# Dark Vector-Gauge-Boson Model and the LHC Subhaditya Bhattacharya 

## UCRIVERSIDE

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## Vector Boson DM: Introduction

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- Existence of Dark Matter (DM) is well motivated:

- Nature of DM is debated: scalars, fermions or vector bosons?
- Vector Boson DM has already appeared in UED or little Higgs models.
- Our model predicts Vector Boson DM, which is a bit less exotic one!


## The Model: Outline

- A model based on $S U(3)_{C} \times S U(2)_{L} \times U(1)_{Y} \times S U(2)_{N}$
- $S U(2)_{N}$ vector gauge bosons $\left(X_{1}, X_{2}, X_{3}\right)$ are neutral and one of them can be DM


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- See if the DM is really a good one !
- LHC is on $\longrightarrow$ Any possible signatures?

$$
\begin{aligned}
& \binom{u}{d} \sim(3,2,1 / 6,1 ; 0), \quad u^{c} \sim\left(3^{*}, 1,-2 / 3,1 ; 0\right), \\
& \left(h^{c}, d^{c}\right) \sim\left(3^{*}, 1,1 / 3,2 ;-1 / 2\right), \quad h \sim(3,1,-1 / 3,1 ; 1), \\
& \left(\begin{array}{lr}
N & \nu \\
E & e
\end{array}\right) \sim(1,2,-1 / 2,2 ; 1 / 2), \quad\binom{E^{c}}{N^{c}} \sim(1,2,1 / 2,1 ; 0), \\
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- $Q=T_{3 L}+Y$ is electric charge, $L=S+T_{3 N}$ is generalized lepton number!
- $h, N, E, n^{c}$ are odd under $R$, SM ones are even.

Higgs sector: $S U(3)_{C} \times S U(2)_{L} \times U(1)_{Y} \times S U(2)_{N} \times S$
The Higgs sector consists of one bidoublet, two doublets, and one triplet:

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\begin{aligned}
& \left(\begin{array}{ll}
\phi_{1}^{0} & \phi_{3}^{0} \\
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\Delta_{2}^{0} / \sqrt{2} & \Delta_{3}^{0} \\
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- $\left\langle\phi_{1}^{0}\right\rangle=v_{1},\left\langle\phi_{2}^{0}\right\rangle=v_{2},\left\langle\Delta_{1}^{0}\right\rangle=u_{1}$, and $\left\langle\chi_{2}^{0}\right\rangle=u_{2},\left\langle\Delta_{3}^{0}\right\rangle=u_{3}$

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- $\phi_{3}, \chi_{1}{ }^{0}$ are odd under $R$


## Gauge Bosons

- Spontaneous breaking of $S U(2)_{N} \times S U(2)_{L} \times U(1)_{Y}$ yields:

$$
\begin{aligned}
& m_{W}^{2}=\frac{1}{2} g_{2}^{2}\left(v_{1}^{2}+v_{2}^{2}\right), \quad m_{X_{1,2}}^{2}=\frac{1}{2} g_{N}^{2}\left[u_{2}^{2}+v_{1}^{2}+2\left(u_{1} \mp u_{3}\right)^{2}\right] \\
& m_{Z, X_{3}}^{2}=\frac{1}{2}\left(\begin{array}{cc}
\left(g_{1}^{2}+g_{2}^{2}\right)\left(v_{1}^{2}+v_{2}^{2}\right) & -g_{N} \sqrt{g_{1}^{2}+g_{2}^{2}} v_{1}^{2} \\
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- SM gauge bosons and $X_{3}\left(=Z^{\prime}\right)$ are even $R, X_{1,2}$ have odd $R$
- All those odd under R : N, E, $n^{c}, h, \phi_{3}, \chi_{1}{ }^{0}, X_{1,2}$


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- SM gauge bosons and $X_{3}\left(=Z^{\prime}\right)$ are even $R, X_{1,2}$ have odd $R$
- All those odd under R : $N, E, n^{c}, h, \phi_{3}, \chi_{1}{ }^{0}, X_{1,2}$
- If $X_{1}$ is lighter than $X_{2}$, it is a DM !!


## $X_{1} X_{1}$ annihilation to SM model



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$$
\begin{aligned}
\sigma v_{r e l} & =\frac{g_{N}^{4} m_{X}^{2}}{72 \pi}\left[\sum_{h} \frac{3}{\left(m_{h}^{2}+m_{X}^{2}\right)^{2}}+\sum_{E} \frac{2}{\left(m_{E}^{2}+m_{X}^{2}\right)^{2}}\right. \\
& \left.+\frac{2}{\left(m_{\phi_{3}}^{2}+m_{X}^{2}\right)^{2}}+\frac{1}{m_{X}^{2}\left(m_{\phi_{3}}^{2}+m_{X}^{2}\right)}+\frac{3}{8 m_{X}^{4}}\right]
\end{aligned}
$$

## Direct dark matter search



- Spin-independent elastic cross section for $X_{1}$ scattering off a nucleus

$$
\sigma_{0}=\frac{1}{\pi}\left(\frac{m_{N}}{m_{X}}\right)^{2}\left|\frac{Z f_{p}+(A-Z) f_{n}}{A}\right|^{2} .
$$

Use ${ }^{73} G e$ with $Z=32$ and $A-Z=41$ in accordance with CDMS !

## Dark matter constraints on the model



Figure 1: $\delta=m_{h} / m_{X}-1$ versus $m_{X}$ (in TeV ) plot showing relic abundance and CDMS direct search bounds. $\sigma v_{\text {rel }}=0.91 \pm 0.05 \mathrm{pb}$

## Collider search

- A possible mass heirarchy :

$$
m_{h}>m_{X_{2}}>m_{E, N}>m_{X_{1}}
$$

- Consider at LHC:

- $h \longrightarrow X_{2} d$, then $X_{2} \longrightarrow E^{+} l^{-}, E^{-} l^{+}$, and $E^{+} \rightarrow X_{1} l^{+}$, $E^{-} \rightarrow X_{1} l^{-}$


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- $h \longrightarrow X_{2} d$, then $X_{2} \longrightarrow E^{+} l^{-}, E^{-} l^{+}$, and $E^{+} \rightarrow X_{1} l^{+}$, $E^{-} \rightarrow X_{1} l^{-}$
- Hence, $p p \rightarrow h X_{1}$ will end up with $\ell^{+} \ell^{-}+1 j$ et $+\not \mathscr{F}_{T}$

Signal: $\ell^{+} \ell^{-}+1$ jet $+\notin \nmid_{T}$

## How to tackle background?

Signal: $\ell^{+} \ell^{-}+1 j e t+\mathbb{E}_{T}$
How to tackle background?


Figure 2: Normalized signal and background distributions as functions of MET

## Event rates: $\ell^{+} \ell^{-}+1$ jet $+\notin T_{T}$

- $m_{X_{1}}=700 \mathrm{GeV}, m_{E, N}=735 \mathrm{GeV}, m_{X_{2}}=770 \mathrm{GeV}$, and $m_{h}=980 \mathrm{GeV}$

Event rates for $\ell^{+} \ell^{-}+1$ jet $+\mathbb{E}_{T}$ with $p_{T_{\ell}}>20, p_{T_{j}}>50$

| $\not \mathbb{E}_{T}>100$ |  | $\mathbb{E}_{T}>200$ |  | $\mathbb{E}_{T}>300$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Signal | Background | Signal | Background | Signal | Background |
| 3.07 | 237 | 1.6 | 0 | 0.59 | 0 |

Table 1: Event rates (fb) for $\ell^{+} \ell^{-}+1 j e t+\mathbb{E}_{T}$ channel at LHC with $E_{c m}=14 \mathrm{TeV}$

Signal: $\ell^{+} \ell^{-} \ell^{+} \ell^{-}+\not \notin T_{T}$

Consider: $p p \rightarrow X_{2} X_{2}$

and $X_{2} \longrightarrow \ell^{+} \ell^{-} X_{1}$. Hence, $p p \longrightarrow \ell^{+} \ell^{-} \ell^{+} \ell^{-}+\not{\not}_{T}$ (Negligible background through $Z Z Z$ or $4 W$ )

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Subhaditya Bhattacharya, 'Pheno', Pittsbergh, May 8, 2012

## Concluding remarks

- Relatively simpler model with vector boson dark matter !
- Identification of $X_{1}$ as dark-matter constrains the $S U(2)_{N}$ breaking scale to be about 1 TeV
- At LHC with $E_{c m}=14 \mathrm{TeV}$, we expect to see
$\ell^{+} \ell^{-}+1 j e t+\nrightarrow T_{T}$, even with luminosity $10 \mathrm{fb}^{-1}$, or a $\ell^{+} \ell^{-} \ell^{+} \ell^{-}+\not_{T}$, which are unique to models of this sort.


## Thank You

## Yukawa Interactions

$$
\begin{aligned}
& \left(d \phi_{1}^{0}-u \phi_{1}^{-}\right) d^{c}-\left(d \phi_{3}^{0}-u \phi_{3}^{-}\right) h^{c}, \quad\left(u \phi_{2}^{0}-d \phi_{2}^{+}\right) u^{c}, \quad\left(h^{c} \chi_{2}^{0}-d^{c} \chi_{1}^{0}\right) h, \\
& \left(N \phi_{3}^{-}-\nu \phi_{1}^{-}-E \phi_{3}^{0}+e \phi_{1}^{0}\right) e^{c}, \quad\left(E \phi_{2}^{+}-N \phi_{2}^{0}\right) n^{c}-\left(e \phi_{2}^{+}-\nu \phi_{2}^{0}\right) \nu^{c}, \\
& \left(E E^{c}-N N^{c}\right) \chi_{2}^{0}-\left(e E^{c}-\nu N^{c}\right) \chi_{1}^{0}, \quad n^{c} n^{c} \Delta_{1}^{0}+\left(n^{c} \nu^{c}+\nu^{c} n^{c}\right) \Delta_{2}^{0} / \sqrt{2}-\nu^{c} \nu^{c} \Delta_{3}^{0} .
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& \left(N \phi_{3}^{-}-\nu \phi_{1}^{-}-E \phi_{3}^{0}+e \phi_{1}^{0}\right) e^{c}, \quad\left(E \phi_{2}^{+}-N \phi_{2}^{0}\right) n^{c}-\left(e \phi_{2}^{+}-\nu \phi_{2}^{0}\right) \nu^{c}, \\
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- $\left\langle\phi_{1}^{0}\right\rangle=v_{1},\left\langle\phi_{2}^{0}\right\rangle=v_{2},\left\langle\Delta_{1}^{0}\right\rangle=u_{1}$, and $\left\langle\chi_{2}^{0}\right\rangle=u_{2}$, are scalar fields with $L=0$, while $\left\langle\Delta_{3}^{0}\right\rangle=u_{3}$, which breaks $L$ to $(-1)^{L}$. Thus $m_{d}, m_{e}$ come from $v_{1}$, and $m_{u}, m_{\nu \nu^{c}}\left(=-m_{N n^{c}}\right)$ come from $v_{2}$, whereas $m_{h}, m_{E}\left(=-m_{N N^{c}}\right)$ come from $u_{2}$, and $n^{c}, \nu^{c}$ obtain Majorana masses from $u_{1}$ and $u_{3}$. The scalar fields $\phi_{3}^{0,-}$ and $\Delta_{2}^{0}$ have $L=1$, whereas $\chi_{1}^{0}$ has $L=-1$ and $\Delta_{3}^{0}$ has $L=2$.


## Back up: Scalar Potential

$$
\begin{aligned}
V & =\mu_{1}^{2} \operatorname{Tr}\left(\phi_{13}^{\dagger} \phi_{13}\right)+\mu_{2}^{2} \phi_{2}^{\dagger} \phi_{2}+\mu_{\chi}^{2} \chi \chi^{\dagger}+\mu_{\Delta}^{2} \operatorname{Tr}\left(\Delta^{\dagger} \Delta\right) \\
& +\left(\mu_{22} \tilde{\chi} \phi_{13}^{\dagger} \tilde{\phi}_{2}+\mu_{12} \chi \Delta \tilde{\chi}^{\dagger}+\mu_{23} \tilde{\chi} \Delta \chi^{\dagger}+H . c .\right)+\frac{1}{2} \lambda_{1}\left[\operatorname{Tr}\left(\phi_{13}^{\dagger} \phi_{13}\right)\right]^{2}+\frac{1}{2} \lambda_{2}\left(\phi_{2}^{\dagger} \phi_{2}\right)^{2} \\
& +\frac{1}{2} \lambda_{3} \operatorname{Tr}\left(\phi_{13}^{\dagger} \phi_{13} \phi_{13}^{\dagger} \phi_{13}\right)+\frac{1}{2} \lambda_{4}\left(\chi \chi^{\dagger}\right)^{2}+\frac{1}{2} \lambda_{5}\left[\operatorname{Tr}\left(\Delta^{\dagger} \Delta\right)\right]^{2}+\frac{1}{4} \lambda_{6} \operatorname{Tr}\left(\Delta^{\dagger} \Delta-\Delta \Delta^{\dagger}\right)^{2} \\
& +f_{1} \chi \phi_{13}^{\dagger} \phi_{13} \chi^{\dagger}+f_{2} \chi \tilde{\phi}_{13}^{\dagger} \tilde{\phi}_{13} \chi^{\dagger}+f_{3} \phi_{2}^{\dagger} \phi_{13} \phi_{13}^{\dagger} \phi_{2}+f_{4} \phi_{2}^{\dagger} \tilde{\phi}_{13} \tilde{\phi}_{13}^{\dagger} \phi_{2}+f_{5}\left(\phi_{2}^{\dagger} \phi_{2}\right)\left(\chi \chi^{\dagger}\right) \\
& +f_{6}\left(\chi \chi^{\dagger}\right) \operatorname{Tr}\left(\Delta^{\dagger} \Delta\right)+f_{7} \chi\left(\Delta^{\dagger} \Delta-\Delta \Delta^{\dagger}\right) \chi^{\dagger}+f_{8}\left(\phi_{2}^{\dagger} \phi_{2}\right) \operatorname{Tr}\left(\Delta^{\dagger} \Delta\right) \\
& +f_{9} \operatorname{Tr}\left(\phi_{13}^{\dagger} \phi_{13}\right) \operatorname{Tr}\left(\Delta^{\dagger} \Delta\right)+f_{10} \operatorname{Tr}\left(\phi_{13}\left(\Delta^{\dagger} \Delta-\Delta \Delta^{\dagger}\right) \phi_{13}^{\dagger}\right),
\end{aligned}
$$

where

$$
\tilde{\phi}_{2}=\binom{\bar{\phi}_{2}^{0}}{-\phi_{2}^{-}}, \quad \tilde{\phi}_{13}=\left(\begin{array}{cc}
\phi_{3}^{+} & -\phi_{1}^{+} \\
-\bar{\phi}_{3}^{0} & \bar{\phi}_{1}^{0}
\end{array}\right), \quad \tilde{\chi}=\left(\bar{\chi}_{2}^{0},-\bar{\chi}_{1}^{0}\right),
$$

and the $\mu_{23}$ term breaks $L$ softly to $(-1)^{L}$.

## Back up: Neutrino masses through Seesaw

We have five neutral fermions per family. Two have odd $L$ parity, i.e. $\nu$ and $\nu^{c}$. Their $2 \times 2$ mass matrix is of the usual seesaw form, i.e.

$$
\mathcal{M}_{\nu}=\left(\begin{array}{cc}
0 & m_{D} \\
m_{D} & M_{3}
\end{array}\right),
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where $m_{D}$ comes from $v_{2}$ and $M_{3}$ from $u_{3}$. The other three have even $L$ parity, i.e. $N, N^{c}$, and $n^{c}$.

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where $m_{D}$ comes from $v_{2}$ and $M_{3}$ from $u_{3}$. The other three have even $L$ parity, i.e. $N, N^{c}$, and $n^{c}$. $3 \times 3$ mass matrix is given by

$$
\mathcal{M}_{N}=\left(\begin{array}{ccc}
0 & -m_{E} & -m_{D} \\
-m_{E} & 0 & 0 \\
-m_{D} & 0 & M_{1}
\end{array}\right),
$$

where $m_{E}$ comes from $u_{2}$ and $M_{1}$ from $u_{1}$. Since $(-1)^{L}$ is exactly conserved, $\nu, \nu^{c}$ do not mix with $N, N^{c}, n^{c}$.

