Quantum Corrections in the 5D (Supersymmetric) Space

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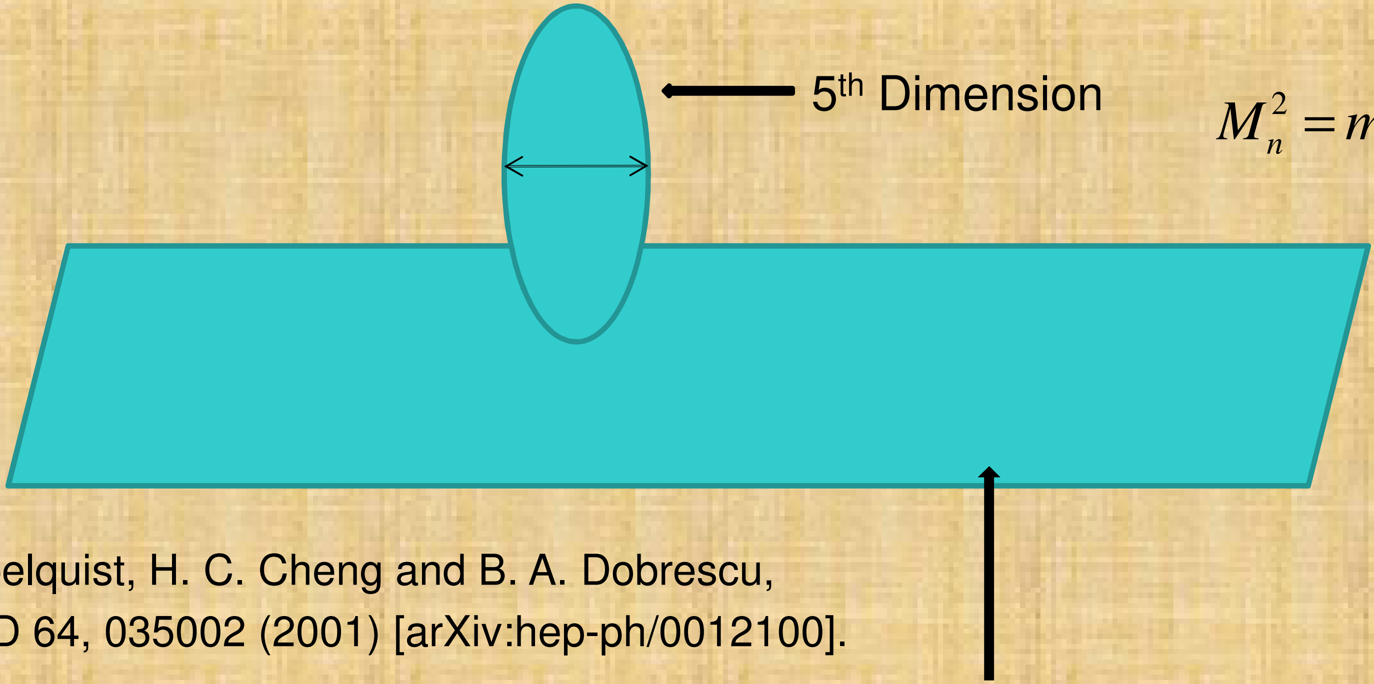
- Lu-Xin Liu, A.S.Cornell, arXiv:1204.0532 [hep-ph]
- A.S. Cornell,A.Deandrea, Lu-Xin Liu, A. Tarhini,arXiv:1110.1942 [hep-ph]
- A. S. Cornell, Lu-Xin Liu, arXiv:1105.1132 [hep-ph] ;
- Lu-Xin Liu, A. S. Cornell, arXiv:1103.1527 [hep-ph] ;
- A. S. Cornell, Lu-Xin Liu, arXiv:1010.5522 [hep-ph];

Universal Extra Dimensional Model

$$y \sim y + 2\pi R$$

$$\phi(x,y) \sim \sum_{n=-\infty}^{\infty} \phi^{(n)}(x)\exp(i\frac{n}{R}y)$$



$$M_n^2 = m^2 + \left(\frac{n}{R}\right)^2$$

$$p_5 = \frac{n}{R}$$

T. Appelquist, H. C. Cheng and B. A. Dobrescu, PRD 64, 035002 (2001) [arXiv:hep-ph/0012100].

Fields in 4D | Fields in 5D

$$A_\mu(x) \qquad A_M(x,y)$$

$$\psi_L(x) \quad \psi_R(x) \qquad \psi(x,y)=\begin{pmatrix}\psi_L(x,y)\\ \psi_R(x,y)\end{pmatrix}$$

$$\phi(x) \qquad \phi(x,y)$$

$$y\to -y \qquad (S_1/Z_2)$$

Kaluza–Klein (KK) expansion of the 5D fields

$$A_\mu(x,y)=\frac{1}{\sqrt{\pi R}}\{A_\mu^0(x)+\sqrt{2}\sum_{n=1}^{\infty}A_\mu^n(x)\cos(\frac{ny}{R})\}$$

$$A_5(x,y)=\sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty}A_5^n(x)\sin(\frac{ny}{R})$$

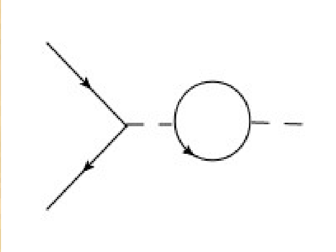
$$\psi_L(x,y)=\frac{1}{\sqrt{\pi R}}\{\psi_L(x)+\sqrt{2}\sum_{n=1}^{\infty}[\psi_L^n(x)\cos(\frac{ny}{R})]\}$$

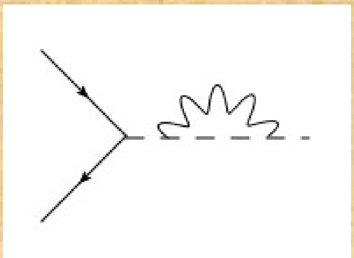
$$\psi_R(x,y)=\frac{1}{\sqrt{\pi R}}\sqrt{2}\sum_{n=1}^{\infty}[\psi_R^n(x)\sin(\frac{ny}{R})]$$

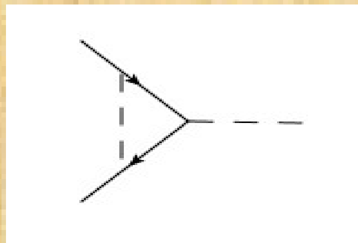
$$\Phi(x,y)=\frac{1}{\sqrt{\pi R}}\{\Phi(x)+\sqrt{2}\sum_{n=1}^{\infty}\Phi_n(x)\cos(\frac{ny}{R})\}$$

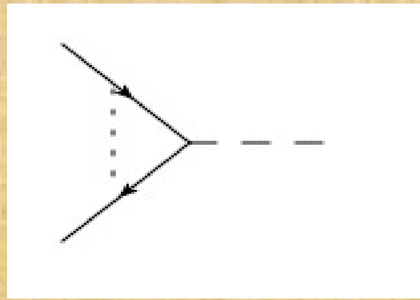
One-Loop Quantum Corrections to Yukawa Couplings in the UED Model $\bar{\psi}_L \psi_R \phi$

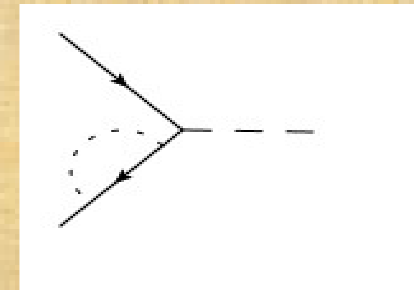
A. S. Cornell, L.X.Liu, arXiv:1010.5522 [hep-ph]

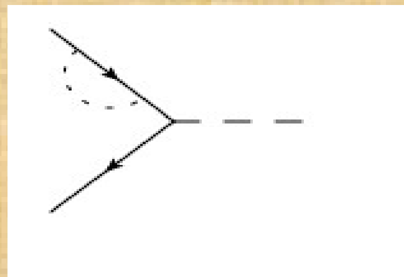


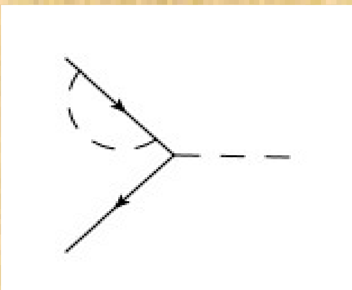


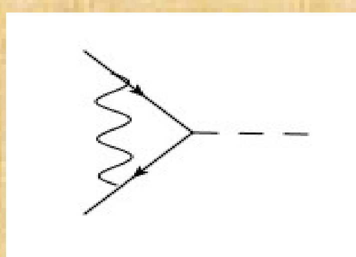


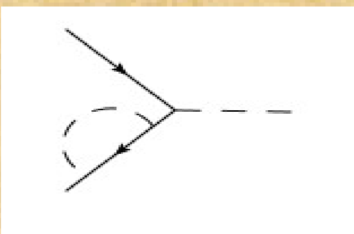












$$D_M u(x, y) = (\partial_M + i g_3^5 G_M + i \frac{Y_u}{2} g_1^5 B_M) u(x, y)$$

$$D_M d(x, y) = (\partial_M + i g_3^5 G_M + i \frac{Y_d}{2} g_1^5 B_M) d(x, y)$$

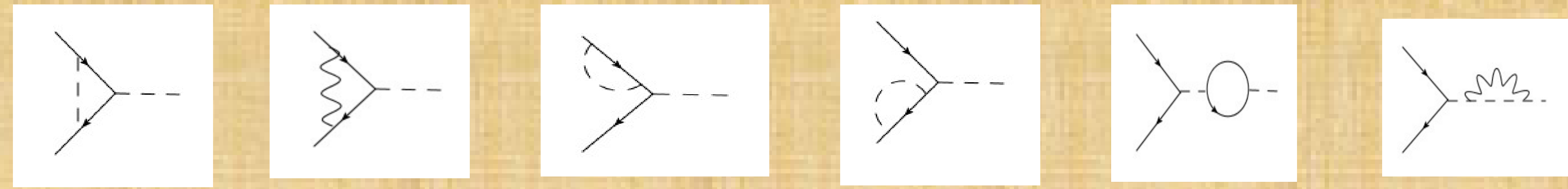
$$\ell_{Quarks} = \int_0^{\pi R} dy \{ i\bar{Q}(x, y)\Gamma^M D_M Q(x, y) + i\bar{u}(x, y)\Gamma^M D_M u(x, y) + i\bar{d}(x, y)\Gamma^M D_M d(x, y)\}$$

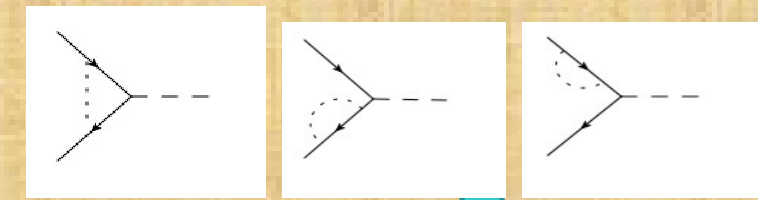
$$D_M Q(x, y) = (\partial_M + i g_3^5 G_M + i g_2^5 W_M + i \frac{Y_Q}{2} g_1^5 B_M) Q(x, y)$$

Standard Model

T. P. Cheng, E. Eichten and L. F. Li, PRD9, 2259,(1974).

Beta functions of the Yukawa Couplings





$$Y^o \bar{\psi}_L^o \psi_R^o \phi^o = Y^R Z_{couping} \bar{\psi}_L^R \psi_R^R \phi^R$$

$$\mu \frac{\partial}{\partial \mu} Y^o = 0$$

$$\mu \frac{\partial}{\partial \mu} \ln Y^R = \frac{1}{2} \mu \frac{\partial}{\partial \mu} \ln Z_{\psi_L} + \frac{1}{2} \mu \frac{\partial}{\partial \mu} \ln Z_{\psi_R} + \frac{1}{2} \mu \frac{\partial}{\partial \mu} \ln Z_{\phi} - \mu \frac{\partial}{\partial \mu} \ln Z_{couping}$$

$$16\pi^2 \frac{dY_U}{dt} = \beta_U^{SM} + \beta_U^{UED} \qquad 16\pi^2 \frac{dY_D}{dt} = \beta_D^{SM} + \beta_D^{UED}$$

Cumulative Contributions from n=1 to ΛR level of KK states

$$\beta_U^{UED} = Y_U \{ (S-1)[-(\frac{28}{3} g_3^2 + \frac{15}{8} g_2^2 + \frac{101}{120} g_1^2) + \frac{3}{2}(Y_U^\dagger Y_U - Y_D^\dagger Y_D) +$$

$$+2(S-1) Tr[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E] \}$$

$$S(\Lambda) = \Lambda R$$

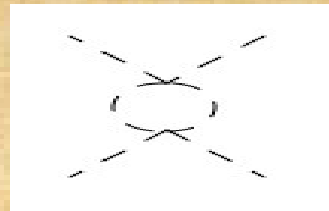
$$\beta_D^{UED} = Y_D \{ (S-1)[-(\frac{28}{3} g_3^2 + \frac{15}{8} g_2^2 + \frac{17}{120} g_1^2) + \frac{3}{2}(Y_D^\dagger Y_D - Y_U^\dagger Y_U) +$$

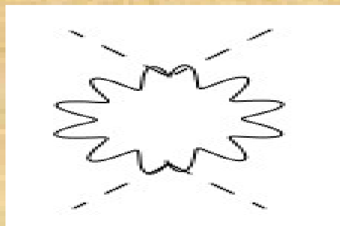
$$+2(S-1) Tr[3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D + Y_E^\dagger Y_E] \}$$

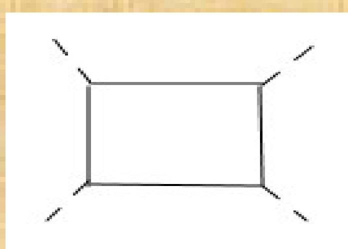
One-Loop Quantum Corrections to Higgs Quartic Coupling in the UED Model

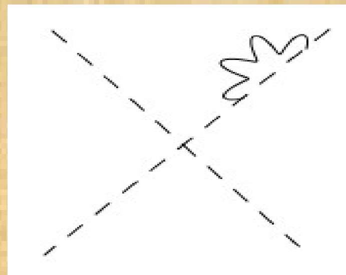
$$\frac{\lambda}{2}(\Phi^\dagger \Phi)^2$$

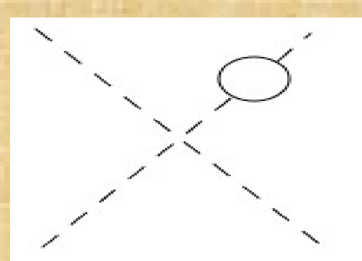
A. S. Cornell, L.X.Liu, arXiv:1105.1132 [hep-ph]

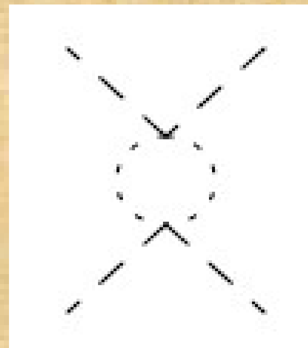












$A_5^{(n)}(x)$

$$\frac{\lambda^0}{2}(\Phi^{0\dagger}\Phi^0)^2 = \frac{\lambda^R}{2} Z_{Vertex}(\Phi^{R\dagger}\Phi^R)^2$$

$$\mu \frac{\partial}{\partial \mu}\lambda^o = 0$$

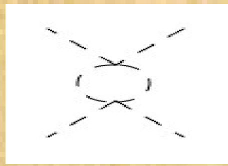
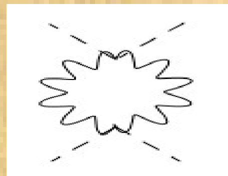
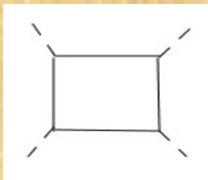
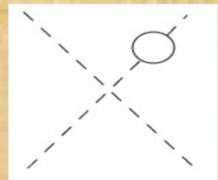
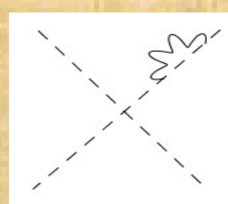
T. P. Cheng, E. Eichten and L. F. Li, PRD9, 2259,(1974).

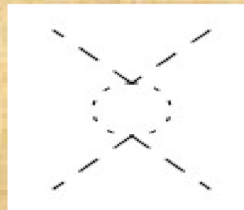
Standard Model

$$\mu \frac{\partial}{\partial \mu}\ln \lambda^R = \mu \frac{\partial}{\partial \mu}\ln Z_\Phi^2 - \mu \frac{\partial}{\partial \mu}\ln Z_{Vertex}$$

RGE of Higgs Quartic Coupling

Beta Functions of the Higgs Quartic Coupling



$$\mu \frac{\partial}{\partial \mu} \ln \lambda^{R} = \mu \frac{\partial}{\partial \mu} \ln Z_{\Phi}^{2} - \mu \frac{\partial}{\partial \mu} \ln Z_{Vertex}$$

$$16\pi^2 \frac{d\lambda}{dt} = \beta_{\lambda}^{SM} + \beta_{\lambda}^{UED}$$

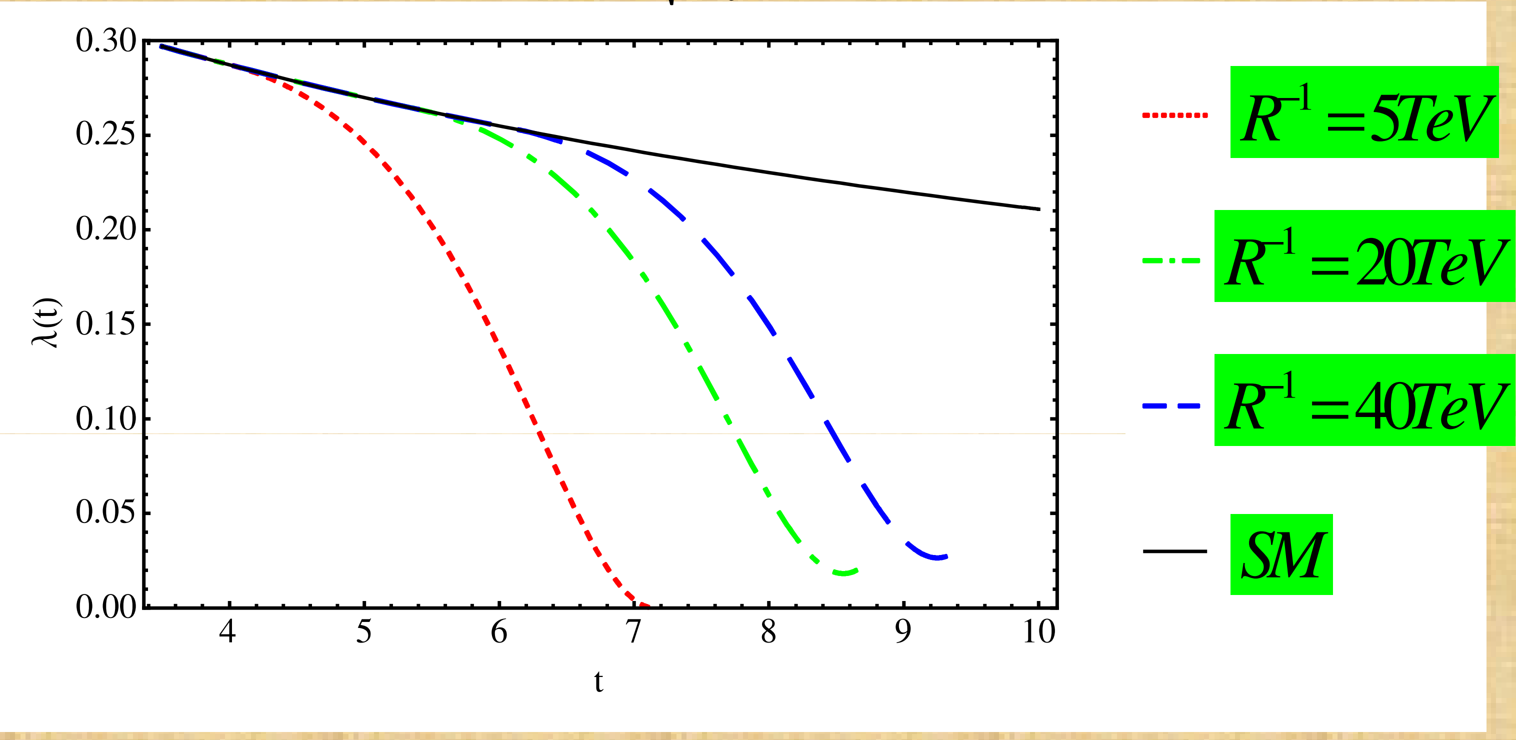
$$\beta_{\lambda}^{UED} = (S(t)-1)\left\{12\lambda^2 - 3\left(\frac{3}{5}g_1^2 + 3g_2^2\right)\lambda + \left(\frac{9}{25}g_1^4 + \frac{6}{5}g_1^2 g_2^2 + 3g_2^4\right)\right\}$$
$$+2(S(t)-1)\left\{4\lambda Tr[3Y_U^{\dagger}Y_U + 3Y_D^{\dagger}Y_D + Y_E^{\dagger}Y_E] - 4Tr[3(Y_U^{\dagger}Y_U)^2 + 3(Y_D^{\dagger}Y_D)^2 + (Y_E^{\dagger}Y_E)^2]\right\}$$

Extra dimension effects: Cumulative Contributions
from n=1 to ΛR level of KK states

Vacuum Stability Condition for the UED Model

$$< VEV >= \sqrt{\frac{\mu^2}{\lambda}} = 246 GeV$$

$$\lambda(M_Z) = 0.394$$



$$V = -\mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$$

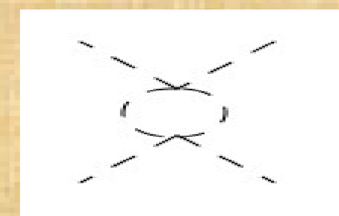
$\frac{\lambda}{2}(\Phi^\dagger \Phi)^2 \longrightarrow \lambda > 0$ for the scalar potential or vacuum stability condition

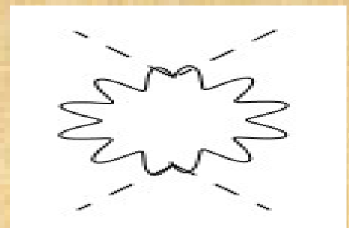
$$152 GeV \sim m_H(M_Z) \sim 157 GeV$$

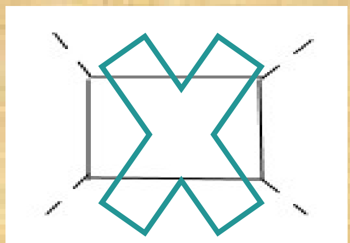
One-Loop Quantum Corrections to Higgs Quartic Coupling in the Brane Localized Matter fields 5D Model

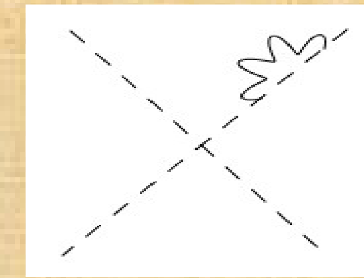
$$\frac{\lambda}{2}(\Phi^{\dagger}\Phi)^{2}$$

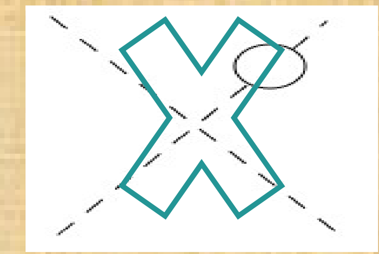
L.X. Liu, A.S.Cornell, arXiv:1204.0532 [hep-ph]



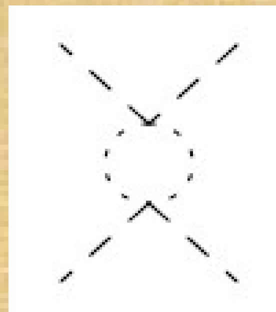








$A_5^{(n)}(x)$



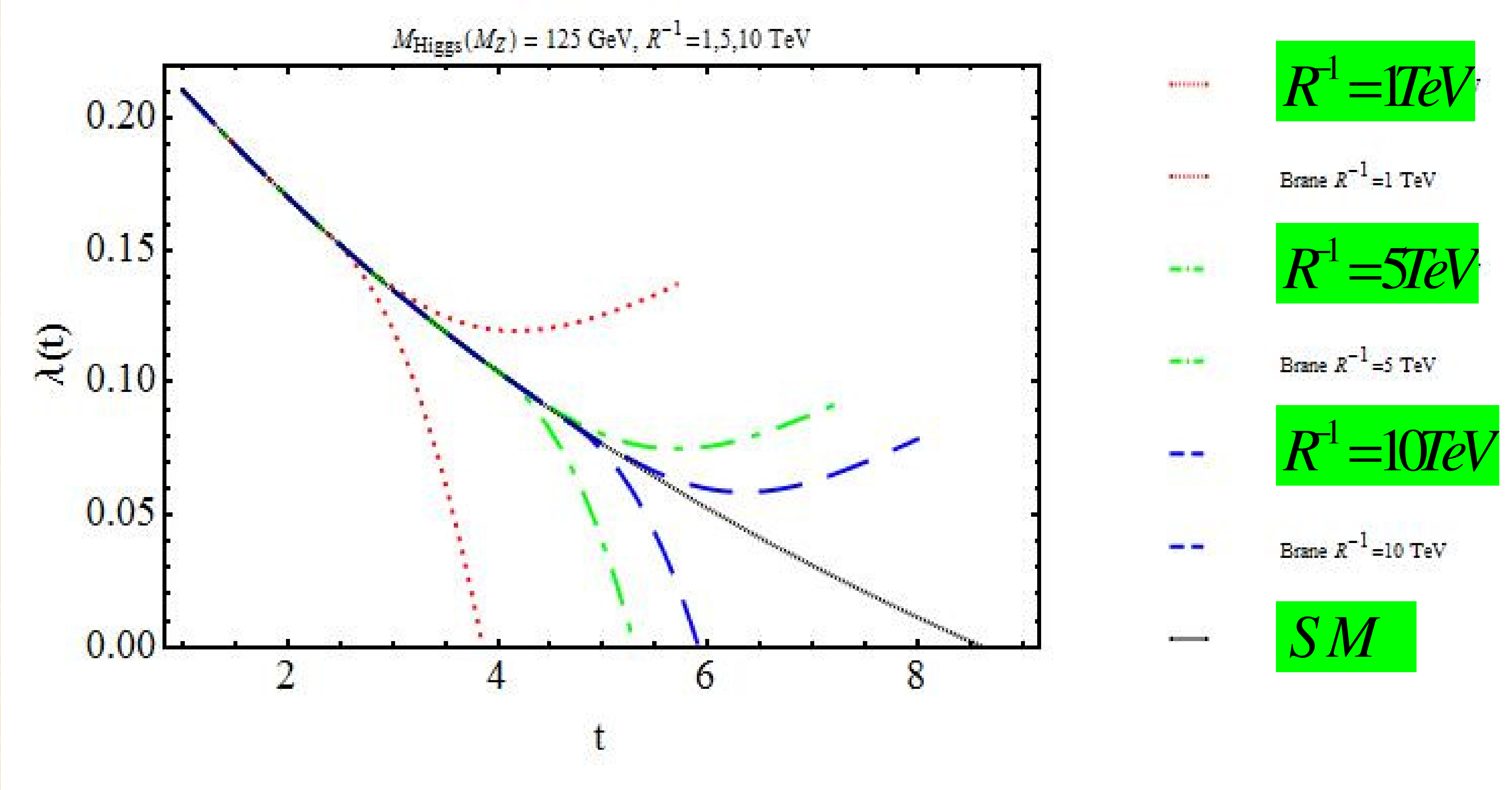
RGE of Higgs Quartic Coupling

$$\beta_{\lambda}^{UED} = (S(t)-1)\left\{12\lambda^{2} - 3\left(\frac{3}{5}g_1^{2} + 3g_2^{2}\right)\lambda + \left(\frac{9}{25}g_1^{4} + \frac{6}{5}g_1^{2}g_2^{2} + 3g_2^{4}\right)\right\}$$

$$+2(S(t)-1)\left\{4\lambda Tr[3Y_U^{\dagger}Y_U + 3Y_D^{\dagger}Y_D + Y_E^{\dagger}Y_E] - 4Tr[3(Y_U^{\dagger}Y_U)^{2} + 3(Y_D^{\dagger}Y_D)^{2} + (Y_E^{\dagger}Y_E)^{2}]\right\}$$

Alternative 5D Scenario-Brane Localized Matter Fields

LHC latest constraints 125GeV



Lu-Xin Liu, A.S.Cornell, Improved vacuum stability in a five dimensional model, arXiv:1204.0532 [hep-ph]

$$m_H = \sqrt{\lambda} v$$

$$v = 246 GeV$$

N=1 Five Dimensional Supersymmetry

(Extra Dimension + Supersymmetry)

Gauge Supermultiplet

A_M 5 vector gauge field

λ Dirac spinor

Σ Real scalar

Scalar Supermultiplet

H^i Scalar fields $SU(2)_R$ doublet

ψ Dirac spinor

Dimensional Reduction

4D vector supermultiplet V

$A_\mu \quad \lambda_L$

4D chiral supermultiplet χ

$\Sigma + iA_5 \quad \lambda_R$

4D chiral supermultiplet Φ

$H^1 \quad \psi_L$

4D antichiral supermultiplet Φ^c

$H^{2*} \quad \psi_R$

Superfield Effective Actions

$$S_{matter} = \int d^8 z \left\{ \bar{\Phi}^{(0)}\Phi^{(0)} + 2g\bar{\Phi}^{(0)}V^{(0)}\Phi^{(0)} + 2g\sum_{n=1}\bar{\Phi}^{(n)}V^{(n)}\Phi^{(0)} + \right.$$

$$\left. + g\sum_{n=1}\Phi^{c(n)}\chi^{(n)}\Phi^{(0)}\delta(\bar{\theta}) + \ldots\ldots \right\}$$

Zero modes → MSSM fields

$y \rightarrow -y$ $\quad S_1 / Z_2$

KK expansion of the Superfields

$$V(x,\theta,\bar{\theta},y) = \frac{1}{\sqrt{\pi R}}\{V^{(0)}(x,\theta,\bar{\theta}) + \sqrt{2}\sum_{n=1}^{\infty}[V^{(n)}(x,\theta,\bar{\theta})\cos(\frac{ny}{R})]\}$$

$$\chi(x,\theta,\bar{\theta},y) = \frac{1}{\sqrt{\pi R}}\sqrt{2}\sum_{n=1}^{\infty}[\chi^{(n)}(x,\theta,\bar{\theta})\sin(\frac{ny}{R})]$$

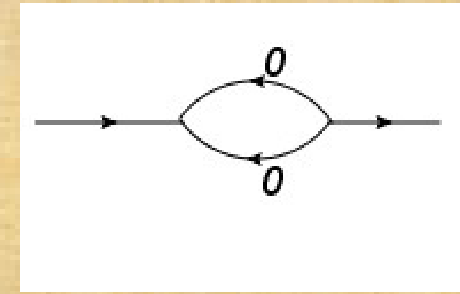
$$S_{Yukawa} = \int d^8 z \left\{ \frac{\lambda_{ijk}}{6}[\Phi_i^{(0)}\Phi_j^{(0)}\Phi_k^{(0)} + 3\sqrt{2}\sum_{n=1}\Phi_i^{(n)}\Phi_j^{(0)}\Phi_k^{(0)} + \right.$$

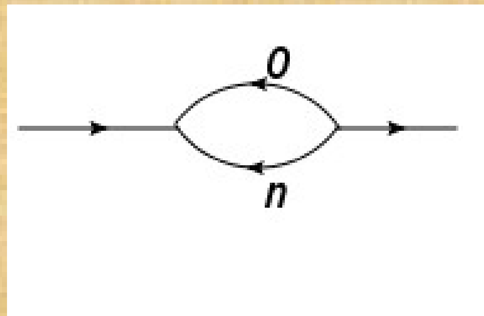
$$\left. + 6\sum_{m,n=1}\Phi_i^{(0)}\Phi_j^{(m)}\Phi_k^{(n)}]\delta(\bar{\theta}) + \ldots\ldots \right\}$$

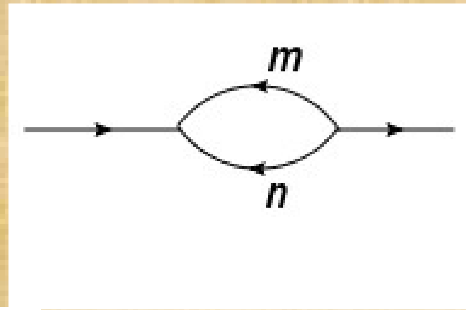
$$\Phi^c(x,\theta,\bar{\theta},y) = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty}\Phi^{c(n)}(x,\theta,\bar{\theta})\sin(\frac{ny}{R})$$

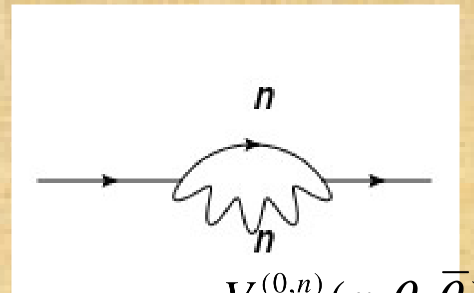
$$\Phi(x,\theta,\bar{\theta},y) = \frac{1}{\sqrt{\pi R}}\{\Phi^{(0)}(x,\theta,\bar{\theta}) + \sqrt{2}\sum_{n=1}^{\infty}\Phi^{(n)}(x,\theta,\bar{\theta})\cos(\frac{ny}{R})\}$$

Quantum Correction to the Superfield Wavefunction Renormalization

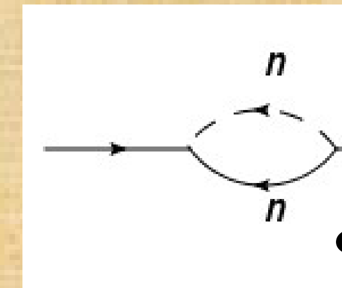








$V^{(0,n)}(x,\theta,\bar{\theta})$



$\chi^{(n)}(x,\theta,\bar{\theta})$

$\Phi^{c(n)}(x,\theta,\bar{\theta})$

Beta Functions of the Yukawa Couplings

$$16\pi^2 \frac{dY_d}{dt} = Y_d(3Tr(Y_d^\dagger Y_d) + Tr(Y_e^\dagger Y_e) + 3Y_d^\dagger Y_d + Y_u^\dagger Y_u)\pi S(t)^2 - Y_d\left(\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)S(t)$$

$$16\pi^2 \frac{dY_u}{dt} = Y_u(3Tr(Y_u^\dagger Y_u) + 3Y_u^\dagger Y_u + Y_d^\dagger Y_d)\pi S(t)^2 - Y_u\left(\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)S(t)$$

$$16\pi^2 \frac{dY_e}{dt} = Y_e(3Tr(Y_d^\dagger Y_d) + Tr(Y_e^\dagger Y_e) + 3Y_e^\dagger Y_e)\pi S(t)^2 - Y_e\left(\frac{9}{5}g_1^2 + 3g_2^2\right)S(t)$$

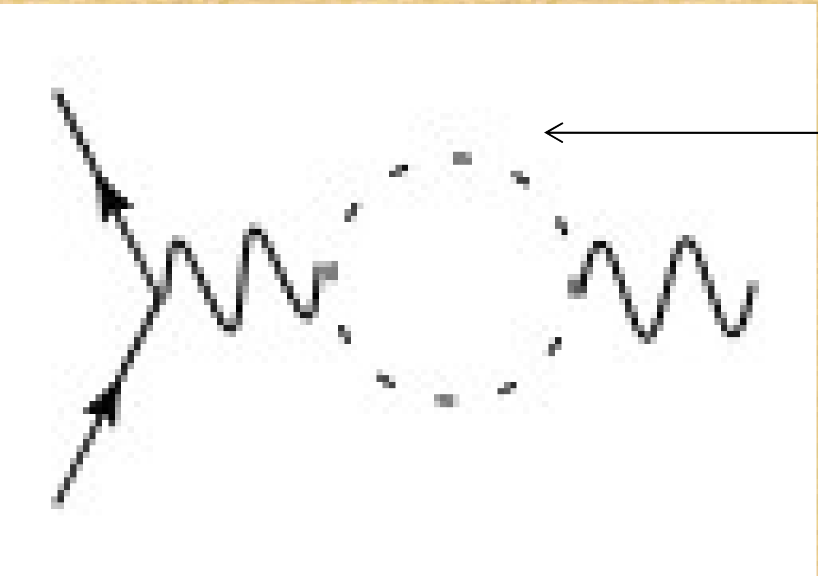
KK number non-conservation effects

Beta functions of the Gauge Couplings

$\chi(x,\theta,\bar{\theta})$ ~the scalar field and its superpartner

$\Phi^{C}(x,\theta,\bar{\theta})$ ~the scalar field and its superpartner for each matter fields

$\Phi^{C}(x,\theta,\bar{\theta})$ ~the scalar field and its superpartner for each Higgs fields



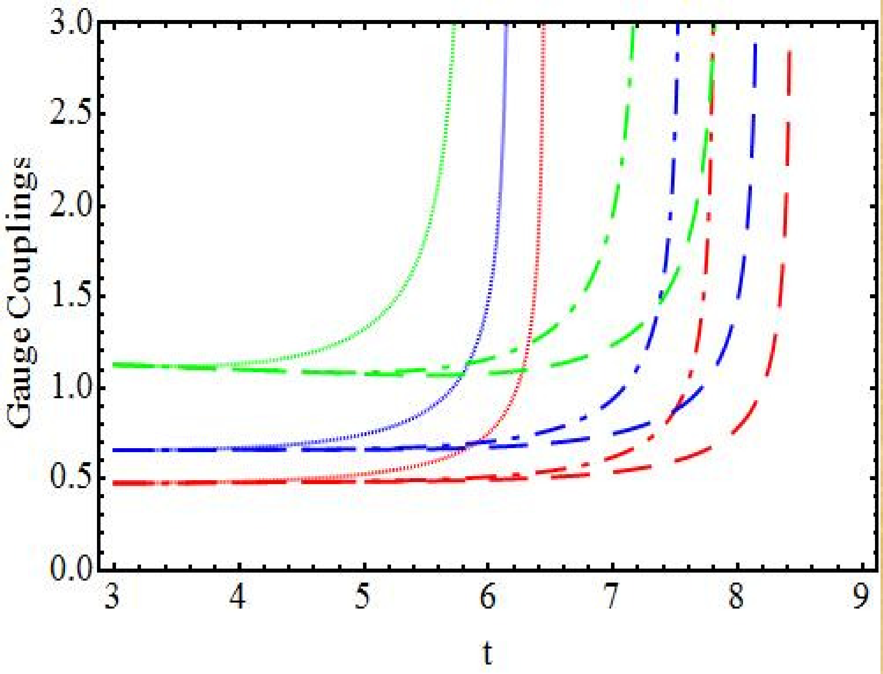
new contributions from these fields to the wave function renormalization of the gauge fields

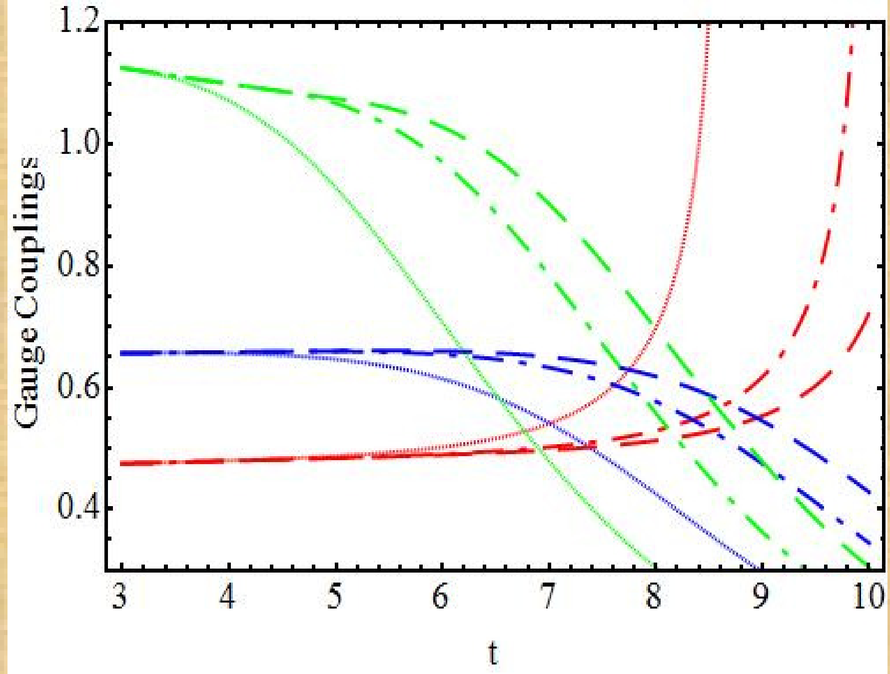
$$16\pi^2 \frac{dg_i}{dt} = [b_i^{MSSM} + (S(t)-1)\tilde{b}_i]g_i^3$$

$$b_i^{MSSM} = (\frac{33}{5},1,-3)$$

$$(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3) = (\frac{6}{5},-2,-6)+4\eta$$

Evolution of the Gauge Couplings





$$(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (\frac{6}{5}, -2, -6) + 4\eta$$

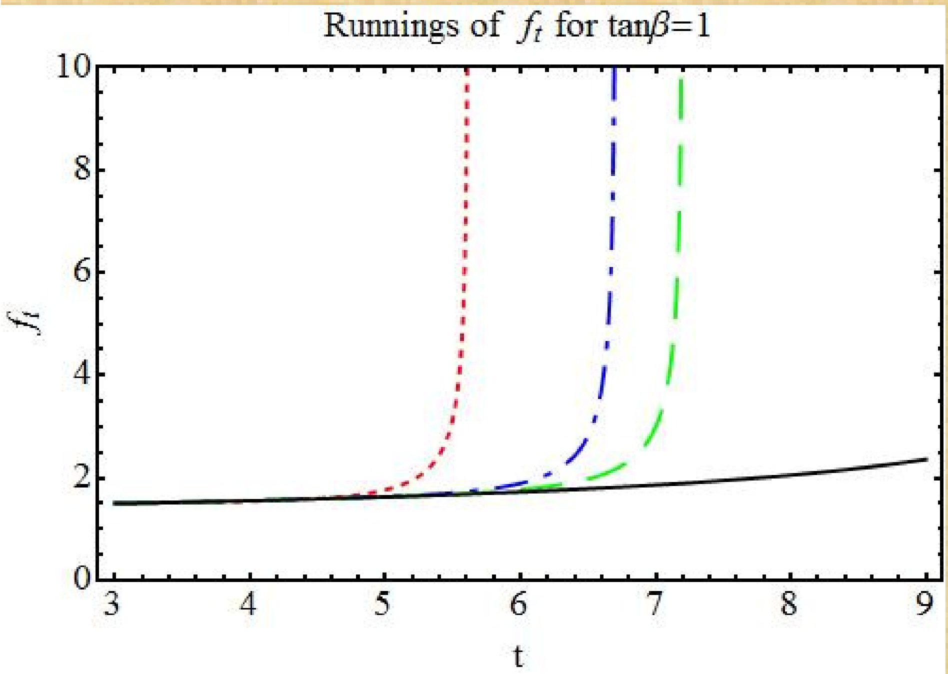
$$\eta = 3$$

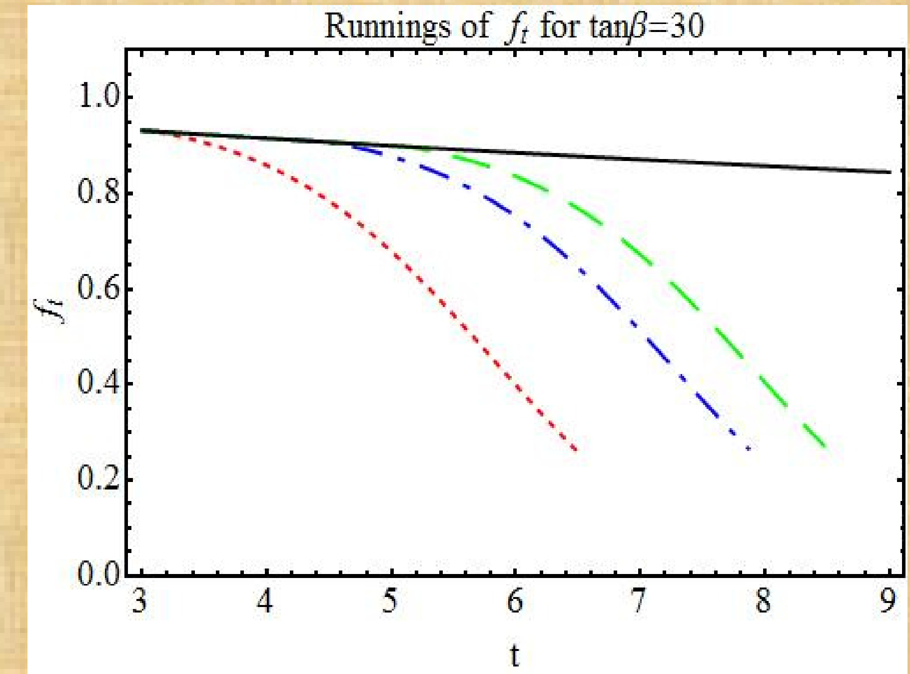
Universal Scenario

$$\eta = 0$$

Brane Localized Matter Fields Scenario

Evolutions of the Top Yukawa Coupling for Different $\tan\beta$

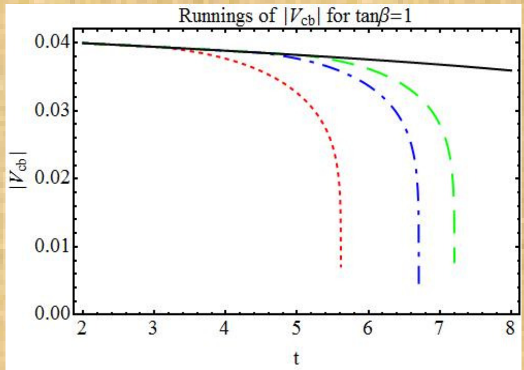


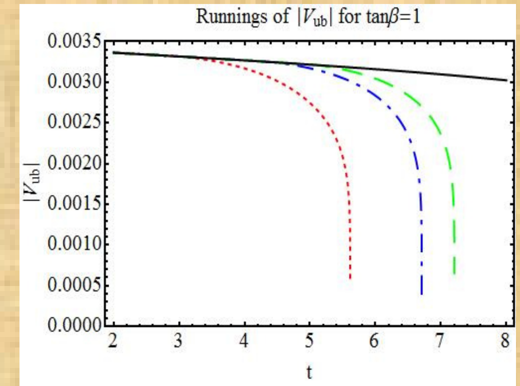


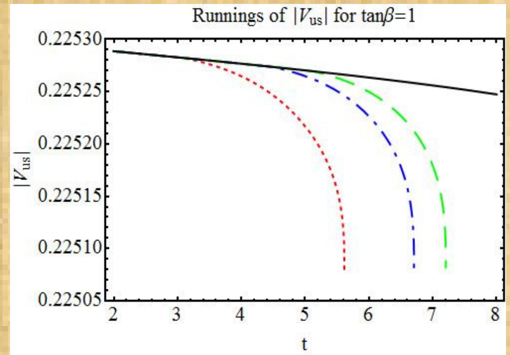
$$\tan\beta = \frac{v_u}{v_d} \qquad v_u{}^2 + v_d{}^2 = \left(\frac{246}{\sqrt{2}}\right)^2 \qquad Y_{d,s,b} = \frac{m_{d,s,b}}{v_d} \qquad Y_{u,c,t} = \frac{m_{u,c,t}}{v_u}$$

A.S. Cornell,A.Deandrea, Lu-Xin Liu, A. Tarhini,arXiv:1110.1942 [hep-ph]

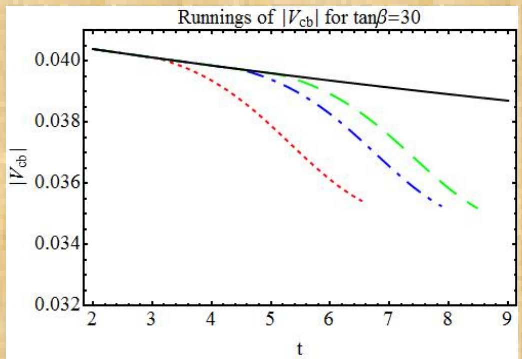
Evolution of the CKM Flavor Mixing Matrix in the 5D MSSM Model

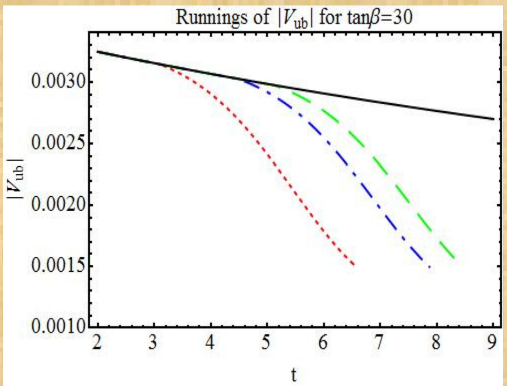


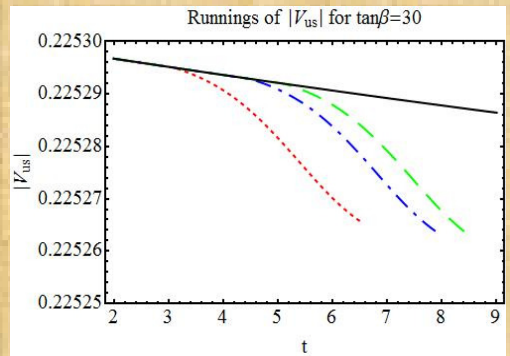




small $\tan\beta$







large $\tan\beta$

$V_{cb} \approx \theta_{23}$ $\quad$ $V_{ub} \approx \theta_{13} e^{-i\delta_{13}}$ $\quad$ $V_{us} \approx \theta_{12}$

$$J_W = \bar{u}_L \gamma_\mu d_L = \bar{u}_L(mass)\gamma_\mu V_U{}^{+} V_D d_L(mass)$$
$$= \bar{u}_L(mass)\gamma_\mu V_{CKM} d_L(mass)$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

Summary

- In the 5D Model, in light of the current LHC Higgs mass bounds, the evolution of the Higgs self-coupling has an improved vacuum stability condition for brane localized matter fields scenario.

- In the 5D MSSM, the brane localized matter field scenario with a large $\tan\beta$ has a preferable gauge unification and better Yukawa evolution behavior.

- As for the flavor mixing matrix, the CKM evolves towards no mixing (or small mixing) scenario at high energy scale.