Lepton Private Higgs and Σ(81)

Yoni BenTov
A. Zee
UCSB
arxiv:1202.4234

Motivation

- $\mathscr{L}_{\text{Yuk}} = -y_\ell(\ell_e \phi_e e^c + \ell_\mu \phi_\mu \mu^c + \ell_\tau \phi_\tau \tau^c) + \text{h.c.}$

- $\mathbb{Z}_{3,\alpha}$: $(\ell_\alpha, \phi_\alpha, \alpha^c) \rightarrow e^{i2\pi/3} (\ell_\alpha, \phi_\alpha, \alpha^c)$

- $\mathbb{Z}_{3,C}$: $(e, \mu, \tau) \rightarrow (\tau, e, \mu) \rightarrow (\mu, \tau, e)$

$\Sigma(81) = (\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_3$

- $$\begin{pmatrix} \omega^{p_1} & & \\ & \omega^{p_2} & \\ & & \omega^{p_3} \end{pmatrix} , \quad \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}^p$$

- $$(\varphi_e, \varphi_\mu, \varphi_\tau) \sim 3 \; , \quad (\varphi_{\mu\tau}, \varphi_{\tau e}, \varphi_{e\mu}) \sim 3'$$
$$(\varphi_\tau^\mu, \varphi_e^\tau, \varphi_\mu^e) \sim 3'' \; , \quad (\varphi_e^{\mu\tau}, \varphi_\mu^{\tau e}, \varphi_\tau^{e\mu}) \sim 3'''$$

- $$\varphi_e \varphi'^e + \varphi_\mu \varphi'^\mu + \varphi_\tau \varphi'^\tau \sim 1 \; , \quad \varphi_e \varphi'^e + \varphi_\mu \varphi'^\mu + \varphi_\tau \varphi'^\tau \sim 1'$$

- $$\varphi_e \varphi'_{\mu\tau} + \varphi_\mu \varphi'_{\tau e} + \varphi_\tau \varphi'_{e\mu} \sim \widetilde{1}$$
$$\varphi_e \varphi'_{\mu\tau} + \omega\, \varphi_\mu \varphi'_{\tau e} + \omega^* \varphi_\tau \varphi'_{e\mu} \sim \widetilde{1}'$$
$$\varphi_e \varphi'_{\mu\tau} + \omega^* \varphi_\mu \varphi'_{\tau e} + \omega\, \varphi_\tau \varphi'_{e\mu} \sim \widetilde{1}''$$

- $$1 + (4 \times 1^2 + 4 \times 3^2) \times 2 = 81 \qquad \checkmark$$

Scalar potential

- $$V = V_{\Sigma(81)\times\mathbb{Z}_3^{\rm aux}} + V_{\rm soft}$$

- $$V_{\rm soft} = -\sqrt{2} \sum_{\alpha=e,\mu,\tau} \mu_\alpha S_\alpha \phi_\alpha^\dagger \phi_W + h.c.$$

- $$M_{\phi_e}^2 = M_{\phi_\mu}^2 = M_{\phi_\tau}^2 \approx \frac{\mu_\alpha}{v_\alpha} v_S\, v_W$$

Neutrino masses I

- $\mathscr{L} = \frac{1}{2} Y(S_e N_e N_e + \text{cyclic}) + y(\ell_e \phi_{\nu e} N_e + \text{cyclic}) + \text{h.c.}$

- Seesaw ⇒ diagonal ~ $\langle\phi^0_{\nu\alpha}\rangle^2/\langle S_\alpha\rangle$

Neutrino masses II

- $\mathscr{L} = f(h^{+}_{\mu\tau}\, \ell_\mu \bullet \ell_\tau + \text{cyclic})$ + h.c.

- 1-loop = off-diagonal ~ $\#(4\pi M_h)^{-2} \langle PH_W \rangle \langle S \rangle\, m_\tau \log(M_h/M_{PH})^2$

$$(m_\nu)_{\mu\tau} \approx \frac{m_\tau}{m_\mu}(m_\nu)_{e\mu} + \frac{m_\mu}{m_\tau}(m_\nu)_{e\tau}$$

Magnetic moments

- $$\frac{a_\alpha}{a_\alpha^{\rm SM}} \sim \left(\frac{y_\ell}{y_\alpha^{\rm SM}} \, \frac{m_H}{M_{\rm PH}} \right)^2$$

- $$a_\mu \sim (10^{-10} - 10^{-9}) \left(\frac{\rm TeV}{M_{\rm PH}} \right)^2$$

Quark masses (?)

- $(d,s,b) \sim (d_e{}^{\mu\tau}, d_\mu{}^{\tau e}, d_\tau{}^{e\mu}) \sim 3'''$ √

- $(\ell, e^c, N, \phi_\ell, \phi_\nu) \longleftrightarrow (q, d^c, u^c, \phi_D, \phi_U)$ √

- Flavor...?

Summary

- $\Sigma(81) \Rightarrow$ PH + ν mixing
- $\delta a_\mu \sim 10^{-9}$ for $M_{PH} \sim$ TeV
- Challenge: $\Sigma(81) < U(3)$