Supersymmetric Non-linear Sigma Model In The Warped Space

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Outlook

- Non-linear σ Model in Flat Extra Dimension
- Non-linear σ Model in Warped Extra Dimension
- Examples
- Summary

Flat Extra Dimension

Flat Extra Dimension (d=5)

- Hypermultiplet

$$(\mathrm{A}^i, \Psi^i), \quad i = 1, \ldots 2n \qquad \Psi^i = (\chi^i, \Omega^i{}_{j^*} \bar{\chi}^{j^*})$$

- Action

$$S = \int dx^5 \{ -g_{ij^*} \partial^M A^i \partial_M A^{*j^*} - \frac{i}{2} g_{ij^*} \bar{\Psi}^{j^*} \gamma^M D_M \Psi^i - V$$

$$+ \frac{i}{2} g_{ij^*} \nabla_k X^i \bar{\Psi}^{j^*} \Psi^k + \frac{1}{8} R_{ij^*kl^*} \bar{\Psi}^{j^*} \Psi^i \bar{\Psi}^{l^*} \Psi^k \}$$

$$V = g_{ij^*} X^i \bar{X}^{j^*}$$

Flat Extra Dimension (d=5)

- Transformation

$$\delta A^i = \bar{\varepsilon}_+ \Psi^i$$

$$\delta \Psi^i = i\gamma^M \varepsilon_+ \partial_M A^i + iX^i \varepsilon_+ - \Gamma^i_{jk} \delta A^j \Psi^k + \ldots$$

- Constraints

$$\nabla_i X_{j^*} + \nabla_{j^*} \bar{X}_i = 0$$

$$\nabla_j X^i + \Omega^{k^*}{}_j \nabla_{k^*} \bar{X}^{l^*} \Omega^i{}_{l^*} = 0$$

Warped Extra Dimension

From Flat to Warped Space

1. $\varepsilon \rightarrow$ Killing Spinor
2. New Scalar Potential V
3. New Constraints on X

5-d Anti-de Sitter

- Metric

$$ds^2 = e^{-2kz}\eta_{mn}dx^m dx^n + dz^2$$

- Killing Spinor

$$\tilde{D}_M \varepsilon_\pm = \partial_M \varepsilon_\pm + \frac{1}{4}\omega_M^{AB}\gamma_{AB}\varepsilon_\pm \mp \frac{i}{2}k\gamma_M \varepsilon_\pm = 0$$

5-d Anti-de Sitter

- Hypermultiplet

$$(\mathrm{A}^i, \Psi^i), \quad i = 1, \ldots 2n \qquad \Psi^i = (\chi^i, \Omega^i{}_{j^*} \bar{\chi}^{j^*})$$

- Action

$$S = \int dx^5 \sqrt{-G} \{ -g_{ij^*} \partial^M A^i \partial_M A^{*j^*} - \frac{i}{2} g_{ij^*} \bar{\Psi}^{j^*} \gamma^M D_M \Psi^i - V + \frac{i}{2} g_{ij^*} \nabla_k X^i \bar{\Psi}^{j^*} \Psi^k + \frac{1}{8} R_{ij^* kl^*} \bar{\Psi}^{j^*} \Psi^i \bar{\Psi}^{l^*} \Psi^k \}$$

$$V = g_{ij^*} X^i \bar{X}^{j^*} + kG(A, A^*)$$

5-d Anti-de Sitter

- Transformation

$$\delta A^i = \bar{\varepsilon}_+ \Psi^i$$

$$\delta \Psi^i = i\gamma^M \varepsilon_+ \partial_M A^i + iX^i \varepsilon_+ - \Gamma^i_{jk} \delta A^j \Psi^k + \dots$$

- Constraints

$$\nabla_i X_{j^*} + \nabla_{j^*} \bar{X}_i = 0$$

$$\nabla_j X^i + \Omega^{k^*}{}_j \nabla_{k^*} \bar{X}^{l^*} \Omega^i{}_{l^*} = -3ik\delta^i_j$$

$$4ig_{ij^*} \bar{X}^{j^*} = \frac{\partial}{\partial A^i} G$$

Model Building Procedure

1

- Pick up ($\mathcal{M}$, g, Ω)

2

- Solve X

3

- Construct G, V

Examples

Flat HyperKahler Space

- Flat Geometry

$$g_{ij^*} = \delta_{ij^*} \quad , \; \Omega_{ij} = \varepsilon_{ij} \quad , \; i = 1,2$$

- Solve Constraints

$$X^i \equiv K^i - \frac{3ik}{2}\left(\phi_1, \phi_2\right) \quad \Rightarrow K \text{ is tri-holomorphic Killing}$$

$$K = c_1\left(i\phi_2, i\phi_1\right) + c_2\left(\phi_2, -\phi_1\right) + c_3\left(i\phi_1, -i\phi_2\right)$$

- Determine Scalar Potential

Flat HyperKahler Space

- "Massless" Case

$$\text{If } K = 0, \quad \Rightarrow G_0 = -6k(\phi_1\phi_1^* + \phi_2\phi_2^*),$$

$$\Rightarrow V = \frac{-15}{4}k^2(\phi_1\phi_1^* + \phi_2\phi_2^*),$$

Mass Spectrum

$$m_1^2 = m_2^2 = \frac{-15k^2}{4}, \qquad m_{1/2} = 0$$

Flat HyperKahler Space

- Massive Case

$$K = c_1\left(i\phi_2, i\phi_1\right) + c_2\left(\phi_2, -\phi_1\right) + c_3\left(i\phi_1, -i\phi_2\right)$$

$$\Rightarrow G = G_0 + 4c_1\left(\phi_1\phi_2^* + \phi_2\phi_1^*\right)$$

$$+4ic_2\left(\phi_1\phi_2^* - \phi_2\phi_1^*\right) + 4c_3\left(\phi_1\phi_1^* - \phi_2\phi_2^*\right)$$

If $c_1 = c_2 = 0,\quad c_3 = \mu$

$$\Rightarrow V = \left(\mu^2 + \mu k - \frac{15k^2}{4}\right)\phi_1\phi_1^* + \left(\mu^2 - \mu k - \frac{15k^2}{4}\right)\phi_2\phi_2^*$$

$$m_1^2 = \mu^2 + \mu k - \frac{15k^2}{4},\quad m_2^2 = \mu^2 - \mu k - \frac{15k^2}{4},\quad m_{1/2} = \mu$$

HyperKahler Cone

- $$\begin{pmatrix} g_{u\bar{u}} & g_{u\bar{v}} \\ g_{v\bar{u}} & g_{v\bar{v}} \end{pmatrix} = e^{v+\bar{v}} \begin{pmatrix} 1 & \bar{u} \\ u & 1+u\bar{u} \end{pmatrix}, \quad \Omega_{uv} = e^{v+\bar{v}} = -\Omega_{vu}$$

- Solve Constraints

$$\begin{cases} \nabla_j Y^i = \delta^i_j \\ X \equiv K - \frac{3}{2} iY \end{cases} \Rightarrow \begin{cases} Y = (0,1) \\ K = c_1\left(-iu, \frac{i}{2}\right) + c_2\left(\frac{-u^2-1}{2}, \frac{u}{2}\right) + c_3\left(\frac{iu^2-i}{2}, \frac{-iu}{2}\right) \end{cases}$$

- If $X = -\frac{3}{2} ikY \quad \Rightarrow G = -6ke^{v+\bar{v}}(1+u\bar{u})$

$$m_u^2 = m_v^2 = \frac{-15k^2}{4}, \quad m_{1/2} = 0$$

Summary

- The Target Space of Non-linear Sigma Model in AdS_5 is HyperKahler Manifold.
- Killing Vector X Satisfies Inhomogeneous Tri-holomophic Condition.
- Target Space Tri-holomorphic Isometries Can be Gauged.
- Ready For Phenomenological Study.

Thank You!