## Asymmetric Dark Matter in a Stueckelberg Extension

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#### with Wan-Zhe (Vic) Feng and Pran Nath arXiv: 1204.5752 [hep-ph]

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3 Depleting the Symmetric Component

4 Model Detection



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#### Motivation

- Baryon and anti-Baryon asymmetry with a non-vanishing B-L
  - Possible mechanisms include: Baryogenesis, Leptogenesis
  - For this talk, we assume a B L excess has been generated already in the early universe
- Cosmic Coincidence
  - $100\Omega_{
    m B}h_0^2=2.255\pm0.054$  [WMAP-7; Astrophys. J. Suppl. 192, 18 (2011)]
  - $\Omega_{
    m DM}\,h_0^2=0.1126\pm0.0036$  [WMAP-7; Astrophys. J. Suppl. 192, 18 (2011)]

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• 
$$\frac{\Omega_{\rm DM} h_0^2}{\Omega_{\rm B} h_0^2} = 4.99 \pm 0.20 \approx 5$$

AsyDM Overview Stueckelberg Overview AsyDM Model Overview AsyDM Mass

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#### Basic Overview (Part I):

Assuming there is a B - L excess existing already in the early universe.

- One issue is to come up with a mechanism to create the asymmetry in dark mater from the B L asymmetry
- Another issue is how to deplete the symmetric component of dark matter generated via thermal processes
- i.e., asymmetric dark matter (AsyDM)

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## Basic Overview (Part II):

Such a mechanism can be accomplished through a Stueckelberg  $U(1)_X$  extension<sup>1</sup> since it ...

- ... is anomaly free (SM)
- 2 ... can be gauged

• 
$$L_e-L_\mu, \ L_\mu-L_\tau, \ L_e-L_\tau, \ ext{ or } B-L$$

- Summetric component of DM
- ... does not suffer from oscillations of DM to anti-DM
  - Such oscillations could wash out the asymmetric DM via pair annihilation
  - Oue to gauge invariance, oscillations are forbidden using this extension

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## Basic Overview (Part II):

Such a mechanism can be accomplished through a Stueckelberg  $U(1)_X$  extension<sup>1</sup> since it ...

- ... is anomaly free (SM)
- 2 ... can be gauged

 $L_{\mu} - L_{\tau}$  or B - L

- ... leads to terms in the Lagrangian that annihilate the symmetric component of DM
- ... does not suffer from oscillations of DM to anti-DM
  - Such oscillations could wash out the asymmetric DM via pair annihilation
  - Oue to gauge invariance, oscillations are forbidden using this extension

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#### AsyDM Model Overview

In Lagrangian include a term that transfers B - L from matter to dark matter<sup>2</sup>, i.e.

$$\mathcal{L}_{\mathrm{asy}} = rac{1}{M_{\mathrm{asy}}^n} X^k \mathcal{O}_{\mathrm{asy}}$$

- $\bullet~M_{\rm asy}$  is the scale of interaction and decouples at  ${\it T}_{\rm int}$
- $\mathcal{O}_{asy}$  is made up of SM fields with a non-vanishing B-L quantum number
- X<sup>k</sup> is the dark matter fields with an opposite B L quantum number (could be a fermion or boson)
- B L transfer (SM) examples include<sup>3</sup>

• 
$$\frac{1}{M_{asy}^3}\psi^3 LH$$
,  $\frac{1}{M_{asy}^3}\phi^2 (LH)^2$ ,  $\frac{1}{M_{asy}^5}\psi^3 Lqd^c$ , or  $\frac{1}{M_{asy}^5}\psi^3 u^c d^c d^c$ 

<sup>2</sup>Kaplan, Luty, and Zurek, Phys. Rev. D **79**, 115016 (2009)

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## AsyDM Model Overview (cont.)

At temperatures above the decoupling of the interaction, the asymmetry in the number density of particle i is given by

$$\begin{split} n_i - n_{\overline{i}} &= \frac{g_i}{2\pi^2} \int_0^\infty \mathrm{d}q q^2 \left[ (e^{(E_i(q) - \mu_i)/T)} \mp 1)^{-1} - (e^{(E_i(q) + \mu_i)/T)} \mp 1)^{-1} \right] \\ &\equiv \frac{g_i T^3}{6} \times \begin{cases} \beta \mu_i c_i(b) & \text{bosons} \,, \\ \beta \mu_i c_i(f) & \text{fermions} \,. \end{cases} \end{split}$$

In the ultra relativistic limit ( $\beta m_i \ll 1$ ) and a weakly interacting plasma ( $\beta \mu_i \ll 1$ ), where  $\beta \equiv 1/T$ , the asymmetry in the number density becomes

$$n_i - n_{\overline{i}} \sim \frac{g_i T^3}{6} \times \begin{cases} 2\beta \mu_i + \mathcal{O}((\beta \mu_i)^3) & \text{bosons}, \\ \beta \mu_i + \mathcal{O}((\beta \mu_i)^3) & \text{fermions}. \end{cases}$$

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#### AsyDM Mass

Letting *B* define the total baryon number in the Universe and *X* to be the total dark matter number, from the cosmic coincidence, we have  $C = \frac{12}{3} + \frac{12}{3}$ 

$$\frac{\Omega_{\rm DM} h_0^2}{\Omega_{\rm matter} h_0^2} = \frac{X \cdot m_{\rm DM}}{B \cdot m_{\rm B}} \approx 5$$

Thus we get the DM mass to be

$$m_{
m DM} \approx 5 \cdot \frac{B}{X} \cdot 1 \ {
m GeV}$$

B and X can be written in terms of B - L by solving the chemical potentials by using:

- Conservation of charge or hypercharge
- Yukawa and gauge interactions
- Sphaleron interactions
- Conservation of B-L in  $\mathcal{L}_{asy}$

# X is determined based on the scale of $T_{\rm int}$ . Possible Models include:

Model A		$T_{ m int} > T_{ m EWPT}$
Model B	SM	$T_{ m EWPT} > T_{ m int} > M_t$
Model C		$M_t > T_{ m int} > M_W$
Model D	2HD	$T_{ m int} > T_{ m EWPT}$
Model E	MSSM	$T_{ m int} > M_{ m SUSY}$
Model F		$T_1 > T_{ m int} > M_2 > T_{ m EWPT}$

- $T_{\rm EWPT}$  is the Electroweak phase transition scale
- $M_t$  ( $M_W$ ) is where the top (W) mass
- $M_{\rm SUSY}$  is the (largest) soft breaking mass
- $T_1$  is where the first two generation of sparticles drop out of the thermal bath
- *M*<sub>2</sub> is the mass of the third generation sparticles, the gauginos, the Higgses and the Higgsinos
- These model classes lead to DM mass  $\lesssim 20 \text{ GeV}$

Basics Z' Constraints Symmetric Component Relic Density MSSM Extension

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The Relic Density produced thermally is

$$\Omega_{\mathrm{DM}} h_0^2 = \Omega_{\mathrm{DM}}^{\mathrm{asy}} h_0^2 + \Omega_{\mathrm{DM}}^{\mathrm{sym}} h_0^2$$

To significantly deplete symmetric component require

 $\Omega_{\rm DM}^{\rm sym} h_0^2/\Omega_{\rm DM} h_0^2 < 0.1$ 

Accomplished in the Stueckelberg formalism by gauging  $L_{\mu}-L_{\tau}$ 

 $\bullet$  This requires DM to have non-vanishing  $\mu$  or  $\tau$  lepton number and the Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{U(1)} + \mathcal{L}_{\mathrm{Stueckelberg}}$$

where in the unitary gauge for the massive Z' vector boson  $\mathcal{L}_{\mathrm{int}}$  is given by

$$\mathcal{L}_{\rm int} = \frac{1}{2} g_C Q_C^{\psi} \bar{\psi} \gamma^{\mu} \psi Z_{\mu}' + \frac{1}{2} g_C Q_C^{f} \bar{f} \gamma^{\mu} f Z_{\mu}'$$

where f denotes  $\mu, \tau$  families

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Z' Constraints  $(g_{\mu} - 2)$ 

In our case we have a strong constraint on  $g_\mu - 2$  and the Z' contribution is

$$\Delta a_{\mu} \equiv \Delta (g_{\mu}-2)/2 = rac{1}{2} \left(rac{1}{2} g_C Q_C^{\mu}
ight)^2 rac{m_{\mu}^2}{6\pi^2 M_{Z'}^2} = (3.0\pm0.8) imes 10^{-9}$$

We impose the constraint that the Z' boson contribution be less than the experimental (4 $\sigma$ ) deviation<sup>4</sup>

<sup>4</sup>K. Nakamura *et al.* J. Phys. G G37, 075021 (2010); Miller *et al.* Rept. Prog. Phys. 70, 1795 (2007) → 📃 = ∽ < , ~

This allows the symmetric component to be depleted via the process  $\psi \bar{\psi} \rightarrow Z' \rightarrow f \bar{f}$ . Relic Densities of  $\psi, \bar{\psi}$  are governed by Boltzmann equations, i.e.

$$\frac{\mathrm{d}n_{\psi}}{\mathrm{d}t} + 3Hn_{\psi} = \langle \sigma v \rangle \left( n_{\psi} n_{\bar{\psi}} - n_{\psi}^{\mathrm{eq}} n_{\bar{\psi}}^{\mathrm{eq}} \right)$$
$$\frac{\mathrm{d}n_{\bar{\psi}}}{\mathrm{d}t} + 3Hn_{\bar{\psi}} = \langle \sigma v \rangle \left( n_{\psi} n_{\bar{\psi}} - n_{\psi}^{\mathrm{eq}} n_{\bar{\psi}}^{\mathrm{eq}} \right)$$

where  $\langle \sigma v \rangle$  is the thermally averaged cross section. Allowing for AsyDM one can obtain significant effects to the relic density. However, in our model we are helped by the Breit-Wigner pole. Using

• 
$$f_{\psi} \equiv n_{\psi}/(hT^3)$$

• 
$$f_{\bar{\psi}} \equiv n_{\bar{\psi}}/(hT^3)$$

• *h* is the entropy degrees of freedom

• 
$$x \equiv T/m_{\psi}$$

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Thus the Boltzmann equations become

$$\begin{aligned} \frac{\mathrm{d}f_{\psi}}{\mathrm{d}x} &= \alpha \langle \sigma v \rangle (f_{\psi}f_{\bar{\psi}} - f_{\psi}^{\mathrm{eq}}f_{\bar{\psi}}^{\mathrm{eq}}) \\ \frac{\mathrm{d}f_{\bar{\psi}}}{\mathrm{d}x} &= \alpha \langle \sigma v \rangle (f_{\psi}f_{\bar{\psi}} - f_{\psi}^{\mathrm{eq}}f_{\bar{\psi}}^{\mathrm{eq}}) \\ \gamma &\equiv f_{\psi} - f_{\bar{\psi}} \\ \alpha(T) &= \sqrt{90} m_{\psi} M_{\mathrm{Pl}} \frac{h}{\sqrt{g}\pi} \left(1 + \frac{1}{4}\frac{T}{g}\frac{\mathrm{d}g}{\mathrm{d}T}\right) \end{aligned}$$

- $\gamma$  is a constant and numerically it is  $1.3 imes 10^{-10}$
- g is the degrees of freedom in the energy per unit volume at the photon temperature, i.e.,  $\rho = \frac{\pi^2}{30}gT^4$

• 
$$\alpha(T) = 6.7 \times 10^{20} \text{ GeV}^2$$
 for  $g = h = 68$  at  $T = 0.5 \text{ GeV}$ 

for calculation of numerical constants see Feng, Nath, and GP, arXiv:1204.5752\_ \_

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Thus the relic densities are

$$\Omega_{\psi} h_0^2 = 2.2 \times 10^{-11} \sqrt{g(x_f)} h(x_0, x_f) \left(\frac{T_{\gamma}}{2.73}\right)^3 \left(\frac{1}{\xi} - \frac{f_{\bar{\psi}}(x_f)}{\xi f_{\psi}(x_f)} e^{-\xi J(x_f)}\right)^{-1}$$
$$\Omega_{\bar{\psi}} h_0^2 = 2.2 \times 10^{-11} \sqrt{g(x_f)} h(x_0, x_f) \left(\frac{T_{\gamma}}{2.73}\right)^3 \left(\frac{f_{\psi}(x_f)}{\xi f_{\bar{\psi}}(x_f)} e^{\xi J(x_f)} - \frac{1}{\xi}\right)^{-1}$$

where

• 
$$J(x_f) \equiv \int_{x_0}^{x_f} \langle \sigma v \rangle \, \mathrm{d}x$$
  
•  $h(x_0, x_f) \equiv \frac{h(x_0)}{h(x_f)} \left[ 1 + \frac{1}{4} \left( \frac{T}{g} \frac{\mathrm{d}g}{\mathrm{d}T} \right)_{x_f} \right]^{-1}$   
•  $\xi \equiv \alpha(x_f)\gamma$ 

In the limit  $\gamma, \xi 
ightarrow 0$  one has  $f_\psi(x_f)/f_{ar\psi} 
ightarrow 1$  and thus

$$\Omega_{\bar{\psi}}h_0^2 = \Omega_{\psi}h_0^2 = 2.2 \times 10^{-11} \sqrt{g(x_f)}h(x_0, x_f) \left(\frac{T_{\gamma}}{2.73}\right)^3 \frac{1}{J(x_f)}$$

recovering the well-known result

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An exhibition of the thermal relic density of the symmetric component of the relic density as a function of Z' mass for different couplings with  $\gamma = 0$  (left) and  $\gamma = 1.3 \times 10^{-10}$  (right). Analysis shows that the symmetric DM can be efficiently annihilated.<sup>5</sup>

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#### AsyDM in MSSM

We can extend this to a multicomponent DM model<sup>6</sup> in the MSSM case. The superpotential for generating AsyDM becomes

$$\mathcal{W}_{\mathrm{asy}} = rac{1}{M_{\mathrm{asy}}^n} X^k \mathcal{O}_{\mathrm{asy}}$$

where examples of  $\mathcal{O}_{asy}$  include:

• LH<sub>u</sub>, LLE<sup>c</sup>, LQD<sup>c</sup>, U<sup>c</sup>D<sup>c</sup>D<sup>c</sup>

The Stueckelberg Lagrangian becomes

$$\mathcal{L}_{\mathrm{St}} = \int \mathrm{d} heta^2 \mathrm{d} ar{ heta}^2 \left[ M \mathcal{C} + \mathcal{S} + ar{\mathcal{S}} 
ight]^2$$

<u>*C* is the  $U(1)_C$  vector multiplet</u> and  $S, \overline{S}$  are the chiral multiplets <sup>6</sup>Feldman, Liu, Nath, and GP, Phys. Rev. D **81**, 095017 (2010) G. Peim (Northeastern University) AsyDM in Stueckelberg Ext.

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#### Depletion of Neutralino



A display of the thermal relic density contributed by the neutralino. Parameter points are displayed by their light CP even Higgs mass and pass experimental constants. The yellow band shows 10% of the WMAP value. Thus we have a two component dark matter picture with the conventional neutralino component being subdominant.<sup>7</sup>

 7 See Feng, Nath, and GP, arXiv:1204.5752 for more details
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#### Detection of Subdominant Neutralino Dark Matter



An exhibition of the neutralino-proton spin-independent cross section as a function of the neutralino mass for mSUGRA (left) and non-universal gauginos (right). To account for the reduced relic density of the neutralino component of dark matter the spin-independent cross section has been corrected by a factor  $\mathcal{R} = \Omega_{\tilde{\chi}_1^0} h_0^2 / (\Omega_{\rm DM} h_0^2)$ .<sup>8</sup>

<sup>8</sup>See Feng, Nath, and GP, arXiv:1204.5752 for more details

#### Muon Collider Smoking Gun



Smoking gun signal for Z' in the  $\tau^-\tau^+$  channel compared to  $e^-e^+$  channel at a muon collider. Such a smoking gun signal could be an indication of  $L_{\mu} - L_{\tau}$  gauge symmetry.

## Conclusion

- One issue for AsyDM models is depleting the symmetric component of DM
- Using a Stueckelberg U(1) extension we are able to explain the cosmic coincidence and deplete the symmetric component of DM
- This Stueckelberg model has interesting signals at a muon collider
  - In the MSSM case, the neutralino component can still be seen at direct detection DM experiments

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Additional Slides

## Additional Slides

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 $|\psi\bar{\psi}
ightarrow f\bar{f}|$ 

$$\begin{split} \sigma_{\psi\bar{\psi}\to f\bar{f}} &= a_{\psi} \left| \left( s - M_{Z'}^{2} + i\Gamma_{Z'}M_{Z'} \right) \right|^{-2} \\ a_{\psi} &= \frac{\beta_{f} \left( \frac{1}{2}g_{C}^{2}Q_{C}^{\psi}Q_{C}^{f} \right)^{2}}{64\pi s\beta_{\psi}} \left[ s^{2} \left( 1 + \frac{1}{3}\beta_{f}^{2}\beta_{\psi}^{2} \right) + 4M_{\psi}^{2} \left( s - 2m_{f}^{2} \right) + 4m_{f}^{2} \left( s + 2M_{\psi}^{2} \right) \right] \right] \\ \Gamma_{Z'\to f\bar{f}} &= \left( \frac{1}{2}g_{C}Q_{C}^{f} \right)^{2} r_{f} \frac{M_{Z'}}{12\pi}, \quad f = \mu, \nu_{\mu}, \tau, \nu_{\tau} \\ \Gamma_{Z'\to\psi\bar{\psi}} &= \left( \frac{1}{2}g_{C}Q_{C}^{\psi} \right)^{2} \frac{M_{Z'}}{12\pi} \left( 1 + \frac{2M_{\psi}^{2}}{M_{Z'}^{2}} \right) \left( 1 - \frac{4M_{\psi}^{2}}{M_{Z'}^{2}} \right)^{1/2} \Theta \left( M_{Z'} - 2M_{\psi} \right) \end{split}$$

• 
$$\beta_{f,\psi} = (1 - 4m_{f,\psi}^2/s)^{1/2}$$
  
•  $r_f = 1$  for  $f = \mu, \tau$   
•  $r_f = 1/2$  for  $f = \nu_\mu, \nu_\tau$ 

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#### Freeze-out Temperature

Define  $x_f$  such that for  $x = x_f = T_f/m_{\psi}$  we have

$$\frac{\mathrm{d}f_{\bar{\psi}}^{\mathrm{eq}}}{\mathrm{d}x} = \alpha \langle \sigma \mathbf{v} \rangle f_{\psi}^{\mathrm{eq}} f_{\bar{\psi}}^{\mathrm{eq}}$$

where

• 
$$f_{\bar{\psi}}^{\text{eq}}(x) = a_{\bar{\psi}} x^{-3/2} e^{-1/x}$$
  
•  $a_{\bar{\psi}} = g_{\bar{\psi}} (2\pi)^{-3/2} h^{-1}(T) \approx 9.3 \times 10^{-4} g_{\bar{\psi}}$  around  
 $T = 0.5 \text{ GeV}$ 

•  $g_{\bar{\psi}}$  denotes the degrees of freedom of the dark particle Plugging this in and solving to first order we get

$$x_f \simeq x_f^0 \left[ 1 - a_{\bar{\psi}}^{-1} \gamma(x_f^0)^{5/2} e^{1/x_f^0} 
ight]$$