DARK MATTER, INFINITE STATISTICS & QUANTUM GRAVITY

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THE "MISSING MASS" PROBLEM : GALACTIC ROTATION CURVES (RUBIN ...)
CLUSTERS OF GALAXIES (ZWICKY ... "BULLET" CLUSTER)
COSMOLOGY (CMB, LARGE SCALE STRUCTURE → ΛCDM)

[4.6% ⇝ SM
23.4% ⇝ DM
72% ⇝ DE

EINSTEIN'S EQUATIONS:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu} + \lambda g_{\mu\nu}$$

$\Lambda \sim \frac{1}{R^2}$; R - SIZE OF THE UNIVERSE (IR) $(R \sim H)$, $\frac{\Lambda}{8\pi G_N} = t_{\mu\nu}^{VACUUM} = \underbrace{\sum_p \frac{1}{2} \hbar \omega_p}_{(UV)}$ → COSMOLOGICAL CONSTANT PROBLEM

(SEE, L.N. CHANG, D.M., TATSU TAKEUCHI)
1004.4220

FOR Λ (C.C ~ "DARK ENERGY" (DE)) NO DECOUPLING BTW UV & IR !!

MISSING MASS ⇝ "DARK MATTER" (DM)

$$T_{\mu\nu} = T_{\mu\nu}^{SM} + T_{\mu\nu}^{DM}$$

$T_{\mu\nu}^{SM}$ ↓ STANDARD MODEL (SM)

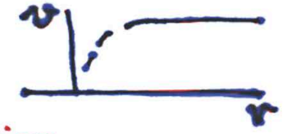
$T_{\mu\nu}^{DM}$ ↓ "DARK MATTER"

} DECOUPLING ?
(I.E. EFFECTIVE FIELD THEORY ?)

CDM ⇝ YES !

"A FLY IN THE OINTMENT": CDM WORKS SPECTACULARLY WELL ON CLUSTER & COSMIC SCALES

HOWEVER: – GALACTIC ROTATION CURVES OBTAINED BY FITTING (USUALLY WITH 2 PARAMETERS) FOR INDIVIDUAL GALAXIES



– CDM NOT BETTER IN EXPLAINING THE OBSERVED BARYONIC TULLY-FISHER RELATION ($v^4 \sim M$)

– PROBLEM WITH CDM ON SMALL SCALES (1-1000 kpc)

WHAT WOULD BE THE ALTERNATIVE: CHANGE EINSTEIN? (EQUIV. PRINCIPLE!)

$$G_{\mu\nu} \to F(G_{\mu\nu})$$

FOR EXAMPLE: GALACTIC ROTATION CURVES $\quad a = \begin{cases} a_N, & a \gg a_c \\ \sqrt{a_N a_c}, & a \ll a_c \end{cases} \quad a_N = \frac{G_N M}{r^2}$

HOWEVER: – PROBLEMS ON CLUSTER SCALES (DIFFERENT a_c!)

$$\underbrace{a_c \sim \frac{cH}{(2\pi)} \sim 1.2 \times 10^{-8}\,\frac{cm}{s^2}}_{\text{MILGROM'S SCALING (MOND)}} \Rightarrow \text{VIOLATION OF DECOUPLING!!}$$

– PROBLEMS ON COSMOLOGICAL SCALES

– NO REALLY FUNDAMENTAL RELATIVISTIC THEORY OF MOND

Ho, D.M. NG: <u>NEW IDEA</u> (ANALOGOUS TO WAVE-PARTICLE DUALITY)

DM WITH MOND SCALING: $\left.\begin{array}{l}\text{AT LARGE SCALES CDM} \\ \text{AT SMALL SCALES MOND}\end{array}\right\} \Rightarrow$ DM WITH UNUSUAL INERTIAL PROPERTIES

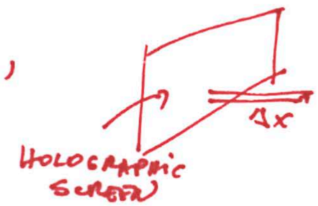
RADICALLY NEW NON-EFT PHYSICS (QUANTUM GRAVITY) $\Leftarrow$ (VIOLATION OF DECOUPLING)

IN ORDER TO COMBINE CDM ("PARTICLE") & MOND ("WAVE") INTO MONDIAN DARK MATTER

USE THE RELATIONSHIP BTW GRAVITY & THERMODYNAMICS

IN PARTICULAR USE VERLINDE'S ENTROPIC REINTERPRETATION:

1) $F_{entropic} = T\frac{\Delta S}{\Delta x}$,



$$\left.\begin{array}{l} \Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x \\ \text{UNRUH:} \quad k_B T = \frac{\hbar a}{2\pi c} \end{array}\right\} \Rightarrow \vec{F}_{entropic} = m\vec{a}$$

$\nabla_x S$ VECTOR

2) QUASI-LOCAL HOLOGRAPHIC SCREEN (S^2): $A = 4\pi r^2$ WITH T!

EQUIPARTITION: $E = \frac{1}{2} N k_B T$, N — # OF DOF ; $N = \frac{Ac^3}{G_N \hbar}$ – HOLOGRAPHIC

USE $k_B T = \frac{\hbar a}{2\pi c}$, $E = Mc^2$ $\Rightarrow$ $a_N = \frac{G_N M}{r^2}$

HO, D.M., NG APPLY 1) & 2) TO de SITTER SPACE (ACCELERATING UNIVERSE $\Lambda > 0$)
($\hbar = c = 1$)

$$\left.\begin{array}{l} T_{dS} = \frac{a_0}{2\pi k_B}, \quad a_0 = \sqrt{\frac{\Lambda}{3}} ; \quad \frac{\Lambda}{3} \equiv H^2 \\ T_{dS+a} = \frac{1}{2\pi k_B}\underbrace{\sqrt{a^2+a_0^2}}_{\text{proper acceleration in } M^5} \end{array}\right\} \Rightarrow \tilde{T} \equiv T_{dS+a} - T_{dS}$$

(ds^4 IN M^5 EMBEDDING COORDINATES)

$$\boxed{\tilde{T} = \frac{1}{2\pi k_B}\left[\sqrt{a^2+a_0^2} - a_0\right]}$$

CDM/MOND DUALITY:

$$F_{entropic} = \tilde{T}\, \nabla_x S = m \left[\sqrt{a^2 + a_0^2} - a_0 \right]$$

$a \ll a_0$, $\quad F_{entropic} \simeq m \dfrac{a^2}{2a_0}$

FIT GALACTIC ROTATION CURVES: $\quad F_{entropic} \simeq m \dfrac{a^2}{2a_0} = F_{Milgrom} \simeq m\sqrt{a_N a_c}$

1) $$\Leftrightarrow \quad a = \left(4 a_N a_0^2 a_c\right)^{1/4} = \left(2 a_N \frac{a_0^3}{4\pi}\right)^{1/4}$$

NUMERICALLY $2\pi a_c \simeq a_0$; THUS SET $a_c = \dfrac{a_0}{2\pi}$

FOR $a < a_0$ $\quad F_{centripetal} = \dfrac{m a^2}{2a_0} \quad \left(= \dfrac{m v^2}{r} \right) \Rightarrow$ CONSTANT v

FLAT ROTATION CURVES !!

2) $$2\pi k_B \tilde{T} = 2\pi k_B \left(\frac{2\tilde{E}}{N k_B} \right) = 4\pi \left(\frac{\tilde{M}}{A/G_N} \right) = \frac{G_N \tilde{M}}{r^2}$$

EQUIPARTITION

$\tilde{M}$ TOTAL MASS IN $V = 4\pi \dfrac{r^3}{3}$.

CONSISTENCY WITH OBSERVATION $\quad \tilde{M} = M + M'$ $\quad M'$ – MISSING MASS (DARK MATTER!)

NOTE $\quad F_{entropic} = m \left[\sqrt{a^2 + a_0^2} - a_0 \right]$ $\quad$ Let $\boxed{M' = \frac{1}{\pi} \left(\frac{a_0}{a} \right)^2 M}$

$$F_{entropic} = m a_N \left[1 + \frac{1}{\pi} \left(\frac{a_0}{a} \right)^2 \right] \quad \left(\frac{G_N m \tilde{M}}{r^2} \right)$$

$a \gg a_0 \Rightarrow F_{entropic} \simeq m a \simeq m a_N \Rightarrow \underline{a = a_N}$

$a \ll a_0 \rightarrow F_{entropic} \simeq m \dfrac{a^2}{2a_0}$

Derive MOND !!

NOW WE CAN REALIZE CDM-MOND duality:

INTERPRET $F_{entropic} = m\left[\sqrt{a^2+a_0^2} - a_0\right] = m a_N \left[1 + \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2\right]$

NO DARK MATTER

OR

WE CAN REWRITE $F_{entropic} = m \frac{G_N (M+M')}{r^2}$ WITH MASS PROFILE

"MONDIAN DARK MATTER" $\equiv$ THIS DARK MATTER BEHAVES AS IF THERE IS NO DARK MATTER BUT THERE IS MOND $\Leftarrow$ $\boxed{M' = \frac{1}{\pi}\left(\frac{a_0}{a}\right)^2 M}$

CONSISTENT COSMOLOGY & LARGE SCALE STRUCTURE:

FRIEDMANN'S EQS:

$$ds^2 = -dt^2 + R^2(t)\,(dr^2 + r^2 d\Omega^2)$$

ASSUME PERFECT FLUID : $T_{\mu\nu} = (\rho+p)\, u_\mu u_\nu + p\, g_{\mu\nu}$, $u_\mu = (1, \vec{0})$

CONSIDER VERLINDE'S IMAGINARY HOLOGRAPHIC SCREEN OF COMOVING RADIUS r (OR, THE PHYSICAL RADIUS $\hat{r} = r R(t)$)

$a_{eff} = \sqrt{a^2+a_0^2} - a_0$

$a_{eff} = -\frac{d^2(rR)}{dt^2} = -\ddot{R} r$, USE $2\pi k_B \tilde{T} = \frac{G_N \tilde{M}}{r^2 R^2} \Rightarrow \ddot{R} = -\frac{G_N \tilde{M}}{r^2 R^2}$

Fully relativistic treatment : $\tilde{M} \Rightarrow \mathcal{M} = \frac{1}{4\pi G_N}\int dV\, R_{\mu\nu} u^\mu u^\nu \xRightarrow{\text{EINSTEIN}} \frac{4\pi}{3} r^3 R^3 \left[(\rho+3p) - \frac{\Lambda}{4\pi G_N}\right]$

(Tolman-Komar mass)

$\Rightarrow H^2 = \frac{8\pi G_N}{3}\rho + \frac{\Lambda}{3} \Rightarrow$ FRIEDMANN'S EQS. !!

NOTE, MONDIAN DARK MATTER HAS UNUSUAL INERTIAL PROPERTIES IN de SITTER SPACE:

INERTIAL PROPERTIES OF MASSIVE PARTICLES WITH MASS m BECOME ACCELERATION & Λ - DEPENDENT

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu} + \Lambda g_{\mu\nu} \Leftarrow$$

QUANTUM GRAVITY
$G_{\mu\nu}$ & $T_{\mu\nu}, \Lambda$ TREATED ON THE SAME FOOTING !!

HOW ABOUT PARTICLE PHYSICS OF THESE QUANTA?

FIRST NOTE; MOND CAN BE REWRITTEN NON-RELATIVISTICALLY AS A GRAVITATIONAL "DIELECTRIC" MEDIUM OF A BORN-INFELD TYPE:

$$\mathcal{L}_g = b^2 \left(1 - \sqrt{1 - \frac{E_g^2}{b^2}} \right)$$

g- "GRAVITATIONAL"

$$\Rightarrow 4\pi H_g = b^2 \left(\sqrt{1 + \frac{D_g^2}{b^2}} - 1 \right) \Rightarrow \frac{1}{4\pi} \left(\sqrt{b^4 + b^2 D_g^2} - b^2 \right)$$

WITH $A_0 \equiv b^2$, $A \equiv b D_g$

$$\Rightarrow H_g = \frac{1}{4\pi} \left(\sqrt{A^2 + A_0^2} - A_0 \right)$$

$$H_g = \frac{1}{2} k_B T_{eff} \quad \Rightarrow \quad a_{eff} = \sqrt{A^2 + A_0^2} - A_0$$

$$\& \quad T_{eff} = \frac{\hbar}{2\pi k_B} a_{eff}$$

NOTE: B-E & F-D $\not\Rightarrow$ $H_g = \frac{1}{2} k_B T_{eff}$ at low T $(\sim O(\hbar))$

$\Rightarrow$ QUANTUM BOLTZMANN STATISTICS
(INFINITE STATISTICS)

INFINITE STATISTICS: $\boxed{a_i a_j^+ = \delta_{ij}}$ (CUNTZ ALGEBRA)

(OR $\underline{q=0}$, $a_i a_j^+ - q\, a_j^+ a_i = \delta_{ij}$; $q=1$ B-E ; $q=-1$ F-D)

GREENBERG PROVED THAT INFINITE STATISTICS

is CONSISTENT WITH LORENTZ SYMMETRY (& CPT)

<u>BUT</u> <u>NOT</u> REALIZABLE AS <u>LOCAL QFT</u>

(<u>NO</u> EFT PICTURE; AGAIN, VIOLATION OF DECOUPLING!)

SIMPLE MODEL:

$$(\partial^2 + m^2)\,\phi(x) = 0 \qquad d\omega_k = \frac{d^3k}{(2\pi)^3\, 2k_0} \text{ as usual}$$

$$\phi(x) = \int d\omega_k \left(a(\vec{k})\, e^{-ikx} + a^+(k)\, e^{ikx} \right)$$

$$a(k)\, a^+(k') = 2k^0 (2\pi)^3\, \delta^3(\vec{k} - \vec{k}') \; , \quad k^0 = \sqrt{\vec{k}^2 + m^2}$$

$$a(k)\,|0\rangle = 0$$

THE WIGHTMAN FUNCTION: $\Delta^+(x-y) = \int d\omega_k\, e^{-ik(x-y)}$

THE FEYNMAN PROPAGATOR $\Delta_F(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik(x-y)}}{k^2 - m^2 + i\epsilon}$

<u>HOWEVER:</u> $[\phi(x), \phi(y)]\big|_{x^0 = y^0} \neq 0$! (Ho, D.M. Ng, Takeuchi)

(VIOLATION OF MICROCAUSALITY !!) $\Rightarrow$ <u>NON-LOCALITY</u>

FINAL COMMENT: INFINITE STATISTICS DOES APPEAR IN QUANTUM GRAVITY.

(i.e. MATRIX THEORY) : <u>Is MONDIAN DARK MATTER QUANTUM GRAVITATIONAL?</u>

BH BH (D0 brane)