Holographic walking technicolor and stability of technibranes[1]

Tonnis ter Veldhuis

Department of Physics and Astronomy
Macalester College
Saint Paul, MN 55105

terveldhuis@Macalester.edu

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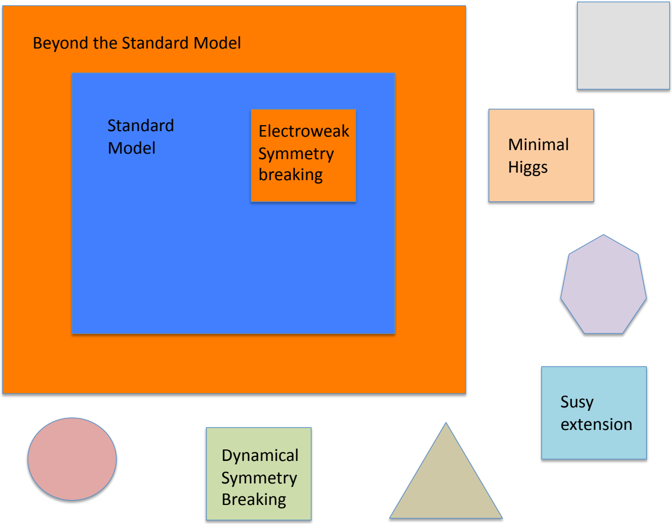
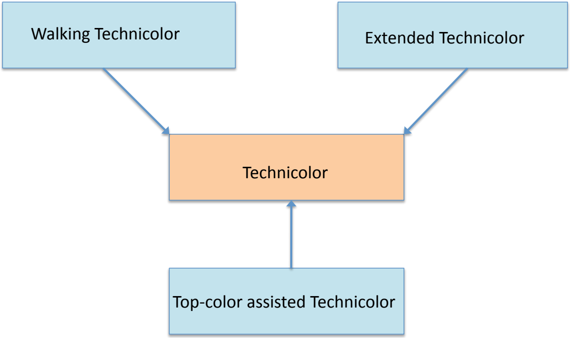
[1]work in collaboration with Tom Clark and Sherwin Love

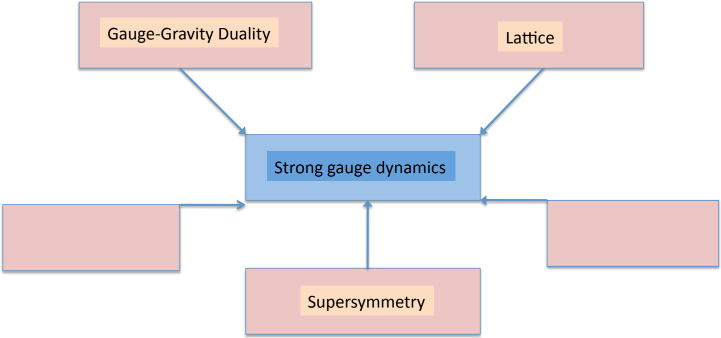
Introduction



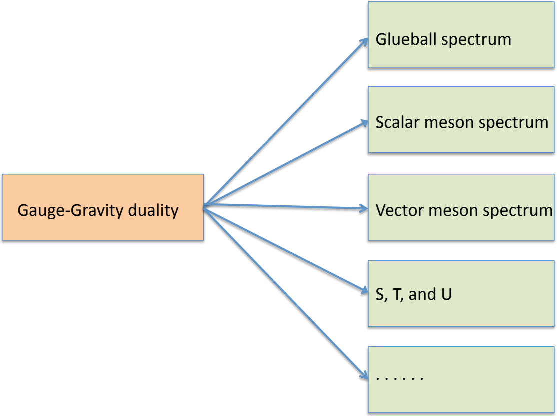
dynamical symmetry breaking



Strong gauge dynamics



Calculability



Walking gauge coupling

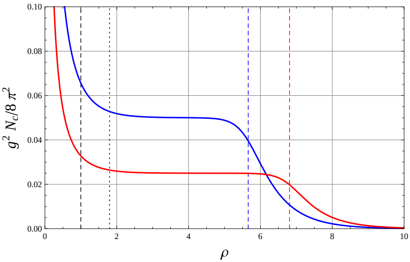
- Walking technicolor requires a non-Abelian gauge theory exhibiting nearly-conformal behavior over a certain finite energy range.
- At high energy, the gauge coupling constant exhibits asymptotic freedom.
- As the distance scale increases the coupling enters an approximate scale invariance region of strong coupling where it slows down ("walking" region).
- At still larger distances the gauge coupling begins to grow rapidly until it enters the confinement region where there is no longer any vestige of the scale symmetry.

Supergravity background

- Gauge-gravity duality provides an alternative means to calculate the running in non-perturbative regions of coupling constant space.
- Nunez, Papadimitriou and Piai [2] constructed a Type IIB supergravity background dual to a strongly coupled $\mathcal{N} = 1$ SUSY gauge theory which exhibits the above described running behavior.
- The background is obtained by considering a supergravity limit of a stack of N_C $D5$ branes wrapping a 2-cycle resulting in a specific ten-dimensional space-time.
- The invariant interval is further characterized by the integration constant parameters C and T.

[2] Int. J. Mod. Phys. **A25**, 2837-2865 (2010).

Walking gauge coupling



$N_C = 10$

blue: $C = 50,\ T = 10^{-10}$
red: $C = 100,\ T = 10^{-12}$

- The running t'Hooft gauge coupling constant $g^2 N_C/8\pi^2$ shows walking behavior. The logarithm of the renormalization scale is related to the distance ρ into the bulk.
- The smaller the value of C, the stronger the gauge coupling in the walking region.
- The smaller the value of T, the greater the length of the walking region.

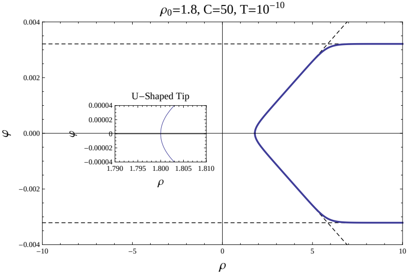
chiral symmetry breaking

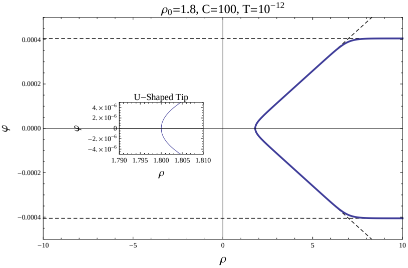
- Anguelova, et al.[3] added techni-flavors to the model by embedding N_F stacks of $D7$ and $\overline{D7}$ branes in a U-shaped geometry necessary for $U(N_F) \times U(N_F) \to U(N_F)$ symmetry breakdown.
- They employed holographic techniques in order to study the vector and axial vector meson contributions to the electroweak precision parameters.
- The purpose of our work is to determine the stability of such a $D7$-$\overline{D7}$ techni-brane embedding.
- A $D7$ probe brane profile is obtained by solving the field equations obtained from the Dirac-Born-Infeld (DBI) action of the $D7$ brane with induced metric $g_{(8)}$.

[3] Nucl. Phys. **B852**, 39-60 (2011)

- The $D7$ branes were chosen to have complementary coordinates as functions of the ρ coordinate only, $\theta = \theta(\rho)$ and $\varphi = \varphi(\rho)$.
- The location ρ_0 of the tip of the branes is an integration constant.

	x^0	x^1	x^2	x^3	θ	φ	ρ	$\tilde{\theta}$	$\tilde{\varphi}$	ψ
	0	1	2	3	4	5	6	7	8	9
$D5$	x	x	x	x	x	x				
$D7$-$\overline{D7}$	x	x	x	x			x	x	x	x





brane stability

- In order to determine the stability of this probe brane embedding, small oscillations about the $D7$ brane profile are considered in the (x^μ, ρ) directions: $\theta(x,\rho) = \pi/2 + \Delta\theta(x,\rho)$, $\varphi(x,\rho) = \varphi(\rho) + \Delta\varphi(x,\rho)$.
- Expanding the DBI action for the $D7$ brane to quadratic order in the fluctuations $\Delta\theta$ and $\Delta\varphi$ yields the fluctuation Lagrangian.
- Separating the x^μ and ρ variables gives the equations $\partial^2\Delta\varphi = M_\varphi^2\Delta\varphi$ and similarly for $\Delta\theta$ for the 4-dimensional behavior.
- The $D7$ and $\overline{D7}$ branes stack separately on each side of the U-shaped embedding profile. In order to distinguish these separate locations a transformation of coordinates from ρ to ζ is introduced with the location of the $D7$ branes along $\zeta > 0$ branch while the $\overline{D7}$ branes are along the $\zeta < 0$ branch.

- The fluctuation spectrum values of M^2 for the techni-scalar and techni-pseudoscalar meson modes must in general be determined numerically for various values of ρ_0, C and T.
- Insight into the spectrum can be obtained by considering values of ρ in the walking region where the metric simplifies.
- The Schrödinger equations for the fluctuations in this case become

$$\begin{aligned} -\phi''_\varphi(\zeta) + V_\varphi(\zeta)\phi_\varphi(\zeta) &= E_\varphi\phi_\varphi(\zeta) \\ -\phi''_\theta(\zeta) + V_\theta(\zeta)\phi_\theta(\zeta) &= E_\theta\phi_\theta(\zeta). \end{aligned}$$

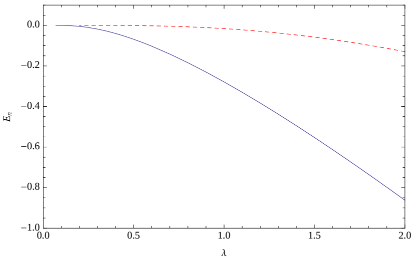
- The meson masses are related to the energy eigenvalues as

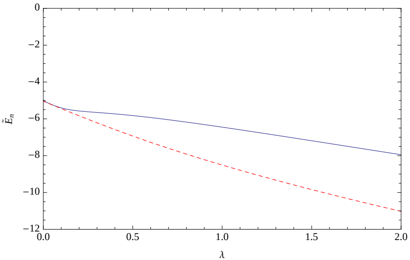
$$E_\varphi = CTM_\varphi^2 e^{4\rho_0} \qquad , \qquad E_\theta = CTM_\theta^2 e^{4\rho_0}.$$

$$
\begin{aligned}
V_\varphi(\zeta) &= -\frac{1}{4}\frac{(\zeta^2+6)}{(\zeta^2+1)^2} - \frac{\lambda_\varphi}{(\zeta^2+1)} \\
\lambda_\varphi &= 0 \\
W_\varphi(\zeta) &= \frac{1}{\zeta} - \frac{1}{2}\frac{\zeta}{1+\zeta^2}
\end{aligned}
\qquad
\begin{aligned}
V_\theta(\zeta) &= -\frac{1}{4}\frac{(\zeta^2-2)}{(\zeta^2+1)^2} - \frac{\lambda_\theta}{(\zeta^2+1)} \\
\lambda_\theta &= Te^{4\rho_0} \\
W_\theta(\zeta) &= \frac{1}{2}\frac{\zeta}{1+\zeta^2}
\end{aligned}
$$

- The spectra can be understood in terms of approximate supersymmetry and conformal symmetry.
- Since the superpotential $W_\varphi(\zeta)$ is singular, supersymmetry does not preclude the existence of negative energy states.
- Both models can be regarded as conformal quantum mechanics with regulated singularity.
- The second term in each potential breaks supersymmetry.
- The parameter λ_θ is very small for values of the parameter T that yield a substantial range of walking behavior.
- For large ζ the potentials asymptote to $-\frac{1/4+\lambda}{\zeta^2}$.

$\Delta\theta$ fluctuation spectrum

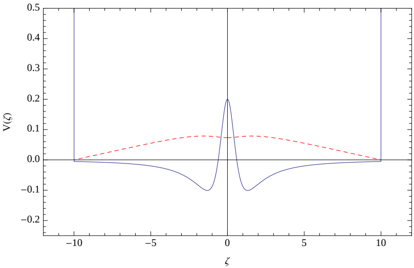


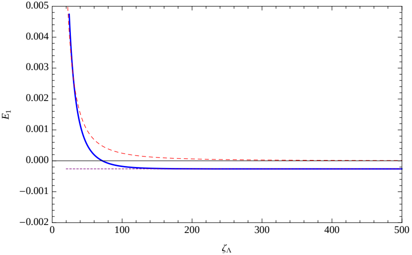


- Ground state ($n = 1$): solid blue curve
- First excited state ($n = 2$): dashed red curve
- There is an infinite number of negative energy states.

- Rescaled energy $\tilde{E}_n \equiv E_n / e^{-\frac{n\pi}{\sqrt{\lambda}}}$
- Energy spectrum approaches $E_n = -C_1 e^{-\frac{n\pi}{\sqrt{\lambda}}}$ for small λ.
- Numerically determined: $C_1 = 5.02$

$\Delta\theta$ fluctuation spectrum: with cut-off

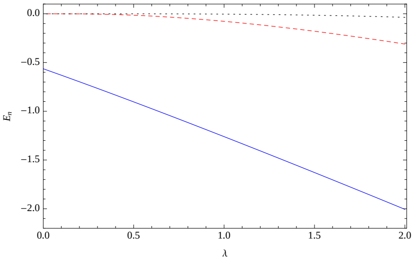


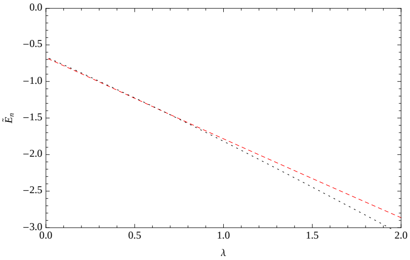


- large distance cut-off further breaks conformal invariance
- dashed red curve: ground state eigenfunction (n=1)
- cut-off: $\zeta_\Lambda = 10$
- $\lambda = 0.1$

- purple dotted curve: ground state energy value in the absence of a cut-off
- red dashed curve: ground state energy of an infinite square well

$\Delta\phi$ fluctuation spectrum

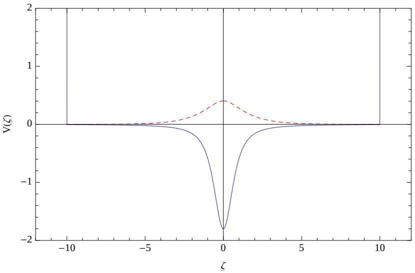


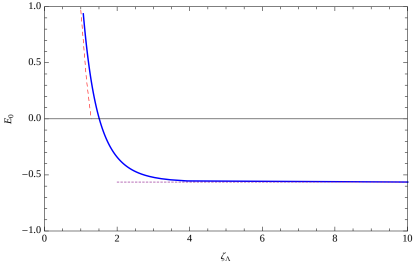


- Ground state ($n = 0$): solid blue curve
- First excited state ($n = 1$): dashed red curve
- Second excited state ($n = 2$): dotted black curve
- Infinite number of bound states for $\lambda > 0$

- one deep lying bound state (n=0) remains even at $\lambda = 0$: $E_0 = -0.56$
- Energy spectrum approaches $E_n = -C_2 e^{-\frac{n\pi}{\sqrt{\lambda}}}$ for small λ and $n > 0$.
- Numerically determined: $C_2 = 0.67$

$\Delta\phi$ fluctuation spectrum: with cut-off





- dashed red curve: ground state eigenfunction (n=0)
- cut-off: $\zeta_\Lambda = 10$
- cut-off does not affect the ground state energy unless it is as low as the width of the ground state wave-function in the absence of a cut-off.

- $\lambda = 0.0$
- purple dotted curve: ground state energy value in the absence of a cut-off
- red dashed curve: ground state energy of an infinite square well

Summary

- The gravity dual to a $\mathcal{N} = 1$ SUSY gauge theory which exhibits an approximately conformal ("walking") region while strongly interacting was considered.
- Stacks of $D7$-$\overline{D7}$ techni-branes corresponding to the addition of techni-fermions in the gauge theory were embedded in the 10 dimensional space-time.
- Fluctuations of the embedded branes into complementary φ and θ space were considered.
- The equations of motion took the form of one-dimensional Schrödinger equations with factorizable Hamiltonians.

- The scalar and pseudoscalar meson mass squared spectrum was obtained by numerically analyzing the Schrödinger equations.
- Both of the $\Delta\varphi$ and $\Delta\theta$ pseudoscalar meson fluctuations and the $\Delta\theta$ scalar meson fluctuations were found to be stable.
- However, the $\Delta\varphi$ scalar meson fluctuation was found to be unstable having a negative mass squared value even for low values of the cut-off.
- Whether a more general embedding of the stacks of $D7$-$\overline{D7}$ branes involving the 2-cycle Σ_2 used to wrap the stacked $D5$ branes of the gauge theory made from the twisting of the S^2 coordinates θ, φ and the S^3 coordinates $\tilde{\theta}$, $\tilde{\varphi}$ and ψ will lead to a stable techni-brane embedding remains to be investigated.