1}{2}\left[(1+\frac{1}{\epsilon})M_{11} + (1-\epsilon-\frac{1}{\epsilon})M_{33}\right]$$
$$m_2^2 = \frac{1}{2}\left[(1-\frac{1}{\epsilon})M_{11} + (1+\epsilon+\frac{1}{\epsilon})M_{33}\right]$$

$\Rightarrow$ Bounds on matrix elements:

- Region-III: $\sin^2\theta \approx 0.4, 100 \lesssim m_2 \lesssim 110, m_1 = 110$
  $107^2 \lesssim M_{11} \lesssim 111^2, 103^2 \lesssim M_{33} \lesssim 109^2, 33^2 \lesssim M_{13} \lesssim 46^2$

Comments for large mixing case:

- Large mass of lighter mass eigenstate as required by LEP bound $\Rightarrow$ *Large mixing is mostly due to near-degeneracy* $M_{11} \approx M_{33}$–a new type of tuning at $< 5\%$ level
- Still need $0.6 \lesssim \lambda \lesssim 0.65$, small $M_{13} \Rightarrow$ in case of $A_\lambda = A_\kappa = 0$, $0.55 \lesssim \kappa \lesssim 0.6$, violates pushup condition. Discussion about 3 cases of possible A-terms mostly follows that for small mixing case...