NLL soft and Coulomb resummation for squark and gluino production at the LHC

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Based on: P. Falgari, C. Schwinn, CW [arXiv: 1202.2260 [hep-ph]]

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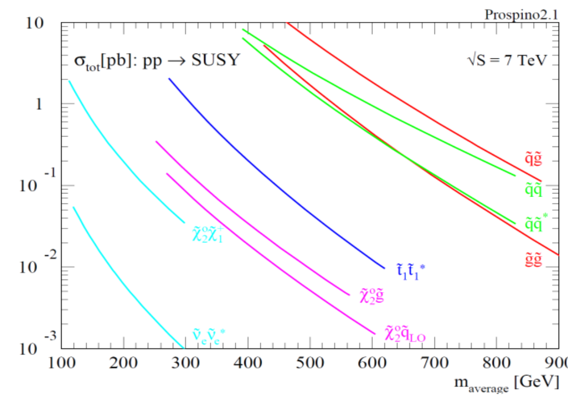
Motivation

- SUSY searches important at LHC
- In MSSM SUSY particles are pair produced
- Main production: squark and gluino pairs
- Strong exclusion bounds on masses



[Plehn, Prospino 2.1]

- Hadronic processes: $$PP \to \tilde{s}\tilde{s}' X \qquad \tilde{s}, \tilde{s}' = \text{squarks, gluinos}$$

- Partonic processes:

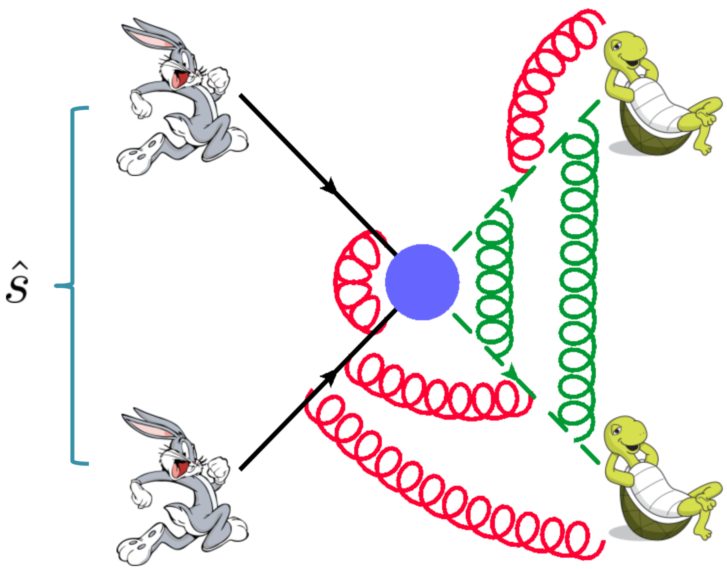
$$gg,\ q_i\bar{q}_j \to \tilde{q}\bar{\tilde{q}}$$
$$q_i q_j \to \tilde{q}\tilde{q} \qquad \bar{q}_i\bar{q}_j \to \bar{\tilde{q}}\bar{\tilde{q}}$$
$$g q_i \to \tilde{g}\tilde{q} \qquad g\bar{q}_i \to \tilde{g}\bar{\tilde{q}}$$
$$gg,\ q_i\bar{q}_i \to \tilde{g}\tilde{g}$$

- Primarily proceed through strong interactions ⟶ focus on QCD interactions

• Analytic LO calculations [Kane, Leveille ’82; Harisson, Smith ’83; Dawson, Eichten, Quigg ’85]
• Numeric NLO calculations [Beenakker et al. ’95-’97; Plehn, Prospino 2.1]

Threshold

- Partonic processes: $pp' \to \tilde{s}\tilde{s}' X$ $\quad p, p' = \text{partons}$, $\tilde{s}, \tilde{s}' = \text{squarks,gluinos}$
- Threshold region: $\beta := \sqrt{1 - \frac{(2M)^2}{\hat{s}}} \ll 1, \quad M := \frac{m_{\tilde{s}} + m_{\tilde{s}'}}{2}, \quad \hat{s} = \tau s = \text{partonic cm energy}$



Relevant modes at threshold:

Collinear: $k_- \sim M, k_+ \sim M\beta^2, k_\perp \sim M\beta$

Hard: $k \sim M$

Soft gluons: $k_0 \sim |k| \sim M\beta^2 \longrightarrow \alpha_s^n \ln^m \beta$

Potential (gluons): $k_0 \sim M\beta^2, |k| \sim M\beta \longrightarrow (\alpha_s/\beta)^n$

[Catani et al. '96; Becher, Neubert '06; Kulesza, Motyka '08; …] [Fadin, Khoze '87-'89; Fadin et al. '90; Kulesza, Motyka '09; …]

- Partonic cs enhanced near threshold by soft and coulomb corrections $\longrightarrow$ need to resum
- Threshold enhanced terms also approximate well away from threshold

$\alpha_s \ln \beta, \left(\frac{\alpha_s}{\beta}\right) \sim 1 \longrightarrow$

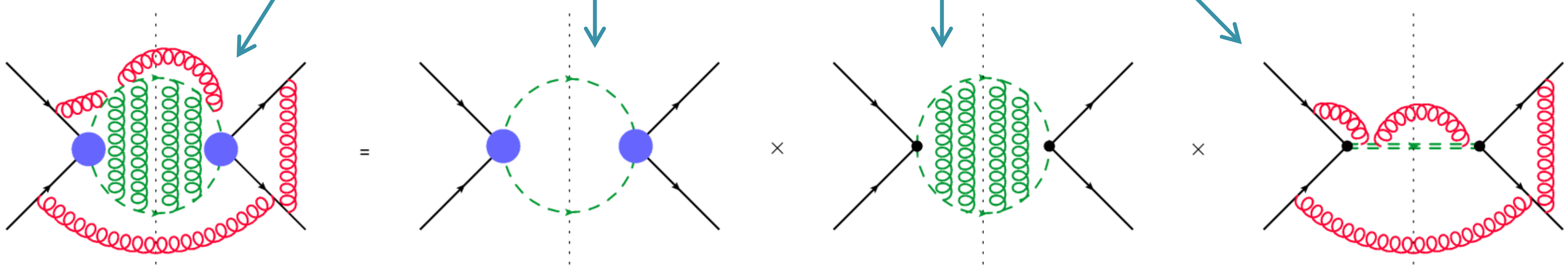
$$\hat{\sigma}_{pp'}(\hat{s}) \sim \hat{\sigma}^{(0)}_{pp'} \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \exp[\underbrace{g_0 \ln\beta(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{g_2\alpha_s(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \ldots]$$
$$\times \{1(\text{LL,NLL}); \alpha_s, \beta(\text{NNLL}); \alpha_s^2, \alpha_s\beta, \beta^2(\text{NNNLL}); \ldots\}$$

Factorization using EFT

- Hierarchy in scales: $M \gg M\beta \gg M\beta^2$ ⟶ use EFT
- Effective lagrangian: $\mathcal{L}_{EFT} = \underbrace{\mathcal{L}_{SCET}}_{\text{Collinear-soft}} + \underbrace{\mathcal{L}_{PNRQCD}}_{\text{Potential-soft}}$
- Field redefinitions: soft gluons decouple from collinear and potential modes at LO in β ⟶

$$\hat{\sigma}_{pp'}(\hat{s},\mu_f) = \sum_{R_\alpha} H^{R_\alpha}_{pp'}(m_{\tilde{q}}, m_{\tilde{g}}, \mu_f) \int d\omega \, J_{R_\alpha}(E - \frac{\omega}{2}) \, W^{R_\alpha}(\omega,\mu_f)$$

[Beneke, Falgari, Schwinn'10]



Partonic cross section | Hard contributions | Coulomb contributions (potential gluons exchange) | Soft contributions (soft gluons exchange)

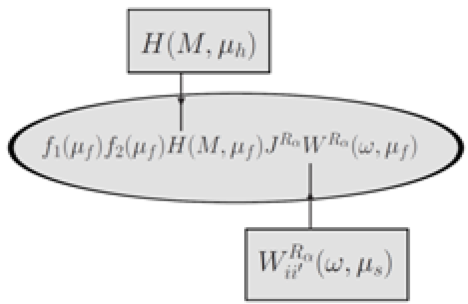
- Coulomb contributions also contain bound-state effects below threshold ($E = \sqrt{\hat{s}} - 2M < 0$)
- Factorization valid up to NNLL for S-wave processes

The factorization is also valid for P-wave processes (qqbar fusion in stop production) [FSW '12]

Resummation using RG flow

- H and W satisfy evolution equations ⟶ choose scales to minimize higher order corrections:

$\sigma(s) \sim$



$$\mu_h = 2k_h M$$

- RG flow resums logs $\ln(\frac{\mu_f}{\mu_h}), \ln(\frac{\mu_f}{\mu_s})$

$$\mu_f = \mu_R = k_f M, \quad \mu_C = k_C \mathrm{Max}\{2\alpha_s(\mu_C)m_r|D_{R_\alpha}|, 2\sqrt{2m_r M}\beta\}$$

$$\mu_s = k_s \mathrm{Max}\{M\beta^2, M\beta_{cut}^2\} \qquad k_f, k_h, k_C, k_s \sim 1$$

- Theoretical errors:

1) Scale variations: $\left\{ \begin{array}{l} \frac{1}{2} \leq k_f, k_h, k_C, k_s \leq 2 \\ 0.8\beta_{cut}^{(0)} \leq \beta_{cut} \leq 1.2\beta_{cut}^{(0)} \end{array} \right.$

2) Parameterization errors: $E = \sqrt{\hat{s}} - 2M, \quad \beta$

3) PDF and α_s errors

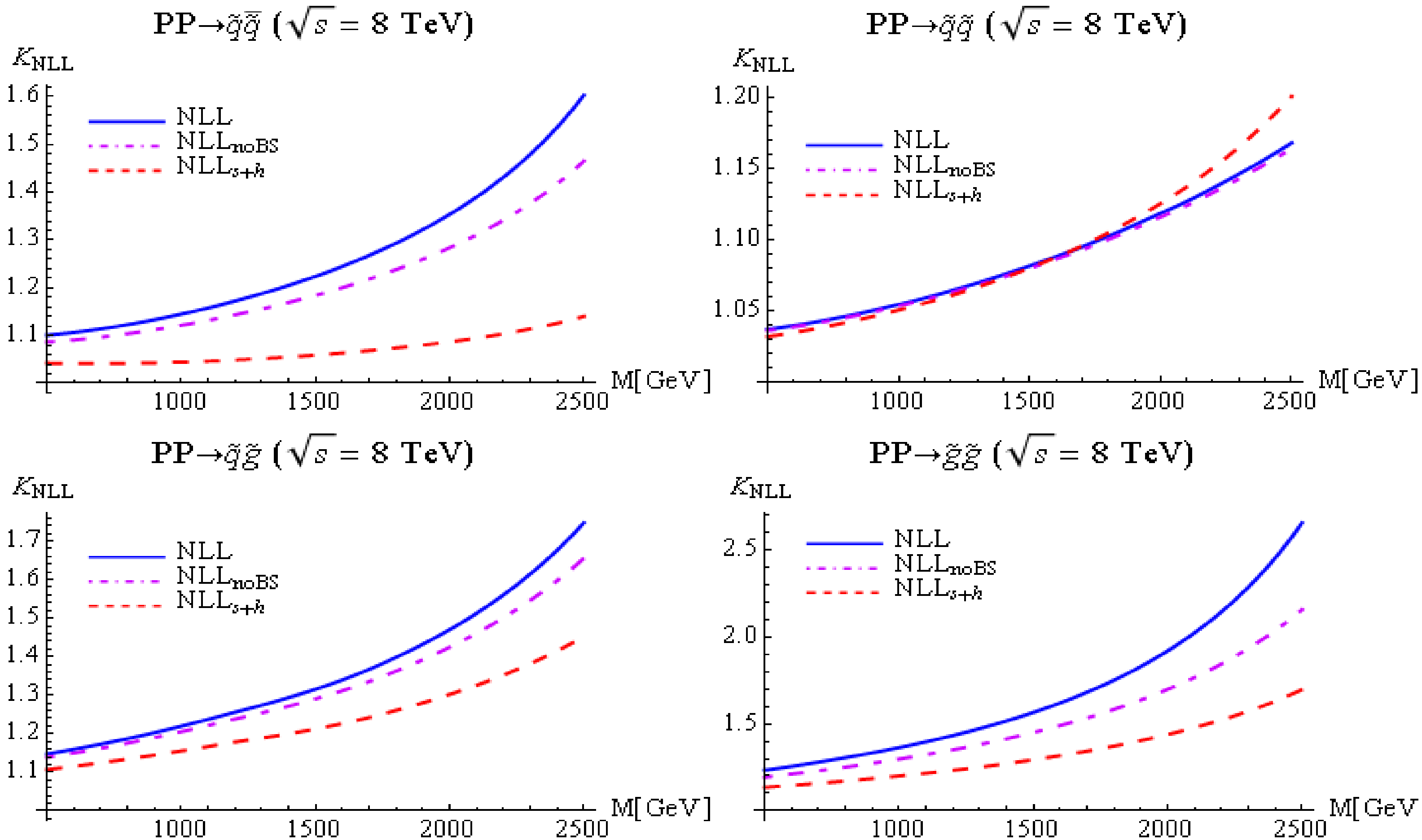
- Use MSTW2008NLO PDF's and match the cs to the full NLO result from Prospino 2.1 [Plehn]:

$$\hat{\sigma}_{pp'}^{\text{matched}}(\hat{s}) = [\hat{\sigma}_{pp'}^{\text{NLL}}(\hat{s}) - \hat{\sigma}_{pp'}^{\text{NLL(1)}}(\hat{s})] + \hat{\sigma}_{pp'}^{\text{NLO}}(\hat{s})$$

- NLL resummation ingredients: for all 4 processes known
- NNLL ingredients: squark-antisquark [Beenakker et al. '11] and gluino-gluino [Kauth et al. '11]

Next we present our results resumming both <u>soft and Coulomb</u> terms at NLL

$$K_{\mathrm{NLL}} = \frac{\sigma^{\mathrm{matched}}}{\sigma^{\mathrm{NLO}}}$$



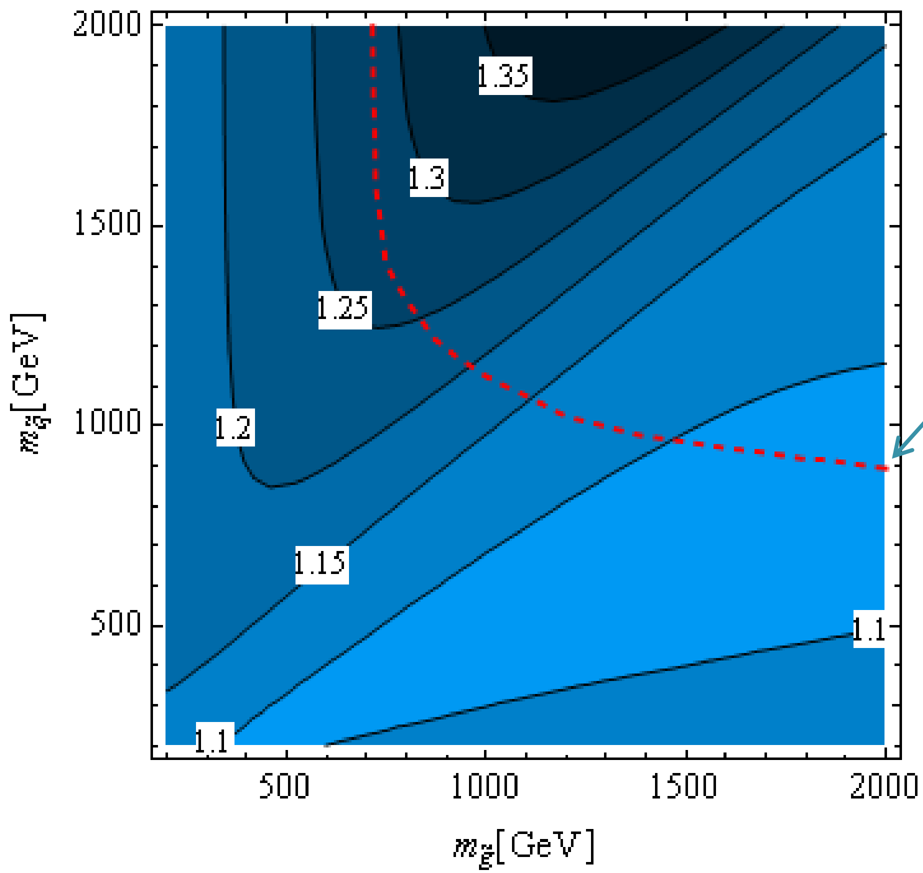
Equal squark and gluino masses:

- NLL: combined soft and Coulomb resummation
- $\mathrm{NLL}_{\mathrm{noBS}}$: no bound-state effects
- NLL_{s+h}: no Coulomb resummation

- Large corrections: 5-160% of NLO
- Coulomb corrections: 0-90%
- Bound state corrections: 0-40%

Contour plot K_{NLL}

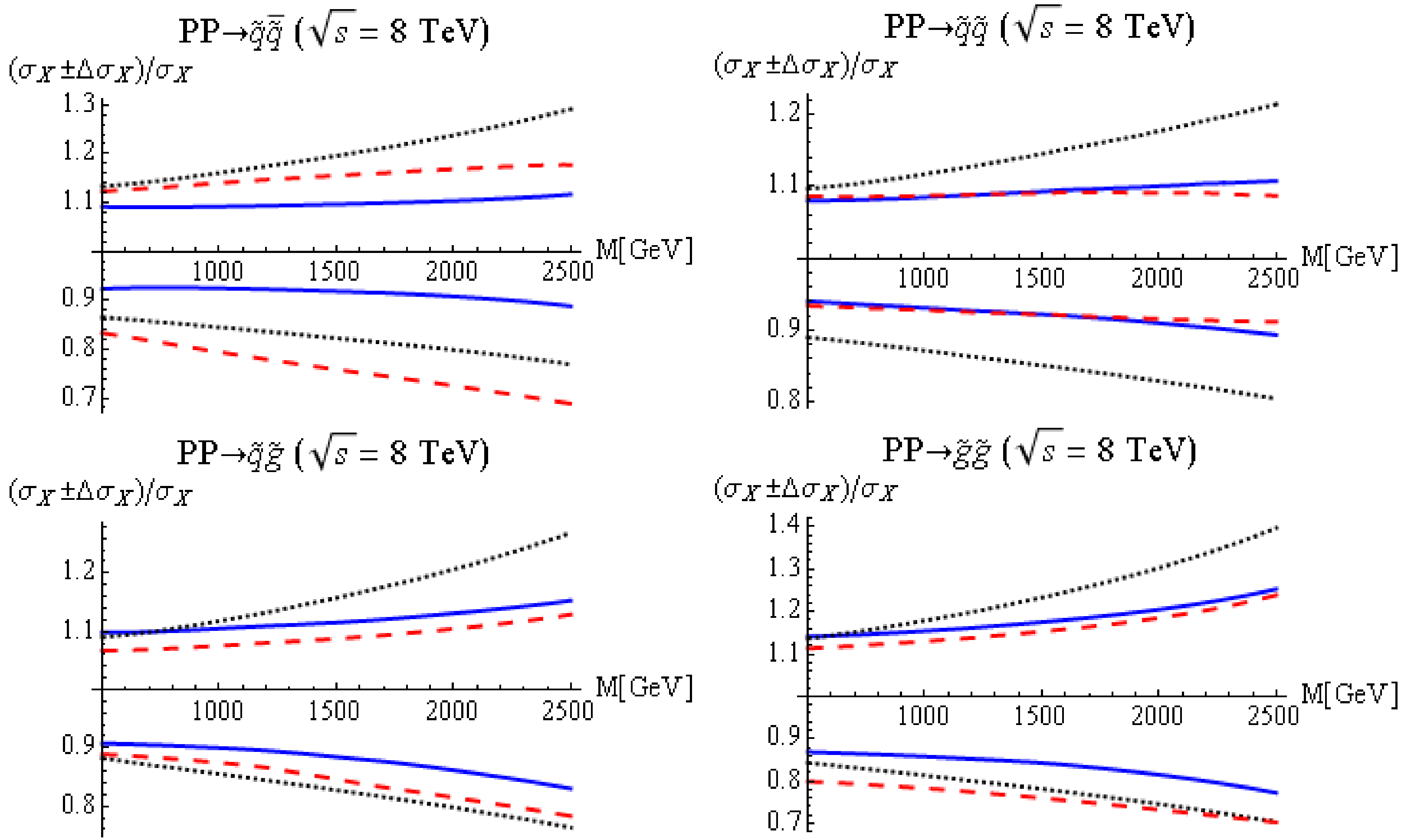
$$PP \to \tilde{q}\bar{\tilde{q}} + \tilde{q}\tilde{q} + \tilde{q}\tilde{g} + \tilde{g}\tilde{g} \ (\sqrt{s} = 8 \text{ TeV})$$



An (old) ATLAS 7 TeV exclusion bound

- Corrections can become as large as 40%, if squark mass is larger than gluino mass
- Exclusion bound goes through large K_{NLL} regions

Uncertainties



Scale and parameterization errors of:

- NLL: combined soft and Coulomb resummation
- NLL_{s+h}: no Coulomb resummation
- NLO: fixed order calculation to α_s^3

- Equal squark and gluino masses
- Corrections reduce NLO errors to $\pm$10%
- Soft-Coulomb interference reduces errors

Summary

- The corrections on total SUSY process can be as large as 15-30%
- Errors reduced to ±10%
- Coulomb corrections can be as large as soft corrections ⟶ need to resum them
- Corrections need to be taken into account for setting more accurate squark-gluino mass (bounds)

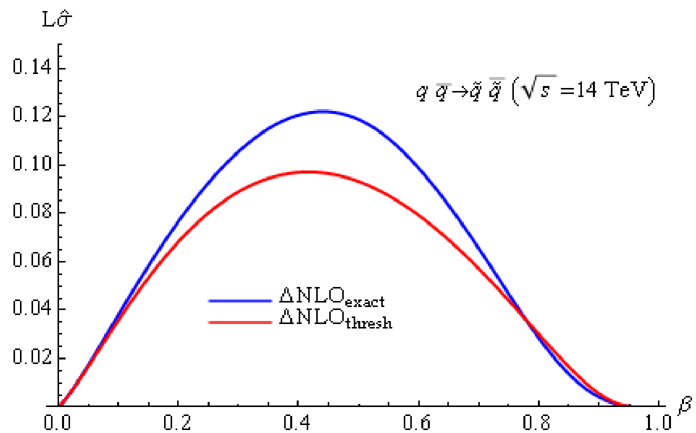
Outlook

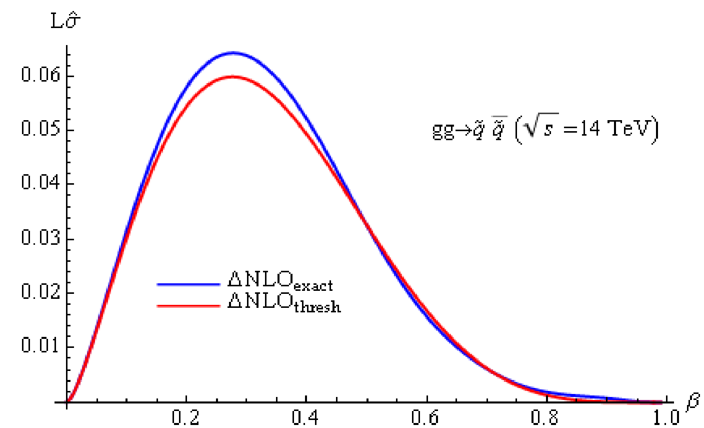
- Extend results to non-degenerate squark masses
- Include finite width effects
- NNLL resummation

Backup slides

Need for resummation

$$\sigma_{PP\to\tilde{s}\tilde{s}'X}(s) = \int_0^{\beta_1} d\beta \sum_{p,p'=q,\bar{q},g} \left(\frac{\partial\tau}{\partial\beta}\right) L_{pp'}(\tau,\mu_f)\hat{\sigma}_{pp'\to\tilde{s}\tilde{s}'X}(\tau s,\mu_f)$$





- Sizeable contribution from small β region ⟶ need to resum at threshold
- Threshold enhanced terms also approximate well away from threshold

• Soft logarithms resummation [Catani et al. '96; Becher, Neubert '06; Kulesza, Motyka '08; Langenfeld, Moch '09; Beenakker et al. '09]
• Coulomb resummation [Fadin, Khoze '87-'89; Fadin, Khoze, Sjostrand '90; Kulesza, Motyka '09]
• Simultaneous soft and Coulomb resummation for squark-antisquark at NLL [Beneke, Falgari, Schwinn '10] and top-quark pairs at NNLL [Beneke et al. '11]

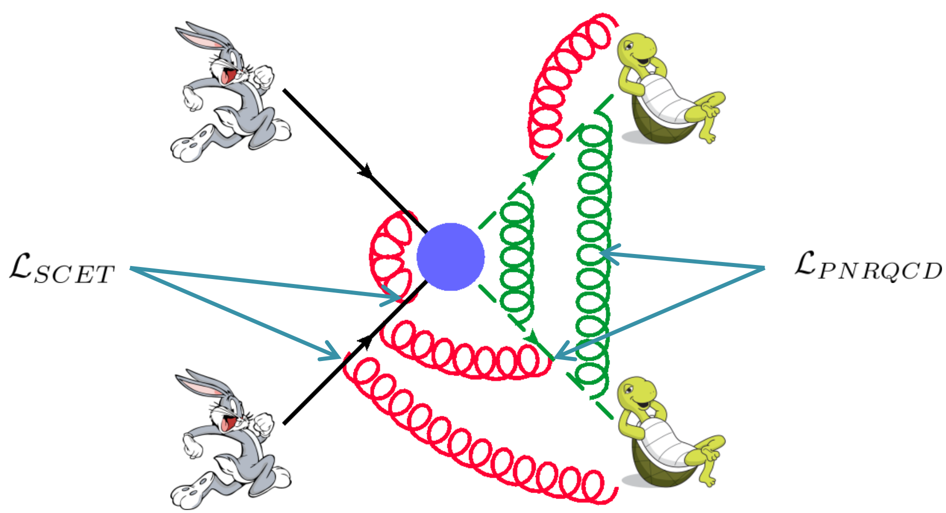
Effective lagrangian

- Effective lagrangian: $\mathcal{L}_{EFT} = \mathcal{L}_{SCET} + \mathcal{L}_{PNRQCD}$

Collinear-soft:
$$\mathcal{L}_{SCET} = \bar{\xi}_c \Big(in.D + i\not{D}_{\perp c} \frac{1}{i\bar{n}D_c} i\not{D}_{\perp c} \Big) \frac{\not{\bar{n}}}{2} \xi_c - \frac{1}{2} tr\Big(F_c^{\mu\nu} F^c_{\mu\nu} \Big)$$

Potential-soft:
$$\mathcal{L}_{PNRQCD} = \psi^\dagger \Big(iD_s^0 + \frac{\vec{\partial}^2}{2m_{\tilde{s}}} + \frac{i\Gamma_{\tilde{s}}}{2} \Big) \psi + \psi'^\dagger \Big(iD_s^0 + \frac{\vec{\partial}^2}{2m_{\tilde{s}'}} + \frac{i\Gamma_{\tilde{s}'}}{2} \Big) \psi' + \int d^3\vec{r} [\psi^\dagger \mathbf{T}^{(R)a} \psi](\vec{r}) \Big(\frac{\alpha_s}{r} \Big) [\psi'^\dagger \mathbf{T}^{(R')a} \psi'](0)$$

$$pp' \to \tilde{s}\tilde{s}' X$$



Potential function

- The potential function sums the Coulomb terms: $(\alpha_s/\beta)^n$

  The potential function equals twice the imaginary part of the LO Coulomb Green's function:

$$G_C^{R_\alpha(0)}(0,0;E) = -\frac{(2m_r)^2}{4\pi}\Big\{\sqrt{-\frac{E}{2m_r}} + (-D_{R_\alpha})\alpha_s\Big[\frac{1}{2}\ln(-\frac{8m_r E}{\mu^2}) - \frac{1}{2} + \gamma_E + \psi\Big(1 - \frac{(-D_{R_\alpha})\alpha_s}{2\sqrt{-E/(2m_r)}}\Big)\Big]\Big\}$$

- Potential function J:

$$J_{R_\alpha}(E) = \frac{(2m_r)^2\pi D_{R_\alpha}\alpha_s}{2\pi}\left(e^{\pi D_{R_\alpha}\alpha_s\sqrt{\frac{2m_r}{E}}} - 1\right)^{-1} \qquad E > 0$$

$$J_{R_\alpha}^{\text{bound}}(E) = 2\sum_{n=1}^{\infty}\delta(E - \Big(-\frac{2m_{\rm r}\alpha_s^2 D_{R_\alpha}^2}{4n^2}\Big))\left(\frac{2m_{\rm r}(-D_{R_\alpha})\alpha_s}{2n}\right)^3 \qquad E < 0$$

  It depends on the Casimir coefficients: $D_{R_\alpha} = \frac{1}{2}(C_{R_\alpha} - C_p - C_{p'})$

NLL resummation formula

- NLL partonic cross section is a sum over the total color representations of final state:

$$\hat{\sigma}^{\rm NLL}_{pp'}(\hat{s},\mu_f) = \sum_{R_\alpha} H^{(0),R_\alpha}_{pp'}(\mu_h)\, U_i(M,\mu_h,\mu_s,\mu_f)\, \frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)} \int_0^\infty d\omega\, \frac{J_{R_\alpha}(M\beta^2-\frac{\omega}{2})}{\omega} \left(\frac{\omega}{2M}\right)^{2\eta}$$

[Beneke, Falgari, Schwinn'10]

Hard function H is determined by Born cross section at threshold: $\hat{\sigma}^{(0),R_\alpha}_{pp'}(\hat{s}) \underset{\hat{s}\to 4M^2}{\approx} \frac{(2m_r)^2}{2\pi}\sqrt{\frac{E}{2m_r}}\, H^{(0),R_\alpha}_{pp'}$

- The function Ui follows from the evolution equations of H and W:

$$U_i(M,\mu_h,\mu_f,\mu_s) = \left(\frac{4M^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_s)} \left(\frac{\mu_h^2}{\mu_s^2}\right)^{\eta} \times \exp\Big[4(S(\mu_h,\mu_f)-S(\mu_s,\mu_f)) - 2a_i^V(\mu_h,\mu_s) + 2a^{\phi,p}(\mu_s,\mu_f) + 2a^{\phi,p'}(\mu_s,\mu_f)\Big]$$

$$S(\mu_i,\mu_j) = \frac{C_p+C_{p'}}{2\beta_0^2}\left[\frac{4\pi}{\alpha_s(\mu_i)}\left(1-\frac{1}{r}-\ln r\right) + \left(2K-\frac{\beta_1}{\beta_0}\right)(1-r+\ln r) + \frac{\beta_1}{2\beta_0}\ln^2 r\right]$$

$$a_\Gamma(\mu_i,\mu_j) = \frac{C_p+C_{p'}}{\beta_0}\ln r\,, \quad a_i^V(\mu_i,\mu_j) = \frac{\gamma_i^{(0),V}}{2\beta_0}\ln r, \quad a^{\phi,p}(\mu_i,\mu_j) = \frac{\gamma^{(0)\phi,p}}{2\beta_0}\ln r$$

γ's are the one-loop anomalous-dimension coefficients, β's the beta coefficients and C's are the Casimir invariants, while other constants are:

$$\eta = 2a_\Gamma(\mu_s,\mu_f), \quad r = \alpha_s(\mu_j)/\alpha_s(\mu_i), \quad K = \left(\frac{67}{18}-\frac{\pi^2}{6}\right)C_A - \frac{10}{9}T_F n_f$$

Determination of β_{cut}

- β_{cut} is determined by minimizing the width of the envelope created by:

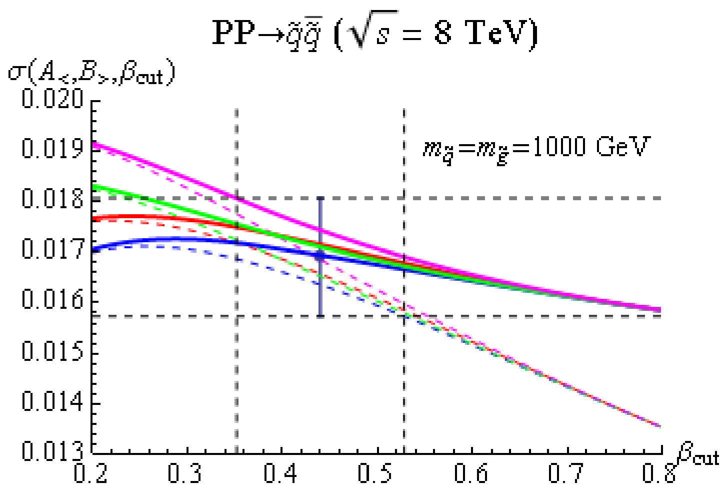
$$\hat{\sigma}_{s\tilde{s}'}(A_<, B_>, \beta_{\text{cut}}) = \hat{\sigma}^{A_<}_{s\tilde{s}'}\ \theta(\beta_{\text{cut}} - \beta) + \hat{\sigma}^{B_>}_{s\tilde{s}'}\ \theta(\beta - \beta_{\text{cut}})$$

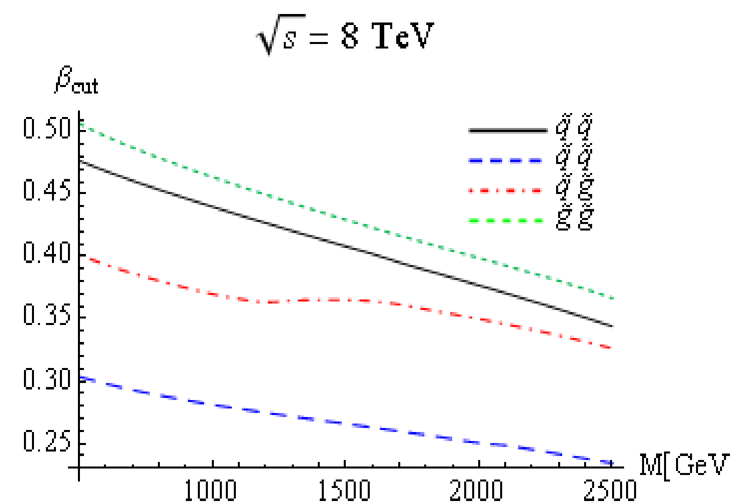
[Beneke et al. '11]

Eight different implementations:

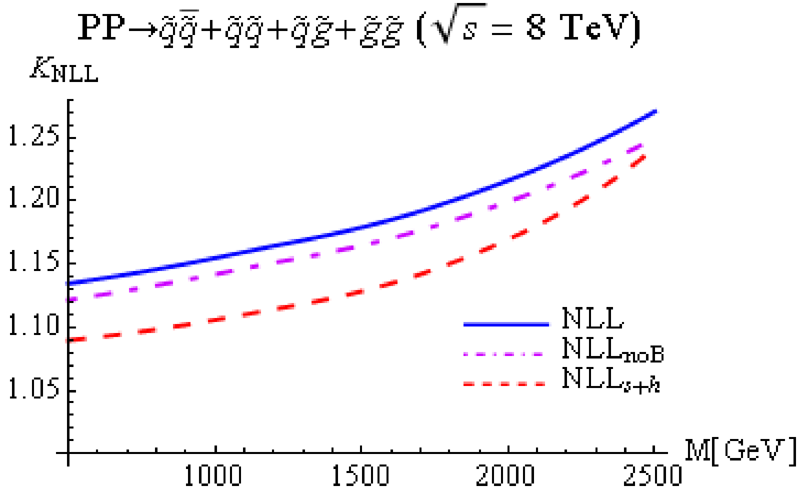
$$A_< \in \{\text{NLL}_1, \text{NLL}_2\}, \qquad B_> \in \{\text{NLL}_2, \text{NLO}_{\text{app}}, \text{NNLO}_{\text{A}}, \text{NNLO}_{\text{B}}\}$$

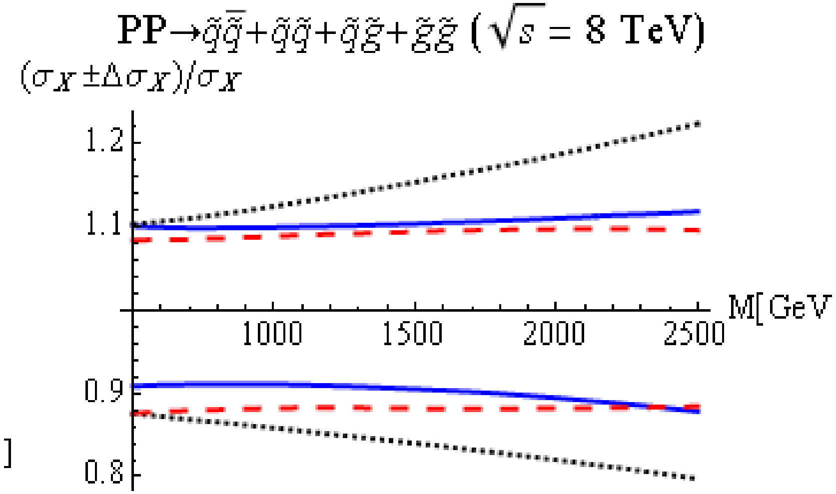
- Default implementation is NLL_2
- Error from envelope when varying β_{cut} by 20% around its default value

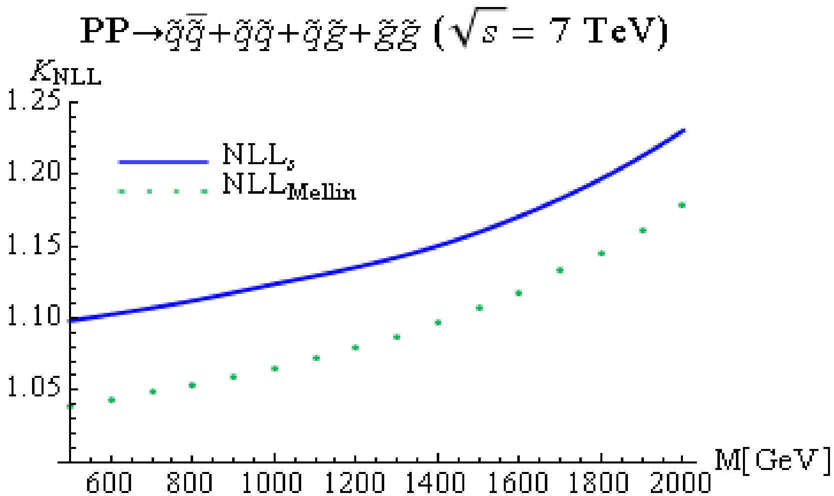


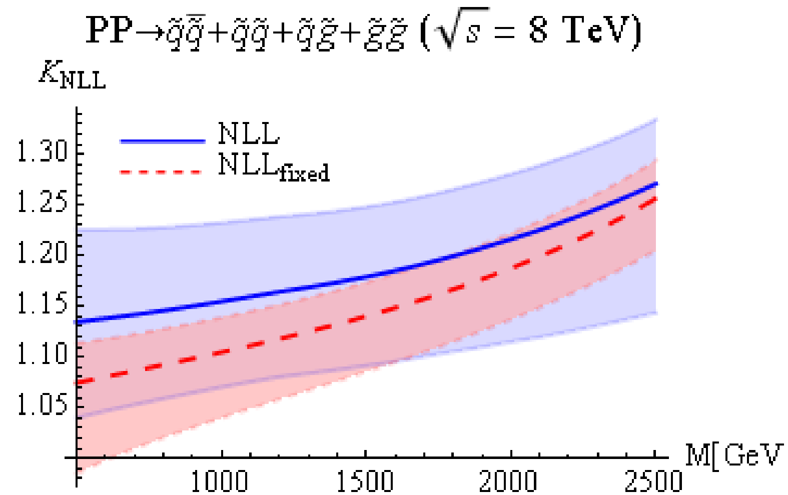


Total SUSY cross section



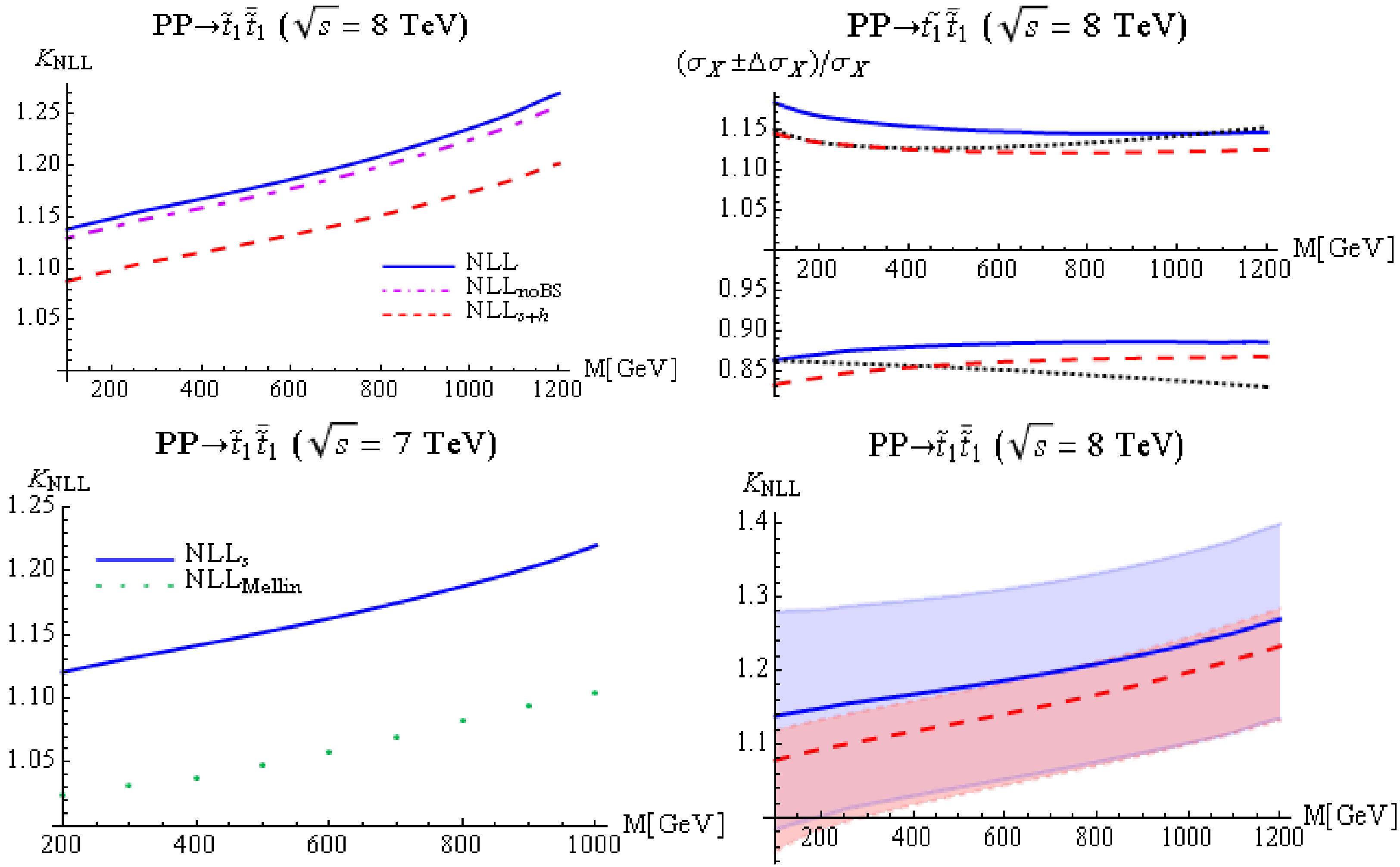






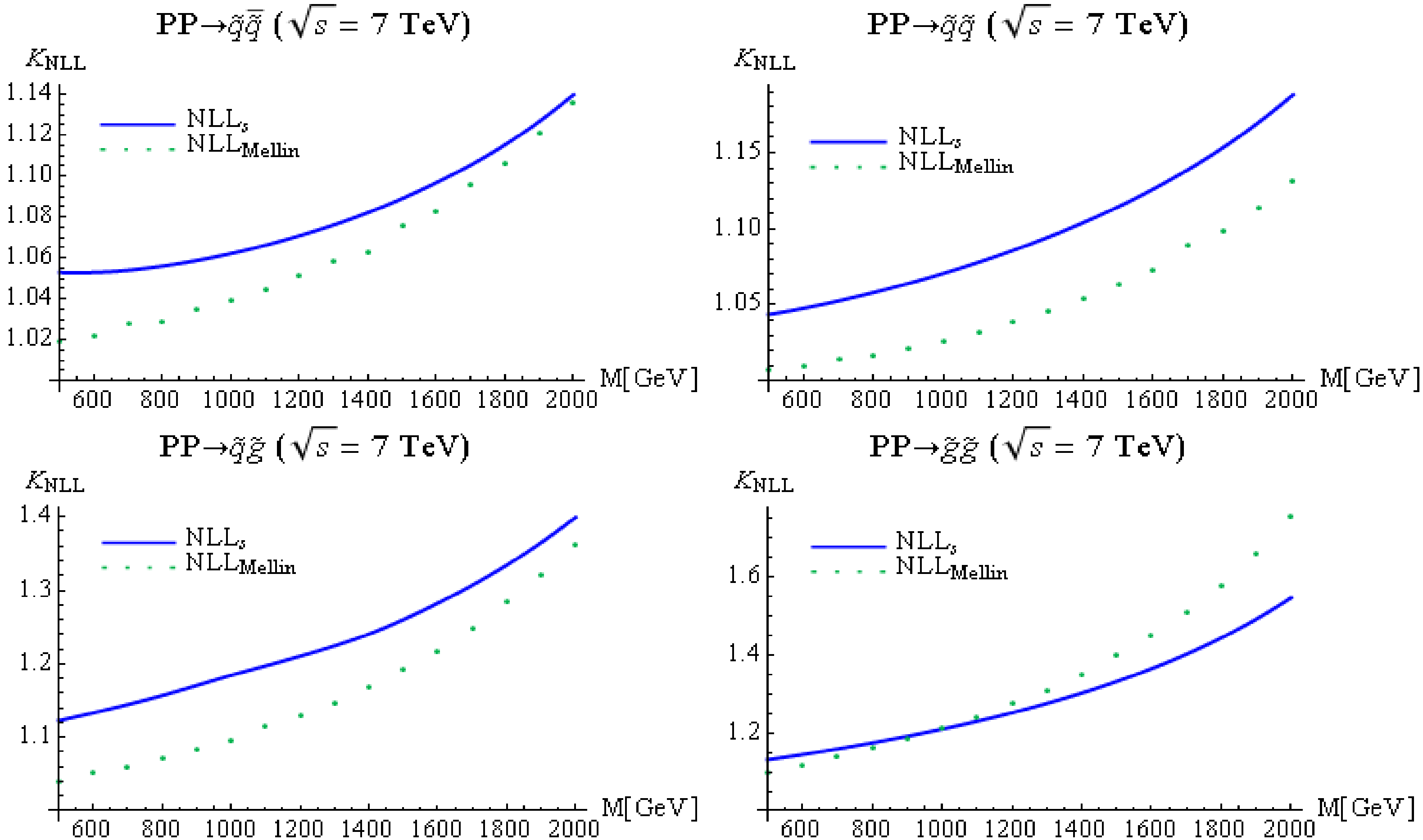
- The corrections are about 15-30%
- Errors reduced to $\pm$10%

Stops



- Q-qbar fusion P-wave suppressed compared to gluon fusion
- Relatively larger soft corrections from gluon fusion than squark-antisquark

Mellin space comparison



- Mellin transf.: $f(x) \to \tilde{f}(N) = \int_0^\infty dx\, f(x) x^{N-1}$

  $N \sim \frac{1}{\beta^2}$ ⟶ Threshold: $N \gg 1$
- Resum and transform back to momentum space
- Only soft resummation: NLL$_s$
- Compare with NLL$_{\rm Mellin}$ [Beenakker et al. '09, NLLFast]
- Agreement is within error bars