Probing New Physics in Neutrino Oscillations

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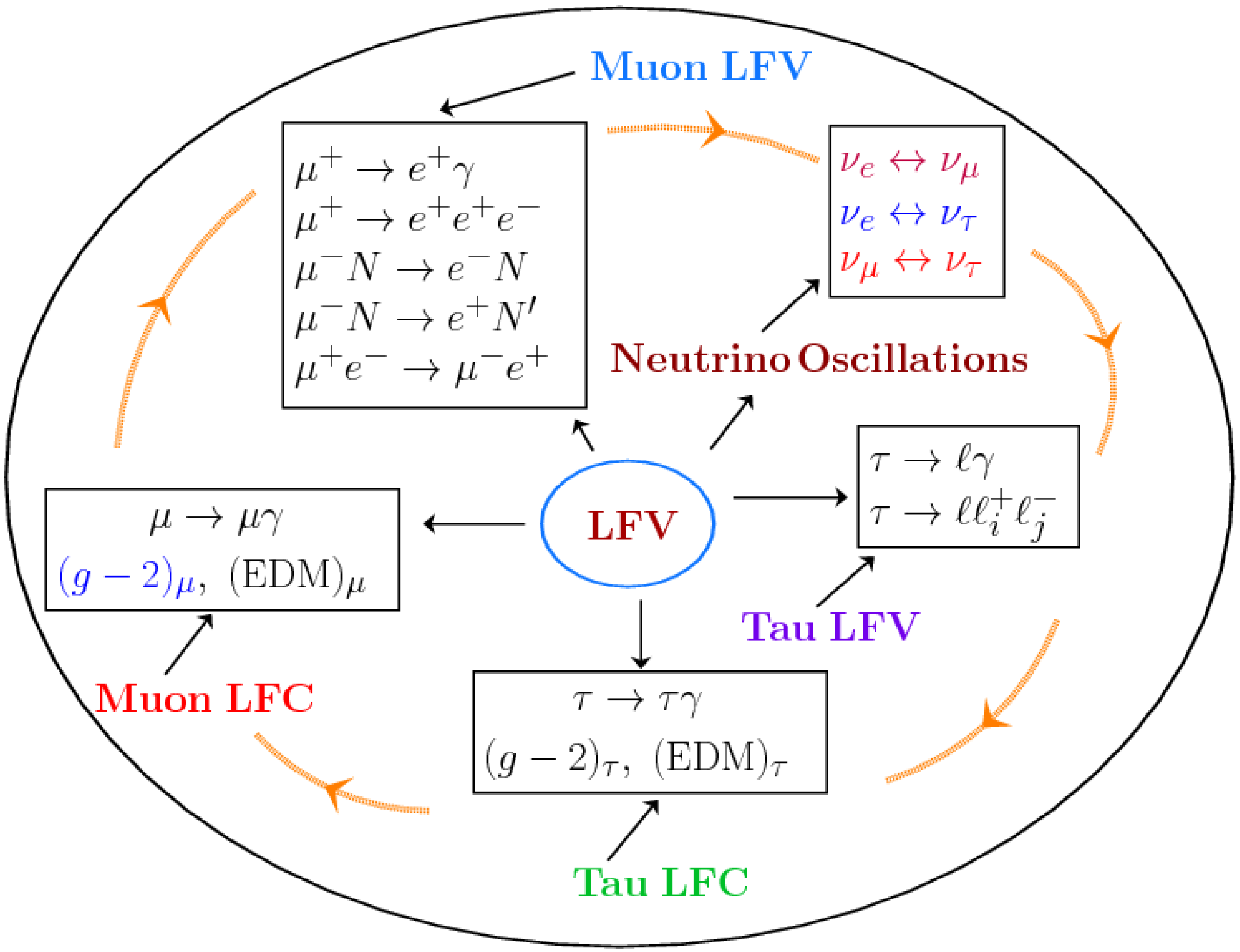
Lepton Flavor Physics: The Big Picture



Lepton Flavor Violation Prospects

Table 1-1. *Evolution of the 95% CL limits on the main CLFV observables with initial state muons. The expected limits in the 5 to 10 year range are based on running or proposed experiments at existing facilities. The 10 to 20 year range are based on sensitivity studies using muon rates available at proposed new facilities. The numbers quoted for $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^-e^+$ are limits on the branching fraction. The numbers quoted for $\mu^- N \to e^- N$ are limits on the rate with respect to the muon capture process $\mu^- N \to \nu_\mu N'$. Below the numbers are the corresponding experiments or facilities and the year the current limit was set.*

Process	Current limit	Expected limit 5-10 years		Expected limit 10-20 years
$\mu^+ \to e^+\gamma$	2.4×10^{-12} PSI/MEG (2011)	1×10^{-13} PSI/MEG		1×10^{-14} PSI, Project X
$\mu^+ \to e^+e^-e^+$	1×10^{-12} PSI/SINDRUM-I (1988)	1×10^{-15} Osaka/MuSIC	1×10^{-16} PSI/$\mu 3e$	1×10^{-17} PSI, Project X
$\mu^- N \to e^- N$	7×10^{-13} PSI/SINDRUM-II (2006)	1×10^{-14} J-PARC/DeeMee	6×10^{-17} FNAL/Mu2e	1×10^{-18} J-PARC, Project X

Intensity Frontier Workshop White Paper (2012)
http://www.lepp.cornell.edu/~yuvalg/ifw/emt.html

Muon (g-2) Anomaly

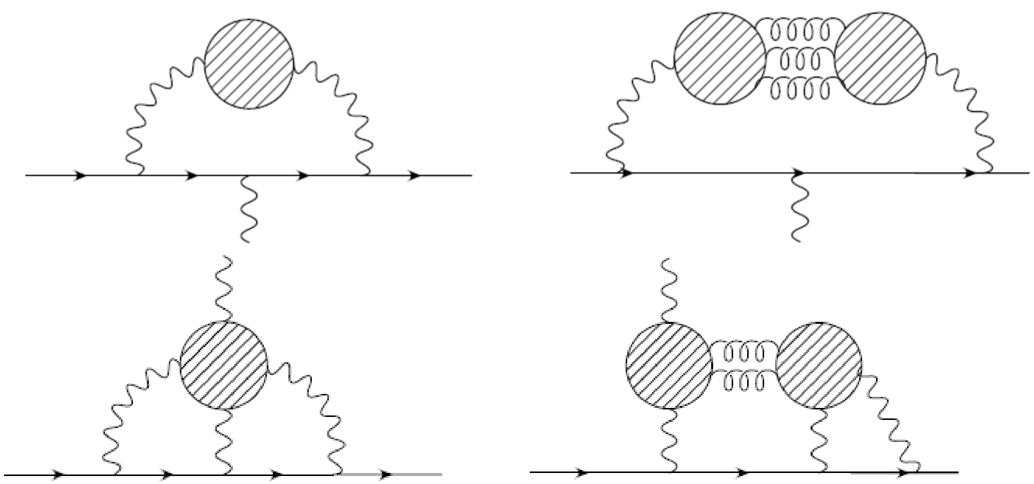
$$a_\mu = (g_\mu - 2)/2$$

$$a_\mu(\text{Expt}) = 116\,592\,089\,(54)(33) \times 10^{-11}$$ BNL E821 (2006)

$$a_\mu(\text{SM}) = 116\,591\,802\,(42)(26)(02) \times 10^{-11}$$

$$\Longrightarrow \quad \Delta a_\mu = 287(80) \times 10^{-11}$$ 3.6σ discrepancy

Major thoery uncertainty in hadronic vacuum polarization



$$a_\mu(\text{HVP}) = (692.3 \pm 4.2) \times 10^{-10}$$

$$= (701.5 \pm 4.7) \times 10^{-10}$$

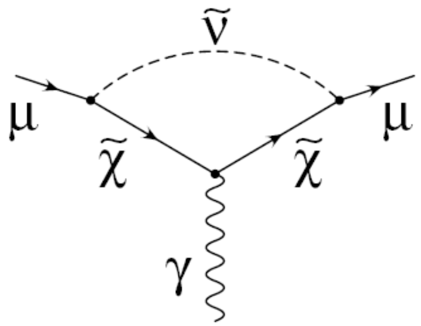
($\tau \to$ hadrons data)

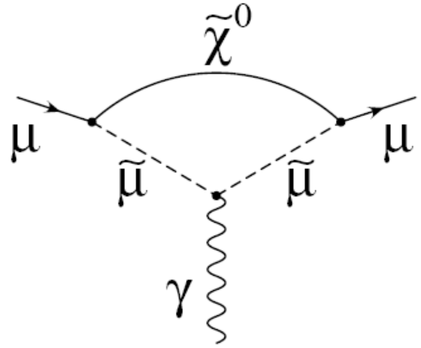
$$a_\mu(\text{HLbL}) = 105\,(26) \times 10^{-11}$$

Muon (g-2) Anomaly and New Physics

New physics should appear below a TeV

Supersymmetry a natural candidate



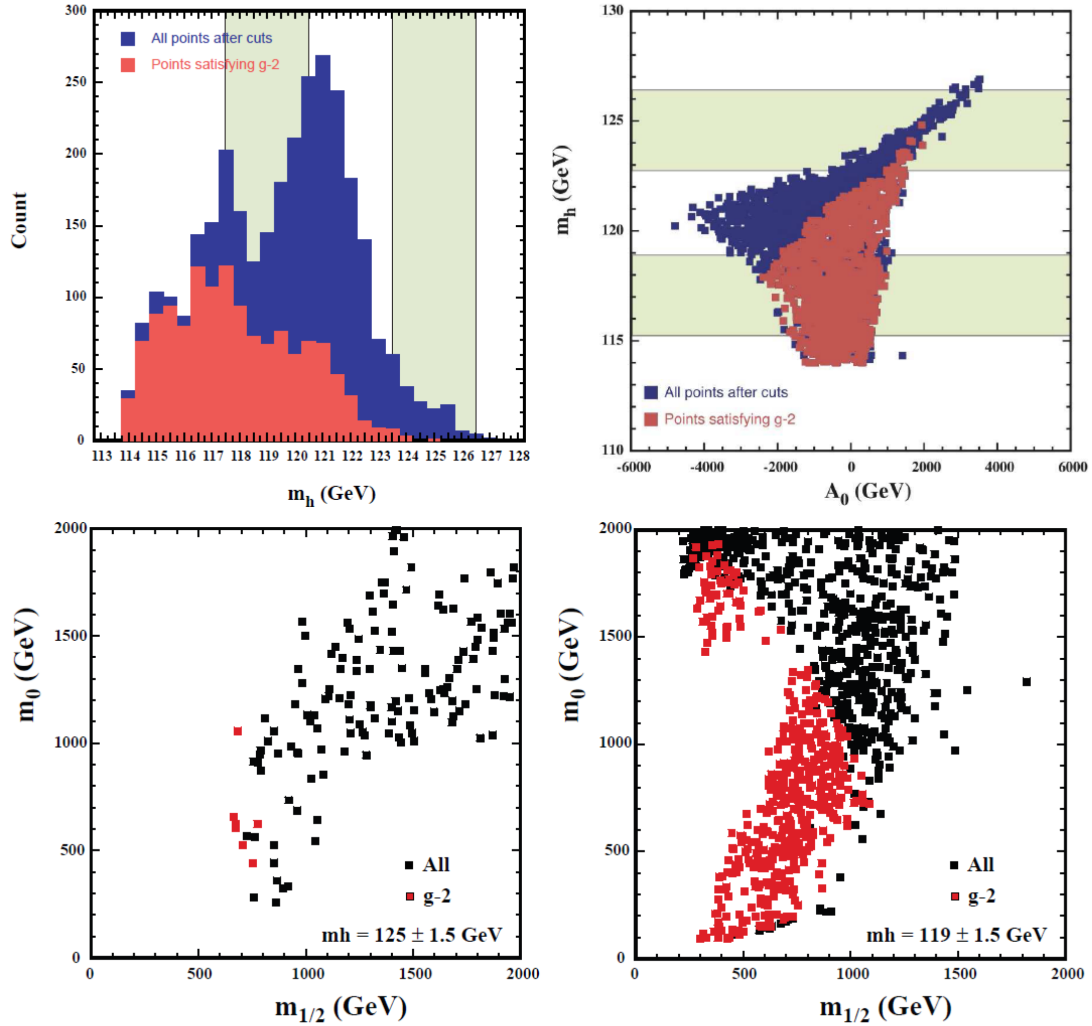


$$|a_\mu(\mathrm{SUSY})| \simeq 130 \times 10^{-11} \left(\frac{100\ \mathrm{GeV}}{\tilde{m}}\right)^2 \tan\beta$$

Czarnecki, Marciano (2001)

Large $\tan\beta$ prefered in constrained SUSY models

CMSSM Scan



Red dots explain $(g-2)_\mu$ anomaly

Ellis, Olive (2012)

Neutrino Oscillations: Present Status

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.58	0.306	0.021	0.42	2.35
		(0.312)	(0.025)		
1σ range	7.32 – 7.80	0.291 – 0.324	0.013 – 0.028	0.39 – 0.50	2.26 – 2.47
		(0.296 – 0.329)	(0.018 – 0.032)		
2σ range	7.16 – 7.99	0.275 – 0.342	0.008 – 0.036	0.36 – 0.60	2.17 – 2.57
		(0.280 – 0.347)	(0.012 – 0.041)		
3σ range	6.99 – 8.18	0.259 – 0.359	0.001 – 0.044	0.34 – 0.64	2.06 – 2.67
		(0.265 – 0.364)	(0.005 – 0.050)		

Fogli, Lisi, Marrone, Palazzo, Rotunno (2011)

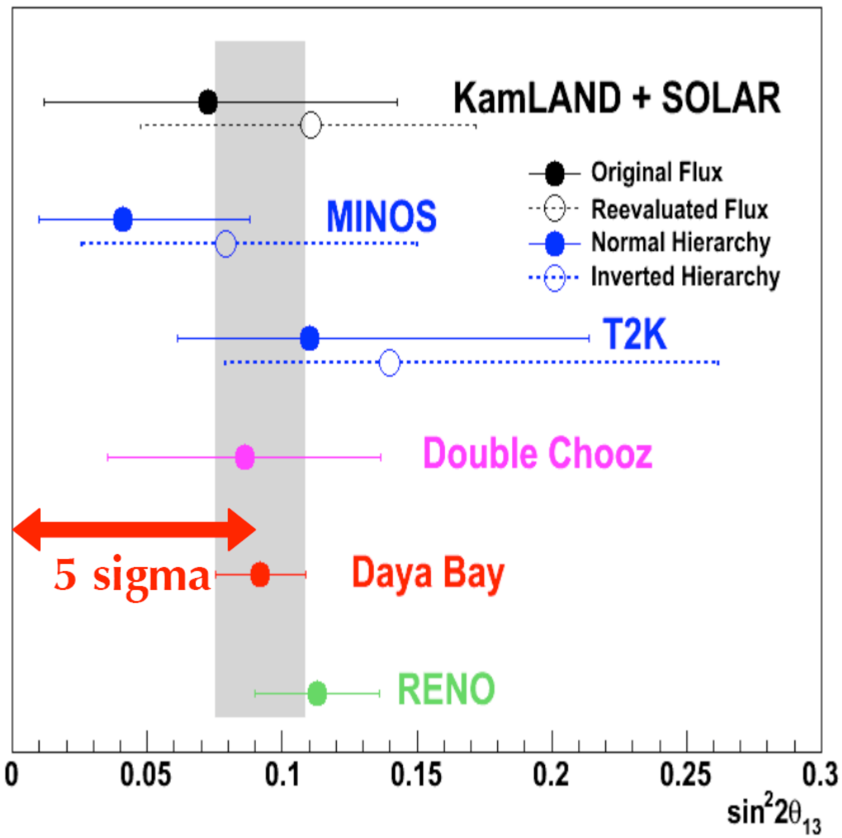
(Pre Daya Bay & RENO)

Neutrino mixing matrix:

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot P$$

$$P = \text{diag.}(e^{i\alpha},\, e^{i\beta},\, 1)$$

Recent Results on Theta(13)

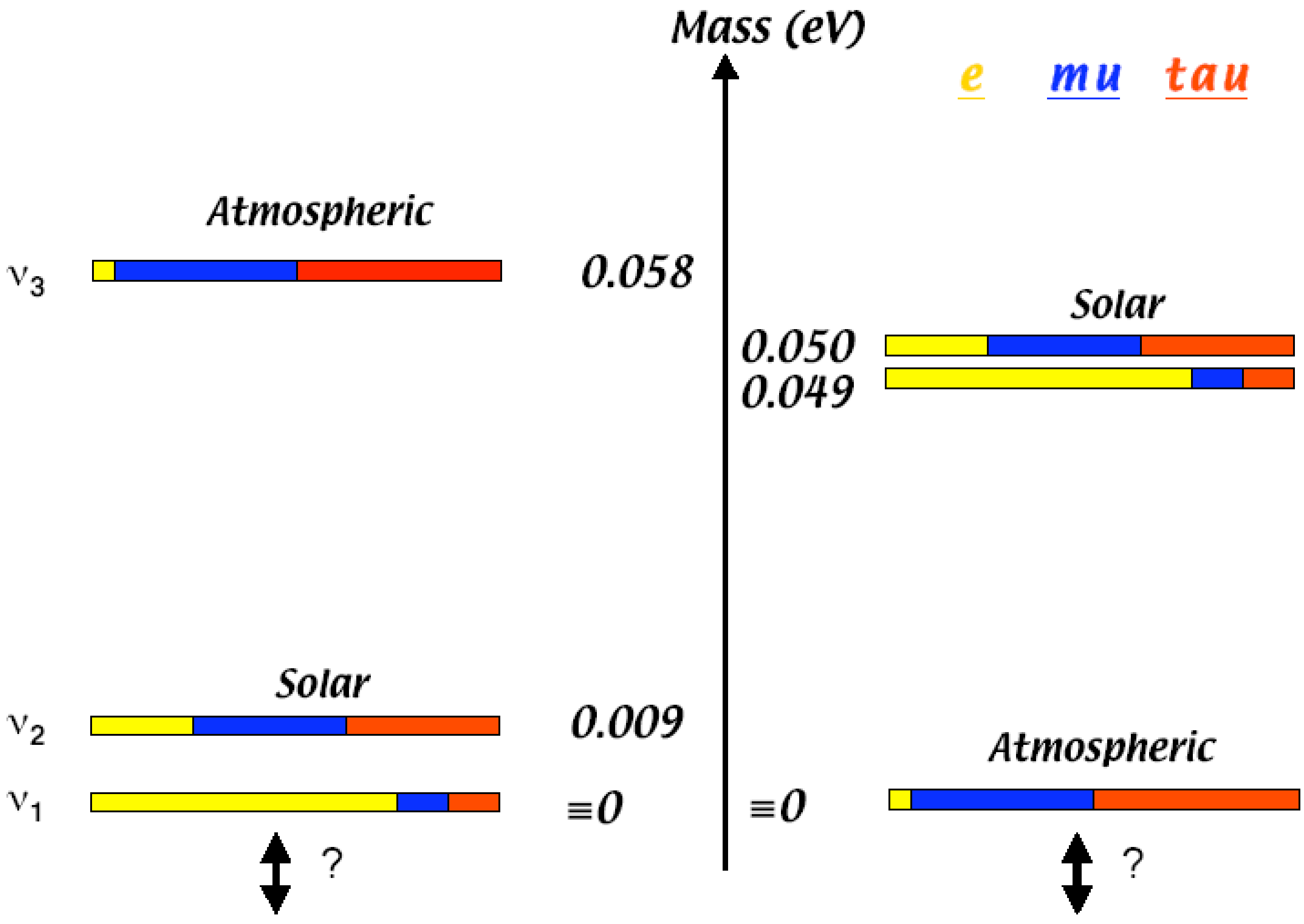


Wei Wang's talk

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.006(\text{syst})$$ **Daya Bay**

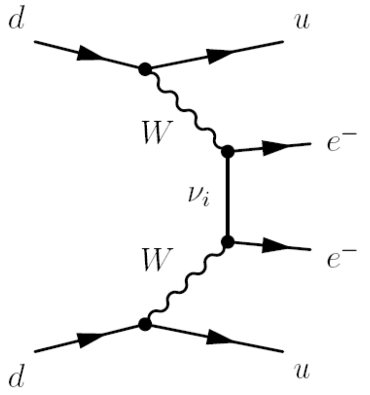
$$\sin^2 2\theta_{13} = 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{syst})$$ **RENO**

Neutrino Mass Ordering

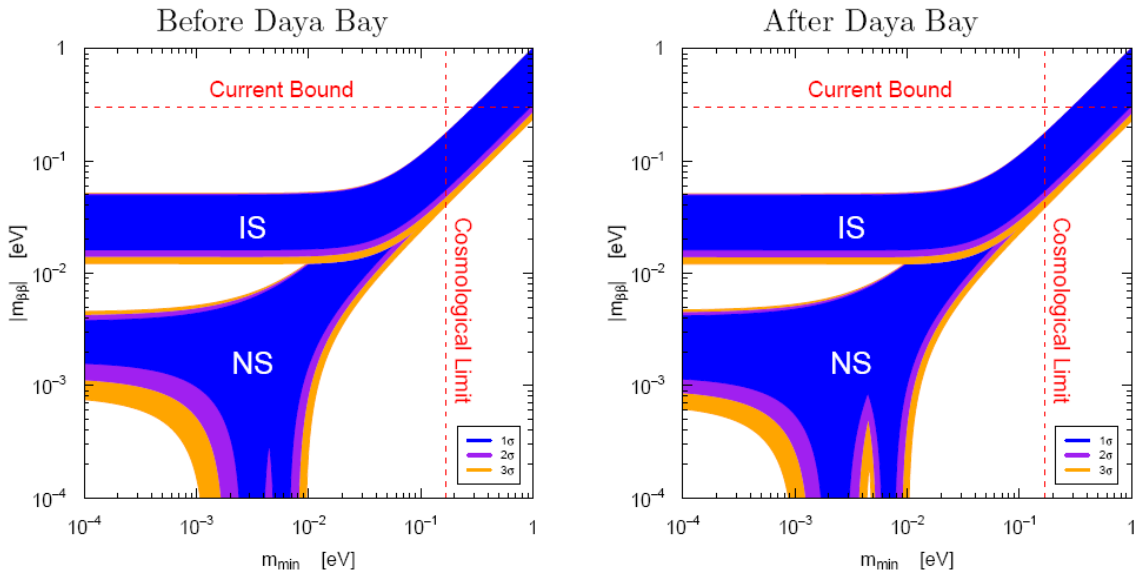


Neutrinoless Double Beta Decay

$$N(A,Z) \to N(A,Z+2) + e^- + e^-$$



$$|m_{\beta\beta}| = \left|\cos^2\theta_{12}\cos^2\theta_{13} m_1 + e^{2i\alpha_{12}}\sin^2\theta_{12}\cos^2\theta_{13} m_2 + e^{2i\alpha_{13}}\sin^2\theta_{13} m_3\right|$$



Bilenky, Giunti, 2012

Pressing Questions for Neutrinos

- Are neutrinos Majorana particles ($\beta\beta_{0\nu}$)
- Normal mass hierarchy or inverted hierarchy?
- Is there CP violation in neutrino oscillations?
- What explains the pattern of neutrino mixings?
- Can neutrinos be unified with quarks?
- Is neutrino CP violation related to baryon asymmetry?

Seesaw Mechanism for Neutrino Mass

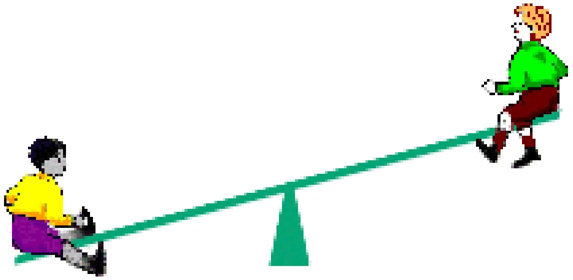
Minkowski (1977)
Yanagida (1979)
Gell-Mann, Ramond, Slansky (1979)
Mohapatra, Senjanovic (1980)

$(\nu,\ \nu^c)$ Mass Matrix:

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

m_D breaks $SU(2)_L \times U(1)_Y$ symmetry $\Rightarrow m_D \leq$ 200 GeV

M_R not protected by any symmetry and can be large, $M_R \sim 10^{14}$ GeV



$$m_\nu(\text{light}) \approx \frac{m_D^2}{M_R} \sim 10^{-2}\ \text{eV}$$

$$m_\nu(\text{heavy}) \approx M_R \sim 10^{14}\ \text{GeV}$$

Tri-bimaximal Neutrino Mixing

$$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot P$$

Harrison, Perkins, Scott (2002)

Neutrino mixing angles are geometrical!

Excluded by Daya Bay and RENO data at 5 σ

A variety of models based on A_4 and other symmetries

Ma, Rajasekaran (2001)
Xing (2002)
Babu, Ma, Valle (2003)
Altarelli, Feruglio (2005)

He, Keum, Volkas (2006)
Mohapatra, Nasri, Yu (2006)
King, Malinsky (2007)
Verzielas, King, Ross (2007)

Chen, Mahanthappa (2007)
Honda, Tanimoto (2008)
Everett, Stuart (2009)
Grimus, Lavoura, Ludl (2009)

There is generically a vacuum alignment problem:

A_4 needs two triplet Higgs: $\langle\chi\rangle = (1,1,1) \quad \langle\phi\rangle = (0,1,0)$

Zeros in fermion mass matrices

CKM mixing angles may be related to quark mass ratios

A two family example:

$$M_u = \begin{pmatrix} 0 & A_u \\ A_u^* & B_u \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & A_d \\ A_d^* & B_d \end{pmatrix}.$$

$$\Rightarrow \quad |\sin\theta_C| \simeq \left| \sqrt{\frac{m_d}{m_s}} - e^{i\psi}\sqrt{\frac{m_u}{m_c}} \right|$$

Weinberg (1977)
Wilczek, Zee (1977)
Fritzsch (1977)

Here ψ is a free phase, but still θ_C is constrained

Two symmetries needed to enforce this structure:

(i) Parity symmetry for hermiticity

(ii) Family $U(1)$ symmetry for the zero in (1,1) entries

Zeros in the neutrino mass matrix

10 interesting possibilites with texture zeros:

7 consistent models with two zeros

3 models with one zero and $m_1 = 0$

Zeros adopted in "flavor basis"

Charged lepton mass matrix assumed diagonal

Normal and inverted hierarchy both supported

θ_{13} is predicted and is typically large

7 Models with two texture zeros

$$A_1 : \begin{pmatrix} 0 & 0 & X \\ 0 & X & X \\ X & X & X \end{pmatrix} \qquad A_2 : \begin{pmatrix} 0 & X & 0 \\ X & X & X \\ 0 & X & X \end{pmatrix}$$

$$B_1 : \begin{pmatrix} X & X & 0 \\ X & 0 & X \\ 0 & X & X \end{pmatrix} \qquad B_2 : \begin{pmatrix} X & 0 & X \\ 0 & X & X \\ X & X & 0 \end{pmatrix}$$

$$B_3 : \begin{pmatrix} X & 0 & X \\ 0 & 0 & X \\ X & X & X \end{pmatrix} \qquad B_4 : \begin{pmatrix} X & X & 0 \\ X & X & X \\ 0 & X & 0 \end{pmatrix}$$

$$C : \begin{pmatrix} X & X & X \\ X & 0 & X \\ X & X & 0 \end{pmatrix}$$

Frampton, Glashow, Marfatia (2002)
Xing (2002)
Merle, Rodejohann (2006)
Goswami et. al (2006)

Phenomenology of Model A2

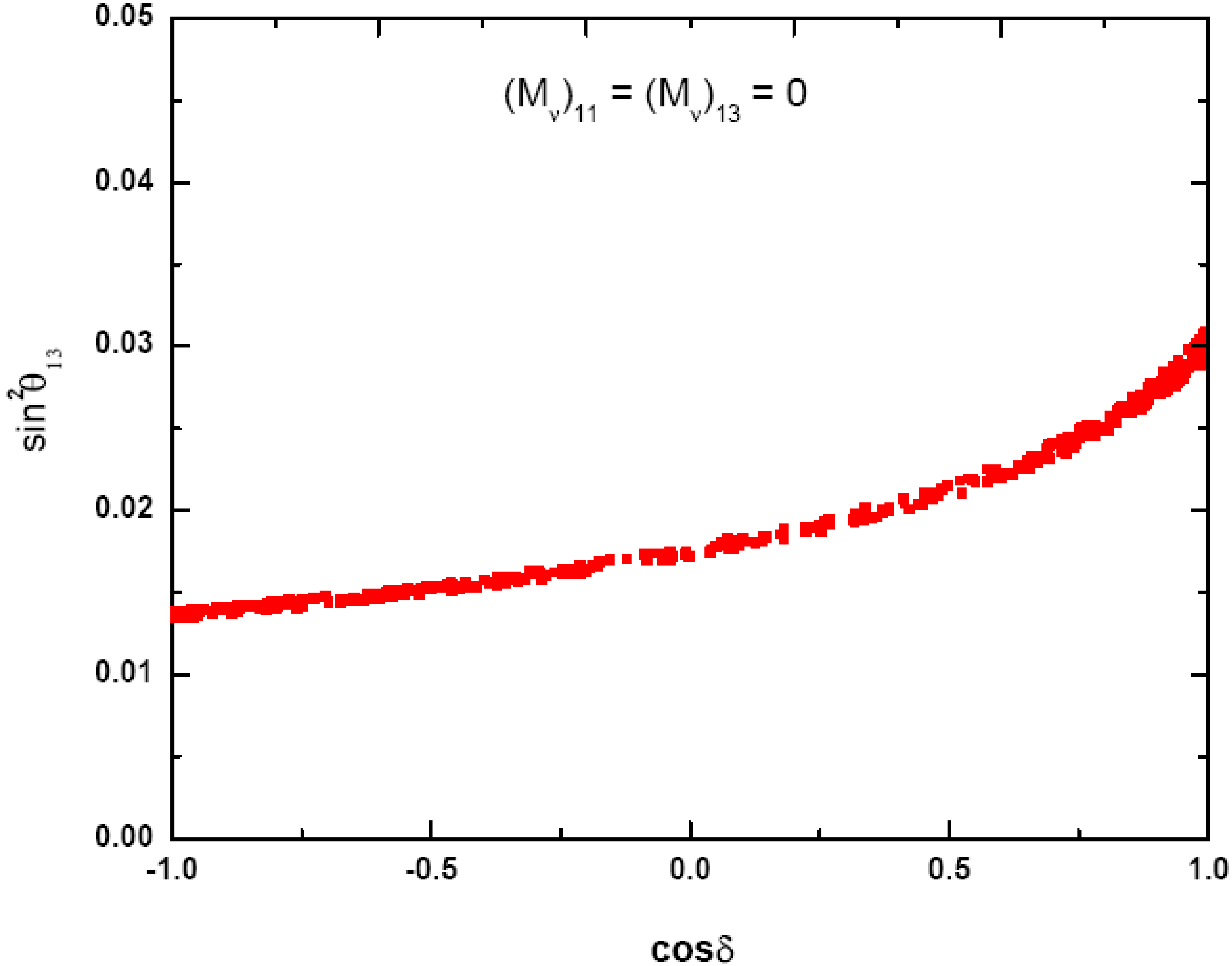
$$A_2 : \begin{pmatrix} 0 & X & 0 \\ X & X & X \\ 0 & X & X \end{pmatrix}$$

Neutrinoless double beta decay amplitude vanishes

$$\left(\frac{m_1}{m_3}\right)^2 = t_{13}^2 \left[t_{12}^2 t_{23}^2 (1 + t_{13}^2) + t_{13}^2 - 2 t_{12} t_{23} t_{13} \sqrt{1 + t_{13}^2} \cos\delta\right]$$

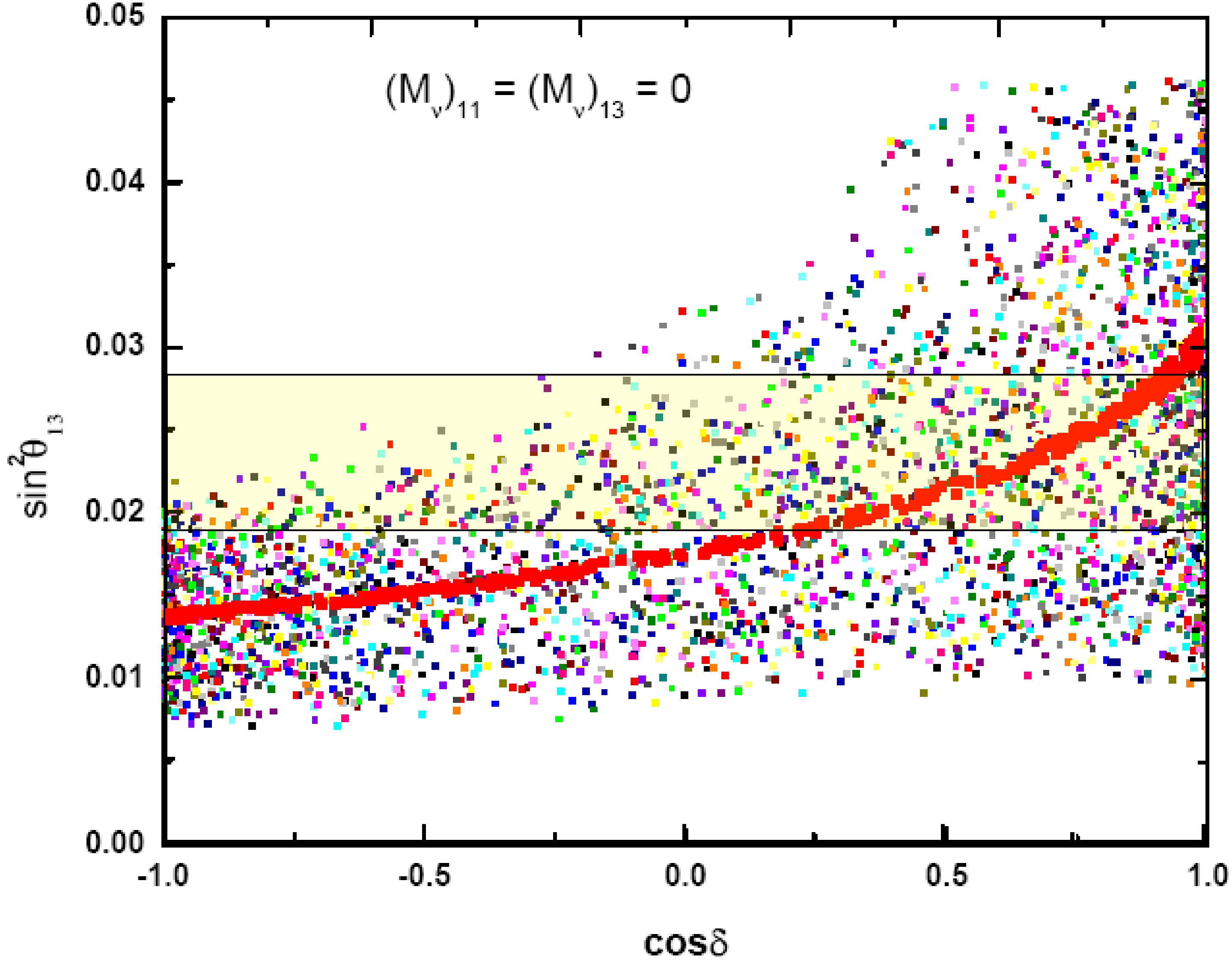
$$\left(\frac{m_2}{m_3}\right)^2 = t_{13}^2 \left[t_{12}^2 + t_{23}^2 / t_{13}^2 (1 + t_{13}^2) + 2 t_{13} t_{23} / t_{12} \sqrt{1 + t_{13}^2} \cos\delta\right]$$

$$(t_{ij} = \tan\theta_{ij})$$

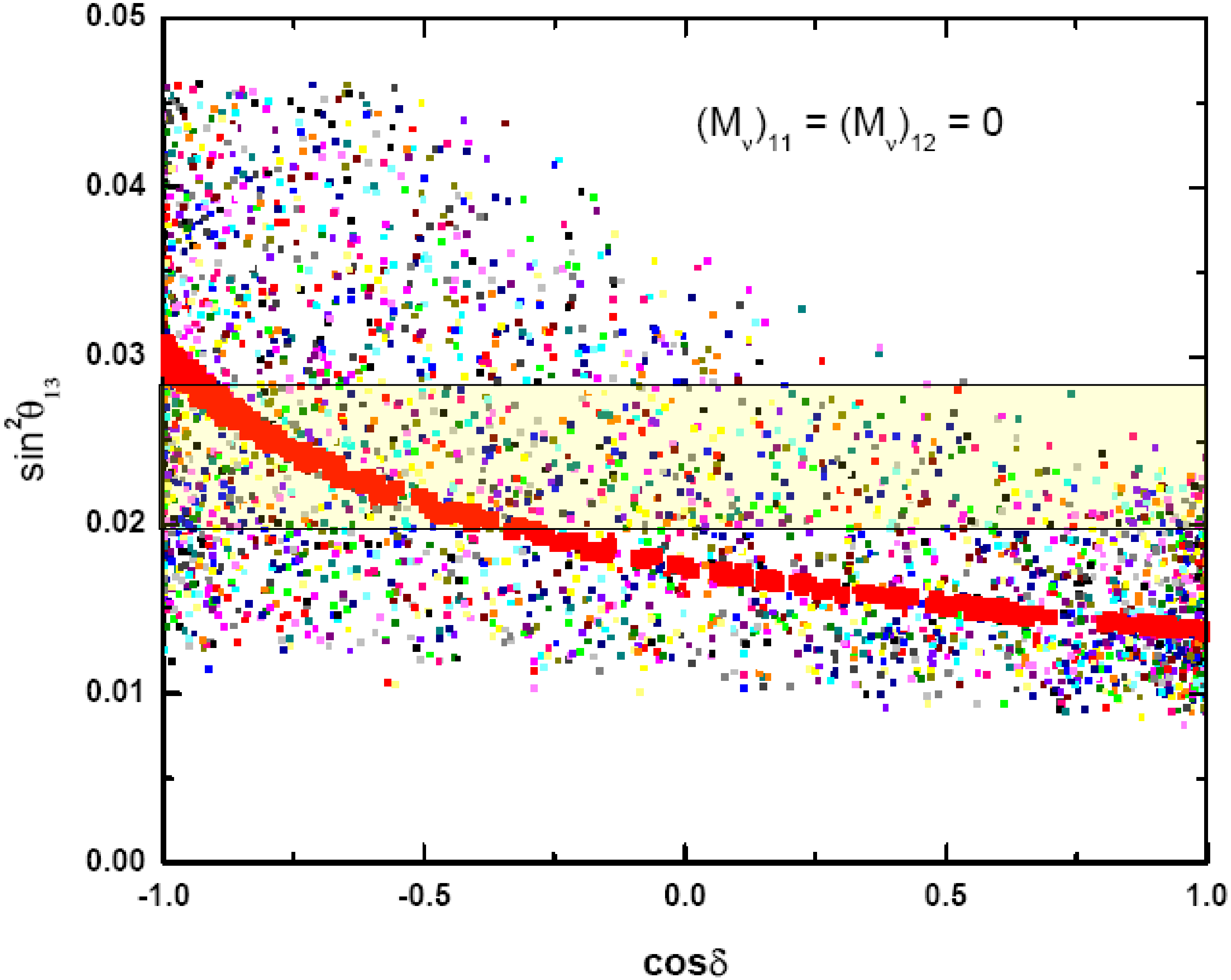


Fortes, KSB

Model A2 (central values)



Model A2 (with 95% CL errors)



Model A1 (with 95% CL errors)

Zeros in Neutrino Mass Matrix (cont.)

Models $B_1 - B_4$ predict large CP violating phase δ

$$\sin\theta_{13}\cos\delta \approx \frac{m_2}{m_3}$$

Model C predicts inverted mass hierarchy with $m_{\beta\beta} = m_3$

All of these textures support large θ_{13}

Texture models with 2 right-handed neutrinos

$$M_\ell = \text{diag}\,\{m_e,\ m_\mu,\ m_\tau\}$$

$$M_N = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} M$$ Heavy 2×2 Majorana matrix

(A) $Y_\nu = \begin{pmatrix} a_1 & 0 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}$ (B$_1$) $Y_\nu = \begin{pmatrix} a_1 & b_1 \\ a_2 & 0 \\ a_3 & b_3 \end{pmatrix}$ (B$_2$) $Y_\nu = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & 0 \end{pmatrix}$

Normal hierarchy   Inverted hierarchy

A single CP violating phase in all 3 cases!

Lightest neutrino mass $m_1 = 0$

Babu, Meng, Tavartkiladze (2008)

Predictions for Texture (B)

(B$_1$) $$M_\nu = \begin{pmatrix} 2a_1b_1 & a_2b_1 & a_1b_3 + a_3b_1 \\ a_2b_1 & 0 & a_2b_3 \\ a_1b_3 + a_3b_1 & a_2b_3 & 2a_3b_3 \end{pmatrix} \frac{(v \sin\beta)^2}{M}$$

$$M_\nu^{\rm diag} = {\rm diag}\,(m_1, m_2, 0)$$ Inverted mass hierarchy

(B$_1$) $$\sin^2\theta_{12} \simeq \frac{1}{2} - \frac{\sin\theta_{13}\tan\theta_{23}\cos\delta}{|\tan^2\theta_{23}\sin^2\theta_{13} + e^{2i\delta}|} + \frac{1}{8}\frac{\Delta m^2_{\rm sol}}{|\Delta m^2_{\rm atm}|}$$

(B$_2$) $$\sin^2\theta_{12} \simeq \frac{1}{2} + \frac{\sin\theta_{13}\tan\theta_{23}\cos\delta}{|\tan^2\theta_{23} + \sin^2\theta_{13}e^{2i\delta}|} + \frac{1}{8}\frac{\Delta m^2_{\rm sol}}{|\Delta m^2_{\rm atm}|}$$

(A) $$\tan\theta_{13} \simeq \sin\theta_{12}\sqrt{\frac{m_2}{m_3}}$$ $$\sin^2\theta_{13} = (0.042 - 0.047)$$

Texture (A) strongly disfavored!

Predictions for Texture (B) – cont.

(B$_1$) $\theta_{13} \gtrsim 0.12$, $\cos\delta \gtrsim 0.573$ $(|\delta| \lesssim 0.96)$

$$m_{\beta\beta} = \sqrt{|\Delta m^2_{\rm atm}|}\, c^2_{13} \frac{|t^2_{12} - 1 + 2 t_{12} t_{23} s_{13} e^{i\delta}|}{|t_{12} + t_{23} s_{13} e^{i\delta}|^2}$$

$\Longrightarrow$ $0.013 \text{ eV} \lesssim m_{\beta\beta} \lesssim 0.023 \text{ eV}$

(B$_2$) $\theta_{13} \gtrsim 0.129$, $\cos\delta \lesssim -0.614$ $(|\pi - \delta| \lesssim 0.91)$

$$m_{\beta\beta} = \sqrt{|\Delta m^2_{\rm atm}|}\, c^2_{13} \frac{|t^2_{12} - 1 - 2 t_{12} c t_{23} s_{13} e^{i\delta}|}{|t_{12} - c t_{23} s_{13} e^{i\delta}|^2}$$

$\Longrightarrow$ $0.013 \text{ eV} \lesssim m_{\beta\beta} \lesssim 0.023 \text{ eV}$

Inverted neutrino mass hierarchy predicted

Finding Order in Ferrmion Mass Spectrum

Fermion masses in units of m_t

$$m_t = 1.0$$
$$m_c = 3.6 \times 10^{-3}$$
$$m_u = 1.3 \times 10^{-5}$$

$$m_b = 1.67 \times 10^{-2}$$
$$m_s = 3.1 \times 10^{-4}$$
$$m_d = 2.3 \times 10^{-5}$$

$$m_\tau = 1.0 \times 10^{-2}$$
$$m_\mu = 6.2 \times 10^{-4}$$
$$m_e = 3.0 \times 10^{-6}$$

$$m_3 = 2.9 \times 10^{-13}$$
$$m_2 = 5.2 \times 10^{-14}$$
$$m_1 = < m_2$$

$$V_q = \begin{pmatrix} 0.976 & 0.22 & 0.004 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{pmatrix}$$

$$U_\ell = \begin{pmatrix} 0.85 & -0.54 & 0.16 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{pmatrix}$$

$$\text{Im}\left(\frac{V_{ub}V_{cs}}{V_{us}V_{cb}}\right) = 0.34$$

Neutrino Masses in Unified Theories

- Electric charge quantization
  - $Q_p = -Q_e$ to better than 1 part in 10^{21}
- Miraculous cancellation of anomalies
- Quantum numbers of quarks and leptons
- **Existence of ν_R and thus neutrino mass via seesaw**
- Unification of gauge couplings with low energy SUSY
- $b - \tau$ unification
- Baryon asymmetry of the universe via leptogenesis

Quantum Numbers of Fermions in SO(10)

$u_r : \{-++ \ +-\}$	$d_r : \{-++ \ -+\}$	$u_r^c : \{+-- \ ++\}$	$d_r^c : \{+-- \ --\}$
$u_b : \{+-+ \ +-\}$	$d_b : \{+-+ \ -+\}$	$u_b^c : \{-+- \ ++\}$	$d_b^c : \{-+- \ --\}$
$u_g : \{++- \ +-\}$	$d_g : \{++- \ -+\}$	$u_g^c : \{--+ \ ++\}$	$d_g^c : \{--+ \ --\}$
$\nu : \{--- \ +-\}$	$e : \{--- \ -+\}$	$\nu^c : \{+++ \ ++\}$	$e^c : \{+++ \ --\}$

16 of $SO(10)$

Frist 3 spins refer to color, last 2 are weak spins

$$Y = \frac{1}{3}\Sigma(C) - \frac{1}{2}\Sigma(W)$$

Eg: $Y(e^c) = \frac{1}{3}(3) - \frac{1}{2}(-2) = 2$

Minimal SO(10) Model

$$\mathcal{L}_{\text{Yukawa}} = f_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H + h_{ij} 16_i 16_j \overline{126}_H$$

Two Yukawa matrices determine all fermion masses and mixings, including the neutrinos

$$\begin{aligned} M_u &= A + B, \quad M_{\nu^D} = A - 3B \\ M_d &= \alpha A + \beta B, \quad M_\ell = \alpha A - 3\beta B \end{aligned}$$

$$M_{\nu^c} = cB$$

Model has only 11 real parameters plus 7 phases

Babu, Mohapatra (1993)
Fukuyama, Okada (2002)
Bajc, Melfo, Senjanovic, Vissani (2004)
Fukuyama, Ilakovac, Kikuchi, Meljanac, Okada (2004)
Aulakh et al (2004)
Bertolini, Frigerio, Malinsky (2004)
Babu, Macesanu (2005)
Bertolini, Malinsky, Schwetz (2006)
Dutta, Mimura, Mohapatra (2007)
Bajc, Dorsner, Nemevsek (2009)

Specific Example

Input at the GUT scale:

$$m_u = 0.00006745, \quad m_c = 0.3308, \quad m_t = 97.335$$
$$m_d = 0.00009726, \quad m_s = 0.02167, \quad m_b = 1,1475$$
$$m_e = 0.0000344, \quad m_\mu = 0.0726, \quad m_\tau = 1.350$$
$$s_{12} = 0.2248, \; s_{23} = 0.0328, \; s_{13} = 0.0022$$
$$\delta_{CKM} = 1.19$$

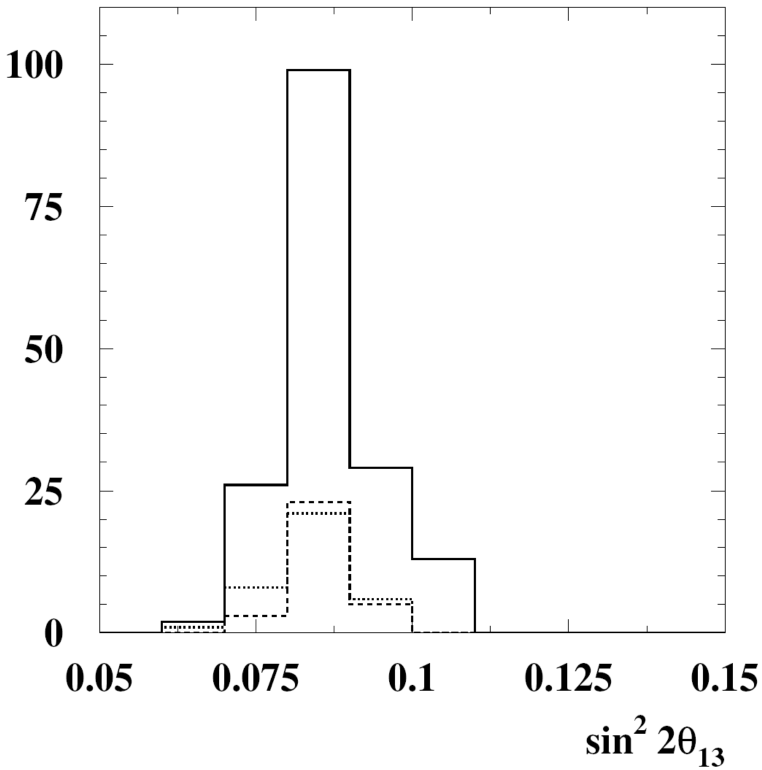
Output for neutrinos:

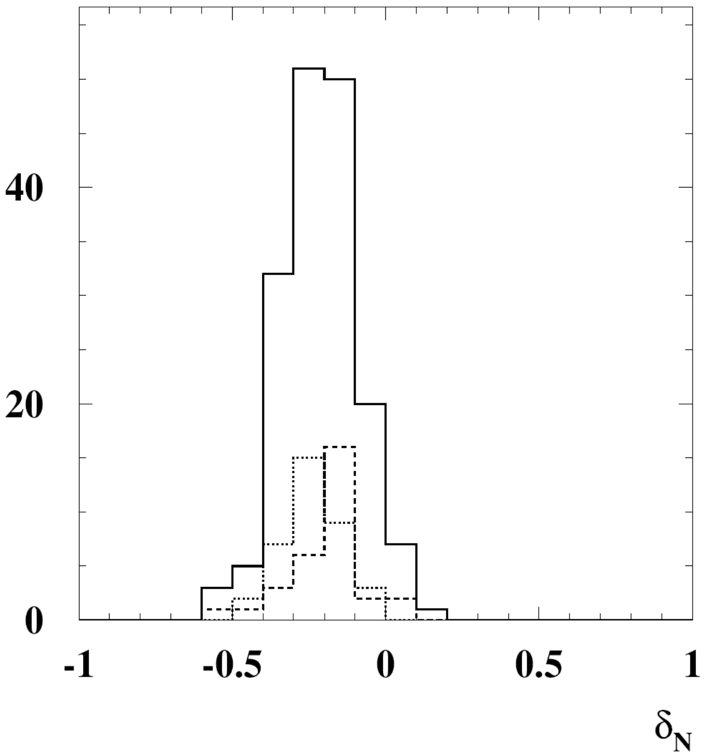
$$\sin^2\theta_{12} \simeq 0.27, \; \sin^2 2\theta_{23} \simeq 0.90, \; \sin^2 2\theta_{13} \simeq 0.08$$

$$m_i = \{0.0021 e^{0.11i}, \; 0.0098 e^{-3.08i}, \; 0.048\} \text{ eV}$$
$$\Delta m^2_{23}/\Delta m^2_{12} \simeq 24$$

K.S. Babu and C. Macesanu (2005)

Theta(13) in Minimal SO(10)





$\sin^2 2\theta_{13}$ and CP violating phase δ_N

K.S. Babu and C. Macesanu (2005)

$\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$ 5.2σ effect

Global Analysis of Minimal SO(10) Model

	A	B	C	D	C1	C2
Observables	Pulls obtained for best fit solution					
(m_u/m_c)	0.0486938	-0.180782	0.0653101	0.0053847	0.0467579	-0.0119661
(m_c/m_t)	1.22599	0.130589	0.246294	0.146932	0.297256	0.273346
(m_d/m_s)	-0.229546	-0.730641	0.223201	-0.748148	-2.2904	-0.689684
(m_s/m_b)	-0.932536	-0.886438	-0.977249	-1.05766	0.735548	0.000467775
(m_e/m_μ)	0.0340323	0.442759	0.103692	-0.476364	0.0649144	-0.0648856
(m_μ/m_τ)	0.310305	-0.526529	0.881934	0.938701	0.705648	0.0178824
(m_b/m_τ)	-0.486477	-0.194215	0.0172182	-0.34079	0.789868	-0.734937
$\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$	0.122267	-0.10063	-0.00563647	-0.120429	-0.180164	0.158557
$\sin\theta^q_{12}$	0.0432634	0.227948	0.0186715	0.084149	0.130301	0.0922391
$\sin\theta^q_{23}$	-0.281221	-0.0401177	-0.167224	0.0649082	-0.273222	-1.17651
$\sin\theta^q_{13}$	1.37864	-0.275689	0.926186	0.559003	1.48675	0.248759
$\sin^2\theta^l_{12}$	-0.0528379	-0.0598219	-0.38133	-0.172148	-0.746107	0.0694831
$\sin^2\theta^l_{23}$	-1.22555	-1.27077	-1.43475	0.0548963	-1.99485	-0.946001
$\delta_{\rm CKM}[^\circ]$	-0.291137	0.397159	-0.350422	-0.755859	-0.956628	-0.3197
χ^2_{min}	**6.3479**	**3.7962**	**5.0715**	**3.8665**	**14.789**	**3.4746**
Observables	Corresponding Predictions at GUT scale					
$\sin^2\theta^l_{13}$	0.0223307	0.0194886	0.0218753	0.0186789	0.0253152	0.0205366
$\delta_{\rm MNS}[^\circ]$	2.41793	4.52493	6.08769	335.07	357.142	14.7651
$\alpha_1[^\circ]$	347.106	8.42838	7.64991	28.0261	14.5679	1.13126
$\alpha_2[^\circ]$	163.759	191.241	188.713	218.586	196.273	177.828
$r_R\left(\frac{m_t^2}{m_\tau}\right)$ [GeV]	1.77×10^{-10}	2.63×10^{-10}	2.50×10^{-10}	4.02×10^{-10}	7.3×10^{-11}	2.82×10^{-10}

Joshipura, Patel (2011)

Large Neutrino Mixing From Lopsided Matrices

Quark and Lepton Mass hierarchy:

$$m_d : m_s : m_b \sim m_e : m_\mu : m_\tau \sim \epsilon_1 : \epsilon_2 : \epsilon_3$$
$$m_u : m_c : m_t \sim \epsilon_1^2 : \epsilon_2^2 : \epsilon_3^2$$

This motivates:

$$\begin{aligned} U &= H^T U_0 H \\ D &= D_0 H \\ L &= H^T L_0 \\ N &= N_0 \end{aligned}$$

$$H = \mathrm{Diag}(\epsilon_1,\ \epsilon_2,\ \epsilon_3) \qquad \epsilon_1 << \epsilon_2 << \epsilon_3$$

10_i of $SU(5)$ carry flavor charge, $\bar{5}_i$ do not.
Leads to large left-handed charged lepton mixing and large right-handed down quark mixing.

K.S. Babu, S. Barr, 1995

Albright, Babu and Barr, 1998

Sato and Yanagida, 1998

Irges, Lavignac, Ramond, 1998

Altarelli, Feruglio, 1998

Flavor $U(1)$ charges:

$10_i : (3, 2, 0)$ $\qquad$ $\overline{5}_i : (p, p, p)$ $\qquad$ $1_i : (q, q, q)$

$$U_{ij} = \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} H_u, \qquad D_{ij} = \begin{pmatrix} \epsilon^3 & \epsilon^3 & \epsilon^3 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ 1 & 1 & 1 \end{pmatrix} \epsilon^p H_d,$$

$$L_{ij} = \begin{pmatrix} \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \epsilon^p H_d, \qquad \nu^D_{ij} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \epsilon^{p+q} H_u$$

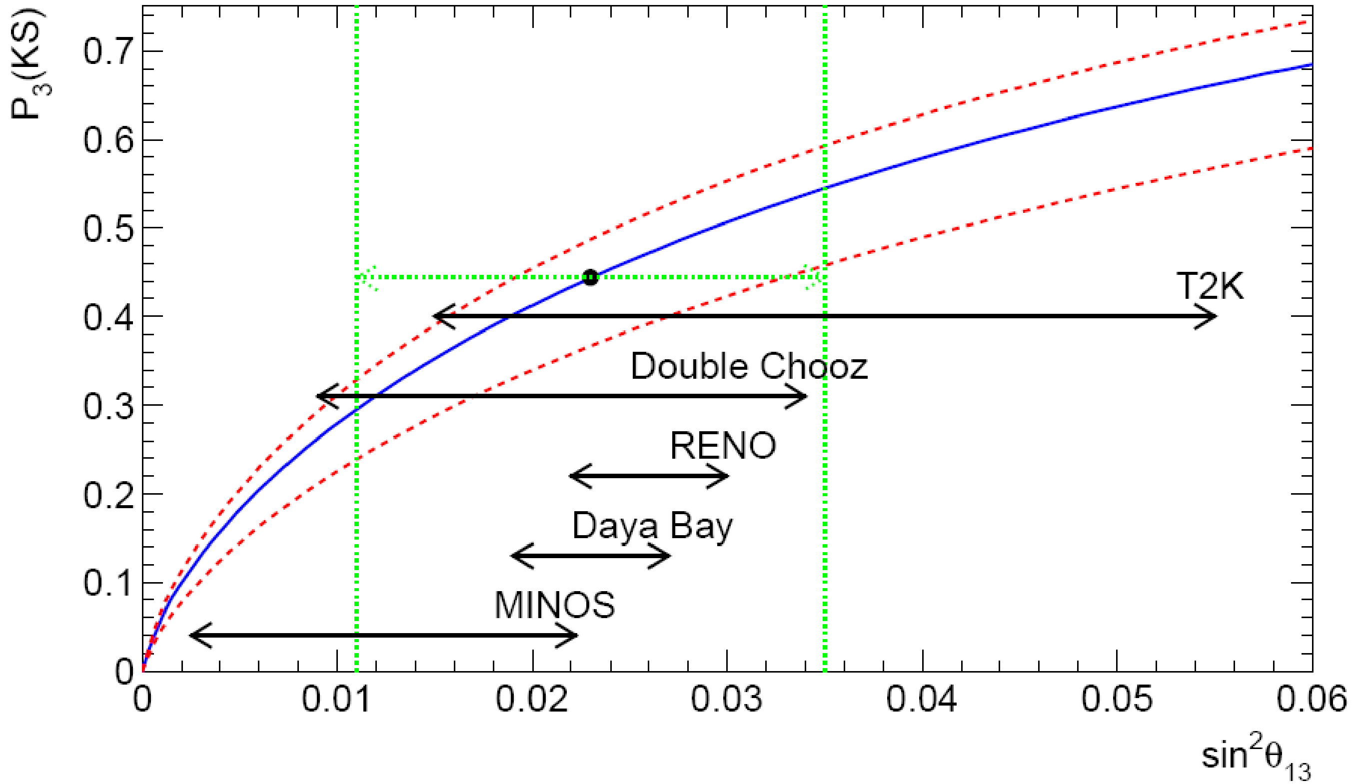
$$(M_\nu)_{ij} \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad \epsilon \sim 0.2$$

All features of fermion masses and CKM mixing reproduced

No particular hierarchy in neutrino masses

θ_{13} predicted to be large

Neutrino Mass Anarchy

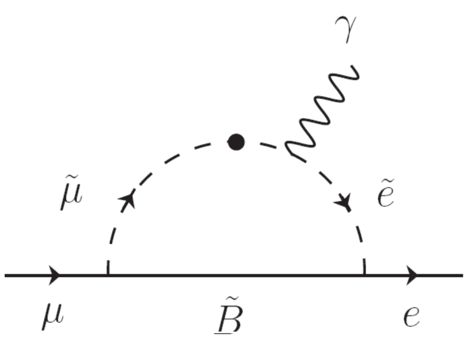


De Gouvea, Murayama (2012)

Lepton mixing matrix described by random draw of numbers in a unitary matrix

Lepton Flavor Violation in SUSY Seesaw Models

$$(m^2_{\tilde{l}_L})_{ij} \simeq -\frac{1}{8\pi^2}(Y_\nu^\dagger Y_\nu)_{ij}(3m_0^2 + |A_0|^2)\ln(\frac{M_P}{M_R})$$



Borzumati, Masiero, 1986
Hall, Kostelecki, Raby, 1986
Hisano et al, 1995

LFV rates depend on neutrino and SUSY parameters

Similar LFV occurs in SUSY GUTs even without m_ν

$$(m^2_{\tilde{e}_R})_{ij} \simeq -\frac{3}{8\pi^2}V_{3i}V^*_{3j}|Y_t|^2(3m_0^2 + |A_0|^2)\ln(\frac{M_P}{M_G})$$

Origin: Top quark ant τ lepton are grouped together in SU(5)

Barbieri, Hall, Strumia, 1995
Hisano et al, 1997

$\mu \longrightarrow e\gamma$ in SUSY seesaw model

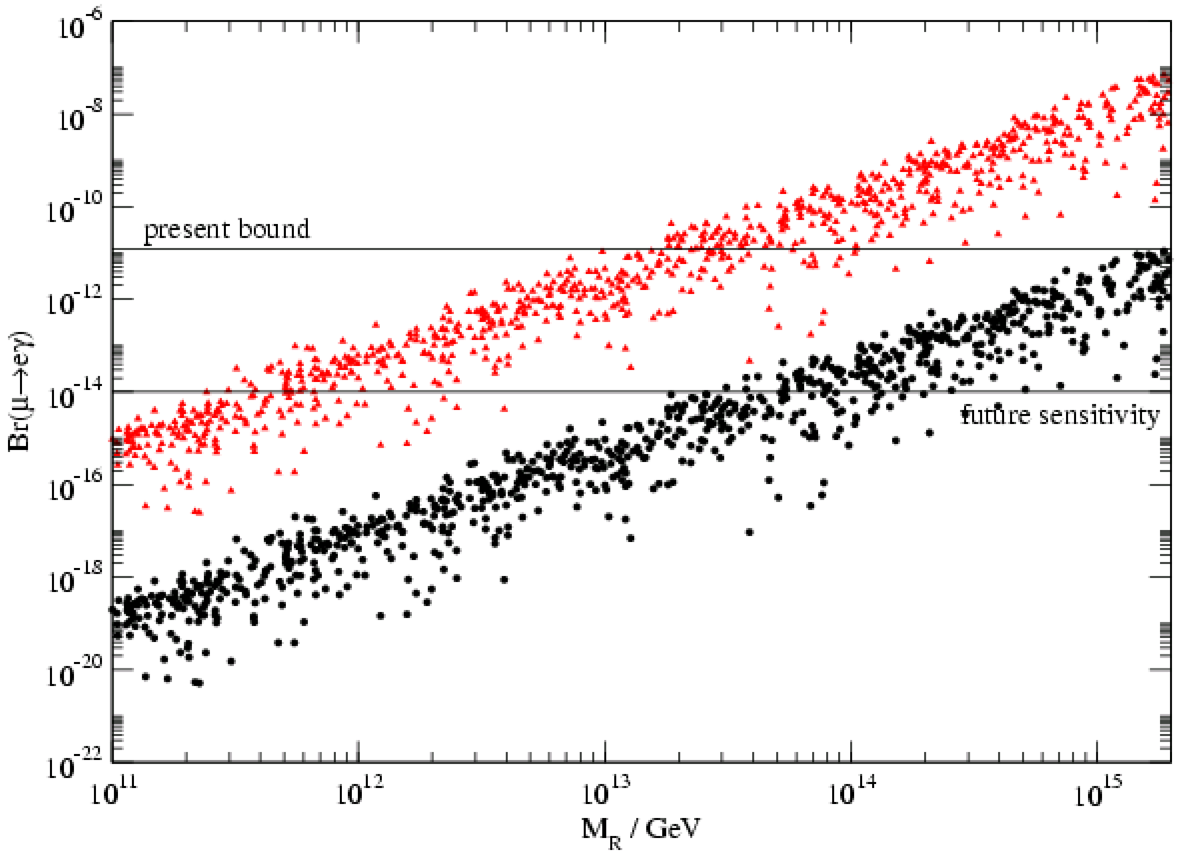


Figure 3: Branching ratio of $\mu \to e\gamma$ for hierarchical neutrinos and uncertainties of future neutrino experiments in the mSUGRA scenarios leading to the largest (L, upper) and the smallest (H, lower) LFV rates.

F. Deppisch, et al, 2002

Radiative Neutrino Mass Generation

Neutrino masses are zero at tree level

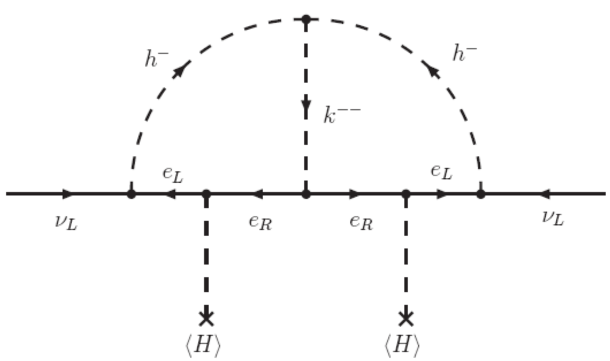
Smallness of m_ν due to loop and chiral suppression

Scale of new physics near TeV

Explicit Example

Two charged scalars h^+ and k^{++} added to standard model

$$\mathcal{L}^{\text{new}} = f_{ij}\ell_i\ell_j h^+ + g_{ij}e_i^c e_j^c k^{--} + \mu h^+ h^+ k^{--} + h.c.$$



A. Zee, 1986
K.S. Babu, 1988

Explains small neutrino mass, gives excellent fit to oscillation data

Requires h^+ and k^{++} to be around TeV

Process	Experiment (90% CL)	Bound (90% CL)
$\mu^- \to e^+e^-e^-$	BR$< 1.0\times10^{-12}$	$\lvert g_{e\mu}g^*_{ee}\rvert < 2.3\times10^{-5}\,(m_k/\mathrm{TeV})^2$
$\tau^- \to e^+e^-e^-$	BR$< 3.6\times10^{-8}$	$\lvert g_{e\tau}g^*_{ee}\rvert < 0.010\,(m_k/\mathrm{TeV})^2$
$\tau^- \to e^+e^-\mu^-$	BR$< 2.7\times10^{-8}$	$\lvert g_{e\tau}g^*_{e\mu}\rvert < 0.006\,(m_k/\mathrm{TeV})^2$
$\tau^- \to e^+\mu^-\mu^-$	BR$< 2.3\times10^{-8}$	$\lvert g_{e\tau}g^*_{\mu\mu}\rvert < 0.008\,(m_k/\mathrm{TeV})^2$
$\tau^- \to \mu^+e^-e^-$	BR$< 2.0\times10^{-8}$	$\lvert g_{\mu\tau}g^*_{ee}\rvert < 0.008\,(m_k/\mathrm{TeV})^2$
$\tau^- \to \mu^+e^-\mu^-$	BR$< 3.7\times10^{-8}$	$\lvert g_{\mu\tau}g^*_{e\mu}\rvert < 0.008\,(m_k/\mathrm{TeV})^2$
$\tau^- \to \mu^+\mu^-\mu^-$	BR$< 3.2\times10^{-8}$	$\lvert g_{\mu\tau}g^*_{\mu\mu}\rvert < 0.010\,(m_k/\mathrm{TeV})^2$
$\mu^+e^- \to \mu^-e^+$	$G_{M\bar{M}} < 0.003G_F$	$\lvert g_{ee}g^*_{\mu\mu}\rvert < 0.2\,(m_k/\mathrm{TeV})^2$

Table I: Constraints from tree-level lepton flavour violating decays.

M. Nebot et al, 2008
K.S. Babu, C. Macesanu, 2003

Experiment	Bound (90%CL)
$\delta a_e = (12\pm10)\times10^{-12}$	$r\left(\lvert f_{e\mu}\rvert^2+\lvert f_{e\tau}\rvert^2\right)+4\left(\lvert g_{ee}\rvert^2+\lvert g_{e\mu}\rvert^2+\lvert g_{e\tau}\rvert^2\right) < 5.5\times10^3\,(m_k/\mathrm{TeV})^2$
$\delta a_\mu = (21\pm10)\times10^{-10}$	$r\left(\lvert f_{e\mu}\rvert^2+\lvert f_{\mu\tau}\rvert^2\right)+4\left(\lvert g_{e\mu}\rvert^2+\lvert g_{\mu\mu}\rvert^2+\lvert g_{\mu\tau}\rvert^2\right) < 7.9\,(m_k/\mathrm{TeV})^2$
$BR(\mu\to e\gamma) < 1.2\times10^{-11}$	$r^2\lvert f^*_{e\tau}f_{\mu\tau}\rvert^2+16\lvert g^*_{ee}g_{e\mu}+g^*_{e\mu}g_{\mu\mu}+g^*_{e\tau}g_{\mu\tau}\rvert^2 < 3.4\times10^{-5}\,(m_k/\mathrm{TeV})^4$
$BR(\tau\to e\gamma) < 1.1\times10^{-7}$	$r^2\lvert f^*_{e\mu}f_{\mu\tau}\rvert^2+16\lvert g^*_{ee}g_{e\tau}+g^*_{e\mu}g_{\mu\tau}+g^*_{e\tau}g_{\tau\tau}\rvert^2 < 1.7\,(m_k/\mathrm{TeV})^4$
$BR(\tau\to\mu\gamma) < 4.5\times10^{-8}$	$r^2\lvert f^*_{e\mu}f_{e\tau}\rvert^2+16\lvert g^*_{e\mu}g_{e\tau}+g^*_{\mu\mu}g_{\mu\tau}+g^*_{\mu\tau}g_{\tau\tau}\rvert^2 < 0.7\,(m_k/\mathrm{TeV})^4$

Table III: Constraints from lepton number violating photon interactions.

LFV constraints and radiative neutrino mass

Summary and Conclusions

- Discovery of θ_{13} constrains theoretical models
- Zeros in neutrino mass matrix leads typically predict large θ_{13}
- A class of GUTs naturally predict large neutrino mixing and large θ_{13}
- New particles at TeV expected in many scenarios of neutrino mass generation and for muon $g-2$ anomaly

Backup Slides

Specific Example for Quark & Lepton masses

Fit

Input at GUT scale

$$\tan\beta = 55$$

$$m_u = 0.85 \text{ MeV} \qquad m_d = 1.08 \text{ MeV}$$
$$m_c = 222.3 \text{ MeV} \qquad m_s = 34.3 \text{ MeV}$$
$$m_t = 85.5 \text{ GeV} \qquad m_b = 1.549 \text{ GeV}$$
$$\delta_{CKM} = 1.508$$
$$V_{us} = 0.22 \quad V_{ub} = 0.0027 \quad V_{cb} = 0.036$$

Output: Type II Seesaw

$$\sin^2 2\theta_\odot = 0.635$$
$$\sin^2 2\theta_{e3} = 0.08$$
$$\sin^2 2\theta_{atm} = 0.892$$
$$\frac{\Delta m^2_{atm}}{\Delta m^2_\odot} = 15.2$$

Babu, Macesanu (2005)

Global fit to fermion masses in minimal SO(10)

	type-II		mixed'		mixed		type-I	
$\|\xi\|$	0		10^{-4}		0.3587		3.59×10^6	
$\arg(\xi)$	–		0.866π		1.018π		1.318π	
$\|r\|$	0.3278		1.9977		0.47896		0.3551	
$\arg(r)$	0.408π		1.849π		0.0013π		0.0057π	
f_u	16.62		11.51		18.77		19.23	
f_ν	1.671×10^{-10}		4.519×10^{-10}		8.732×10^{-10}		3.613×10^{-17}	
observable	pred.	pull	pred.	pull	pred.	pull	pred.	pull
m_d [MeV]	0.7662	**−1.16**	0.4956	**−1.82**	1.122	−0.29	0.4719	**−1.87**
m_s [MeV]	31.33	**1.85**	22.46	0.15	22.85	0.22	19.99	−0.33
m_b [MeV]	1147	0.61	1096	0.25	1078	0.13	1029	−0.35
m_u [MeV]	0.5543	0.02	0.5576	0.03	0.5512	0.00	0.5538	0.02
m_c [MeV]	213.1	0.17	213.5	0.18	210.6	0.03	213.1	0.16
m_t [MeV]	78030	−0.29	77411	−0.34	81659	−0.05	78117	−0.29
$\sin\phi_{23}^{CKM}$	0.0345	−0.43	0.0352	0.08	0.0351	0.03	0.0349	−0.13
$\sin\phi_{13}^{CKM}$	0.00331	0.23	0.00319	−0.02	0.00319	−0.01	0.00323	0.06
$\sin\phi_{12}^{CKM}$	0.2245	0.11	0.2243	0.02	0.2243	0.01	0.2243	0.01
δ_{CKM} [°]	79.35	**1.38**	59.47	−0.04	61.41	0.10	61.11	0.08
$\sin^2\theta_{23}^{PMNS}$	0.3586	**−2.17**	0.5126	0.19	0.5027	0.04	0.4944	−0.09
$\sin^2\theta_{13}^{PMNS}$	0.0145	0.93	0.0106	0.68	0.0066	0.43	0.0095	0.61
$\sin^2\theta_{12}^{PMNS}$	0.2829	**−1.08**	0.3078	−0.09	0.3094	−0.02	0.3078	−0.09
$\Delta m_{21}^2 [10^{-5}\mathrm{eV}^2]$	7.863	−0.12	7.894	−0.02	7.898	−0.01	7.896	−0.01
$\Delta m_{31}^2 [10^{-3}\mathrm{eV}^2]$	2.385	0.50	2.232	0.09	2.210	0.03	2.223	0.06
$m_1/\sqrt{\Delta m_{21}^2}$	0.279		0.478		0.382		0.361	
δ_{PMNS} [°]	−0.70		−59		−0.70		4.9	
α_1 [°]	1.1		30		1.8		−2.1	
α_2 [°]	91		126		−84		90	
χ^2		14.5		4.1		0.35		4.3

$\sin^2 2\theta_{13} \simeq 0.04$

Bertolini, Malinsky, Schwetz (2006)

Discrete example of lopsided matrices

Gogoladze, Wang, KSB (2003)

$$U_{ij} = \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} H_u, \qquad D_{ij} = \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^p H_d,$$

$$L_{ij} = \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon \\ \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \epsilon^p H_d, \qquad \nu^D_{ij} = \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^{a_1} H_u$$

$\epsilon \sim 0.2$

$\theta_{13} \sim \epsilon \sim 0.2$

$$\nu^M_{ij} \propto \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^{a_2} \qquad \sim \mathrm{M}^{\nu}_{\mathrm{light}}$$

Discrete Z_{14} Gauge Symmetry with Green-Schwarz anomaly cancellation

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	H_u	H_d	θ	S	A_2	A_3
A	0,2,6	1,3,7	3,5,5	4,6,6	13,1,5	5,7,7	1	13	7	2	6	13
B	4,6,10	13,1,5	11,13,13	6,8,8	9,11,1	5,7,7	13	1	7	2	13	13
C	6,8,12	5,7,11	1,3,3	0,2,2	7,9,13	5,7,7	9	5	7	2	13	6

	A	**B**	**C**	**D**	**C1**	**C2**
Observables	Pulls obtained for best fit solution					
(m_u/m_c)	−0.00668428	0.0276825	0.0259467	0.120767	−0.0212532	0.0356043
(m_c/m_t)	0.56521	0.157569	0.0201093	0.0730136	0.130288	0.320944
(m_d/m_s)	−1.21642	−0.891034	−0.27664	−1.36265	−1.04724	−1.57673
(m_s/m_b)	0.112798	0.440678	0.163272	0.752408	0.884723	0.789053
(m_e/m_μ)	0.0590249	−0.00627804	0.3944	0.0396087	0.0297987	0.0555931
(m_μ/m_τ)	0.182548	0.103214	0.821485	0.0192305	0.26316	0.121145
(m_b/m_τ)	0.87282	2.20829	2.79368	2.34331	0.26656	0.407798
$\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$	0.256292	0.116314	−0.14908	0.230056	0.0188227	−0.0140039
$\sin\theta^q_{12}$	0.0730813	0.0702755	0.0399788	0.105989	0.0779176	0.127757
$\sin\theta^q_{23}$	−0.0311676	−0.172792	−0.471738	−0.0960437	−0.757038	−0.945821
$\sin\theta^q_{13}$	1.33502	−0.0354198	0.494732	0.606606	0.890741	1.17758
$\sin^2\theta^l_{12}$	0.00836789	−0.106439	−0.599727	−0.27881	−0.63356	−0.510182
$\sin^2\theta^l_{23}$	−1.53367	−4.97038	−4.95673	−4.70944	−2.56294	−1.84412
$\delta_{\mathrm{CKM}}[^\circ]$	−0.345931	−0.163765	−0.600814	−0.214459	−0.650554	−0.75885
χ^2_{min}	**6.9367**	**30.70**	**34.52**	**30.68**	**10.804**	**9.3559**
Observables	Corresponding Predictions at GUT scale					
$\sin^2\theta^l_{13}$	0.0226508	0.0190847	0.0206716	0.0196974	0.0239619	0.0209208
$\delta_{\mathrm{MNS}}[^\circ]$	19.9399	18.9784	19.5619	11.92	358.789	1.78569
$\alpha_1[^\circ]$	337.171	346.627	344.795	350.595	12.4786	349.711
$\alpha_2[^\circ]$	147.364	151.912	146.886	161.702	194.023	168.156
$r_L m_\tau[\mathrm{GeV}]$	8.37×10^{-10}	6.0×10^{-10}	6.49×10^{-10}	6.94×10^{-10}	7.15×10^{-10}	9.1×10^{-10}