Model independent determination of the axial mass in quasielastic neutrino-nucleon scattering.

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PRD **84** 073006 (2011) [arXiv:1108.0423]

Quasiealstic $\nu - N$ scattering

- Quasiealstic $\nu - N$ scattering: $\nu_\ell + n \to \ell^- + p$
  basic signal for ν oscillation experiment

- At the quark level: $\nu_\ell + d \to \ell^- + u$

- Process "folded" twice

Quark: $\nu_\ell + d \to \ell^- + u$

$\Downarrow$ Form factor

Nucleon $\nu_\ell + n \to \ell^- + p$

$\Downarrow$ Nuclear model

Nucleus: $\nu_\ell + \text{nucleus} \to \ell^- + \ldots$

Quark $\to$ Nucleon

- The interaction

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} V_{ud}^{*}\, \bar{\ell}\gamma^{\alpha}(1-\gamma^5)\nu\, \bar{u}\gamma_{\alpha}(1-\gamma^5)d.$$

- Known current: $\bar{u}\gamma_{\alpha}(1-\gamma^5)d$

- Parametrize $\langle p(p')|\bar{u}\gamma_{\alpha}(1-\gamma^5)d|n(p)\rangle$ by form factors:

  F_1, F_2, F_p, F_A, functions of $q^2=(p'-p)^2$

- Of the form factors, only the axial is not constrained

- What do we know about $F_A(q^2)$?

The Axial Mass

- Consider a small q^2 expansion of $F_A(q^2)$

- $F_A(0) = -1.269$ is known from neutron decay

- Define the axial mass m_A as

$$F_A(q^2) = F_A(0)\left[1 + \frac{2}{m_A^2}q^2 + \dots\right] \implies m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$$

- Common **model** for F_A: the dipole model

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\rm dipole})^2]^{-2}$$

- One parameter **model** for F_A

  Known to be inadequate for EM form factors

Nucleon → Nucleus

- Experiments usually scatter ν off nuclei

  Need a nuclear model: how do nucleons behave in the nucleus

- Popular model: "Relativistic Fermi Gas" (RFG)

  [Smith, Moniz, NPB **43**, 605 (1972)]

- Model validity and parameters from quasielastic *e*-nuclei scattering

  Moniz, Sick, Whitney, Ficenec, Kephart, Trower, PRL **26**, 445 (1971)

The axial mass problem

- Neutrino scattering:

  $m_A^{\mathrm{dipole}} = 1.35 \pm 0.17$ GeV

  MiniBooNE Collaboration

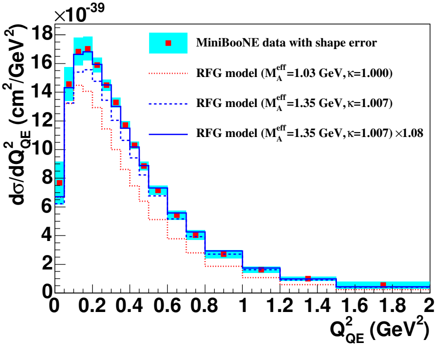
  PRD **81** (2010) 092005

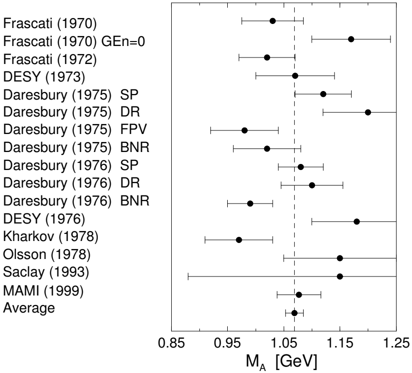
- Pion electro-prodcution:

  $m_A^{\mathrm{dipole}} = 1.07 \pm 0.02$ GeV

  Bernard, Elouadrhiri, Meissner

  J. Phys. G 28, R1 (2002)





Both use dipole ansatz for axial form factor

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\mathrm{dipole}})^2]^{-2}$$

The axial mass problem

- Axial mass $m_A^{\rm dipole} = 1.35 \pm 0.17$ GeV
  [MiniBooNE Collaboration, PRD **81** 092005 (2010)]
- Similar result from other recent ν experiments
- K2K SciFi: $m_A^{\rm dipole} = 1.20 \pm 0.12$ GeV
  [K2K Collaboration, PRD **74** 052002 (2006)]
- K2K SciBar $m_A^{\rm dipole} = 1.144 \pm 0.077({\rm fit})^{+0.078}_{-0.072}({\rm syst})$ GeV
  Espinal, Sanchez, AIP Conf. Proc. **967**, 117 (2007)
- Minos $m_A^{\rm dipole} = 1.19^{+0.09}_{-0.1}({\rm fit})^{+0.12}_{-0.14}({\rm syst})$ GeV
  [MINOS Collaboration, AIP Conf. Proc. **1189**, 133 (2009)]

- Nomad: $m_A^{\rm dipole} = 1.05 \pm 0.02 \pm 0.06$ GeV
  [NOMAD Collaboration, EPJ C **63**, 355 (2009)]
- Pion electro-prodcution: $m_A^{\rm dipole} = 1.07 \pm 0.02$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)
- ν experiments before 1990: $m_A^{\rm dipole} = 1.026 \pm 0.021$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)
- **What could be the source of the discrepancy?**

Theoretical studies focus on nuclear modeling

- Modify nuclear model
  [Butkevich, PRC **82**, 055501 (2010); Benhar, Coletti, Meloni, PRL **105**, 132301 (2010); Juszczak, Sobczyk, Zmuda, PRC **82**, 045502 (2010)]

- Include multi-nucleon emission
  [Martini, Ericson, Chanfray, Marteau
  PRC **80**, 065501 (2009), PRC **81**, 045502 (2010);
  Amaro, Barbaro, Caballero, Donnelly, Williamson
  PLB **696**, 151 (2011), PRD **84**, 033004 (2011);
  Nieves, Ruiz Simo, Vicente Vacas
  PRC **83**, 045501 (2011), arXiv:1106.5374]

- Modify G_M for bound nucleons but not G_E or F_A
  [Bodek, Budd, EPJ C **71**, 1726 (2011)]

- **All** use dipole form factor

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\rm dipole})^2]^{-2}$$

What is the axial mass?

- The axial mass is defined as

$$F_A(q^2) = F_A(0)\left[1 + \frac{2}{m_A^2}q^2 + \dots\right] \implies m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$$

- Everyone extracts $m_A^{\rm dipole}$ from

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- $m_A^{\rm dipole}$ is not m_A!

- When extractions of $m_A^{\rm dipole}$ disagree is it
  - Problem with the dipole model?
  - Real disagreement between experiments?

- Need to extract m_A in a model independent way!

Model independent extraction of m_A

- Need to extract m_A in a model independent way!
- How to do that?

Model independent extraction of m_A

- Need to extract m_A in a model independent way!
- How to do that?
- Let's look at a simpler problem: The charge radius of the proton

Proton Charge Radius

- Matrix element of EM current between nucleon states gives rise to two form factors ($q = p' - p$)

$$\langle N(p')| \sum_q e_q \, \bar{q}\gamma^\mu q |N(p)\rangle = \bar{u}(p') \left[\gamma^\mu F_1^N(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2^N(q^2) q^\nu \right] u(p)$$

- Sachs electric form factor

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2)$$

- Consider a small q^2 expansion of $G_E^p(q^2)$

- $G_E^p(0) = 1$

- The slope of G_E^p

$$r_p^2 = 6 \frac{dG_E^p}{dq^2}\bigg|_{q^2=0}$$

determines the charge radius r_p

Charge Radius Problem

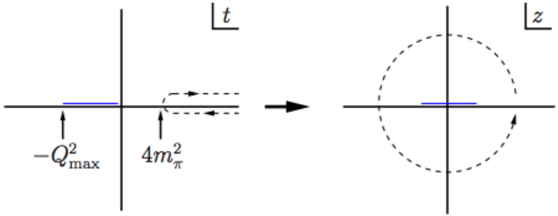
- Lamb shift in **muonic** hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_p = 0.84184(67)$ fm
- **electronic** hydrogen [Mohr et al. RMP **80**, 633 (2008)]
  $r_p = 0.87680(690)$ fm
- **5σ discrepancy!**
- Electron-proton scattering data:
  r_p between $0.8 - 0.9$ fm [PDG 2010]
  Using different models for G_E^p
  How to avoid model dependence?

z expansion

- Standard tool in analyzing **meson** transition form factors
- Analytic properties of $G_E^p(t)$ are known
- Map the domain of analyticity onto the unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

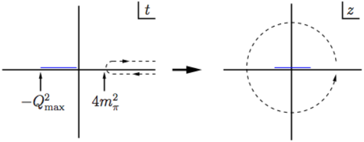
where $t_{\rm cut} = 4m_\pi^2$, $z(t_0, t_{\rm cut}, t_0) = 0$



- Expand G_E^p in a Taylor series in z: $G_E^p(q^2) = \sum_{k=0}^{\infty} a_k \, z(q^2)^k$

Analytic structure and a_k

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$



- Analytic structure implies:
  Information about $\mathrm{Im}\, G_E^p(t + i0) \Rightarrow$ information about a_k

- Can use data to constrain a_k:
  Arbitrary functional form with no change in r_p

- [Hill, GP PRD **82** 113005 (2010)] studied the size of a_k using
  - vector dominance ansatz
  - $\pi\pi$ continuum
  - $e^+e^- \to N\bar{N}$ data

- Found $|a_k| \leq 10$ very conservative
  Results presented for $|a_k| \leq 5$ and $|a_k| \leq 10$

Proton Charge Radius

- [Hill, GP PRD **82** 113005 (2010)]:
- previous extractions are model dependent
  underestimated error by a factor of 2 or more

- Based on a Model-independent approach
  $r_p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$

- CODATA value (extracted mainly from electronic hydrogen)
  [Mohr et al. RMP **80**, 633 (2008)]
  $r_p = 0.8768(69)$ fm

- Lamb shift in muonic hydrogen
  [Pohl et al. Nature **466**, 213 (2010)]
  $r_p = 0.84184(67)$ fm

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  $r_p = 0.84184(67)$ fm

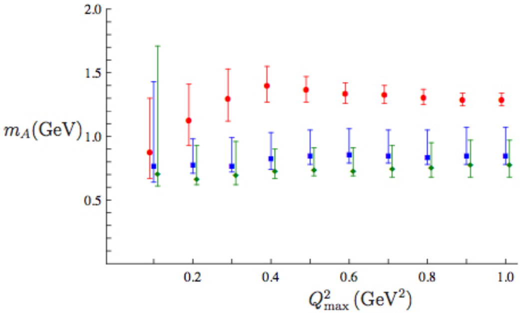
- Theoretical treatment for muonic hydrogen is lacking
  [Richard J. Hill, GP PRL **107** 160402 (2011)]

Goal

- Extract m_A in a model independent way: z expansion
- Following the charge radius analysis: $|a_k| \leq 5$ and $|a_k| \leq 10$
- Extract m_A from fit to MiniBooNE data for $d\sigma/dE_\mu d\cos\theta_\mu$
  [MiniBooNE Collaboration, PRD **81** 092005 (2010)]
- Mostly follow MiniBooNE's analysis: use RFG as nuclear model

Neutrino: Model independent approach

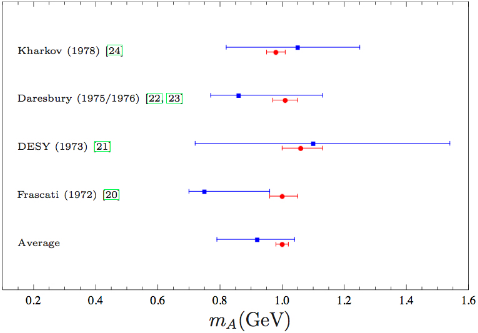
- Our z expansion fit to MiniBooNE data (Assuming RFG):
  Red: dipole, Blue: z, $|a_k| \leq 5$, Green: z, $|a_k| \leq 10$



- Our fit using z expansion: $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV
  Our fit using dipole model: $m_A^{\rm dipole} = 1.29 \pm 0.05$ GeV
  MiniBooNE's fit: $m_A^{\rm dipole} = 1.35 \pm 0.17$ GeV

Pion Electro-production: Model independent approach

- Is there a discrepancy with pion electro-production data?
  Red: dipole, Blue: z, $|a_k| \le 5$



- Our fit using z expansion: $m_A = 0.92^{+0.12}_{-0.13} \pm 0.08$ GeV
  Our fit using dipole model: $m_A^{\rm dipole} = 1.00 \pm 0.02$ GeV
  Bernard et. al. fit using dipole model: $m_A^{\rm dipole} = 1.07 \pm 0.02$ GeV
  Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)

Model independent approach

- MiniBooNE (Assuming RFG):

  $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV

  $m_A^{\rm dipole} = 1.29 \pm 0.05$ GeV

- Pion electro-prodcution:

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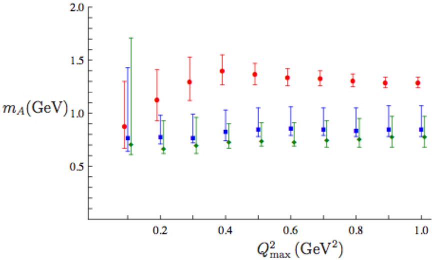
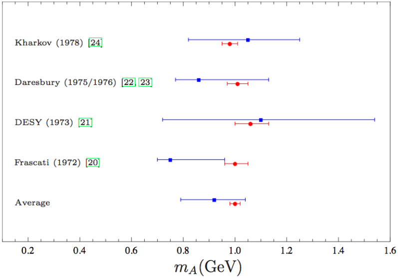


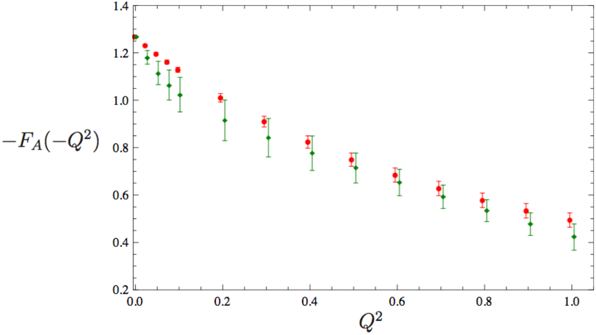


Discrepancy is an artifact of the use of the dipole form factor!

Going beyond m_A

- We can also extract F_A directly from MiniBooNE data

  Red: dipole, Green : z, $|a_k| \leq 10$



Error on F_A underestimated in the dipole model

Conclusions

- Recent $m_A^{\rm dipole}$ extractions from quasielastic $\nu - N$ scattering are typically higher than pre-1990 ν experiments and pion electro-production data

- We presented model-independent extraction of the axial mass from quasielastic $\nu - N$ scattering data using the z expansion

- MiniBooNE (Assuming RFG):
  $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV $\quad m_A^{\rm dipole} = 1.29 \pm 0.05$ GeV

- Pion electro-prodcution:
  $m_A = 0.92^{+0.12}_{-0.13} \pm 0.08$ GeV $\quad m_A^{\rm dipole} = 1.00 \pm 0.02$ GeV

- As far as M_A is concerned:
  discrepancy is an artifact of the use of the dipole form factor!

Future directions

- Extract m_A from other ν experiments, e.g. Minerνa
- Is m_A consistent between experiments?
- m_A from pion electro-production data, extrapolated from soft π limit

  Extract m_A in a model-independent way
- ν experiments need F_A, extract it from another source
- After F_A is under control, discuss nuclear models