Discovering Technicolor

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* Sannino et al. '11, Eur.Phys.J.Plus 126 (2011) 81. arXiv:1104.1255

CP³ - Origins

Particle Physics & Origin of Mass

Pheno 2012, Pittsburgh

QCD & Dynamical EWSB

In QCD at Λ_{QCD} the interaction becomes strong and the quarks form a bound state with non-zero vev:

$$\langle 0|\, \bar{u}_L u_R + \bar{d}_L d_R \,|0\rangle \neq 0,\ T_L^3 + Y_L = Y_R = Q \ \Rightarrow\ SU(2)_L \times U(1)_Y \to U(1)_{EM}$$

By redefining currents in terms of composite peudo-scalars (pions) one finds that the EW bosons acquire masses:

$$M_W^{QCD} = g f_{\pi^\pm}/2,\ \ \rho = \frac{M_W^{QCD}}{M_Z^{QCD}} \cos^{-1}(\theta_W) = 1.$$

Given the experimental value for the pion decay constant

$$f_\pi = 93\,\mathrm{MeV} \quad \Rightarrow \quad M_W^{QCD} = 29\,\mathrm{MeV!}$$

Technicolor

The effective Lagrangian expansion breaks down at

$$\Lambda_{QCD} \simeq 4\pi f_\pi = 1.2\,\mathrm{GeV} \Rightarrow \Lambda_{TC} \simeq 4\pi v = 3\,\mathrm{TeV},\ \ v = 246\ \mathrm{GeV}\,.$$

A Technicolor (TC) model able to give the right masses to the EW gauge bosons is simply "scaled up" QCD (no fundamental scalar ⇒ no fine-tuning!):

$$SU(N)_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y\,.$$

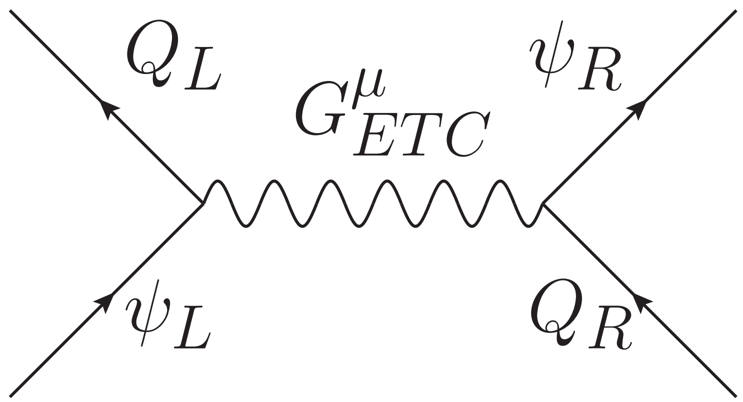
To generate the fermion masses an Extended Technicolor (ETC) interaction is necessary.

* Susskind '79

Extended Technicolor

If the ETC gauge group gets broken at some large scale $\Lambda_{ETC} \gg \Lambda_{TC}$, the massive ETC gauge bosons can be integrated out.

Four fermion interactions, technifermion condensate $\Rightarrow$ SM mass terms



$$\rightarrow \frac{g^2_{ETC}}{M^2_{ETC}} (\bar{Q}_L Q_R)(\bar{\psi}_R \psi_L) \Rightarrow m_\psi \approx \frac{g^2_{ETC}}{M^2_{ETC}} \langle \overline{Q} Q \rangle \ .$$

The lowest ETC scale is determined by the heaviest mass:

$$m_t = 173 \text{ GeV} \approx \frac{\Lambda^3_{TC}}{\Lambda^2_{ETC}} \Rightarrow \Lambda_{ETC} \simeq 10\,\text{TeV}$$

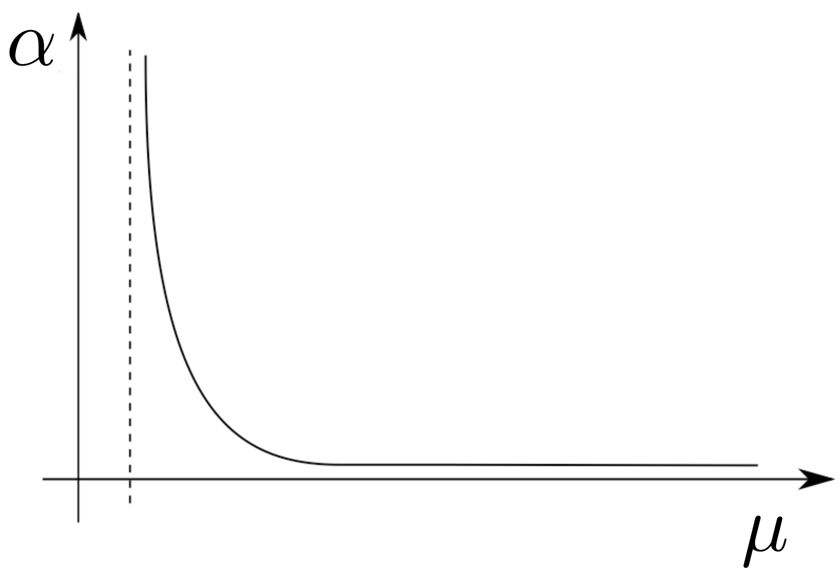
Flavor changing neutral currents bounds though require $\Lambda_{ETC} \gtrsim 10^3 \text{ TeV} \ldots$

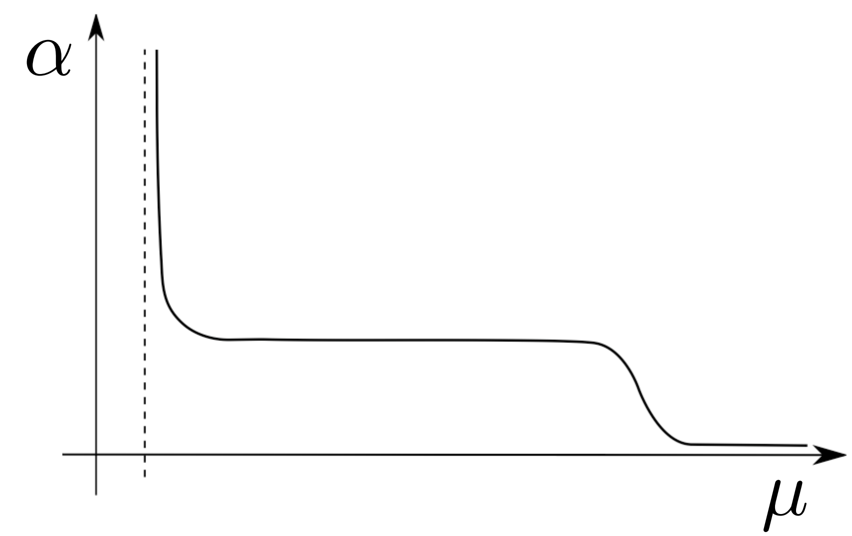
Fermion Mass Renormalization

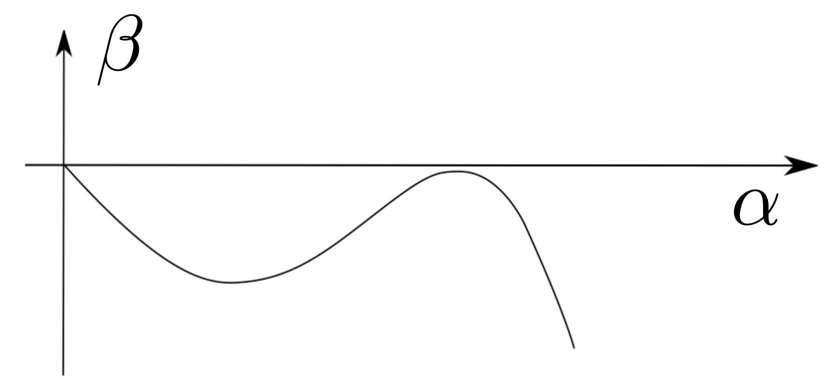
The limits on Λ_{ETC} from the large value of m_t and the FCNC experimental data seem to be incompatible, but that was without taking into account renormalization:

$$\gamma_m = \frac{d\log m}{d\log \mu},\ m^3 \propto \langle \overline{Q}Q \rangle \Rightarrow \langle \overline{Q}Q \rangle_{ETC} = \langle \overline{Q}Q \rangle_{TC} \exp\left(\int_{\Lambda_{TC}}^{\Lambda_{ETC}} \frac{d\mu}{\mu} \gamma_m(\mu) \right)$$

Running vs Walking TC







for $\Lambda_{ETC} > \mu > \Lambda_{TC}$:

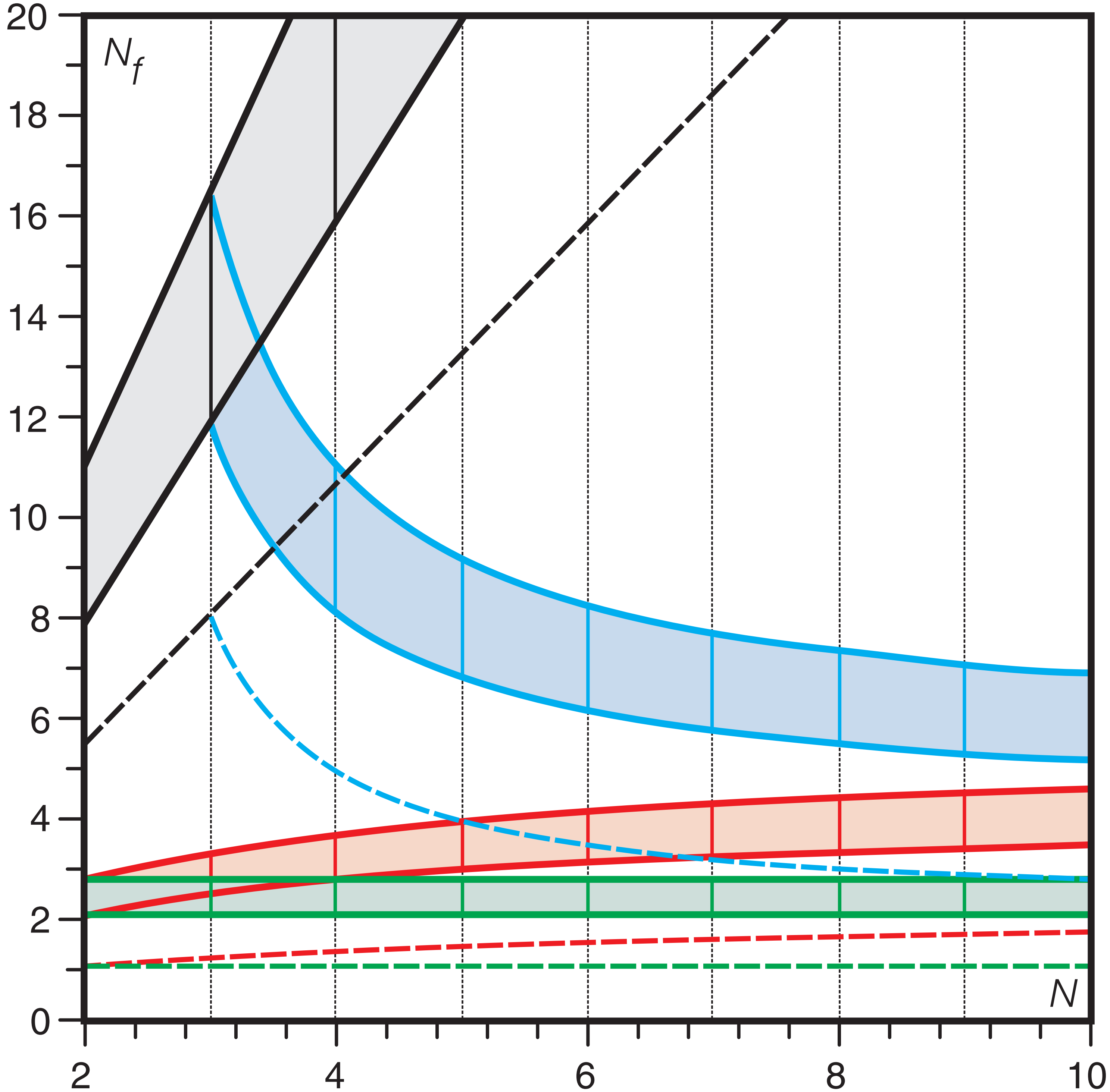
- Running TC: $\alpha(\mu) \propto \frac{1}{\ln \mu}, \Rightarrow \langle \overline{Q}Q \rangle_{ETC} \simeq \langle \overline{Q}Q \rangle_{TC}$

- Walking TC: $\beta(\alpha_*) = 0 \Rightarrow \langle \overline{Q}Q \rangle_{ETC} \simeq \langle \overline{Q}Q \rangle_{TC} \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{\gamma_m(\alpha_*)}$

A Walking TC obtains a big boost to fermion masses, while FCNC are unaffected.

* Yamawaki et al. '86, Appelquist et al '86

Walking in the SU(N)



Phase diagram for theories with fermions in the:

- fundamental representation (grey)
- two-index antisymmetric (blue)
- two-index symmetric (red)
- adjoint representation (green)

The S parameter for a TC model is estimated by:

$$S_{th} = \frac{1}{6\pi}\frac{N_f}{2}d(\mathrm{R}),$$

$$12\pi\, S_{exp} \leq 6 \text{ @ } 95\%$$

* Dietrich, Sannino '06

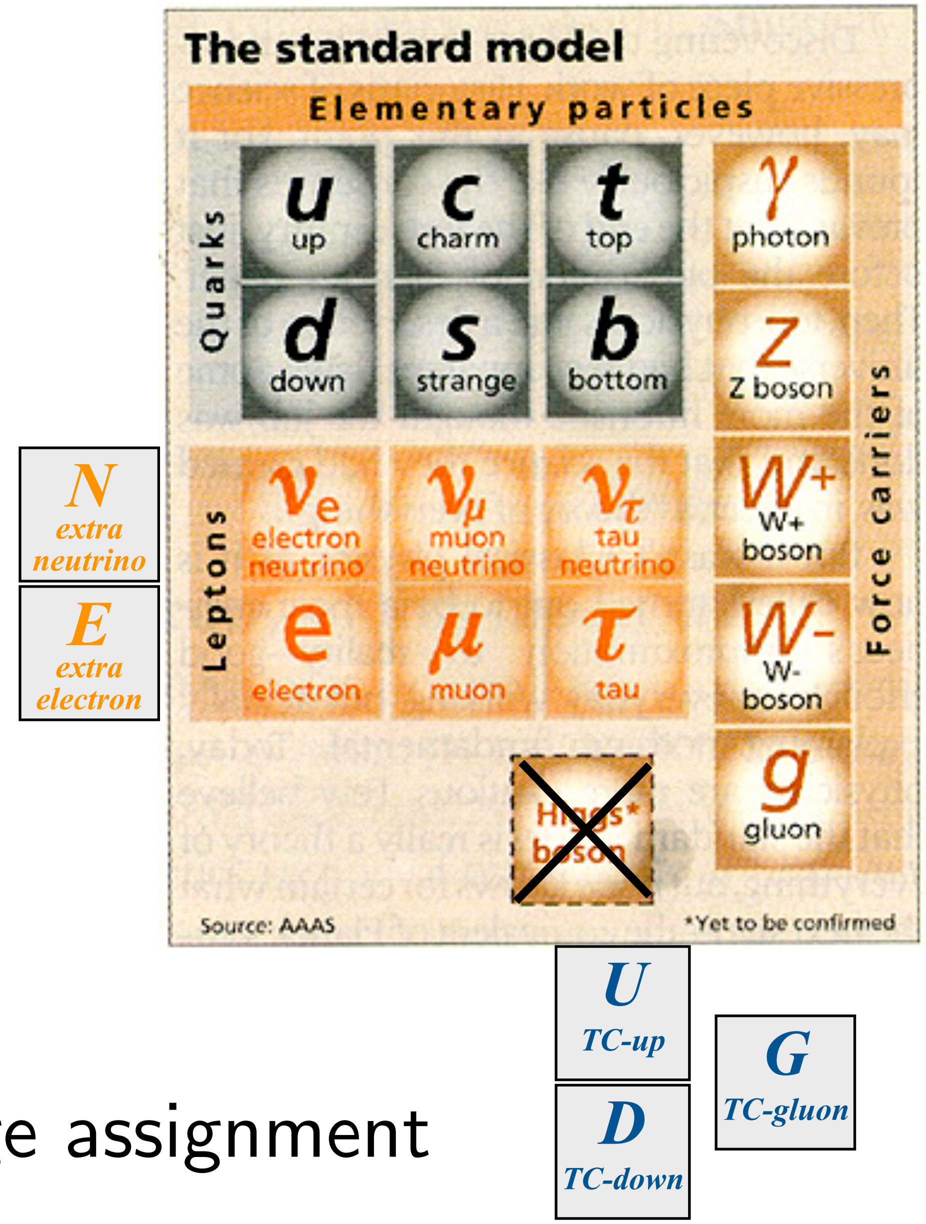
Minimal Walking Technicolor

TC-fermions in the $SU(2)_{TC}$ adjoint representation: $a = 1, 2, 3$;

$$Q_L^a = \begin{pmatrix} U_L^a \\ D_L^a \end{pmatrix}, \; U_R^a, \; D_R^a \; .$$

Heavy leptons to cancel Witten anomaly:

$$L_L = \begin{pmatrix} N_L \\ E_L \end{pmatrix}, \; N_R, \; E_R \; .$$



$U(1)_Y$

$SU(2)_L$

$SU(3)_C$

$SU(2)_{TC}$

Gauge anomalies cancel for hypercharge assignment

$$Y(Q_L) = \frac{y}{2}, \qquad Y(U_R, D_R) = \left(\frac{y+1}{2}, \frac{y-1}{2}\right),$$

$$Y(L_L) = -3\frac{y}{2}, \qquad Y(N_R, E_R) = \left(\frac{-3y+1}{2}, \frac{-3y-1}{2}\right)$$

* Sannino, Tuominen '04

MWT Lagrangian

For $y = \frac{1}{3}$ TC-fields have SM-like hypercharges, for $y = 1$ $\bar{D}_R$ corresponds to a techni-gaugino. The MWT Lagrangian is

$$\begin{aligned}
\mathcal{L}_{MWT} &= \mathcal{L}_{SM} - \mathcal{L}_H + \mathcal{L}_{TC}, \\
\mathcal{L}_{TC} &= -\frac{1}{4}\mathcal{F}^a_{\mu\nu}\mathcal{F}^{a\mu\nu} + i\bar{Q}_L\gamma^\mu D_\mu Q_L + i\bar{U}_R\gamma^\mu D_\mu U_R + i\bar{D}_R\gamma^\mu D_\mu D_R \\
&\quad + i\bar{L}_L\gamma^\mu D_\mu L_L + i\bar{E}_R\gamma^\mu D_\mu E_R + i\bar{N}_R\gamma^\mu D_\mu N_R,
\end{aligned}$$

with the covariant derivatives defined by the fields' quantum numbers. The techniquarks condense and break EW:

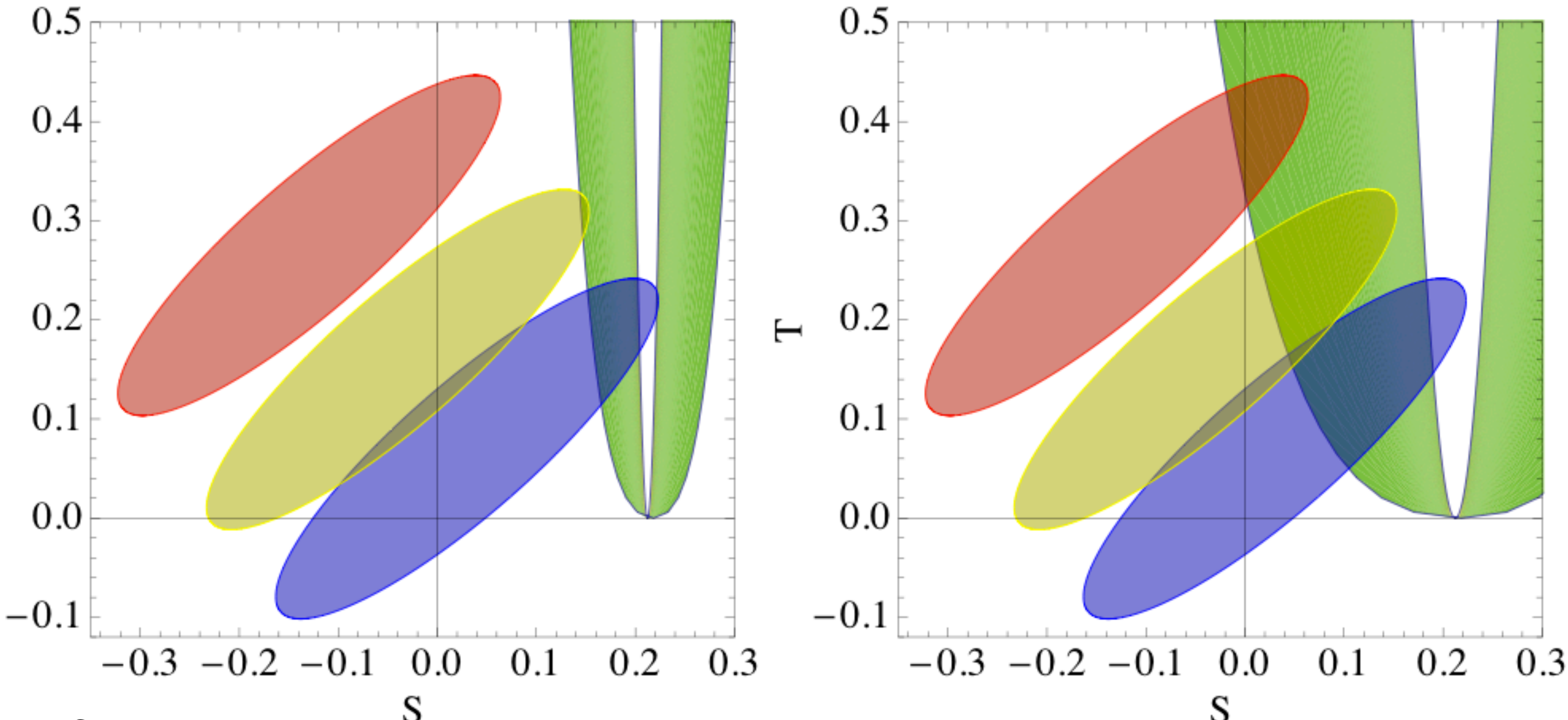
$$\langle Q_i^\alpha Q_j^\beta \epsilon_{\alpha\beta} E^{ij} \rangle = -2\,\langle \overline{U}_R U_L + \overline{D}_R D_L \rangle,\ Q = \begin{pmatrix} U_L \\ D_L \\ -i\sigma^2 U_R^* \\ -i\sigma^2 D_R^* \end{pmatrix},\ E = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}$$

$$\langle Q_i^\alpha Q_j^\beta \epsilon_{\alpha\beta} E^{ij} \rangle \neq 0 \qquad \Rightarrow \qquad SU(2)_L \times U(1)_Y \to U(1)_{EM}$$

* Foadi, Frandsen, Ryttov, Sannino '07

S-T Parameters

The ellipses give the S and T 90% CL region for M_H= 117 GeV (blue), 300 GeV (yellow), 1 TeV (red). MWT's S and T region (green) calculated for $y = \frac{1}{3}$ (left panel), $y = 1$ (right panel) and $M_Z \leqslant M_{E,N} \leqslant 10\ M_Z$.



* Sannino et al. '11

Low Energy Lagrangian

Low energy Lagrangian:

$$\mathcal{L}_{\rm Higgs} = \frac{1}{2}{\rm Tr}\left[D_\mu M D^\mu M^\dagger\right] - \mathcal{V}(M) + \mathcal{L}_{\rm ETC} \ ,$$

where the potential reads

$$\begin{aligned}\mathcal{V}(M) &= -\frac{m_M^2}{2}{\rm Tr}[MM^\dagger] + \frac{\lambda}{4}{\rm Tr}\left[MM^\dagger\right]^2 + \lambda' {\rm Tr}\left[MM^\dagger MM^\dagger\right] \\ &- 2\lambda''\left[{\rm Det}(M) + {\rm Det}(M^\dagger)\right] \ ,\end{aligned}$$

$$M_{ij} \sim Q_i Q_j \ \text{with} \ i,j = 1\ldots 4, \ \langle M\rangle = \frac{v}{2}E.$$

M transforms under the full $SU(4)$ group according to

$$M \rightarrow uMu^T \ , \qquad \text{with} \qquad u \in SU(4) \ .$$

Composite Vector Bosons

Composite vector bosons described by the four-dimensional traceless Hermitian matrix:

$$A^\mu = A^{a\mu} \, T^a \ ,$$

where T^a are the $SU(4)$ generators. Under an arbitrary $SU(4)$ transformation, A^μ transforms like

$$A^\mu \ \rightarrow \ u \ A^\mu \ u^\dagger \ , \quad \text{where } u \in SU(4) \ .$$

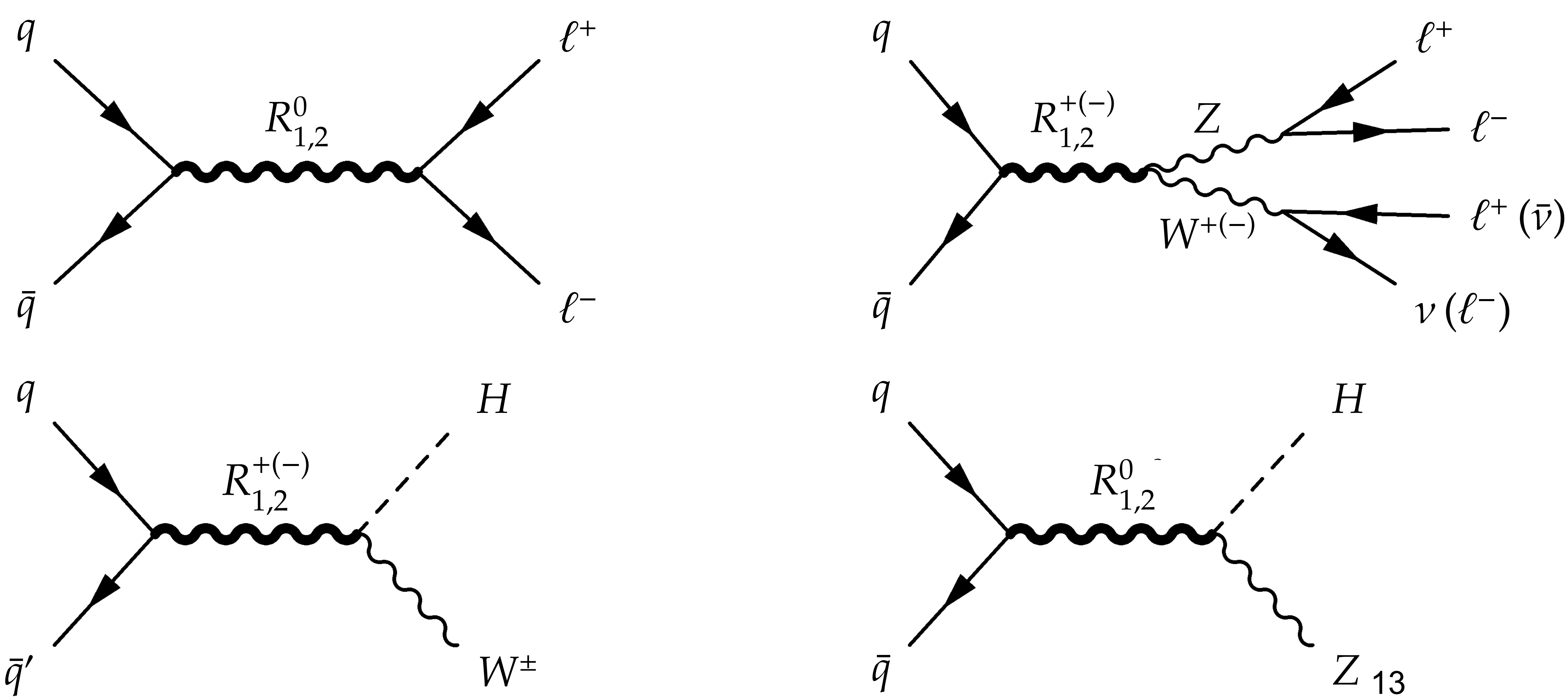
The techniquark content is expressed by the bilinears:

$$A_i^{\mu,j} \sim \ Q_i^\alpha \sigma^\mu_{\alpha\dot{\beta}} \bar{Q}^{\dot{\beta},j} - \frac{1}{4}\delta_i^j Q_k^\alpha \sigma^\mu_{\alpha\dot{\beta}} \bar{Q}^{\dot{\beta},k} \ .$$

LHC Phenomenology

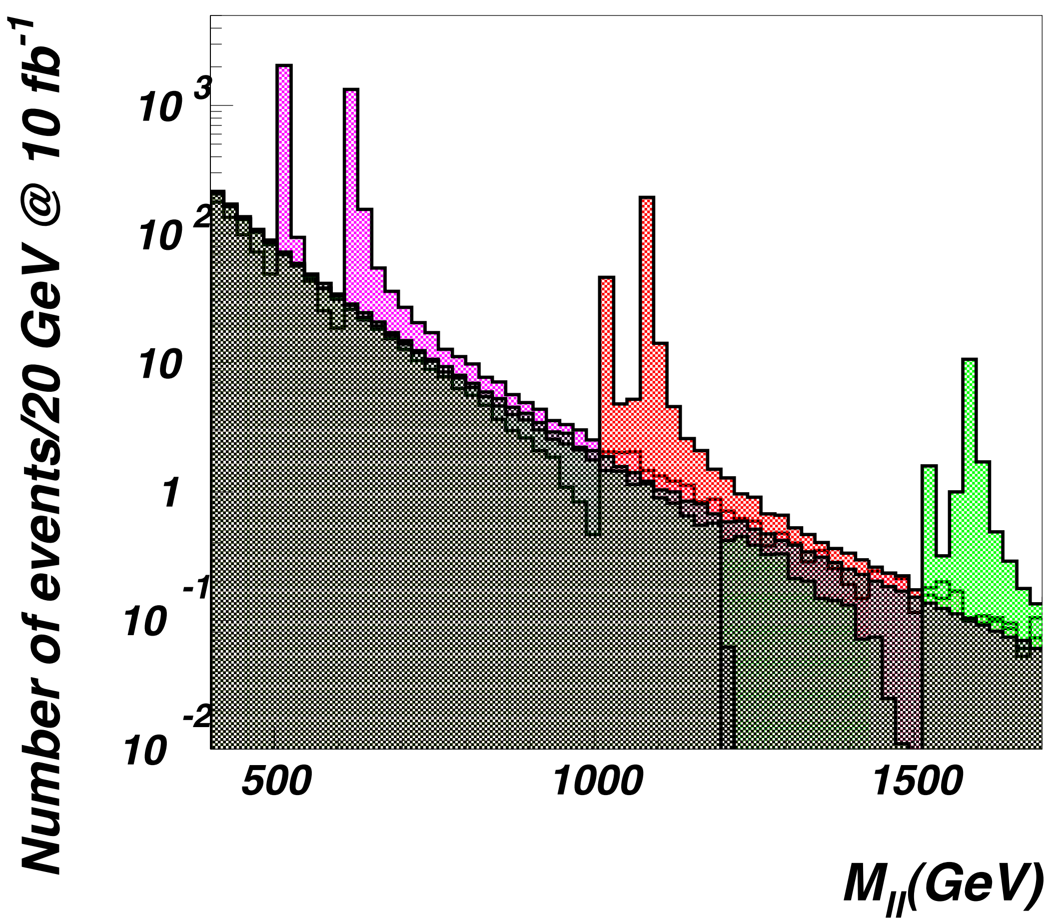
Effective Lagrangian implemented in Madgraph through FeynRules, and following processes studied for $\sqrt{s} = 7$ TeV:

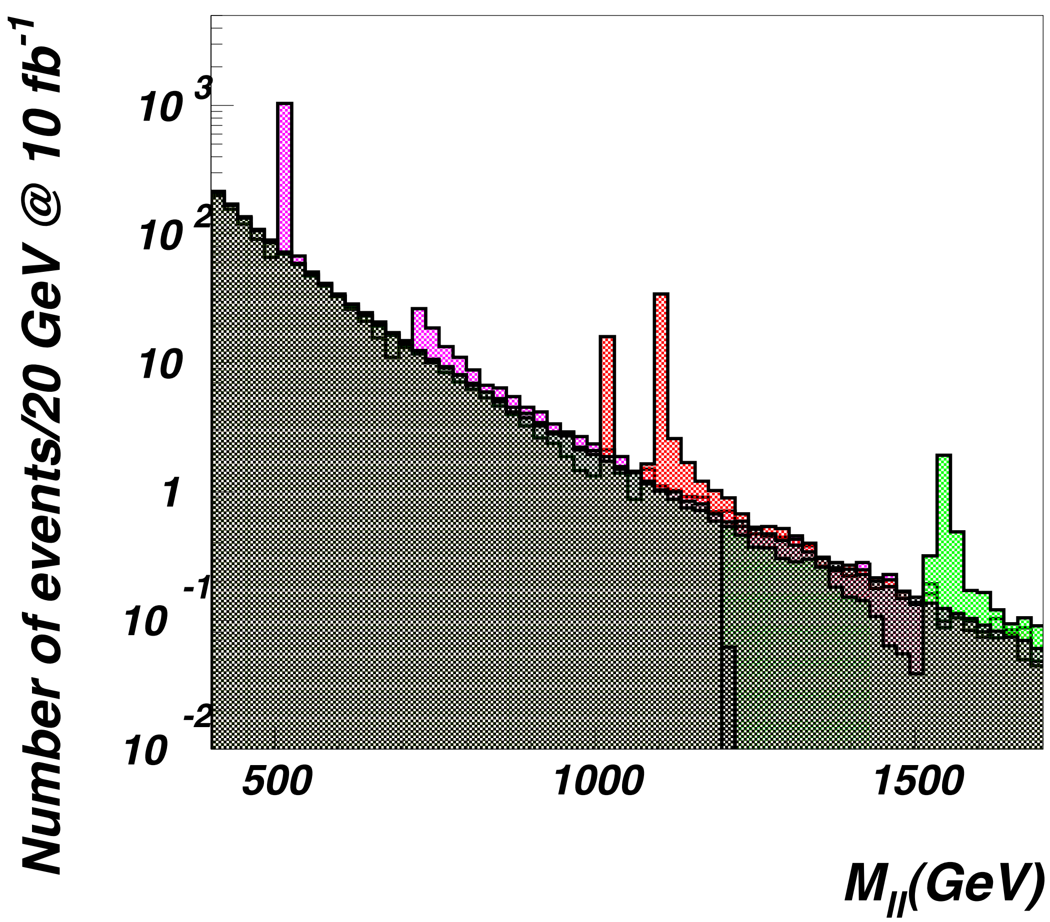
- Heavy vector boson ($R_{1,2}$) production
- Associated composite Higgs production with $W^{\pm}, Z$



Drell-Yan Process

Invariant mass distribution $M_{\ell\ell}$ for $pp \to R_{1,2} \to \ell^+\ell^-$ signal and background processes given by $\tilde{g} = 2$ (left), $\tilde{g} = 3$ (right), and $M_A = 0.5$ TeV (purple), 1 TeV (red), 1.5 TeV (green). $R_1(R_2)$ is the lighter (heavier) vector meson. $\tilde{g}$ = composite vector bosons self-coupling; M_A = axial-vector boson mass.





* Sannino et al. '11

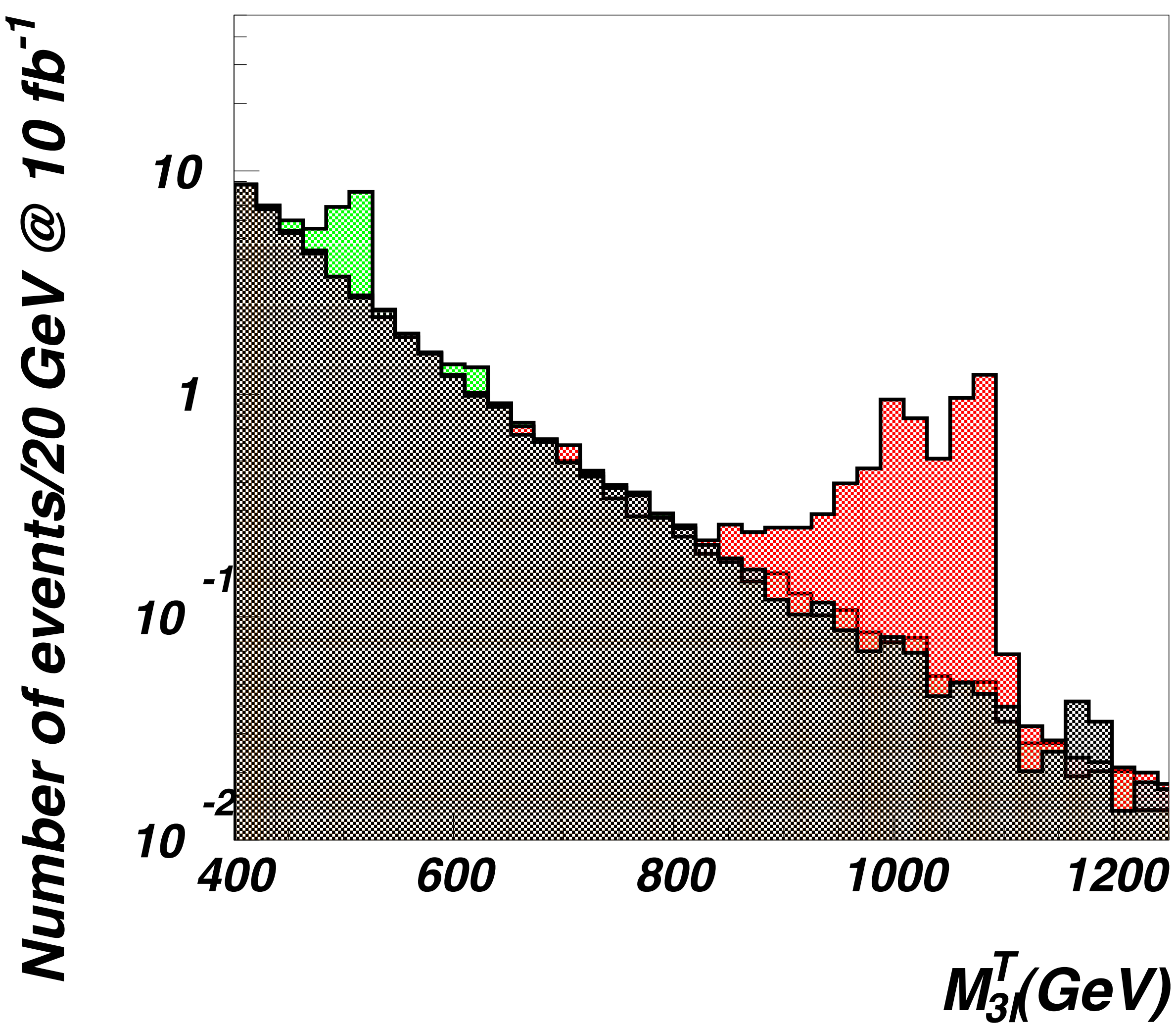
Vector Resonance Signals

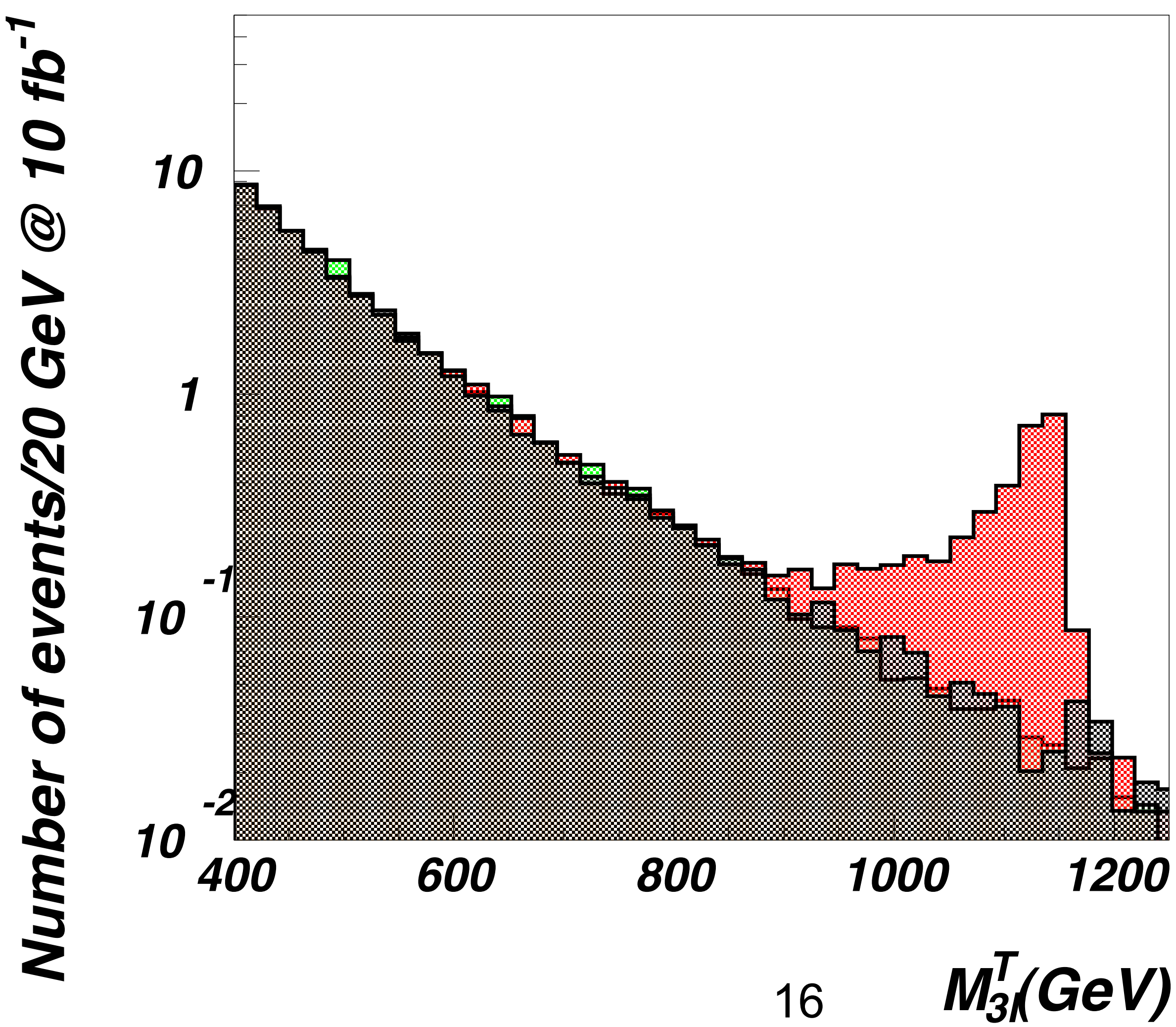
$pp \to R_{1,2} \to \ell^+\ell^-$. Signal and background cross sections for $\tilde{g} = 2, 3, 4$, and required luminosity for 3σ and 5σ signals.

$\tilde{g}$	M_A	$M_{R_{1,2}}$	σ_S (fb)	σ_B (fb)	$\mathscr{L}(\text{fb}^{-1})$ for 3σ	$\mathscr{L}(\text{fb}^{-1})$ for 5σ
2	500	$M_1 = 517$	194	3.43	0.012	0.038
2	500	$M_2 = 623$	118	1.34	0.019	0.056
2	1000	$M_1 = 1027$	4.57	$9.17 \cdot 10^{-2}$	0.53	1.8
2	1000	$M_2 = 1083$	16.4	$5.60 \cdot 10^{-2}$	0.13	0.39
2	1500	$M_1 = 1526$	0.133	$5.91 \cdot 10^{-3}$	26	67
2	1500	$M_2 = 1546$	0.776	$2.81 \cdot 10^{-3}$	2.7	8.2
3	500	$M_1 = 507$	93.5	3.71	0.037	0.090
3	500	$M_2 = 715$	0.447	0.649	39	81
3	1000	$M_1 = 1013$	1.32	$8.81 \cdot 10^{-2}$	2.7	7.4
3	1000	$M_2 = 1097$	2.94	$5.15 \cdot 10^{-2}$	0.79	2.5
3	1500	$M_1 = 1514$	$3.19 \cdot 10^{-3}$	$5.63 \cdot 10^{-3}$	6300	14000
3	1500	$M_2 = 1586$	0.120	$3.94 \cdot 10^{-3}$	29	68
4	500	$M_1 = 504$	34.6	3.85	0.12	0.34
4	500	$M_2 = 836$	0.0	0.649	-	-
4	1000	$M_1 = 1007$	0.234	$8.98 \cdot 10^{-2}$	30	85
4	1000	$M_2 = 1148$	0.0	$5.15 \cdot 10^{-2}$	-	-
4	1500	$M_1 = 1509$	$1.31 \cdot 10^{-3}$	$3.94 \cdot 10^{-3}$	25000	57000
4	1500	$M_2 = 1533$	$1.43 \cdot 10^{-2}$	$3.94 \cdot 10^{-3}$	435	1200

Three Leptons+Missing Et

Transverse mass distribution $M_{3\ell}^T$ for $pp \to R_{1,2}^{\pm} \to ZW^{\pm} \to \ell\ell\ell\nu$ signal and background processes, calculated with $\tilde{g} = 2$ (left), 4 (right), and $M_A = 0.5$ TeV (green), 1 TeV (red). The $R_{1,2}$ coupling to $W^{\pm}, Z$ is enhanced for large values of $\tilde{g}$, balancing the suppression coming from the quark-$R_{1,2}$ couplings.





* Sannino et al. '11

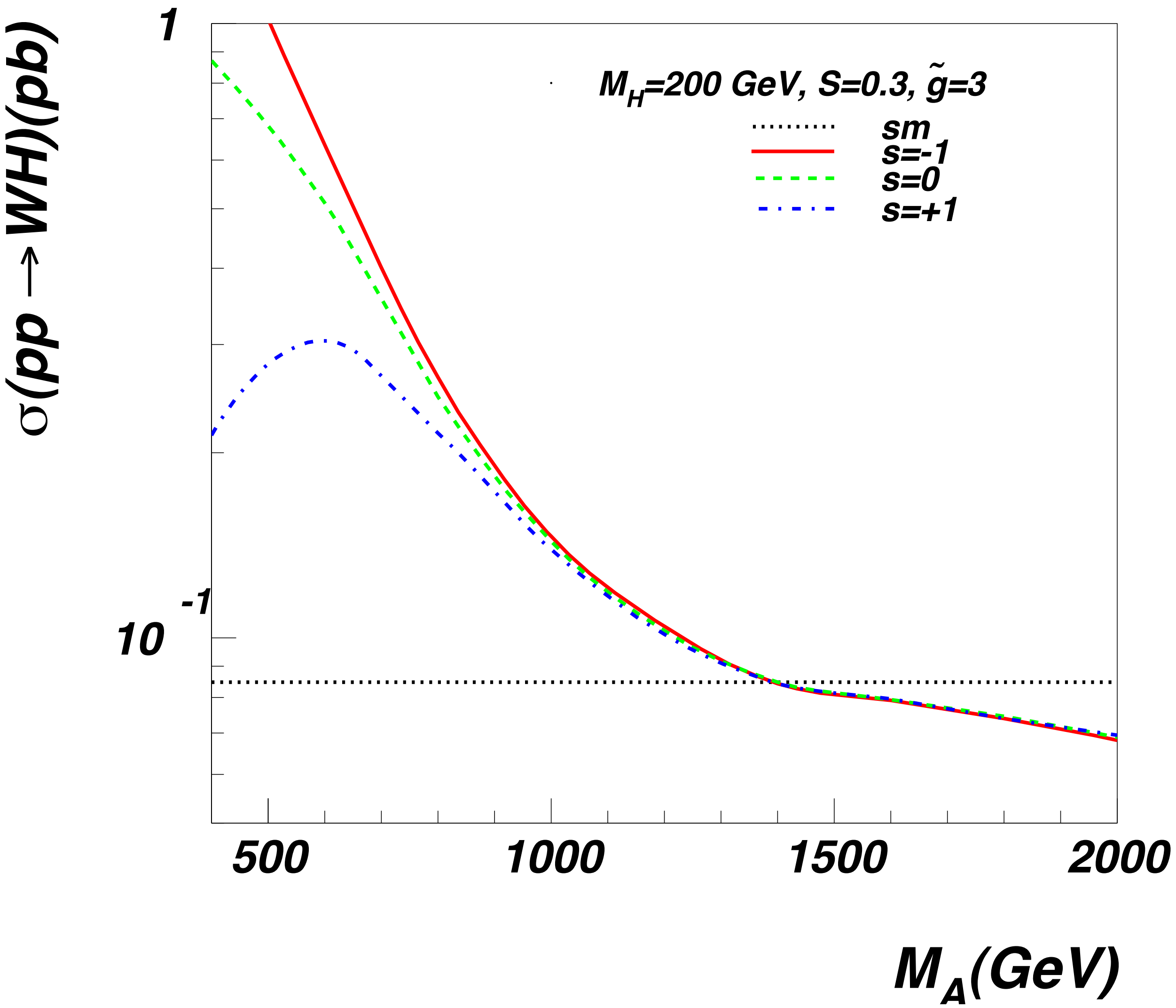
Conclusions

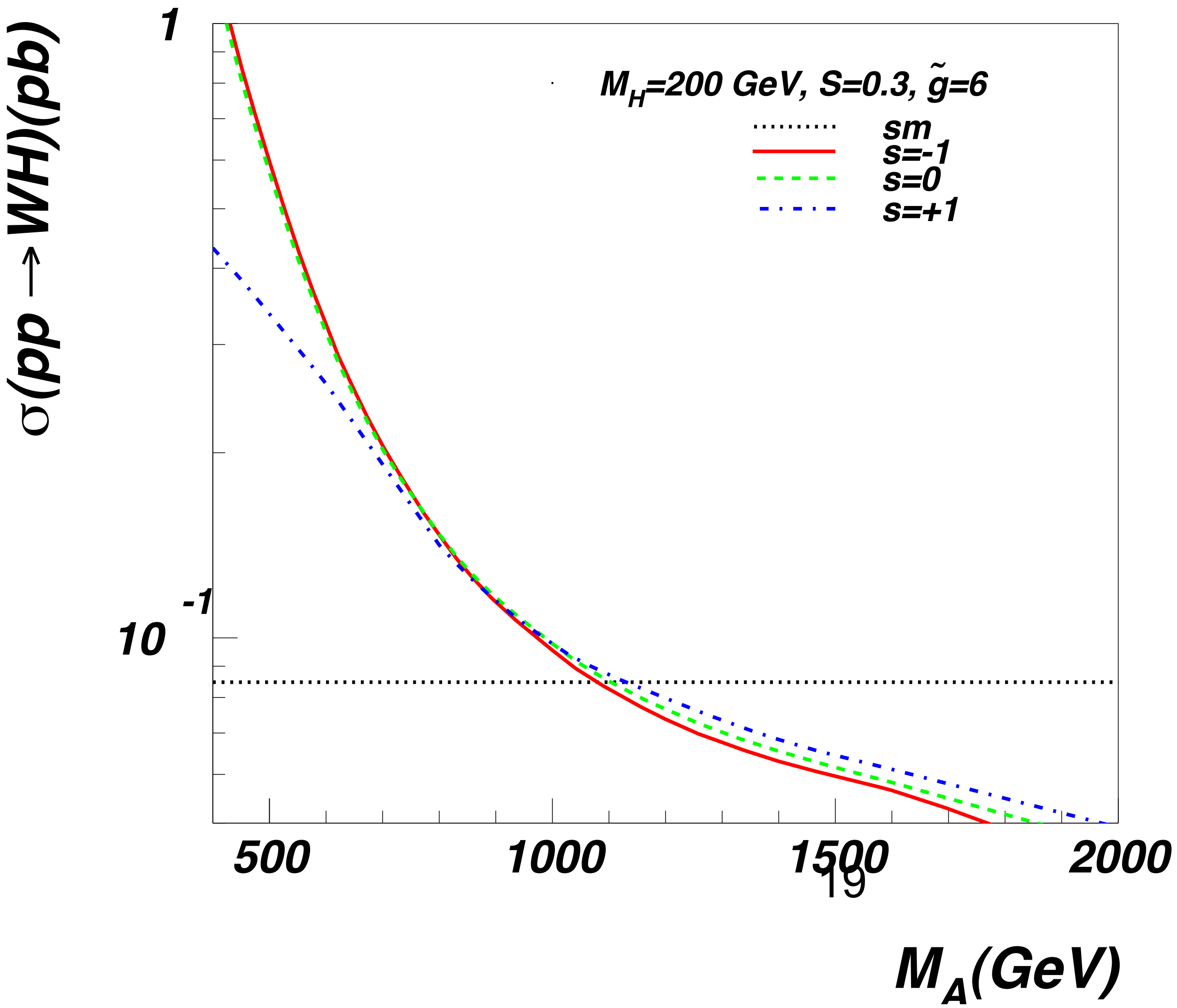
- Technicolor solves fine tuning
- Walking dynamics allow to satisfy experimental constraints
- MWT viable model with interesting LHC phenomenology
- Dark matter, inflation, unification, can all be accommodated within Technicolor

Backup Slides

Composite Higgs Production

The cross section for $pp \to WH$ production at 7 TeV versus M_A for $S = 0.3$, $s = (+1, 0, 1)$ and $\tilde{g} = 3$ (left) and $\tilde{g} = 6$ (right). The dotted line at the bottom indicates the SM cross section level. The resonant production of heavy vectors can enhance HW and ZH production by a factor 10.



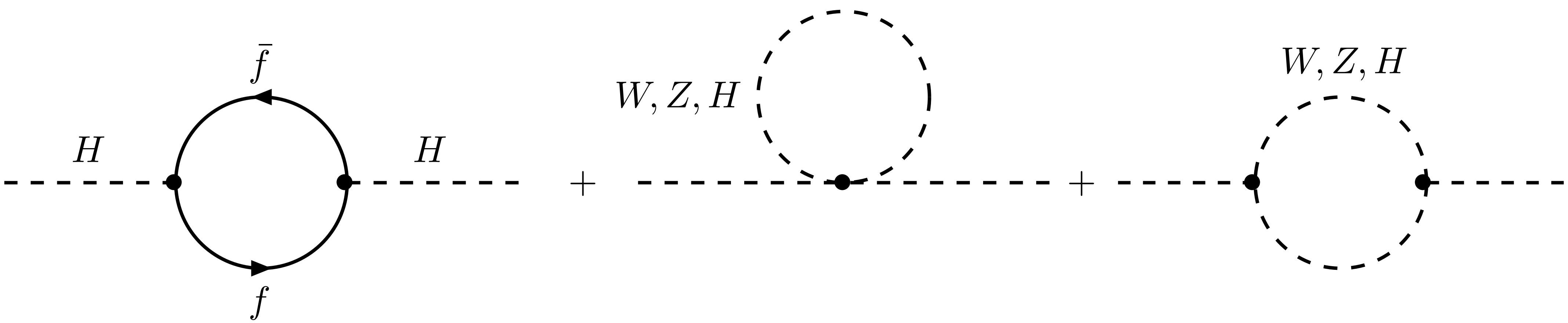


SM Fine Tuning

SM Higgs mass at one loop:

$$M_H^2 = \left(M_H^0\right)^2 + \Delta M_H^2, \left(M_H^0\right)^2 = \frac{\lambda v^2}{2},$$

$$\Delta M_H^2 = \frac{3\Lambda^2}{8\pi^2 v^2}\left(M_H^2 - 4m_t^2 + 2M_W^2 + M_Z^2\right) + O\left(\log\frac{\Lambda^2}{v^2}\right) =$$



If $\Lambda = 2.4 \times 10^{18}$ GeV (Planck scale) $\Rightarrow \frac{\Delta M_H^2}{M_H^2} \simeq 10^{32}$: λ has to be determined up to the 32nd digit to miraculously cancel the quantum correction . . .

One Family ETC

A toy ETC model: each entire family belongs to a single ETC fermion.

$$SU(N_{ETC}) \times SU(3)_C \times SU(2)_W \times U(1)_Y :$$

$$Q_L = (N_{ETC}, 3, 2)_{1/6} \qquad L_L = (N_{ETC}, 1, 2)_{-1/2}$$
$$U_R = (N_{ETC}, 3, 1)_{2/3} \qquad E_R = (N_{ETC}, 1, 1)_{-1}$$
$$D_R = (N_{ETC}, 3, 1)_{-1/3} \qquad N_R = (N_{ETC}, 1, 1)_0$$

The lowest ETC scale is determined by the heaviest mass:

$$m_t = 173\text{GeV} \Rightarrow \Lambda_{ETC} \simeq 10\,\text{TeV}$$

Because of global symmetry breaking there are also massless NGB

$$SU(8)_L \times SU(8)_R \to SU(8)_V \Rightarrow 60\,\text{NGB}$$

$$SU(N_{TC}+3)$$
$$\Lambda_1 \quad \downarrow \quad m_1 \approx \frac{\Lambda_{TC}^3}{\Lambda_1^2}$$
$$SU(N_{TC}+2)$$
$$\Lambda_2 \quad \downarrow \quad m_2 \approx \frac{\Lambda_{TC}^3}{\Lambda_2^2}$$
$$SU(N_{TC}+1)$$
$$\Lambda_3 \quad \downarrow \quad m_3 \approx \frac{\Lambda_{TC}^3}{\Lambda_3^2}$$
$$SU(N_{TC})$$

pNGB Masses

Without specifying an ETC one can write down the most general ETC sector:

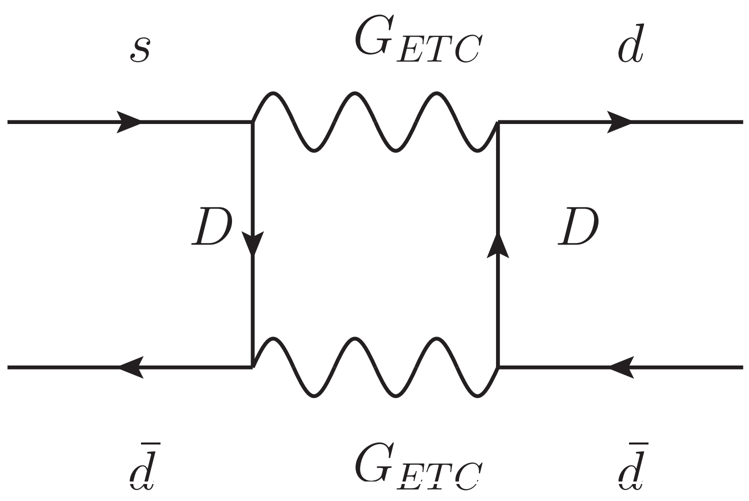
$$\mathcal{L}_{ETC} = \alpha_{ab}\frac{\bar{Q}_L T^a Q_R \bar{Q}_R T^b Q_L}{\Lambda^2_{ETC}} + \beta_{ab}\frac{\bar{Q}_L T^a Q_R \bar{\psi}_R T^b \psi_L}{\Lambda^2_{ETC}} + \gamma_{ab}\frac{\bar{\psi}_L T^a \psi_R \bar{\psi}_R T^b \psi_L}{\Lambda^2_{ETC}}$$

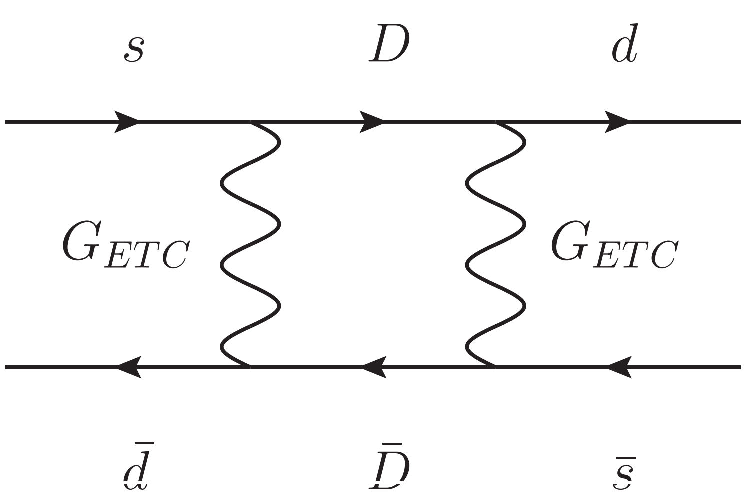
The first terms generate masses for the uneaten NGB. These can be estimated by:

$$\bar{Q}_R Q_L \to \Lambda^3_{TC}\Sigma,\ \Sigma \equiv \exp(i\pi^c \tilde{T}^c / F_T),\ \tilde{T} \in \mathcal{G}_{ETC}$$

$$(M^{cd}_{PNGB})^2 \simeq \frac{\alpha_{ab}\Lambda^6_{TC}}{\Lambda^2_{ETC} F^2_T}\mathrm{Tr}([\tilde{T}^c, T^a][T^b, \tilde{T}^d]) \Rightarrow M_{PNGB} = O\left(\frac{\Lambda^2_{TC}}{\Lambda_{ETC}}\right)$$

FCNC





The second terms generate masses for the SM fermions, while the third terms are responsible for Flavor Changing Neutral Currents (FCNC):

$$\mathcal{L}_{\Delta S=2} = \gamma_{sd} \frac{(\bar{s}\gamma^5 d)\,(\bar{s}\gamma^5 d)}{\Lambda^2_{ETC}} + hc,\ \gamma_{sd} \sim \sin^2\theta_c \simeq 10^{-2}.$$

Measured value of the neutral kaon mass splitting determines tight bound on ETC scale:

$$\frac{\Delta m^2}{m_K^2} \simeq \gamma_{sd} \frac{f_K^2 m_K^2}{\Lambda^2_{ETC}} \lesssim 10^{-14} \Rightarrow \Lambda_{ETC} \gtrsim 10^3 \text{ TeV}\,.$$

Walking TC

Look for Walking TC ($\beta(\alpha_*) = 0$) in theory space (Representation (R), Number of colors (N), Number of flavors (N_f)) by studying

$$\begin{aligned}
\beta(g) &= -\beta_0 \frac{\alpha^2}{4\pi} - \beta_1 \frac{\alpha^3}{(4\pi)^2},\ \alpha_* = -4\pi \frac{\beta_0}{\beta_1},\ \beta_0 = \frac{11}{3} C_2(\mathrm{G}) - \frac{4}{3} T(\mathrm{R}), \\
\beta_1 &= \frac{34}{3} {C_2}^2(\mathrm{G}) - \frac{20}{3} C_2(\mathrm{G}) T(\mathrm{R}) - 4 C_2(\mathrm{R}) T(\mathrm{R}).
\end{aligned}$$

The conformal window is defined by requiring asymptotic freedom, existence of a Banks-Zaks fixed point, and conformality to arise before chiral symmetry breaking:

$$\begin{aligned}
\beta_0 > 0 \quad &\Rightarrow \quad N_f > \frac{11}{4} \frac{d(\mathrm{G}) C_2(\mathrm{G})}{d(\mathrm{R}) C_2(\mathrm{R})}, \\
\beta_1 < 0 \quad &\Rightarrow \quad N_f < \frac{d(\mathrm{G}) C_2(\mathrm{G})}{d(\mathrm{R}) C_2(\mathrm{R})} \frac{17 C_2(\mathrm{G})}{10 C_2(\mathrm{G}) + 6 C_2(\mathrm{R})} \\
\alpha_* < \alpha_c \quad &\Rightarrow \quad N_f > \frac{d(\mathrm{G}) C_2(\mathrm{G})}{d(\mathrm{R}) C_2(\mathrm{R})} \frac{17 C_2(\mathrm{G}) + 66 C_2(\mathrm{R})}{10 C_2(\mathrm{G}) + 30 C_2(\mathrm{R})}.
\end{aligned}$$

TC Models

Walking Technicolor candidate models:

- Fundamental:
  $12\pi S(N=3, N_f=12) = 36,$
  $12\pi S(N=2, N_f=8) = 16$

- Adjoint:
  $12\pi S(N=2, N_f=2) = 6,$
  $12\pi S(N=3, N_f=2) = 16$

- 2 I. Symmetric:
  $12\pi S(N=2, N_f=2) = 6,$
  $12\pi S(N=3, N_f=2) = 12$

- 2 I. Antisymmetric:
  $12\pi S(N=3, N_f=12) = 36$

Alternatives to reduce S:

- Custodial TC ($S=0$)
- Partially Gauged TC
- Split TC

The best (fully gauged) Walking TC candidates are:

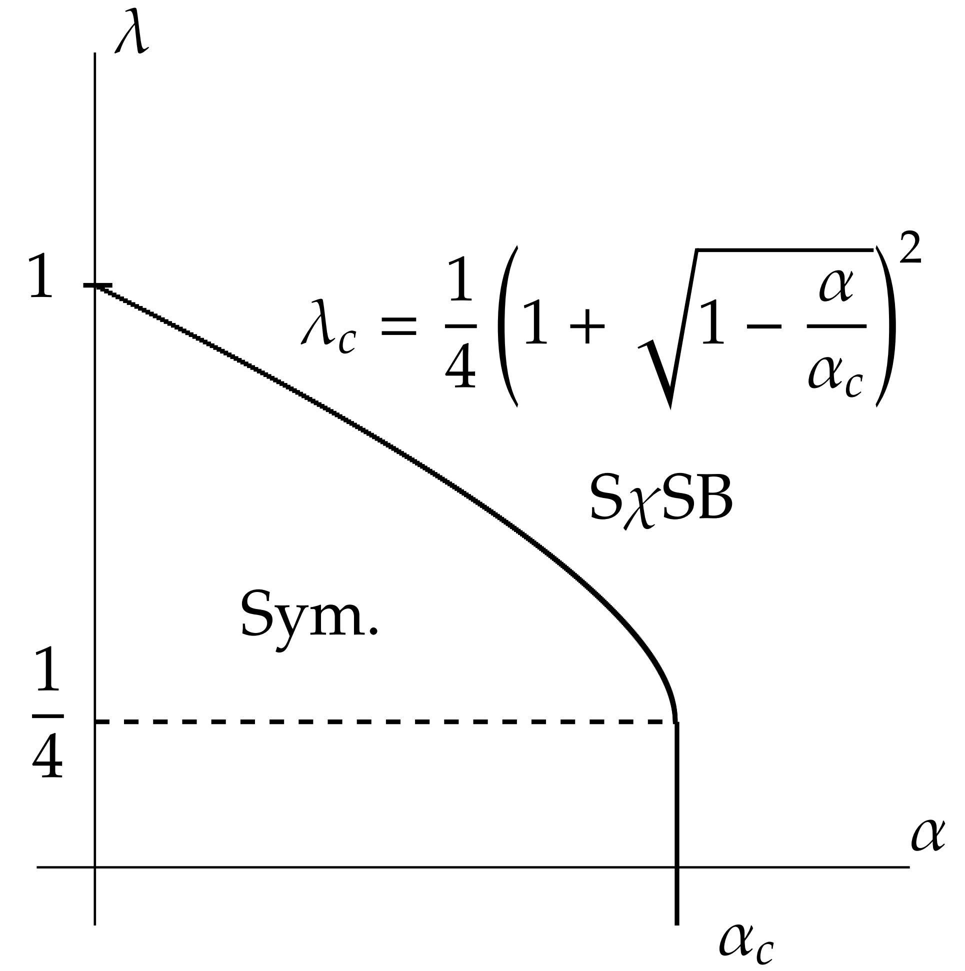
- $\mathrm{Adj}, N=2, N_f=2$
- 2-IS, $N=3, N_f=2$

Ideal Walking

A strong ETC sector increases the value of the fermion mass anomalous dimension. In gauged Nambu-Jona-Lasinio (gNJL):

$$\mathcal{L}_{gNJL} = \mathcal{L}_{TC} + \frac{16\pi^2\lambda}{d[r]N_f\Lambda^2_{ETC}}\bar{\psi}_L\psi_R\bar{\psi}_R\psi_L \Rightarrow$$

$$\gamma_m(\lambda) = 1 - \omega + 2\omega\frac{\lambda}{\lambda_c},\ \omega \equiv \sqrt{1 - \frac{\alpha}{\alpha_c}}$$

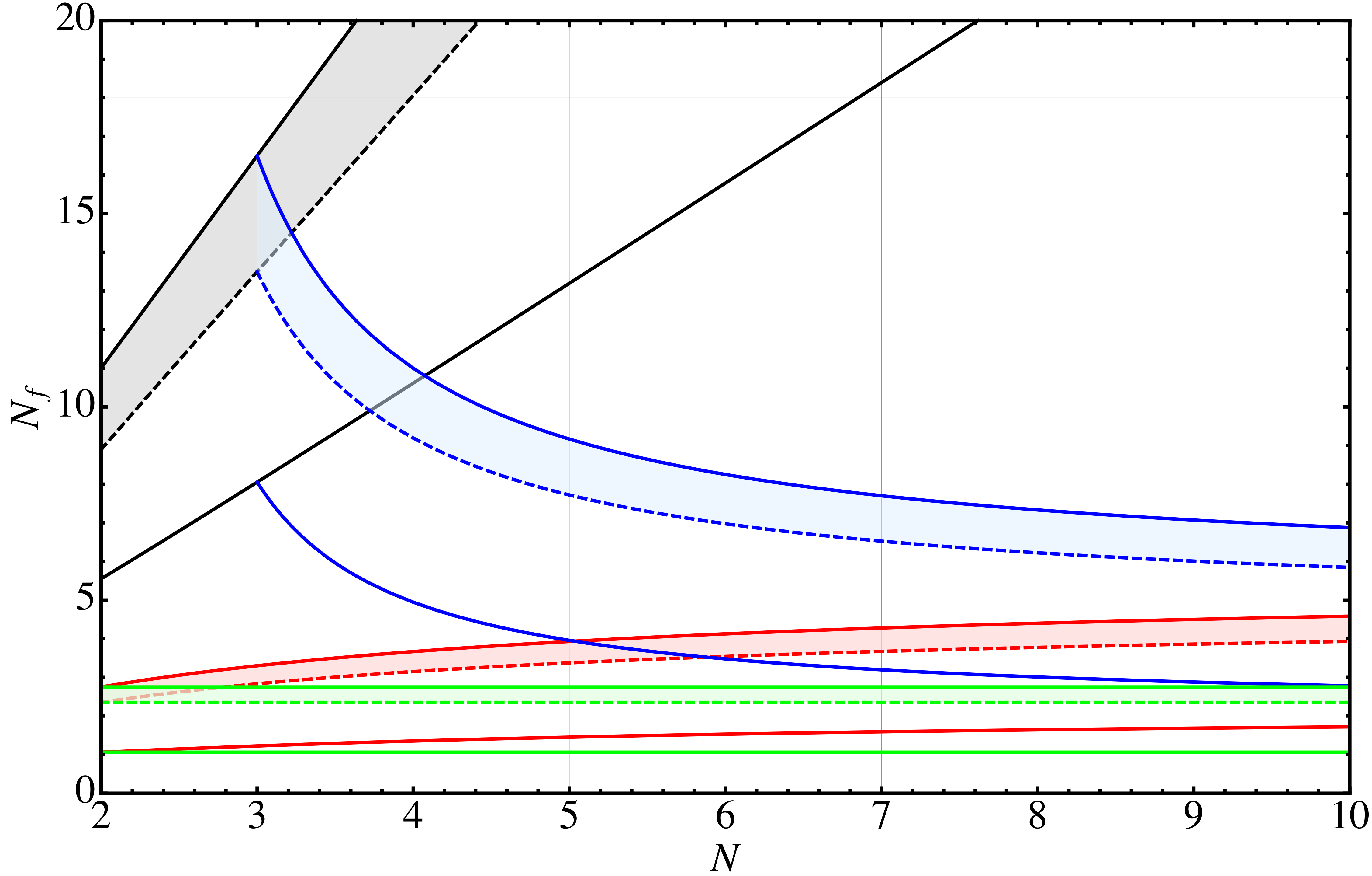


Assuming $\lambda = \lambda_c = 0.75$ one gets $\gamma_m(\lambda = \lambda_c) = 1 + \omega = 1.73 \ \Rightarrow$
By using dimensional analysis $m_t = 172\,\mathrm{GeV}$ for $\Lambda_{ETC} \approx 10^7\,\mathrm{TeV}$!

An accurate estimate of Λ_{TC} and $\langle \bar{T}T \rangle_{\mathrm{TC}}$ is needed to determine Λ_{ETC}.

Phase Diagram with 4F Interaction

Phase diagram for $SU(N)$ representations with chiral symmetry breaking (dashed) line determined for $\lambda_c = 0.75$



ETC Scalar Sector

In order to give masses to the 6 uneaten Goldstone bosons we add the following term which is generated in the ETC sector:

$$\mathcal{L}_{\rm ETC} \supset \frac{m_{\rm ETC}^2}{4} \, {\rm Tr}\left[MBM^\dagger B + MM^\dagger \right] \; ,$$

$$M_{pNGB}^2 = m_{\rm ETC}^2 \; .$$

MWT Gauge Sector

The minimal kinetic Lagrangian is:

$$\mathcal{L}_{\rm kinetic} = -\frac{1}{2}{\rm Tr}\Big[\widetilde{W}_{\mu\nu}\widetilde{W}^{\mu\nu}\Big]-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}-\frac{1}{2}{\rm Tr}\Big[F_{\mu\nu}F^{\mu\nu}\Big]+m^2\,{\rm Tr}\Big[C_\mu C^\mu\Big]\ ,$$

where $\widetilde{W}_{\mu\nu}$ and $B_{\mu\nu}$ are the EW elementary field strength tensors, and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\tilde{g}\left[A_\mu, A_\nu\right]\ .$$

The vector field C_μ is defined by

$$C_\mu \ \equiv\ A_\mu \ -\ \frac{g}{\tilde{g}}\ G_\mu\ ,$$

with G_μ given by

$$G_\mu = g\ W_\mu^a\ L^a + g'\ B_\mu Y.$$

Vector-Scalar Couplings

The C_μ fields couple with M via gauge invariant operators:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{M-C}} &= \tilde{g}^2\ r_1\ \mathrm{Tr}\left[C_\mu C^\mu M M^\dagger\right] + \tilde{g}^2\ r_2\ \mathrm{Tr}\left[C_\mu M C^{\mu T} M^\dagger\right] \\
&+ i\ \tilde{g}\frac{r_3}{2}\ \mathrm{Tr}\left[C_\mu \left(M (D^\mu M)^\dagger - (D^\mu M) M^\dagger\right)\right] \\
&+ \tilde{g}^2\ s\ \mathrm{Tr}\left[C_\mu C^\mu\right] \mathrm{Tr}\left[M M^\dagger\right] .
\end{aligned}
$$

The dimensionless parameters r_1, r_2, r_3, s express interaction strength in units of $\tilde{g}$, and are therefore expected to be of order one.

The fermions are coupled to the low energy effective Higgs through effective SM Yukawa interactions.

Weinberg Sum Rules

The free parameters of the low energy spectrum are: $r_1, r_2, r_3, s, M_A, M_H$, $\tilde{g}$, with A referring to the axial-vector meson. Three of these parameters can in principle be eliminated by using the constraints from the S parameter and the Weinberg Sum Rules (WSR).

The 1st and 2nd WSR are obtained from the vector and axial-vector two-point correlation functions, by assuming partial conservation of the axial current and they read

$$F_V^2 - F_A^2 = F_\pi^2 \ , \ F_V^2 M_V^2 - F_A^2 M_A^2 = a \frac{8\pi^2}{d(R)} F_\pi^4 \ ,$$

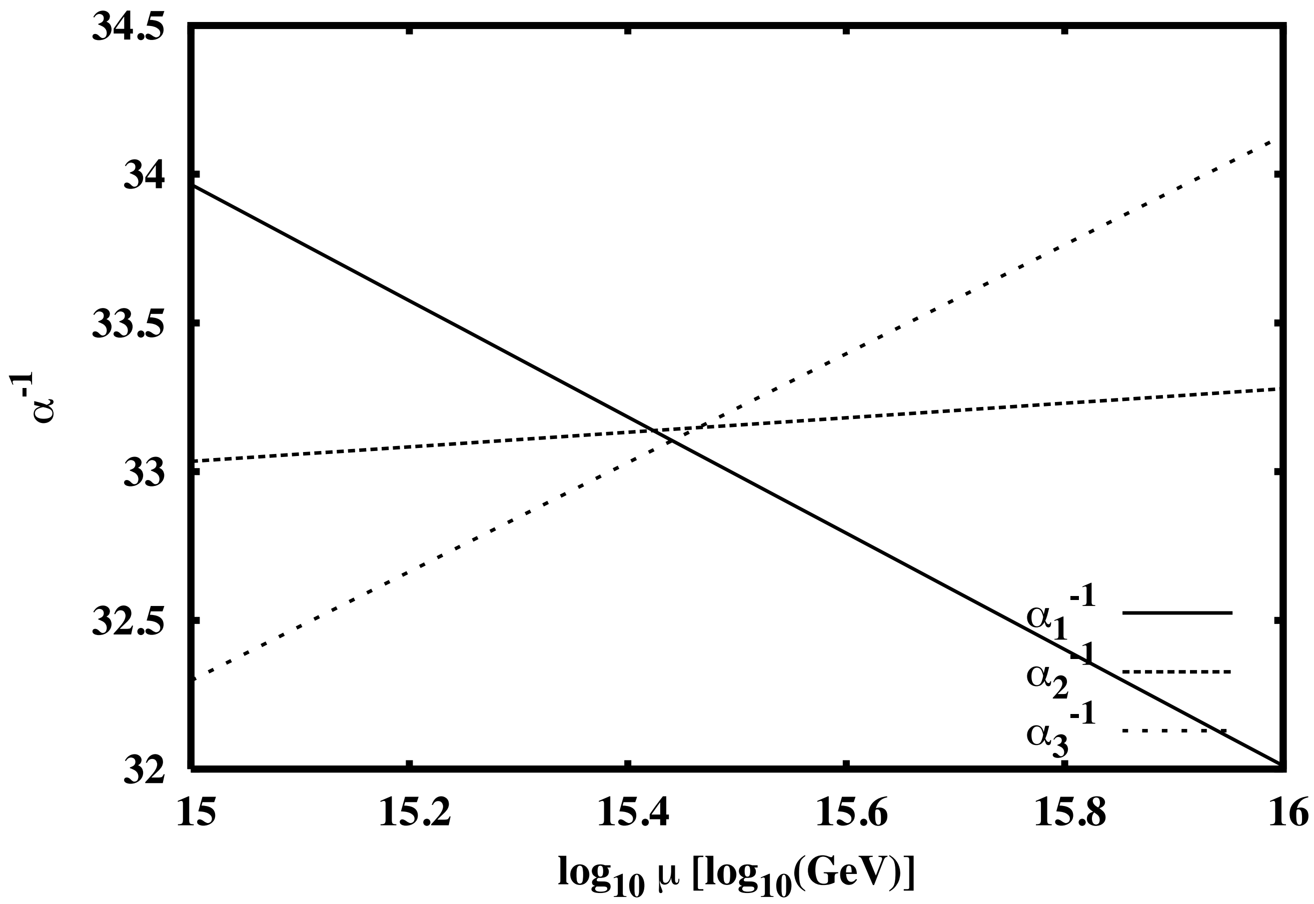
where a is expected to be positive and $\mathcal{O}(1)$ for a walking theory and 0 for a running one.

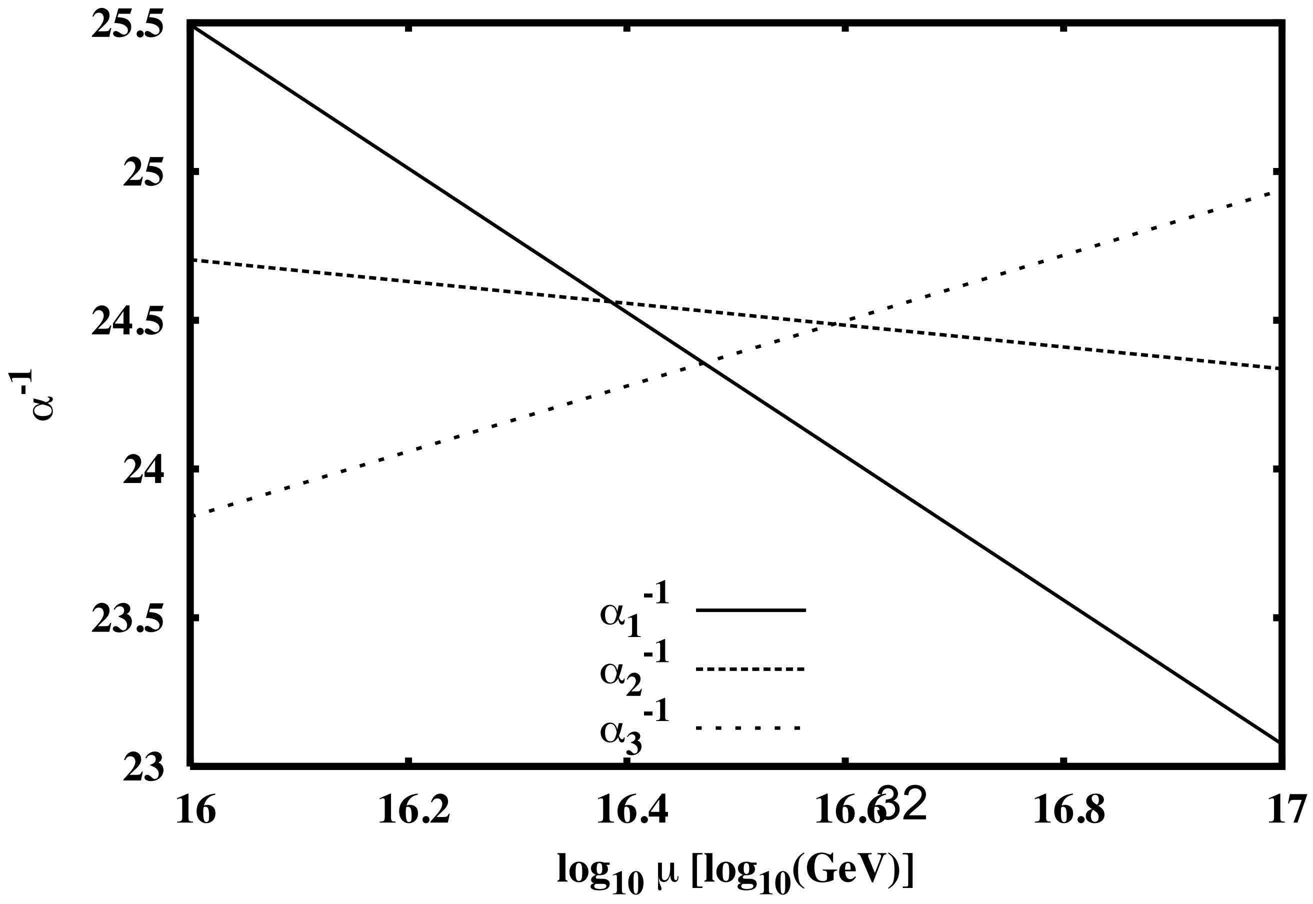
Unification in MWT

Unification ingredients for MWT:

- Make g_{TC} run at scale X by embedding $SU(2)_{Adj}$ in $SU(3)_F$
- Delay unification ($M_{GUT} \gg v$) to avoid the experimental bounds on the proton decay by adding a wino and a bino

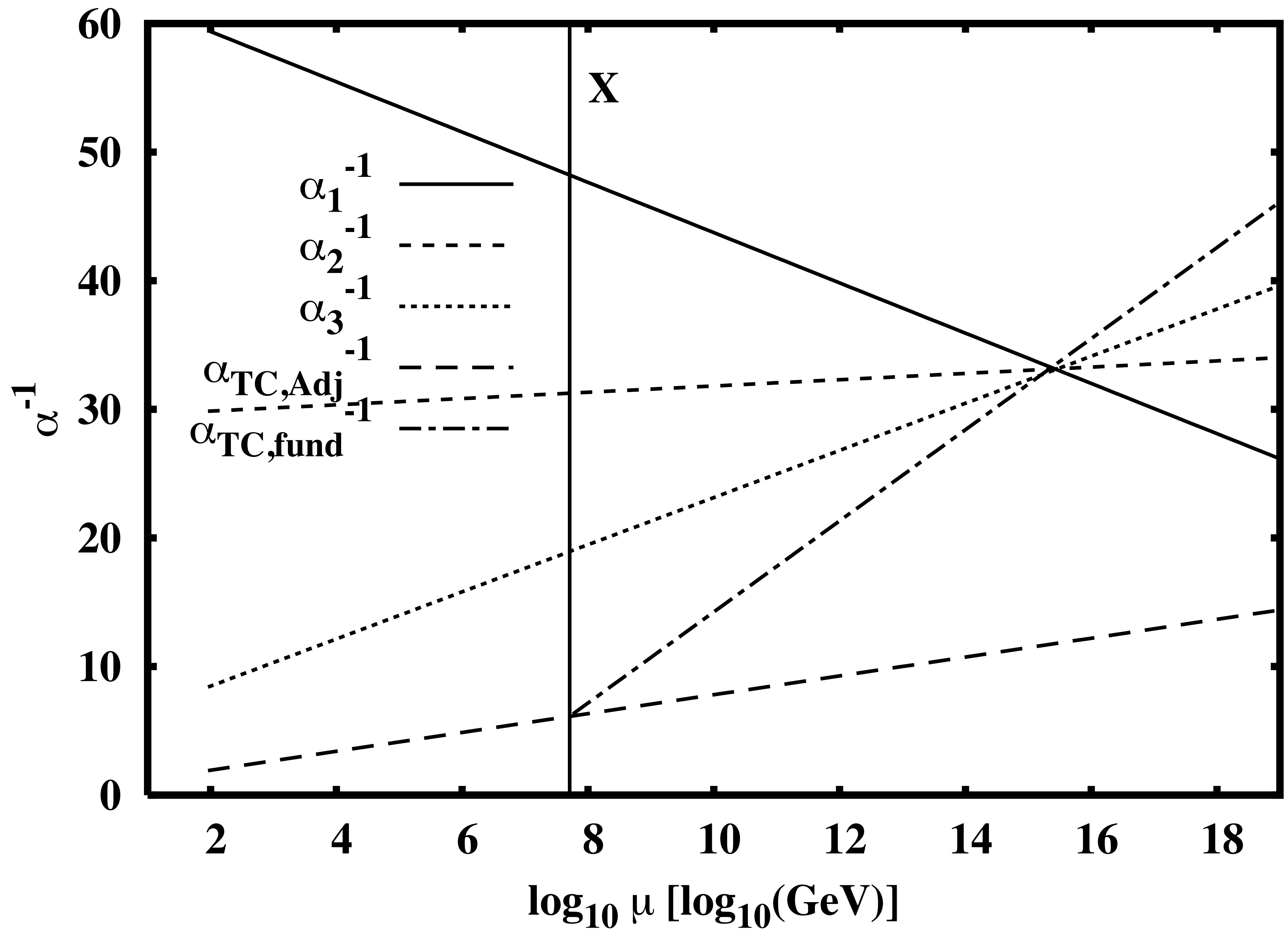
Unification of g_Y, g_L, g_s in uMWT (left) and MSSM (right)



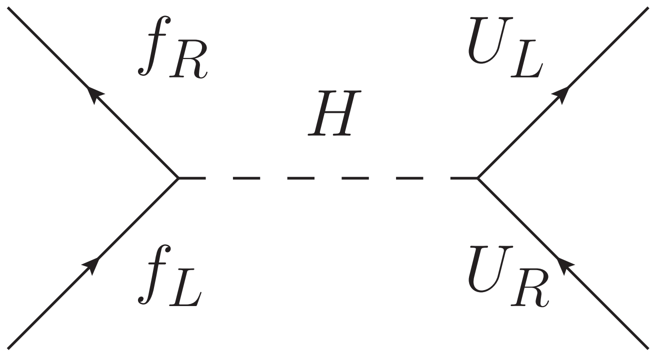


uMWT Gauge Unification

Unification of gauge couplings in the uMWT:



Bosonic Technicolor



$$\to \frac{y_f y_U}{M_H^2}(\bar{U}_L U_R)(\bar{f}_R f_L) \Rightarrow m_f \approx \frac{y_f y_U}{M_H^2}\Lambda_{TC}^3$$

By supersymmetrizing the theory and taking the limit of scalars much heavier than their fermion superpartners, one finds that the theory is not fine tuned:

$$m_{\tilde{f}} \gg m_f \Rightarrow \Delta m_{\tilde{f}}^2 \propto \frac{y}{16\pi^2} m_{\tilde{f}}^2 \left(1 - \log\frac{m_{\tilde{f}}^2}{\mu^2}\right) \Rightarrow \frac{\Delta M_H^2}{M_H^2} = O(1)$$

In the same limit the FCNC generated by scalars are suppressed.

From MWT to N=4 SUSY

MWT	Minimal S-partners	N=1 Multiplets	N=4
G_μ	G_μ, $\bar{D}_R$	V	V, Φ_3, Φ_1, Φ_2
$\bar{D}_R$			
$\bar{U}_R$	$\bar{U}_R$ $\tilde{\bar{U}}_R$	Φ_3	
U_L	U_L $\tilde{U}_L$	Φ_1	
D_L	D_L $\tilde{D}_L$	Φ_2	

Superpotential for $SU(N)$ $\mathcal{N}=4$ Super Yang-Mills (4SYM):

$$f(\Phi) = -\frac{g}{3\sqrt{2}}\epsilon_{ijk} f^{abc} \Phi_i^a \Phi_j^b \Phi_k^c, \;\; i = 1,2,3; a = 1, \cdots, N^2 - 1;$$

Minimal Super Conformal TC

	Superfield	$SU(2)_{TC}$	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$\mathcal{N}=4$	Φ_L	Adj	1	$\square$	1/2
	Φ_3	Adj	1	1	-1
	V	Adj	1	1	0
4^{th} Lepton Family	Λ_L	1	1	$\square$	-3/2
	N	1	1	1	1
	E	1	1	1	2
	H	1	1	$\square$	1/2
	H'	1	1	$\square$	-1/2

MSCT Superpotential

- Spectrum: 4SYM + lepton 4th superfamily + MSSM
- Gauge group: $SU(2)_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y$

$$
\begin{aligned}
f(\Phi)_{TC} &= -\frac{g_{TC}}{3\sqrt{2}}\epsilon_{ijk}\epsilon^{abc}\Phi_i^a\Phi_j^b\Phi_k^c + y_U\epsilon_{ij3}\Phi_i^a H_j \Phi_3^a \\
&+ y_N\epsilon_{ij3}\Lambda_i H_j N + y_E\epsilon_{ij3}\Lambda_i H_j' E + y_R\Phi_3^a\Phi_3^a E .
\end{aligned}
$$

MSCT represents a possible UV completion of MWT.