MEASUREMENT OF θ_{13} VIA ANTINEUTRINO DISAPPEARANCE USING TWO THEORIES

PHENO 2012 SYMPOSIUM

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arXiv:1204.3084/hep-ph,
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1 Antineutrino Disappearance Experiments and θ_{13}

Neutrinos are produced as ν_a, where a is the flavor, $a = e,\ \mu,\ \tau$. However, neutrinos of definite mass are ν_α, with $\alpha = 1, 2, 3$. These forms are connected by

$$\nu_a \ = \ U\nu_\alpha \tag{1}$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix},$$

with notation $sin\theta_{ij} \equiv s_{ij}$. $s_{23} = 0.707$ and $s_{12} = 0.56$. CP angle δ_{CP} is not known.

The goal of the antineutrino disappearance experiments Daya Bay (China), Double Chooz (France), and RENO (Korea) is the accurate determination of the angle θ_{13}.

In the present work we compare the value of θ_{13} extracted from recent Daya Bay data from the formula used by the experimental groups with our formula based on more recent S-matrix theory.

2 Transition Probability $\mathcal{P}(\nu_a \to \nu_b)$ Derived Using S-Matrix Theory

Given the Hamiltonian, H(t), for neutrinos, the neutrino state at time $= t$ is obtained from the state at time $= t_0$ from the matrix, $S(t, t_0)$, by

$$\begin{aligned} |\nu(t)> &= S(t,t_0)|\nu(t_0)> \quad (2) \\ i\frac{d}{dt}S(t,t_0) &= H(t)S(t,t_0)\ . \quad (3) \end{aligned}$$

In the vacuum the $S(t, t_0)$ is obtained from

$$S_{ab}(t,t_0) = \sum_{j=1}^{3} U_{aj} exp^{iE_j(t-t_0)} U^*_{bj}\ . \quad (4)$$

Neutrinos travelling through matter experience a potential $V = \sqrt{2}G_F n_e$, where G_F is the Fermi constant, and n_e is the density of electrons in matter. This is a very small effect for the neutrino disappearance experiments.

As an example, The transition probability $\mathcal{P}(\nu_\mu \to \nu_e)$ is obtained from S_{12}:

$$\mathcal{P}(\nu_\mu \to \nu_e) = |S_{12}|^2 = Re[S_{12}]^2 + Im[S_{12}]^2\ . \quad (5)$$

$$
\begin{aligned}
S_{12} &= c_{23}\beta - is_{23}ae^{-i\delta_{CP}}A & (6)\\
a &= s_{13}(\Delta - s_{12}^2\delta) & (7)\\
\delta &= \delta m_{12}^2/(2E) & (8)\\
\Delta &= \delta m_{13}^2/(2E) & (9)\\
A &\simeq f(t)I_{\alpha}* & (10)\\
I_{\alpha}* &= \int_0^t dt'\alpha^*(t')f(t') & (11)\\
\alpha(t) &= cos\omega t - icos2\theta sin\omega t & (12)\\
f(t) &= e^{-i\bar{\Delta}t} & (13)\\
2\omega &= \sqrt{\delta^2 + V^2 - 2\delta V cos(2\theta_{12})} & (14)\\
\beta &= -isin2\theta sin\omega L & (15)\\
\bar{\Delta} &= \Delta - (V+\delta)/2 & (16)\\
sin2\theta &= s_{12}c_{12}\frac{\delta}{\omega}\ , & (17)
\end{aligned}
$$

where the neutrino mass differences are $\delta m_{12}^2 = 7.6\times 10^{-5}(eV)^2$ and $\delta m_{13}^2 = 2.4\times 10^{-3}(eV)^2$.

One can show that

$$
\begin{aligned}
\mathcal{P}(\nu_\mu \to \nu_e) &\simeq (c_{23}s_{12}c_{12}(\delta/\omega)sin\omega L)^2 + 2(s_{23}s_{13})^2(1 - cos\bar{\Delta}L) & (18)\\
&+2s_{13}s_{12}c_{12}s_{23}c_{23}(\delta/\omega)sin\omega L \\
&(cos(\bar{\Delta}L + \delta_{CP})sin\bar{\Delta}L + sin(\bar{\Delta}L + \delta_{CP})(1 - cos\bar{\Delta}L))\ .
\end{aligned}
$$

We find that there is almost δ_{CP} dependence, which greatly simplifies the extraction of θ_{13} from the data.

3 $\bar{\nu}_e$ Disappearance Derived Using S-Matrix Theory Compared to Daya Bay Evaluation

In this section we present the $\bar{\nu}_e$ disappearance,

$$\mathcal{P}(\bar{\nu}_e \rightarrow \bar{\nu}_e) \;=\; 1 - \mathcal{P}(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) - \mathcal{P}(\bar{\nu}_e \rightarrow \bar{\nu}_\tau) \;, \tag{19}$$

using the S-matrix method, and compare it to the expression used by the Daya Bay, Double Chooz, and RENO, which is

$$\mathcal{P}^{\mathcal{DB}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) \;\simeq\; 1 - 4(s_{13}c_{13})^2 sin^2(\frac{\Delta L}{2}) \tag{20}$$

In the S-matrix method the probability of $\bar{\nu}_e$ oscillation to $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ is given by

$$\begin{aligned} \mathcal{P}^{\mathcal{SM}}(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) \;&=\; |\bar{S}_{21}|^2 \\ \mathcal{P}^{\mathcal{SM}}(\bar{\nu}_e \rightarrow \bar{\nu}_\tau) \;&=\; |\bar{S}_{31}|^2 \;. \end{aligned} \tag{21}$$

We take $\delta_{CP} = 0$, since $|S_{12}|^2$ is essentially independent of δ_{CP}, and find

$$\begin{aligned} \mathcal{P}^{\mathcal{SM}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) \;&=\; 1 - [(.46\delta sin\bar{\omega}L/\bar{\omega})^2 \\ &\quad +2(s_{13})^2(1 - cos\bar{\bar{\Delta}}L)] \;, \end{aligned} \tag{22}$$

with $\bar{\bar{\Delta}} = \Delta + (V - \delta)/2$ and 2
$\bar{\omega} = \sqrt{\delta^2 + V^2 + 2\delta V cos(2\theta_{12})}$

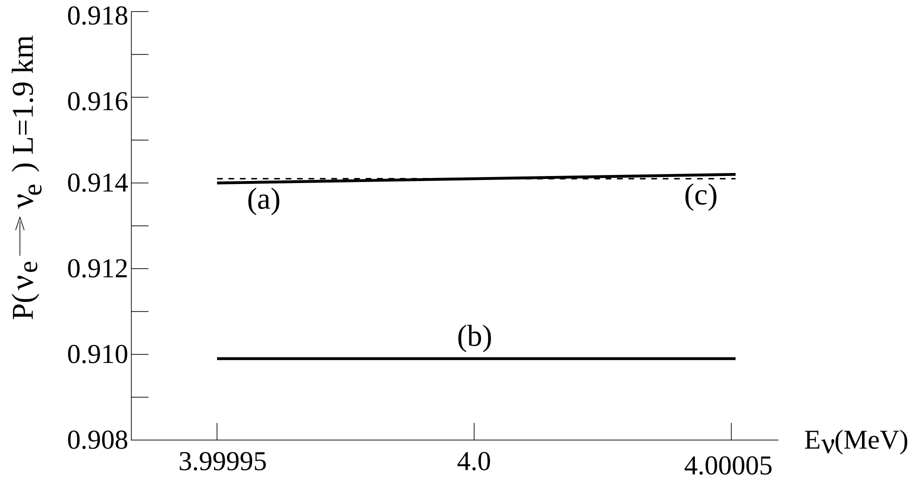

Figure 1: $\mathcal{P}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$. For $s_{13} = .15$ (a) $\mathcal{P}^{\mathcal{DB}}$, (b) $\mathcal{P}^{\mathcal{SM}}$; (c) $\mathcal{P}^{\mathcal{SM}}$ for $s_{13} = .147$

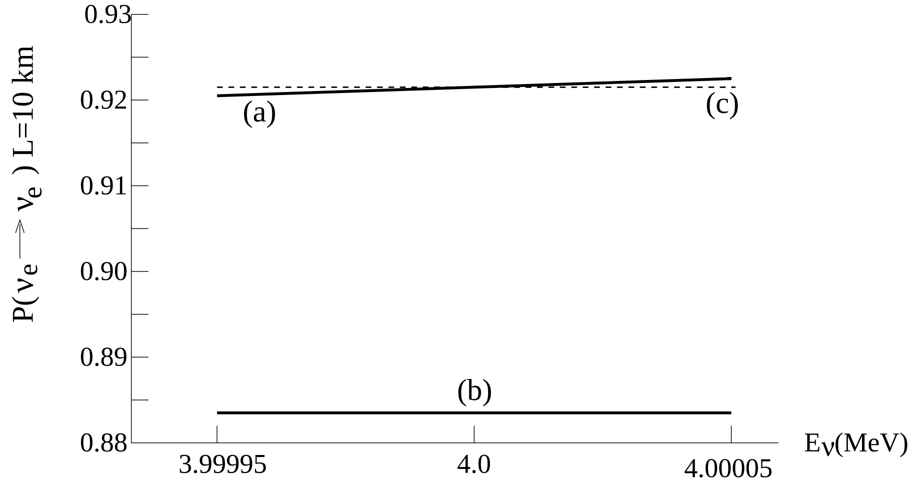

Figure 2: $\mathcal{P}(\bar{\nu}_e \rightarrow \bar{\nu}_e)$. For $s_{13} = .15$ (a) $\mathcal{P}^{\mathcal{DB}}$, (b) $\mathcal{P}^{\mathcal{SM}}$; (c) $\mathcal{P}^{\mathcal{SM}}$ for $s_{13} = .0.097$

In Fig. 1 we use $s_{13} = .15$, the recent Daya Bay result, for $\mathcal{P}^{DB}(\bar{\nu}_e \to \bar{\nu}_e)$ shown via curve (a), and $\mathcal{P}^{SM}(\bar{\nu}_e \to \bar{\nu}_e)$) shown via curve (b).

Curve (c) is the fit to $\mathcal{P}^{SM}(\bar{\nu}_e \to \bar{\nu}_e)$ for $s_{13} = 0.147$ for E=4.0 MeV, L=1.9km. From this we conclude that using the S-Matrix method for fitting $\mathcal{P}(\bar{\nu}_e \to \bar{\nu}_e)$ would give a 2% correction to the value of s_{13} determined by the data.

Fig. 2 is the same as Fig. 1, except we use a longer baseline, L=10 km, as as future project might use a longer baseline for a larger effect given s_{13}. One sees that a fit to the data given by the S-matrix gives s_{13}=0.097 rather than 0.15, a large 35% correction.

It is also important to note that our SM method gives $\mathcal{P}(\bar{\nu}_e \to \bar{\nu}_e) \neq 1.0$ even for s_{13}=0.

4 Conclusions

Our new results for $\bar{\nu}_e$ disappearance, as is being measured the Daya Bay, Double Chooz and RENO projects, make use of a different theoretical formulation than that used by these projects to extract s_{13} from the data. We have shown that the recent result from the Daya Bay collaboration, with E=4 MeV and L=1.9 km, from which it was stated that $s_{13} \simeq .15$, by our analysis is $s_{13} \simeq .147$, a 2% correction.

This is small, but the goal of these projects is 1% accuracy for s_{13}. For a baseline of L=10 km, with E=4 MeV, we find $s_{13} \simeq .097$ using the S-Matrix method, rather than .15, a 35% correction.

Also, our SM method gives $\mathcal{P}(\bar{\nu}_e \to \bar{\nu}_e) \neq 1.0$ even for s_{13}=0.