Gravitational Wave Parameter Estimation as a Test of General Relativity

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Cosmic Censorship Conjecture

- The complete gravitational collapse of a massive body will always result in a singularity that is concealed by an event horizon (a black hole)
- Naked singularities, which are singularities that can be seen by distant observers, do not exist
- Believed to be true in General Relativity, but violations could be ubiquitous in alternative theories of gravity

Kerr Black Hole

- The Kerr geometry describes the spacetime around a stationary, rotating black hole, which is characterized only by its mass M and angular momentum J

- The Kerr metric contains a physical singularity, which is safely concealed by a horizon at radius (G = c = 1)

$$r = M + \sqrt{M^2 - \left(\frac{J}{M}\right)^2}$$

- This horizon only exists if

$$\frac{J}{M} \leq M \Rightarrow J \leq M^2$$

- Therefore, the cosmic censorship conjecture constrains the angular momentum of a Kerr black hole

Testing Cosmic Censorship

- To test cosmic censorship, we need...
  - ...to observe a compact object with $M \geq 3M_{\odot}$, which should be a black hole
  - ...to determine the mass of the compact object
  - ...to determine the angular momentum of the compact object
- What observations can provide all of this information?

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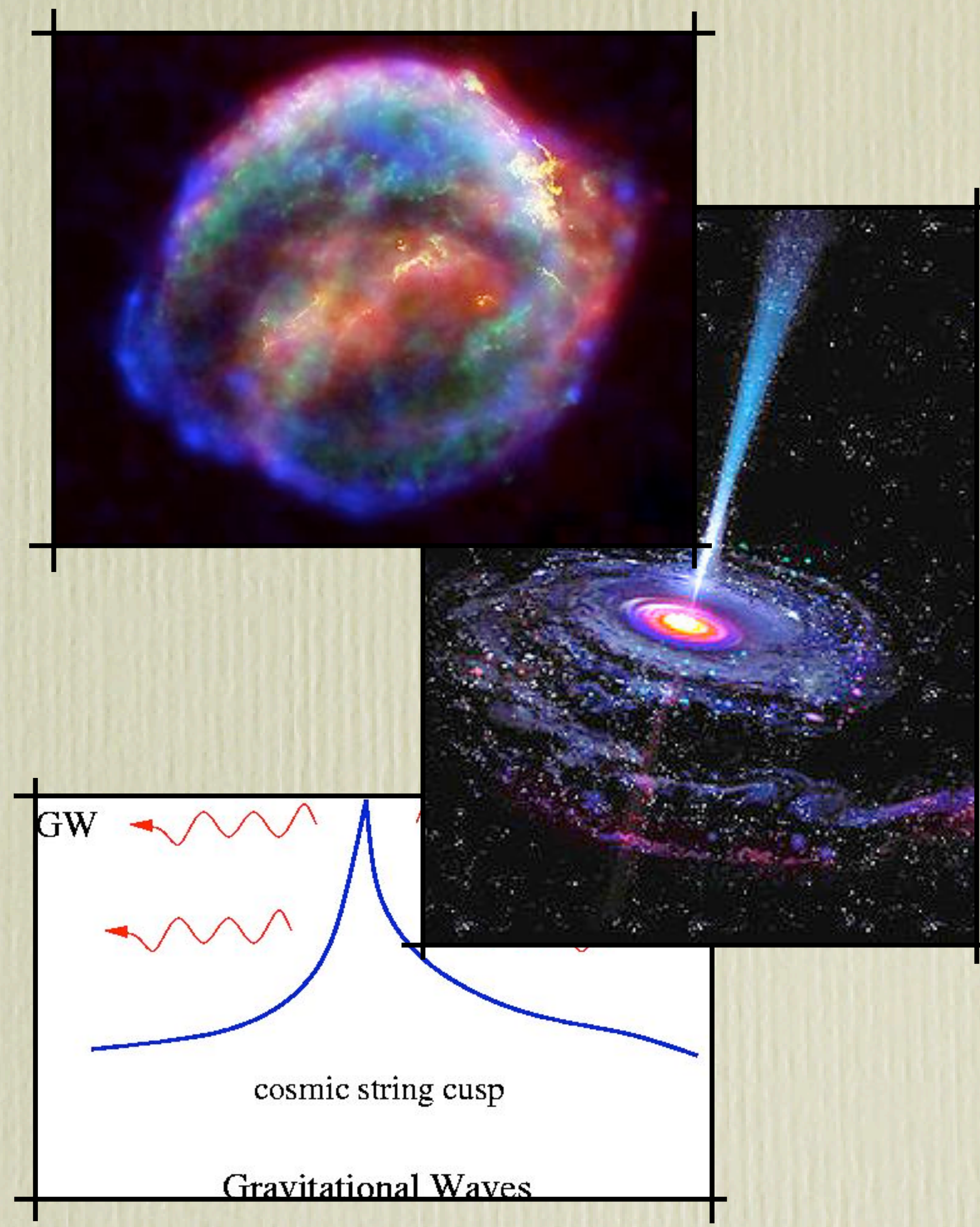
Gravitational wave observations from compact binary coalescence

Gravitational Waves

- Wave-like perturbations in the spacetime metric
- Predicted by metric theories of gravity, including Einstein's General Relativity
- Sources:
    - Rotating, deformed ellipsoidal objects
    - Compact binary coalescence
    - Supernovae
    - Cosmic string cusps



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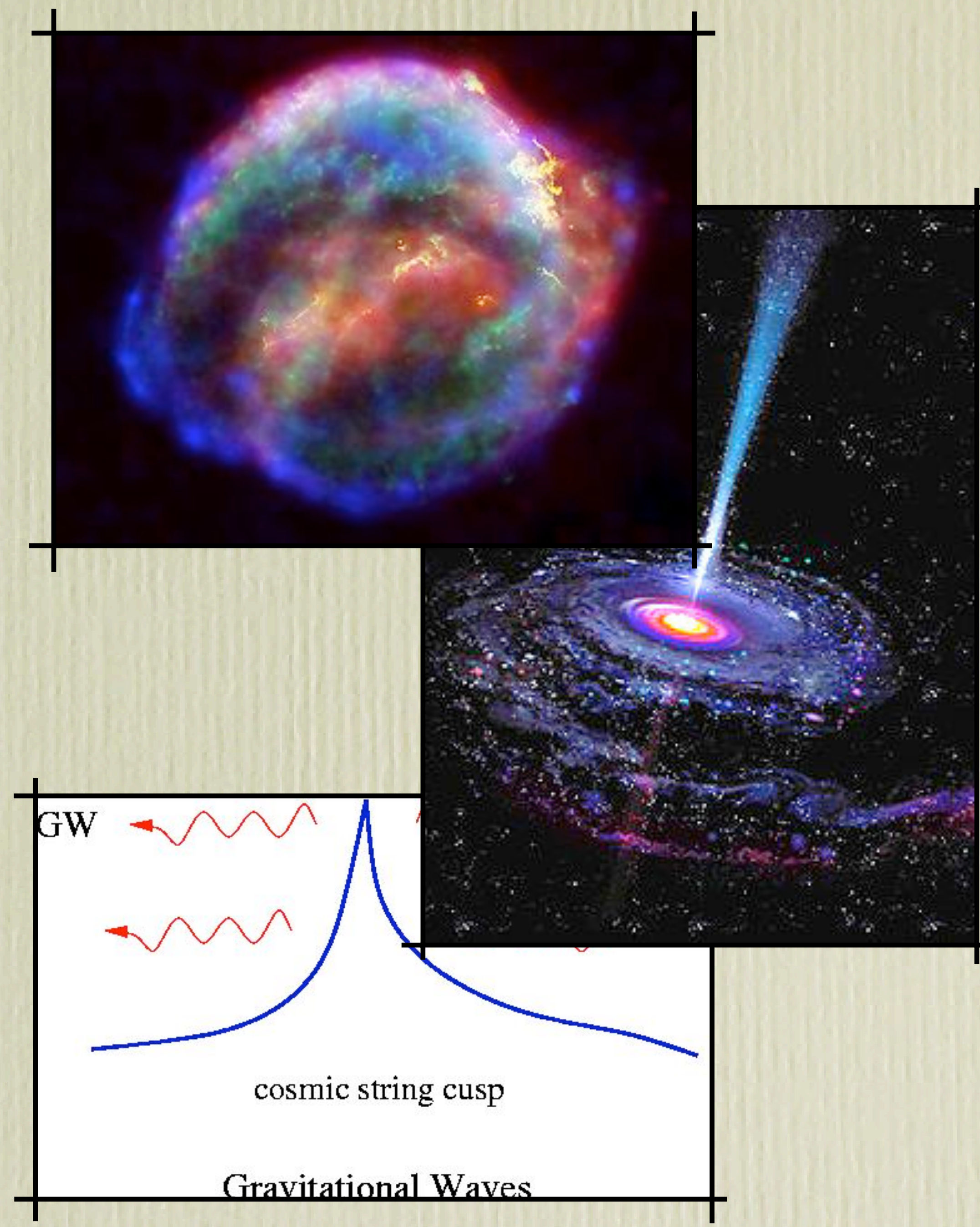
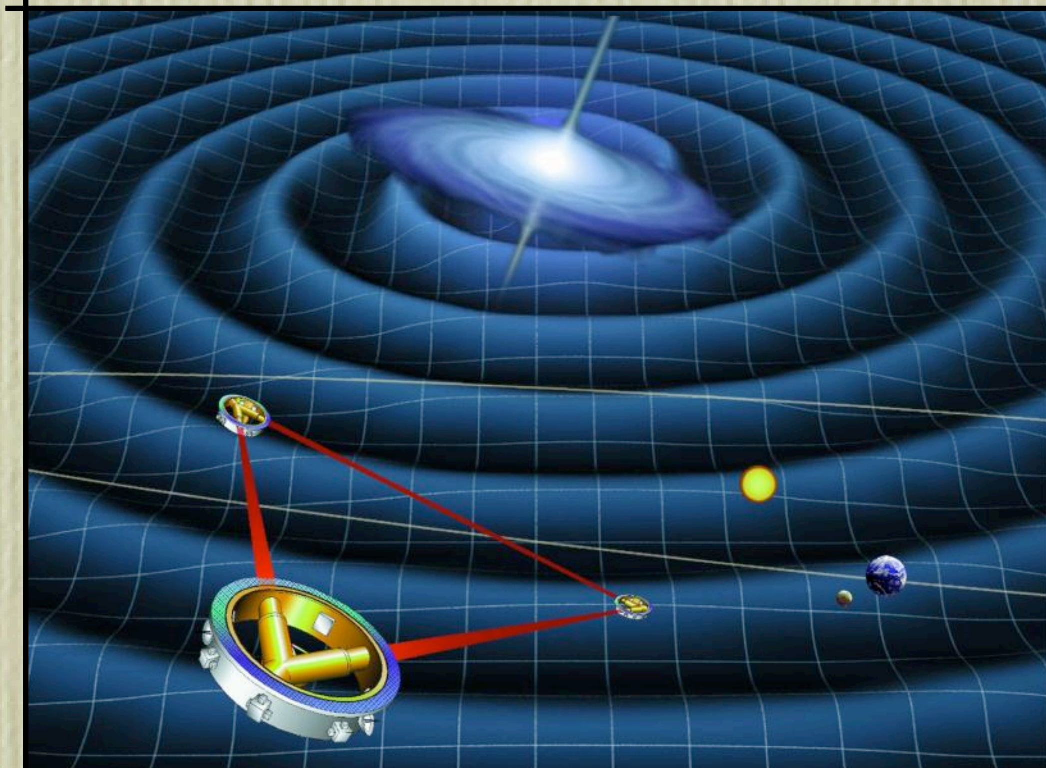
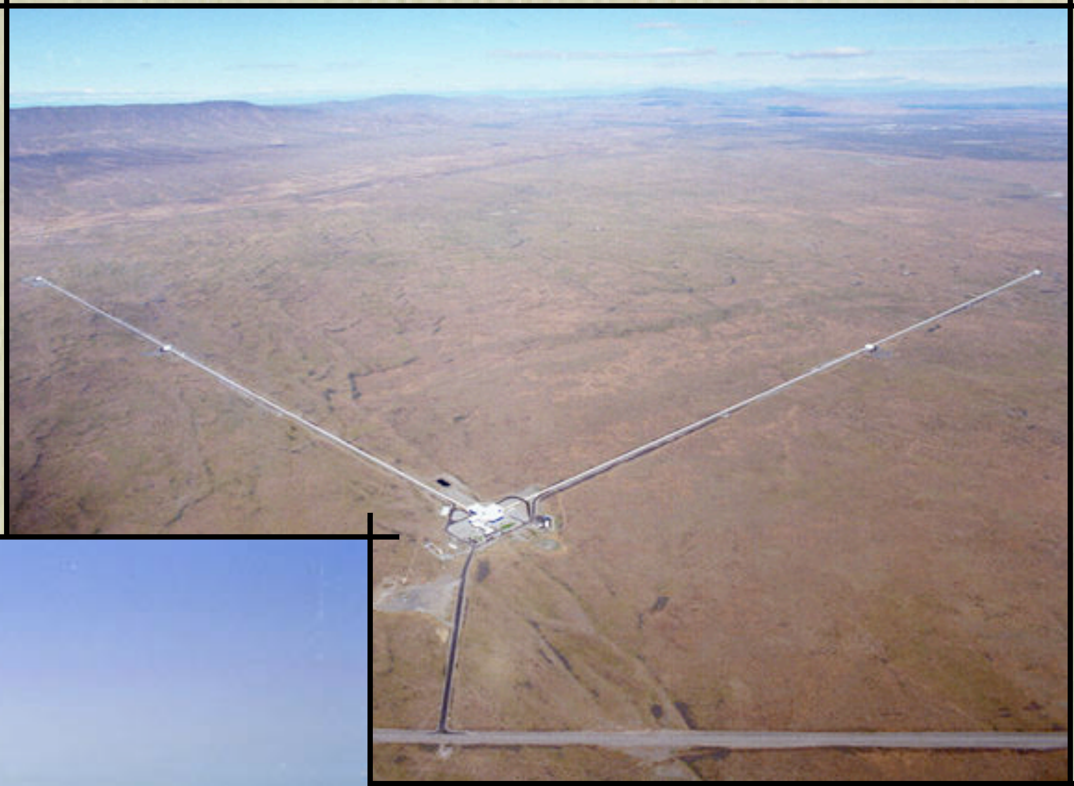


Gravitational Wave Interferometers





Compact Binary Coalescence

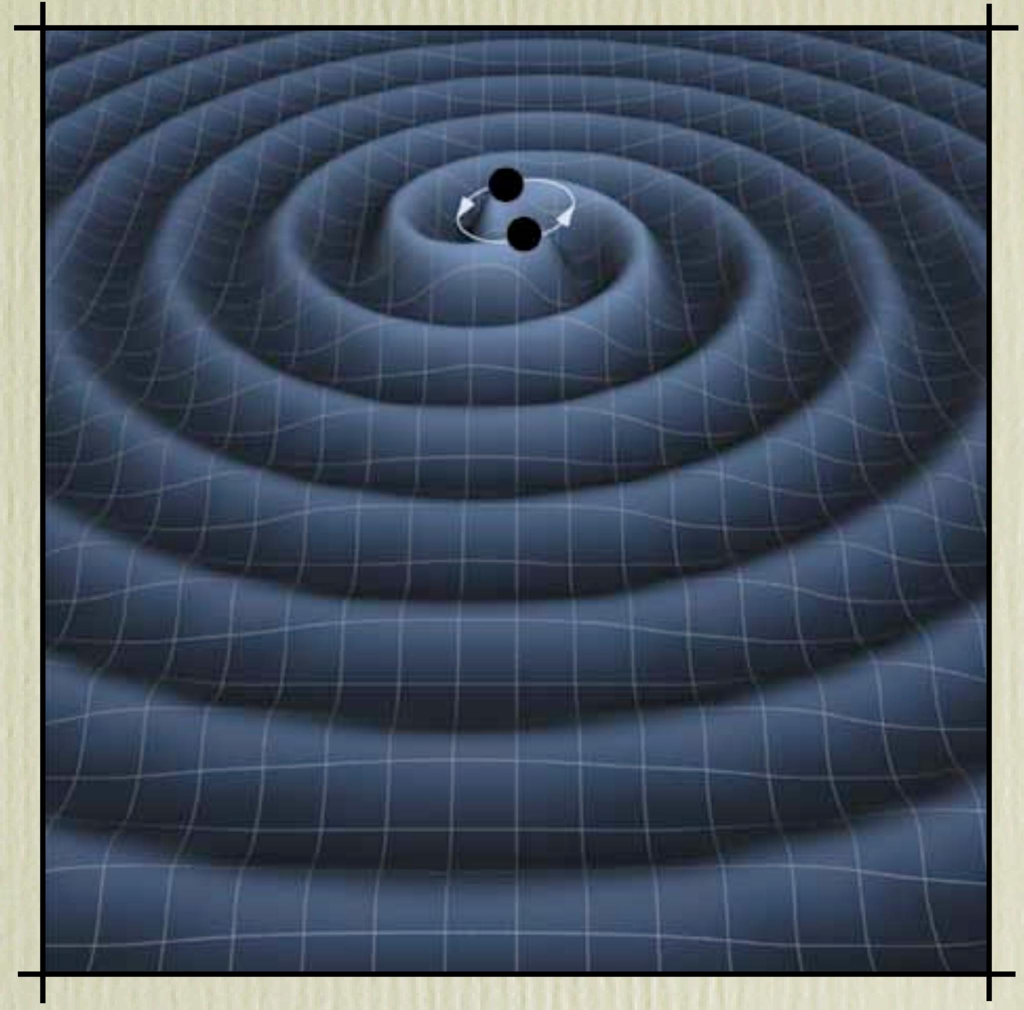
- Compact binary coalescence (CBC) is the *inspiral*, *merger*, and *ringdown* of two compact objects (i.e. neutron stars and black holes)

- The gravitational wave strain of the *inspiral* portion of a CBC event in the frequency domain is

$$\tilde{h}(f) = A(f;\theta_i)e^{i\psi(f;\theta_i)}$$

  - depends on frequency f of the gravitational wave and source/detector parameters

$$\theta_i = \{M_1, M_2, J_1, J_2, \text{ and other parameters}\}$$



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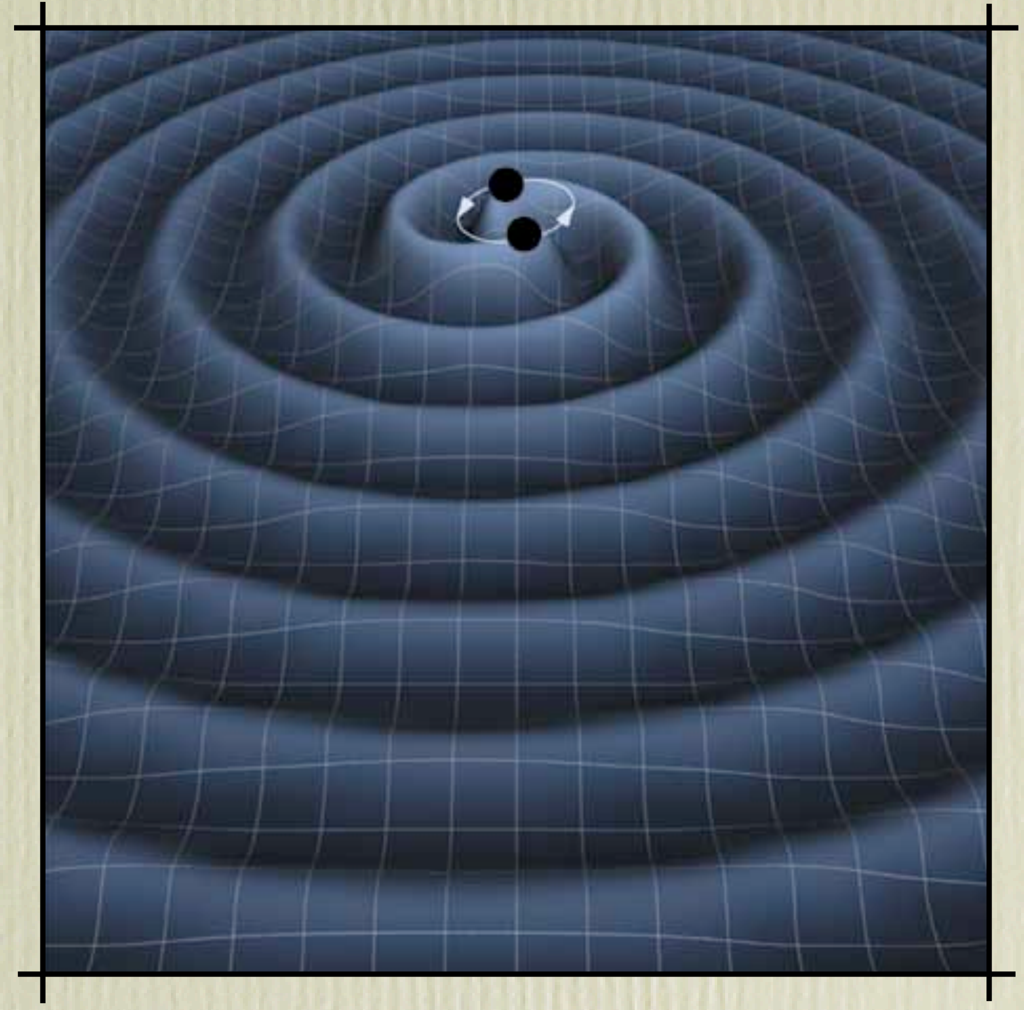

When aLIGO detects a CBC event, we can extract mass and spin of the system from the waveform.

Compact Binary Coalescence Waveform

- Easiest to measure linear combinations of mass and spin that appear in the waveform, i.e.

$$\eta = \frac{M_1 M_2}{(M_1 + M_2)^2} \leq \frac{1}{4}$$

$$\beta = \frac{1}{(M_1 + M_2)^2}\left[\left(\frac{113}{12} + \frac{25}{4}\frac{M_1}{M_2}\right) J_1 + \left(\frac{113}{12} + \frac{25}{4}\frac{M_1}{M_2}\right) J_2\right]$$

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These limits divide the physically realizable area of parameter space into regions that are consistent and inconsistent with cosmic censorship in the Kerr geometry.

Parameter Estimation

- Determine most likely value of source parameters from a detected signal

- The *Fisher information matrix* is a simple method for estimating parameter errors and correlations

$$\Gamma_{ij} = \left(\frac{\partial h}{\partial \theta_i} \middle| \frac{\partial h}{\partial \theta_j} \right)$$

$$(a|\, b) = 4\Re \int \frac{\tilde{a}^*(f)\tilde{b}(f)}{S_h(f)} df$$

$$\sigma_i = \sqrt{(\Gamma^{-1})_{ii}}$$

$$c_{ij} = \frac{\left(\Gamma^{-1}\right)_{ij}}{\sqrt{\left(\Gamma^{-1}\right)_{ii}\left(\Gamma^{-1}\right)_{jj}}}$$

Cutler and E. Flanagan. Gravitational waves from merging compact binaries: How accurately can one extract the binary's parameters from the inspiral waveform?. Phys.Rev.D, 49:2658-2697, 1994.

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Parameter estimation techniques can be used to determine the region of parameter space that a system's β and η lie in.

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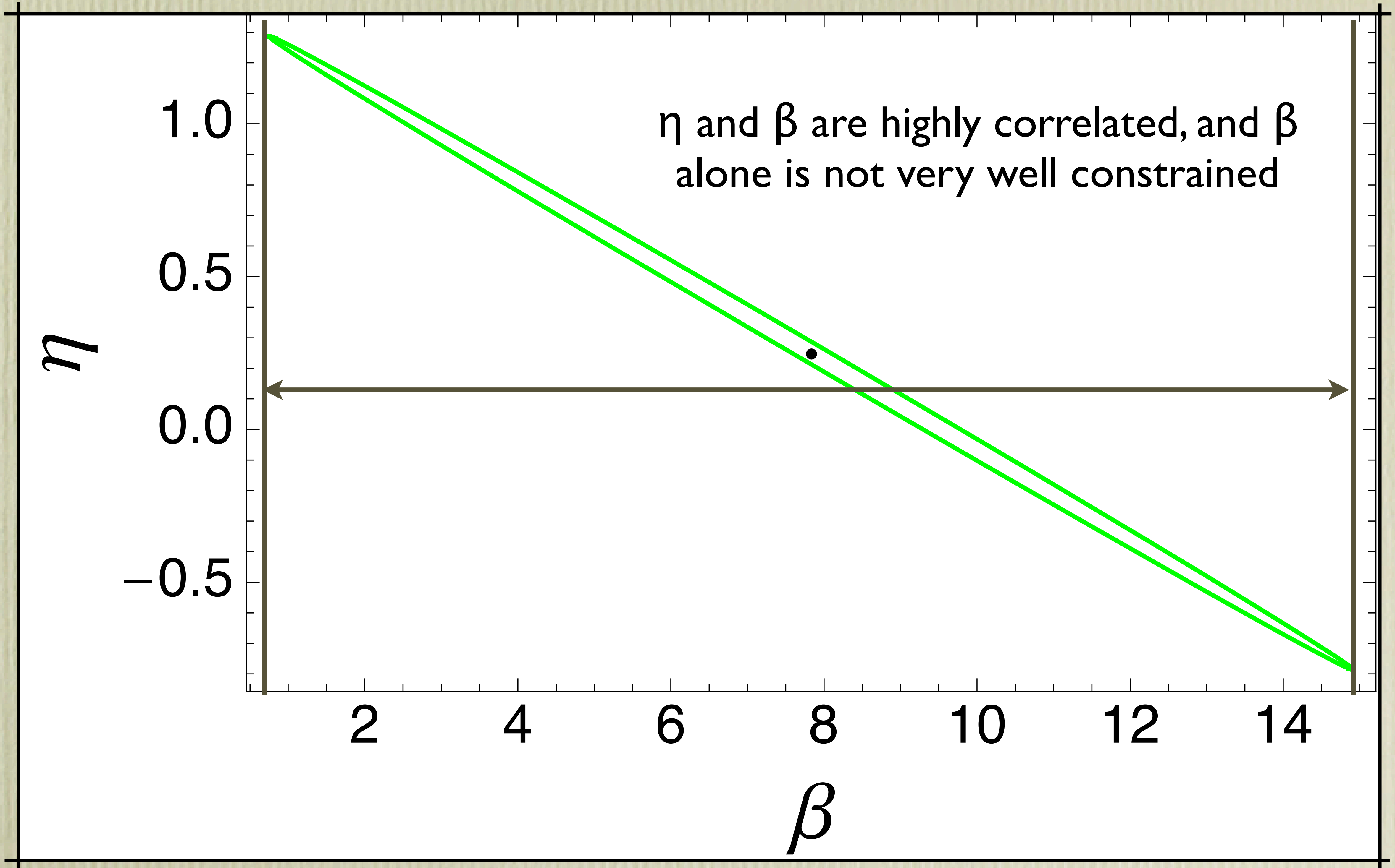
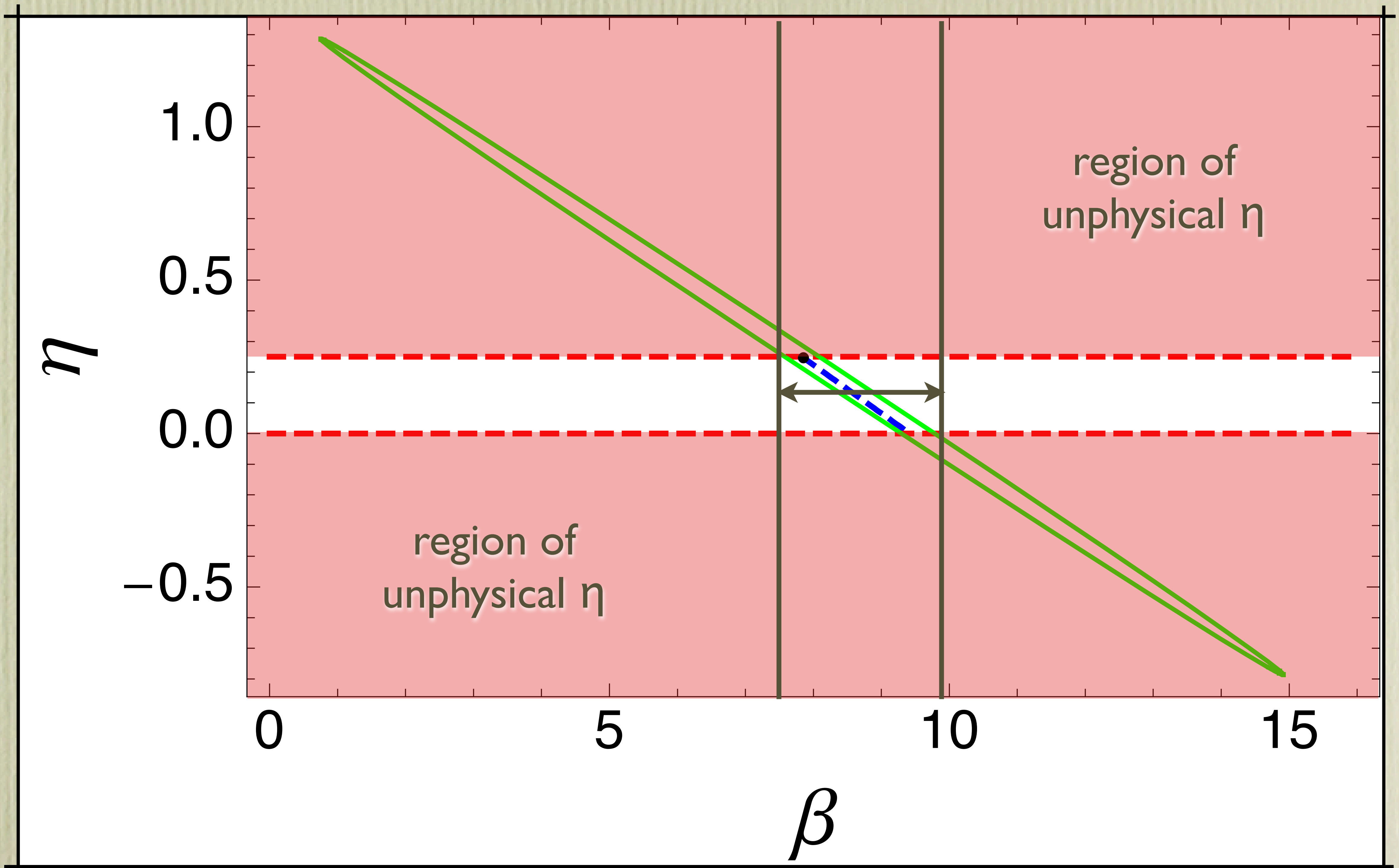


Calculated for SNR $= 50$ using the aLIGO noise curve with $M_1 = M_2 = 10M_\odot$



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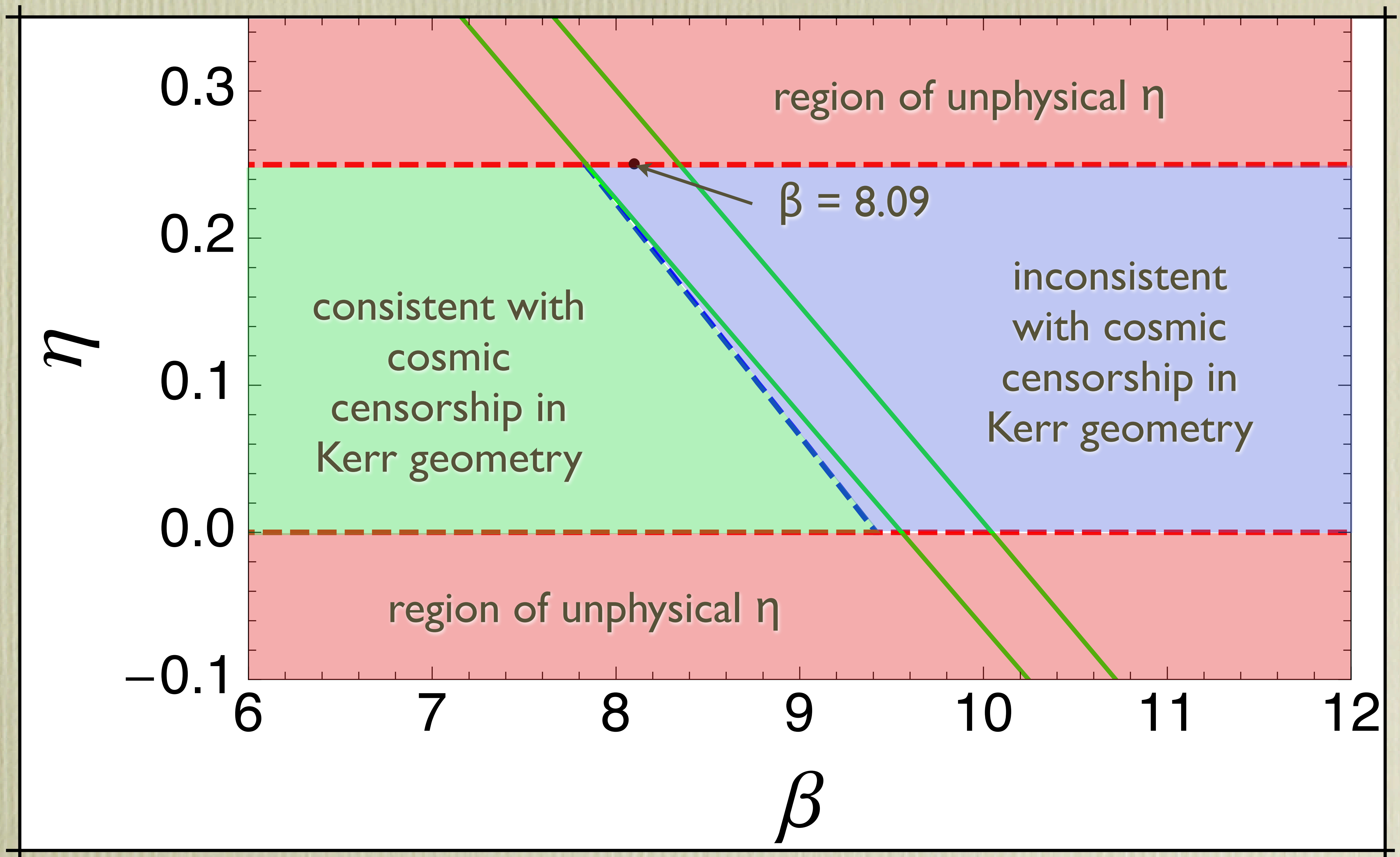

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Current and Future Work

- Other methods for improving parameter estimation
  - Improved waveforms in both phase and amplitude
  - Improved numerical methods (i.e. SVD for matrix inversion)
- Tidal corrections to test the no-hair theorem
  - General Relativity predicts black holes to be "hairless"
  - The tidal parameter that appears in the CBC waveform should be zero
  - Use similar methods described here to improve the measurability of the tidal parameter

Implications for Cosmology

- Advanced ground based detectors should provide the first direct detection of gravitational waves
- Gravitational wave detectors are telescopes to explore a new type of radiation and add to our knowledge of the universe
- Gravitational wave detections will test the cosmic censorship conjecture in the context of General Relativity, as well as other aspects of metric theories of gravity