Insensitive
Unification of Gauge Couplings

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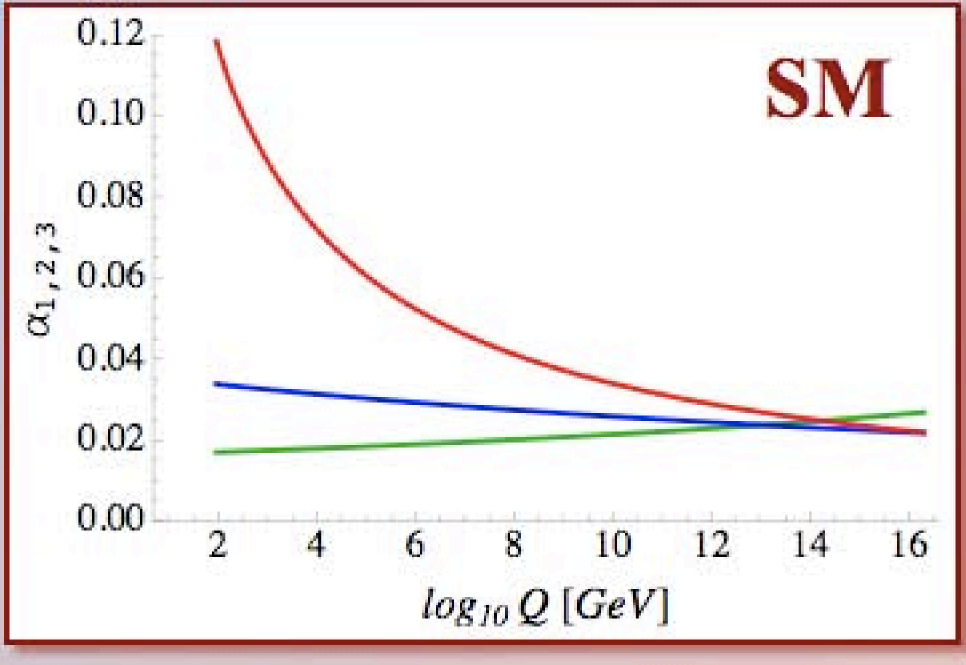
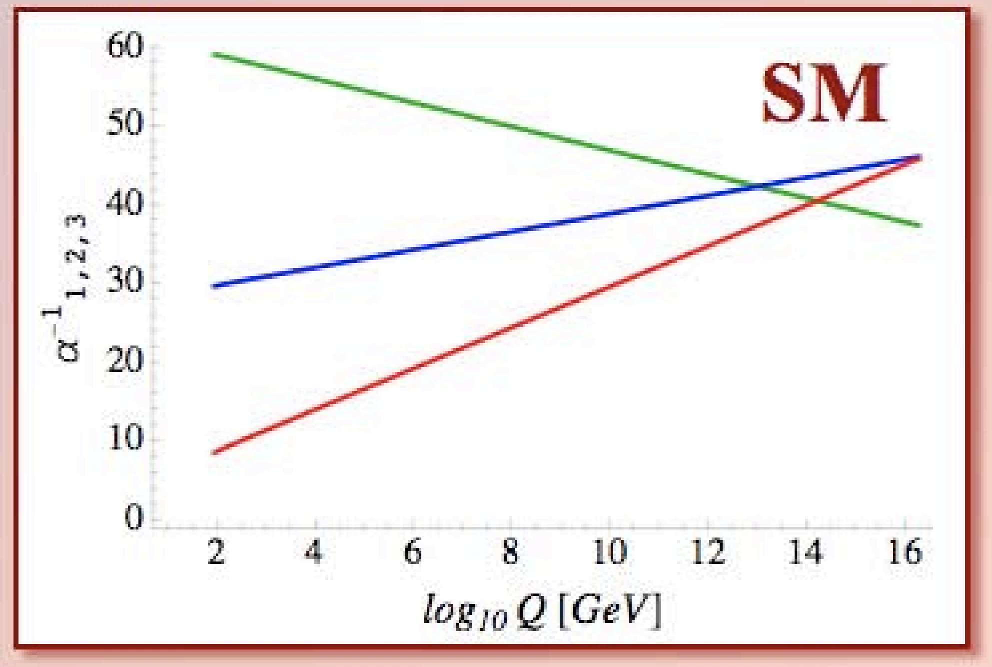
Gauge couplings in the standard model



$\alpha_3(M_Z)_{exp} = 0.1184$
$\alpha_2(M_Z)_{exp} = 0.03380$
$\alpha_1(M_Z)_{exp} = 0.01695$
$\alpha_{EM}(M_Z) = 1/127.916$
$\sin^2\theta_W = 0.2313$



RGEs:

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i$$

$$t = \ln Q/Q_0$$

$$b_i = \left(\frac{1}{10} + \frac{4}{3}n_g,\ -\frac{43}{6} + \frac{4}{3}n_g,\ -11 + \frac{4}{3}n_g\right)$$

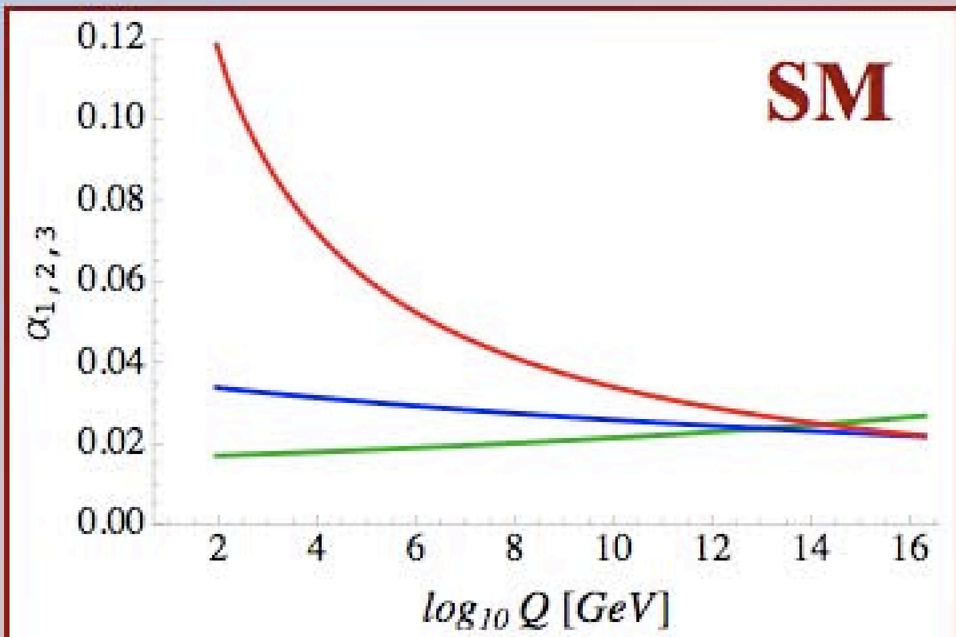
$$b_i = (41/10,\ -19/6,\ -7)$$

solution:

$$\boxed{\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi}\ln\frac{M_G}{M_Z} + \alpha_i^{-1}(M_G)}$$

SM

SM+3VF



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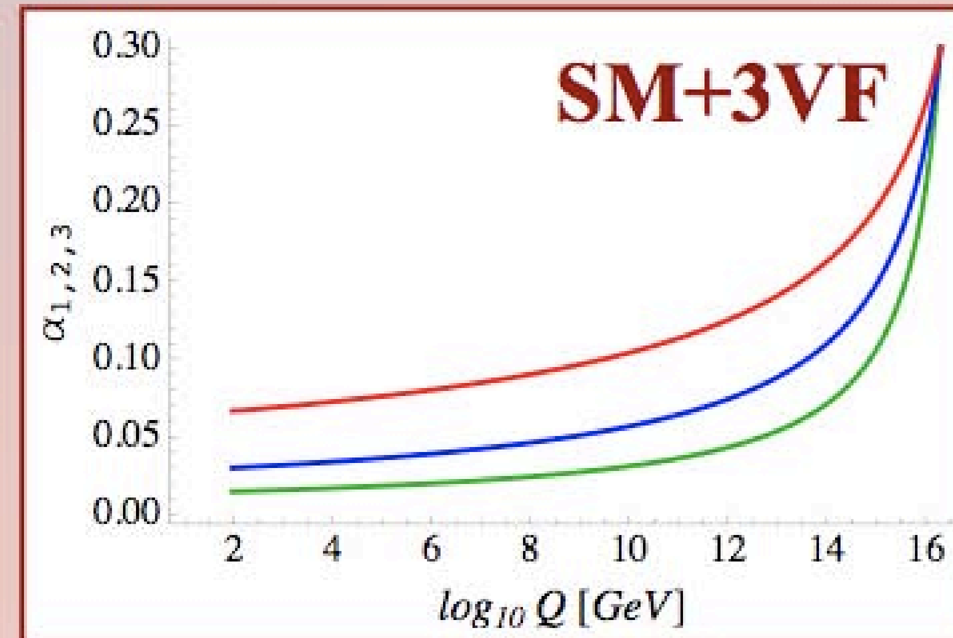

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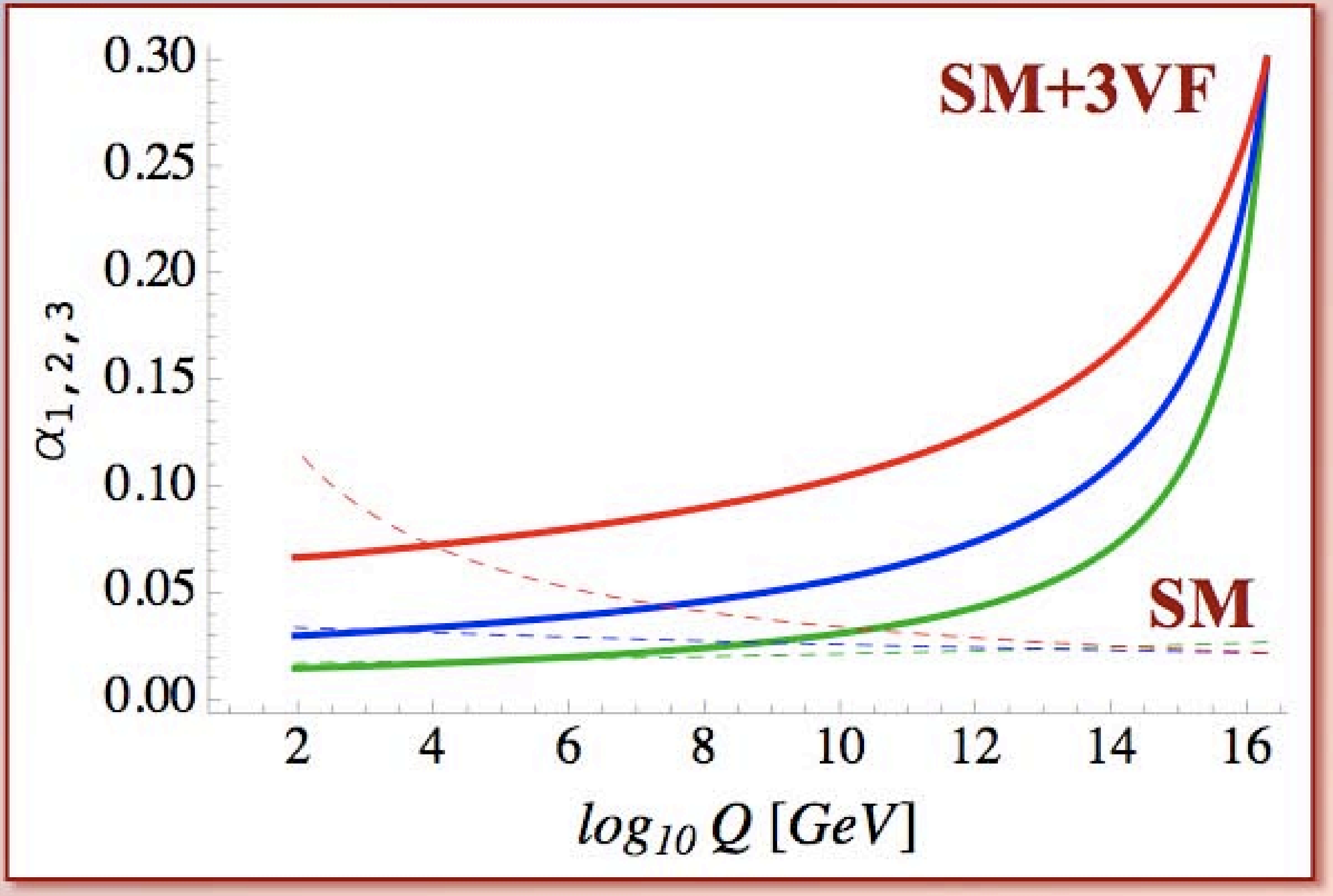
$$b_i = (41/10,\ -19/6,\ -7)$$

$$b_i = (121/10,\ 29/6,\ +1)$$

solution:

$$\boxed{\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi} \ln \frac{M_G}{M_Z} + \alpha_i^{-1}(M_G)}$$

SM + 3 vector-like families



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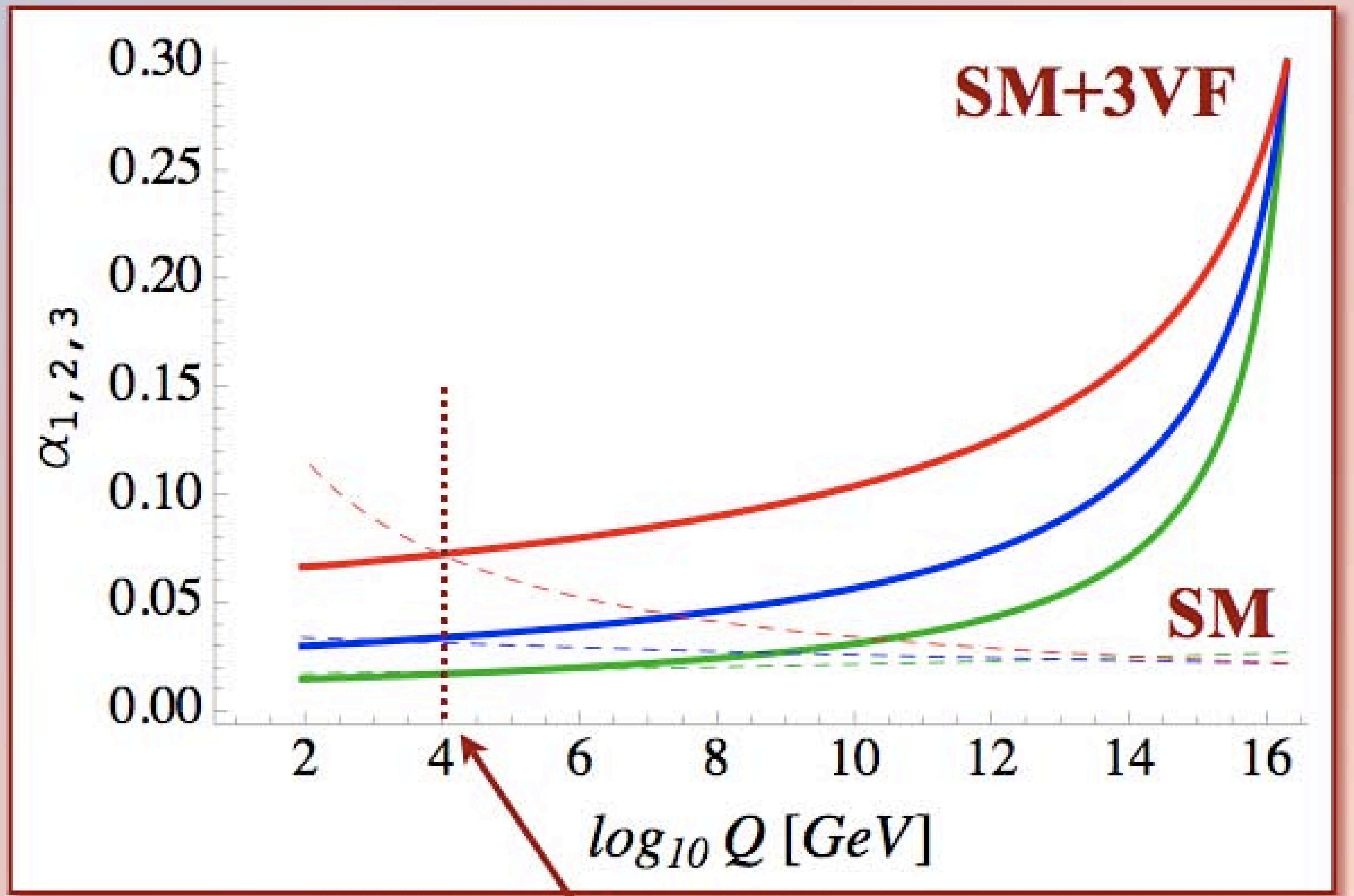
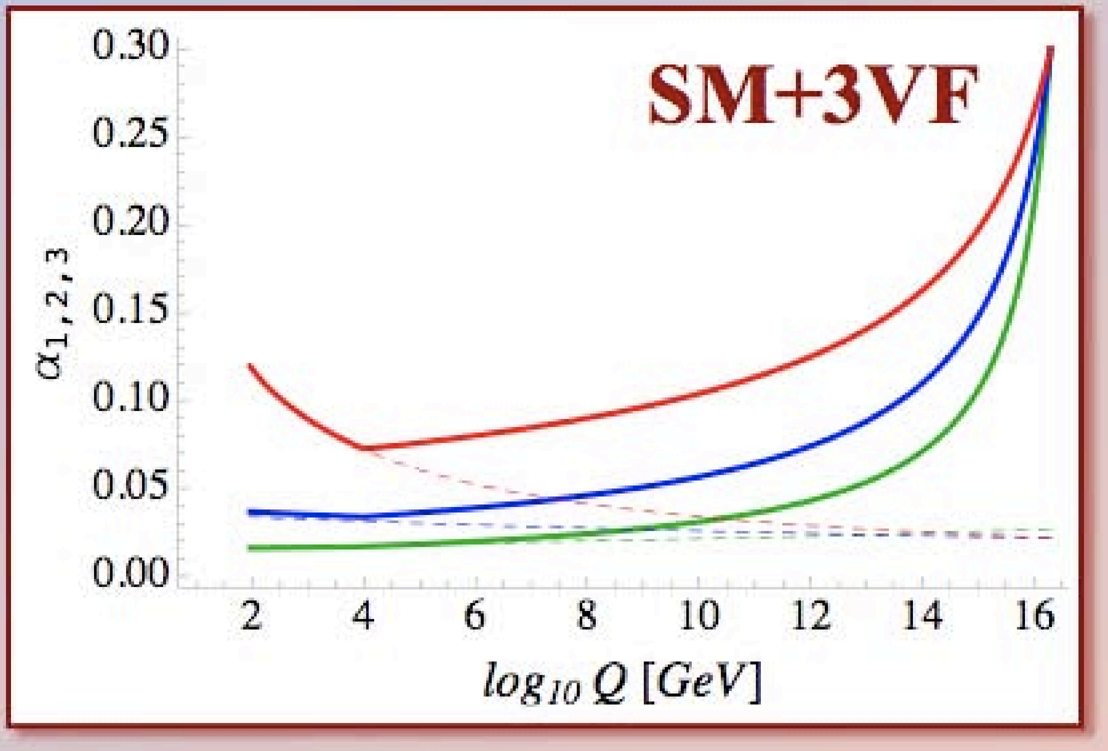
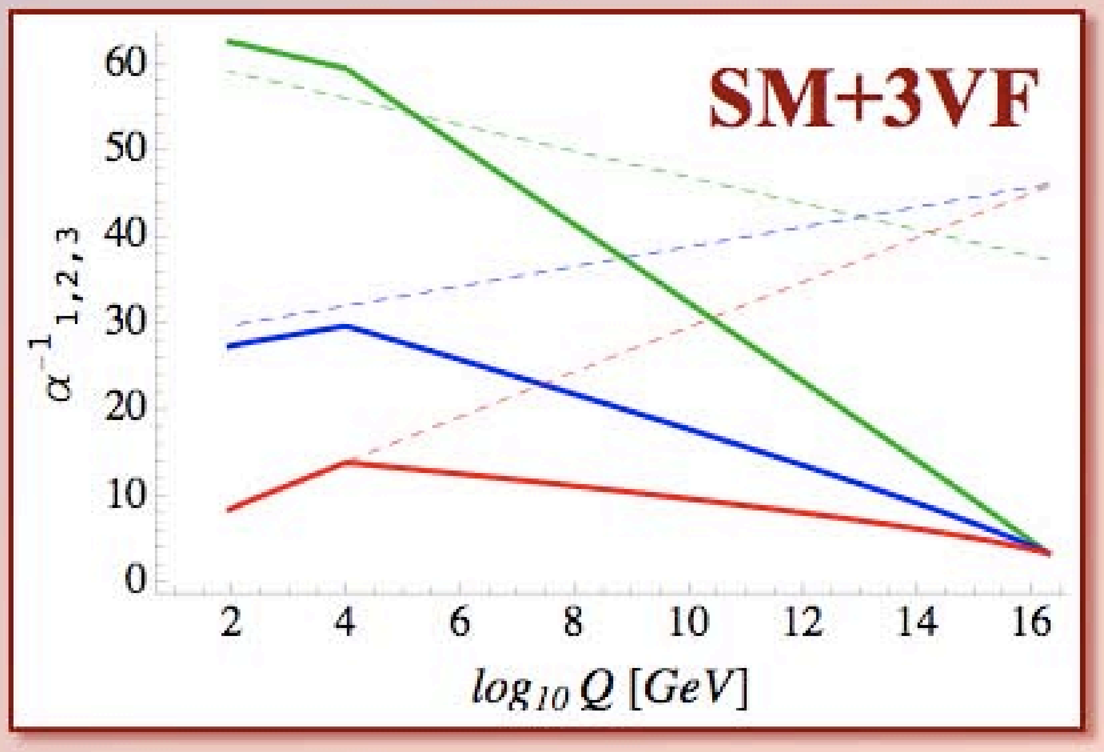


Is this a threshold effect?

SM + 3 vector-like families at 10 TeV





Exp. values of gauge couplings reproduced within 8%

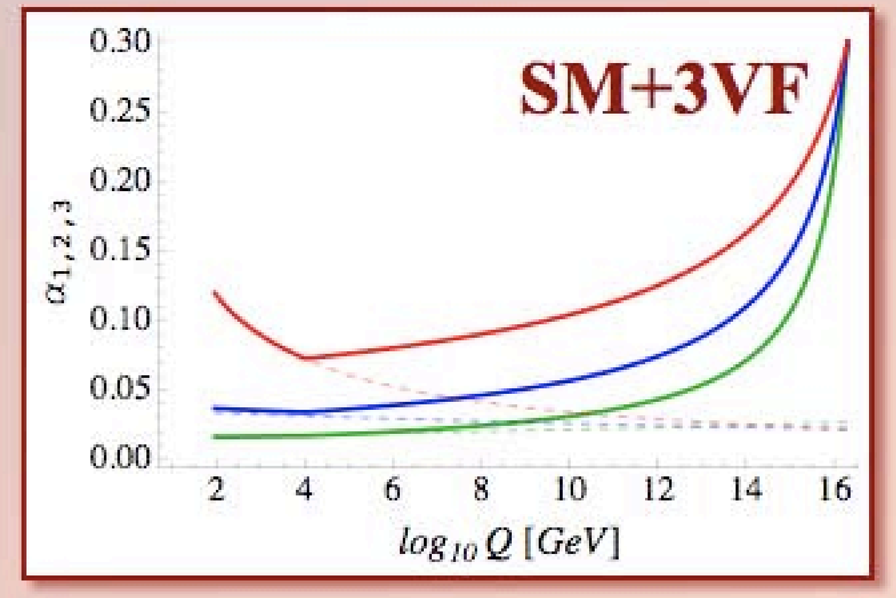
the only relevant parameters are M_G and M_{VF}

predictive, comparable to MSSM unification

Predictions and Sensitivity

RGEs: $$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i$$

$$b_i = (121/10,\ 29/6,\ +1)$$



solution at 1-loop **(good approximation for i=1,2):**

$$\underbrace{\alpha_i^{-1}(M_Z)}_{\sim 60,\,30} = \frac{b_i}{2\pi}\ln\frac{M_G}{M_Z} + \underbrace{\alpha_G^{-1}}_{\sim 3} - \underbrace{T_i}_{\sim 6}$$

$$T_i = \frac{1}{2\pi}\sum_f b_i^f \ln\frac{M_f}{M_Z}$$

~ 60, 30 ~ 3 ~ 6 (for common mass 10 TeV)

neglecting threshold effect and α_G : $$\frac{\alpha_i(M_Z)}{\alpha_j(M_Z)} \simeq \frac{b_j}{b_i}$$

$$\alpha' = \frac{3}{5}\alpha_1, \quad b' = \frac{5}{3}b_1$$

Parameter free prediction:

$$\boxed{\sin^2\theta_W \equiv \frac{\alpha'}{\alpha_2 + \alpha'} = \frac{b_2}{b_2 + b'} = 0.193}$$

$$\alpha_{EM} = \alpha_2 \sin^2\theta_W$$

Maiani, Parisi, and Petronzio (1978)

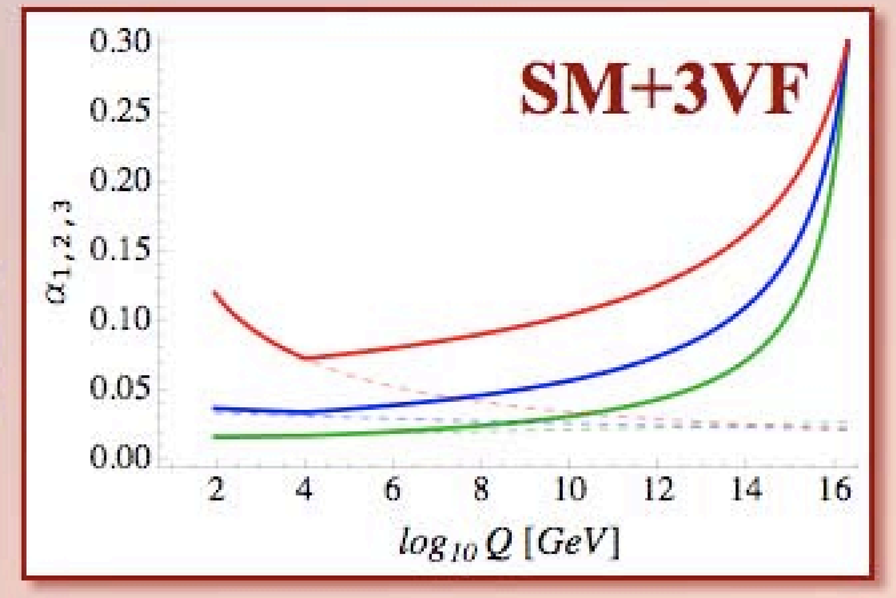
Predictions and Sensitivity

RGEs:

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i + \frac{\alpha_i^3}{8\pi^2} B_i + \ldots$$

$$b_i = (121/10,\ 29/6,\ +1)$$

$$B_3 = -102 + (76/3) n_g = 126$$



neglecting 1-loop (good approximation for i=3):

$$\underbrace{\alpha_3^{-1}(M_Z)}_{\sim 8} \simeq \sqrt{\frac{B_3}{4\pi^2} \underbrace{\ln\frac{M_G}{M_Z}}_{\sim 100} + \underbrace{\alpha_G^{-2}}_{\sim 9}} - \underbrace{T_i}_{\sim 6}$$

$$T_i = \frac{1}{2\pi} \sum_f b_i^f \ln\frac{M_f}{M_Z}$$

~8 ~100 ~9 ~6 (for common mass 10 TeV)

Parameter free prediction:

(neglecting threshold effect and α_G)

$$\boxed{\frac{\alpha_3^2(M_Z)}{\alpha_{EM}(M_Z)} \simeq 2\pi \frac{b_2 + b'}{B_3}}$$

predicts $\alpha_3 = 0.099$

1-loop contribution can be added:

$$\alpha_3(M_Z) \to \frac{\alpha_3(M_Z)}{1 + \frac{1}{3}\frac{\epsilon}{\alpha_3(M_Z)} - \frac{1}{12}\left(\frac{\epsilon}{\alpha_3(M_Z)}\right)^2 + \ldots}$$

$$\epsilon = 4\pi b_3 / B_3$$

predicts $\alpha_3 = 0.073$

Sensitivity to M_G, α_G, and M_{VF}

approximate solutions:

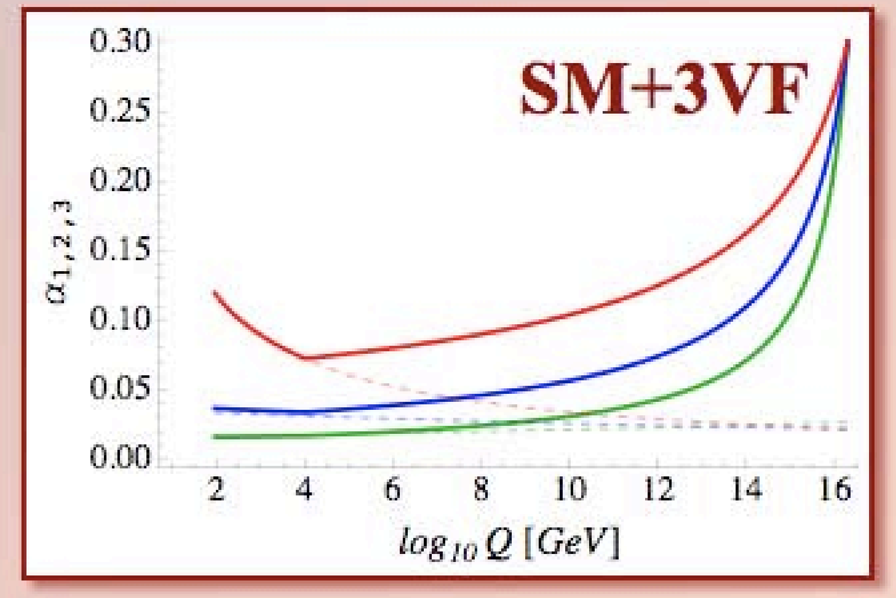
$$\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi}\ln\frac{M_G}{M_Z} + \alpha_G^{-1} - T_i \quad \text{(i=1,2)}$$

~ 60, 30 ~ 3 ~ 6

$$\alpha_3^{-1}(M_Z) \simeq \sqrt{\frac{B_3}{4\pi^2}\ln\frac{M_G}{M_Z} + \alpha_G^{-2}} - T_i$$

~ 8 ~ 100 ~ 9 ~ 6

~ 14



$$T_i = \frac{1}{2\pi}\sum_f b_i^f \ln\frac{M_f}{M_Z}$$

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Changing any of the fundamental parameters by a factor of 2 does not modify predicted values of gauge couplings by more than ~10%.

Realistic example

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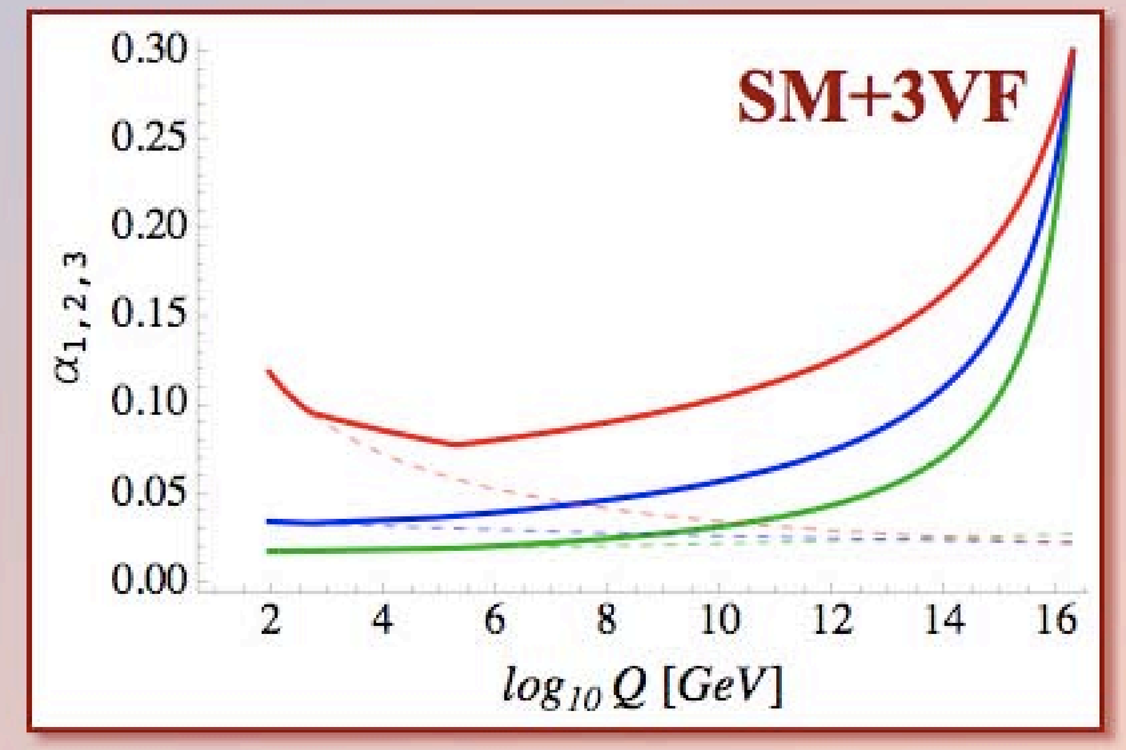
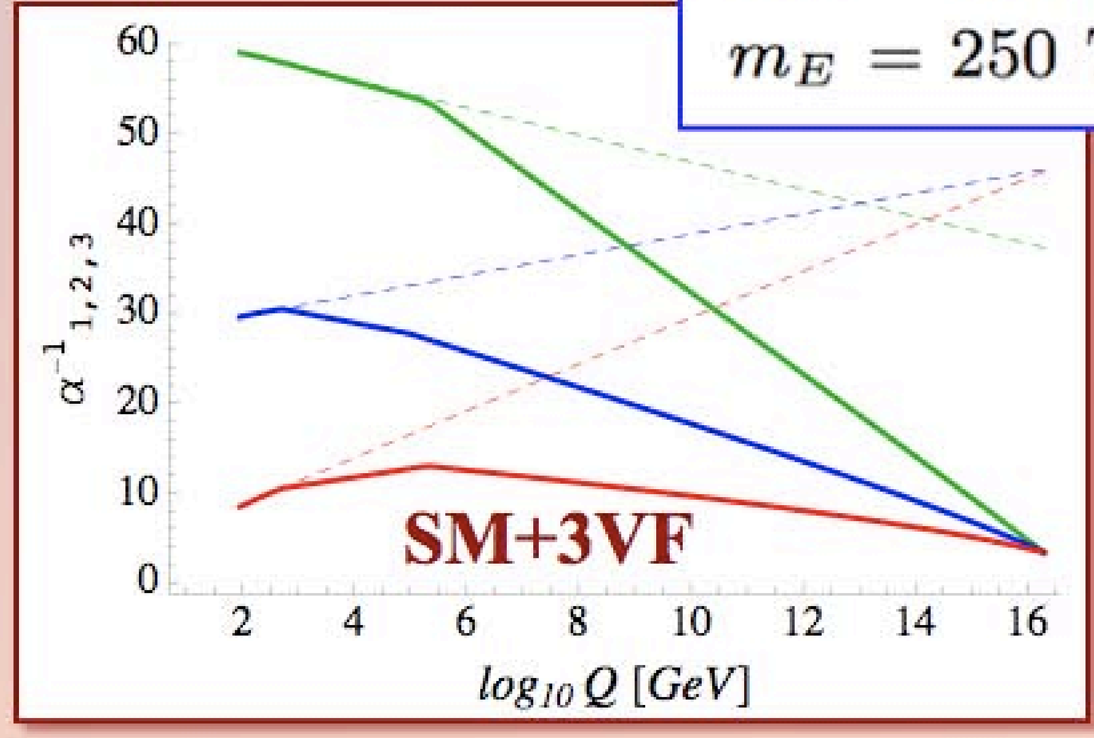
Gauge couplings reproduced (within fractions of exp. uncertainties) for :

$$\begin{aligned} m_Q &= 500 \text{ GeV} \\ m_L &= 95 \text{ TeV} \\ m_U &= 220 \text{ TeV} \\ m_D &= 180 \text{ TeV} \\ m_E &= 250 \text{ TeV} \end{aligned}$$

4 sig. figures

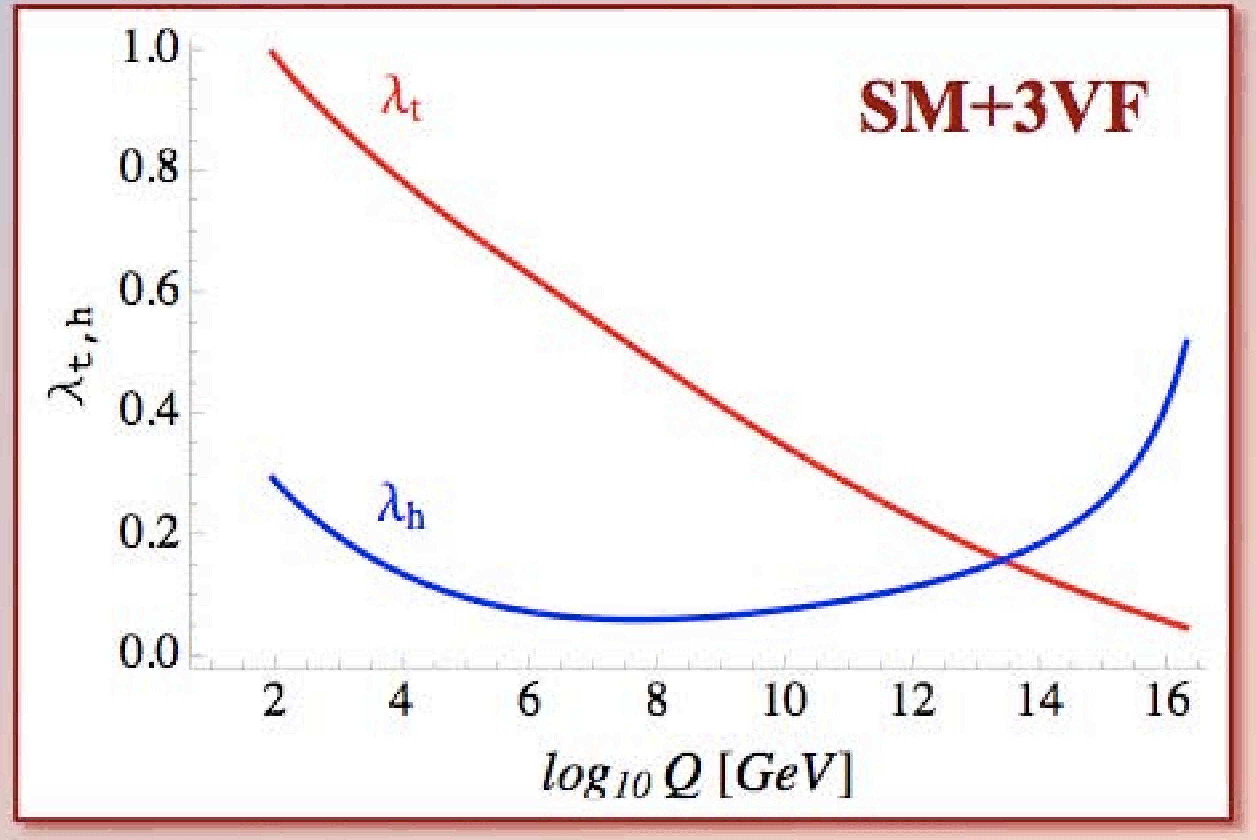
2 sig. figures





Many possible solutions!

Top Yukawa and Higgs quartic couplings



m_H = 125 GeV

- **reduced sensitivity of Yukawa and Higgs quartic couplings to GUT scale boundary conditions**

  (different textures for fermion masses compared to usual GUTs)

- **Electroweak minimum is stable!**

Conclusions

- **3 (or more) pairs of vector-like families allow for insensitive unification of gauge couplings**

  predictive, comparable to SUSY unification

- **resurrects simple non-supersymmetric GUTs (proton decay)**

  the GUT scale is adjustable and could be identified with the string or Planck scale

- **the electroweak minimum is stable all the way to the GUT scale**

- **some of the extra fermions might be within the reach of the LHC**

  and modify phenomenology of the SM: small flavor violation from mixing through Yukawa couplings, contributions in loops, ...