ELECTRON SPIN RESONANCE

&

MAGNETIC RESONANCE TOMOGRAPHY

1. AIM OF THE EXPERIMENT

This is a model experiment for electron spin resonance, for clear demonstration of interaction between the magnetic moment of the electron spin with a superimposed direct or alternating magnetic field. This experiment can also be used to explain the basic mode of operation of Magnetic Resonance Tomography (MRT), sometimes also referred as Magnetic Resonance Imaging (MRI). Further explications on this will be given at the end of this description.

A ball with a central rod magnet, which rotates with low friction on an air cushion, acts as model electron or any element that possesses a nonzero spin. Two pairs of coils generate a constant magnetic field B_o and an alternating magnetic field B_1. The axes of both fields intersect perpendicularly at the centre of the ball.

The table is slightly inclined to start the electron gyroscope with an air draught (Magnus effect). If the direct magnetic field B_o acts on the ball, a precession of the magnet axis is observed. Precession frequency increases with the intensity of field B_o. With a second pair of coils and a pole changeover switch (commutator), a supplementary alternating field B_1 is generated. If the change of poles occurs at the right phase, the angle between the gyroscope axis and the direction of the direct field is continuously increased, until the magnetic axis of the ball is opposed to the field direction (spin flip). This phenomenon is also used to generate a measurable signal for MRI, which will be explained in section 3.

2. THEORETICAL BACKGROUD

2.1. Torque on a Current Loop

The torque on a current-carrying coil, as in a DC motor, can be related to the characteristics of the coil by the "magnetic moment" or "magnetic dipole moment". In general, the torque $\vec{\tau}$ is given by

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where $\vec{r}$ is the length of the lever arm and $\vec{F}$ the force on the lever arm. In this specific case, the magnetiv force is given by $F = ILB$, so that the torque exerted by the magnetic force (including both sides of the coil) is given by

$$\tau = BILW\sin\theta$$

where L and W are the length and the width of the coil. The coil characteristics can be grouped as

$$\mu = IA \quad (or \; \mu = NIA \quad for \; n \; loops)$$

called the **magnetic moment of the loop** (A = L·W is the area of the coil). The torque now can be written as

$$\tau = \mu B\sin\theta$$

The direction of the magnetic moment is perpendicular to the current loop in the right-hand-rule direction, the direction of the normal to the loop in the illustration. Considering torque as a vector quantity, this can be written as the vector product

$$\tau = \mu \; x \; B$$

Since this torque acts perpendicular to the magnetic moment, it can cause the magnetic moment to **precess** around the magnetic field at a characteristic frequency called the Larmor frequency (see below).

If you exerted the necessary torque to overcome the magnetic torque and rotate the loop from angle 0° to 180°, you would do an amount of rotational work given by the integral
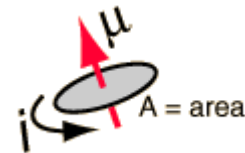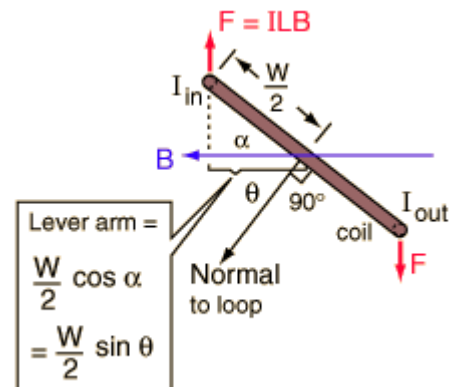
$$W = -\int_0^\pi \tau d\theta = -\int_0^\pi \mu B \sin\theta d\theta = -\mu B \cos\theta\big|_0^\pi = 2\mu B$$

The position where the magnetic moment is opposite to the magnetic field is said to have a higher magnetic potential energy.

As seen in the geometry of a current loop, the torque τ tends to line up the magnetic moment with the magnetic field B, so this represents its lowest energy configuration.
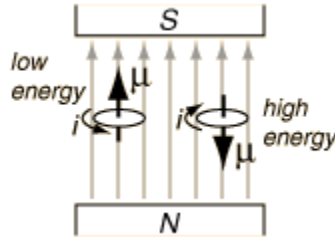
The potential energy associated with the magnetic moment is

$$U(\theta) = -\mu \cdot B$$

so that the difference in energy between aligned and anti-aligned is

$$\Delta U = 2\mu B$$

These relationships for a finite current loop extend to the magnetic dipoles of electron orbits and to the intrinsic magnetic moment associated with electron spin.

2.2   Electron Intrinsic Angular Momentum

Experimental evidence like the hydrogen fine structure and the Stern-Gerlach experiment suggest that an electron has an intrinsic angular momentum S, independent of its orbital angular momentum L. These experiments suggest just two possible states for this angular momentum, such that

$$S_z = \pm \frac{1}{2}\hbar$$

An angular momentum and a magnetic moment could indeed arise from a spinning sphere of charge, but this classical picture cannot fit the size or quantized nature of the electron spin. The property called electron spin must be considered to be a quantum concept without detailed classical analogy.
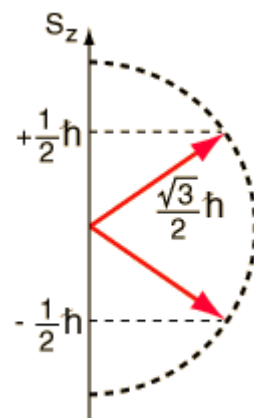
2.3. Electron Spin Magnetic Moment

Since the electron displays an intrinsic angular momentum, one might expect a magnetic moment which follows the form of that for an electron orbit. The z-component of magnetic moment associated with the electron spin would then be expected to be

$$\mu_z = \pm\frac{1}{2}\mu_B$$

(where $\mu_B = \dfrac{e\hbar}{2m_e}$ is the Bohr magneton) but the measured value turns out to be about twice that. The measured value is written

$$\mu_z = \pm\frac{1}{2}g\mu_B$$

where g is called "the electron spin g-factor" and is equal to 2.00232. The precise value of g was predicted by relativistic quantum mechanics and was measured in the Lamb shift experiment.

The electron spin magnetic moment is important in the spin-orbit interaction which splits atomic energy levels and gives rise to fine structure in the spectra of atoms. The electron spin magnetic moment is also a factor in the interaction of atoms with external magnetic fields (Zeeman Eeffect).

2.4 Electron Spin Resonance

When the molecules of a solid exhibit paramagnetism as a result of unpaired electron spins, transitions can be induced between spin states by applying a magnetic field and then supplying electromagnetic energy, usually in the microwave range of frequencies. The resulting absorption spectra are described as electron spin resonance (ESR) or electron paramagnetic resonance (EPR).

ESR was first observed in Kazan State University by the Soviet physicist Yevgeniy Zavoyskiy in 1944, and was developed independently at the same time by Brebis Bleaney at Oxford University.

ESR has been used as an investigative tool for the study of radicals formed in solid materials, since the radicals typically produce an unpaired spin on the molecule from which an electron is removed. Particularly fruitful has been the study of the ESR spectra of radicals produced as

radiation damage from ionizing radiation. Study of the radicals produced by such radiation gives information about the locations and mechanisms of radiation damage.

The interaction of an external magnetic field with an electron spin depends upon the magnetic moment associated with the spin, and the nature of an isolated electron spin is such that two and only two orientations are possible. **The application of the magnetic field then provides a magnetic potential energy which splits the spin states by an amount proportional to the magnetic field (Zeeman effect), and then radio frequency radiation of the appropriate frequency can cause a transition from one spin state to the other**. The energy associated with the transition is expressed in terms of the applied magnetic field B, the electron spin g-factor g, and the constant μ_B.

Magnetic potential energy of electron spin in magnetic field: $U = \mu \cdot B = \pm \frac{1}{2} g \mu_B B$
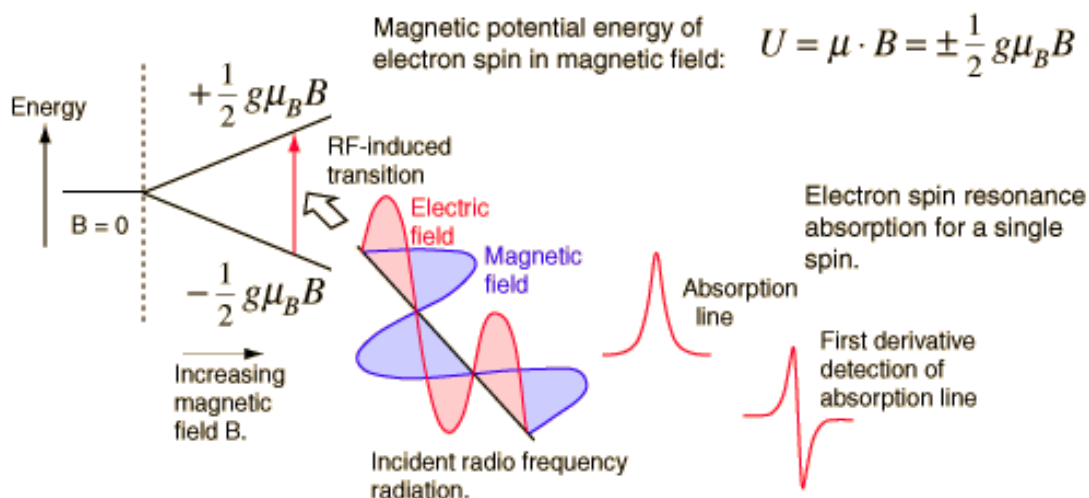
If the radio frequency excitation was supplied by a klystron at 20 GHz, the magnetic field required for resonance would be 0.71 T, a sizable magnetic field typically supplied by a large laboratory magnet.

If you were always dealing with systems with a single spin like this example, then ESR would always consist of just one line, and would have little value as an investigative tool, but several factors influence the effective value of g in different settings. Much of the information obtainable from ESR comes from the splittings caused by interactions with nuclear spins in the vicinity of the unpaired spin, splittings called nuclear hyperfine structure.

2.5. Larmor Precession

When a magnetic moment μ is placed in a magnetic field B, it experiences a torque which can be expressed in the form of a vector product

$$\tau = \mu \ x \ B$$

For a static magnetic moment or a classical current loop, this torque tends to line up the magnetic moment with the magnetic field B, so this represents its lowest energy configuration. But if the magnetic moment arises from the motion of an electron in orbit around a nucleus, the magnetic moment is proportional to the angular momentum of the electron. The torque exerted then produces a change in angular momentum which is perpendicular to that angular momentum, causing the magnetic moment to precess around the direction of the magnetic field rather than settle down in the direction of the magnetic field. This is called Larmor precession.

$$|L| = \sqrt{l(l+1)} \ \hbar$$
$l = orbital \ quantum \ number$
$$\mu = \frac{-e}{2m_e} L$$
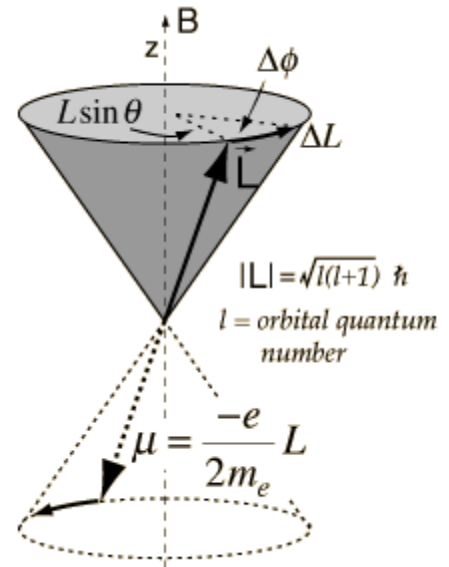
When a torque is exerted perpendicular to the angular momentum L, it produces a change in angular momentum ΔL which is perpendicular to L, causing it to precess about the z axis. Labeling the precession angle as φ, we can describe the effect of the torque as follows:

$$\tau = \frac{\Delta L}{\Delta t} = \frac{L\sin\theta\Delta\phi}{\Delta t} = |\mu B \sin\theta| = \frac{e}{2m_e}LB\sin\theta$$

The precession angular frequency (Larmor frequency) is

$$\omega_{Larmor} = \frac{d\phi}{dt} = \frac{e}{2m_e}B$$

These relationships for a finite current loop extend to the magnetic dipoles of electron orbits and to the intrinsic magnetic moment associated with electron spin. There is also a characteristic Larmor frequency for nuclear spins.

In the case of the electron spin precession, the angular frequency associated with the spin transition is usually written in the general form

$$\omega = \gamma \cdot B$$

where $\gamma = g\dfrac{e}{2m_e}$ is called the gyromagetic ratio (sometimes the magnetogyric ratio). This angular frequency is associated with the "spin flip" or spin transition, involving an energy change of $2 \cdot \mu \cdot B$.

The characteristic frequencies associated with electron spin are employed in electron spin resonance (ESR) experiments, and those associated with the nuclear spin in nuclear magnetic resonance (NMR) experiments.

3. APPLICATION: MRI

3.1. Prerequisites

There are two prerequisites to MRI. One is Nuclear Magnetism and the other one is Nuclear Spin Angular Momentum. (This is the same effect as Electron Spin Angular Momentum described above.)

Nuclear magnetism is a quantum effect and does not occur because the electron or nucleus is actually spinning in a physical sense: they do not. However, there is a real magnetic field local to the nucleus which behaves like a permanent magnet.

Figure 3.1.: MRI of a brain tumor (white spot)

Spin angular momentum is a property of certain nuclei. If an element possesses spin depends on its atomic number (number of protons) and its atomic weight (number of protons and neutrons).

An element has

- no spin (spin = 0) if there is an even number of protons and an even number of neutrons.
- an integer as spin if there is an odd number of protons and an odd number of neutrons.
- a noninteger multiple of ½ as spin for all other cases.

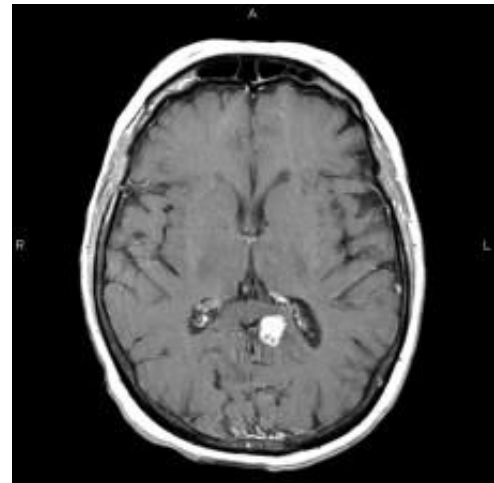In medical applications of MRI, one often focuses on the hydrogen nucleus ^{1}H, since the body is composed of tissues primarily consisting of water and fat, both of which contain hydrogen.

The whole MRI function can be reduced to a reemission phenomenon. Energy is applied and a hort time later, this energy is reemitted and detected.

3.2. Spin alignment and magnetization

When no magnetic field is applied, the spins of the protons and therefore their magnetic moments too, are randomly pointing in all directions (see figure 3.2.).
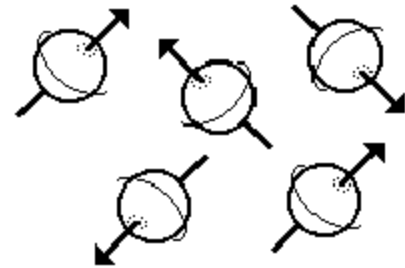
Once a magnetic field B_0 (along the z-axis) is applied, the spin will orientate towards the B_0-axis. In the direction perpendicular to the B_0-axis, there will be no net magnetization because in the sum, all the precessing vectors

Figure 3.2.: Randomly distributed spin

in the xy-plane will cancel each other out. In parallel direction however, there will be an effect (Zeeman Effect), which all in all will cause a net magnetization in the B_0-direction. Of course, there will also be protons aligned against the B_0-axis, which means a higher energy level, however this are only a few (Boltzmann distribution)[1]. In other words, the analyzed tissue will become magnetized in the presence of a magnetic field B_0 with a small nonzero value M_0, called the net magnetization. This net magnetization is constant in time and is the source of signal for all MR experiments. The different sorts of measurement will be discussed later on (see section below).

In MRI, one is usually not dealing with only one proton, but billions of protons. Adding all the longitudinal magnetizations up will deliver us the component M_z of M_0 which is aligned in the direction of B_0. If we look at all the different transverse components of the magnetization vectors of the protons, they will all cancel each other out. In fact, they are precessing with the same

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[1] Boltzmann distribution: $N_{UPPER}/N_{LOWER} = e^{-\Delta E/kT}$
Where N_{UPPER} and N_{LOWER} are the number of protons in the upper and lower energy levels. T is the absolute temperature in K of the volume of tissue, and k is the Boltzmann constant.

larmor frequency $\omega_0 = \gamma B_0$ but not with the same phase, so all in all, there is no transverse component M_t of M_0.

3.3. Measurement of the signal

How can we measure a MRI signal? It is actually not possible to measure the M_z component since it is in the direction of the external magnetic field B_0, which is much stronger than M_z. M_t is zero, so we can't measure that either. However, if we put energy into the system, the spins will start to flip and the net longitudinal magnetization M_z will decrease and the transverse component M_t will increase. Moreover, the phase coherence of spins will increase in a higher energy configuration. With the help of a conductor, the change in the transverse magnetization can be detected (because change of magnetization will induce a current in the conductor). This is the MR signal intensity.

By applying a radiofrequency-pulse (rf-pulse) with the correct larmor frequency ω_0, the net magnetization vector M_0 will start to precess around B_0. Its transverse component will grow and its longitudinal component will decrease.

Now there are different measurement methods. First, by adding energy to the system (rf-pulse), the net magnetization vector will be deflected. How much it will be deflected depends on how long we add energy with the right frequency ω_0. To keep things simple, let's say we want a
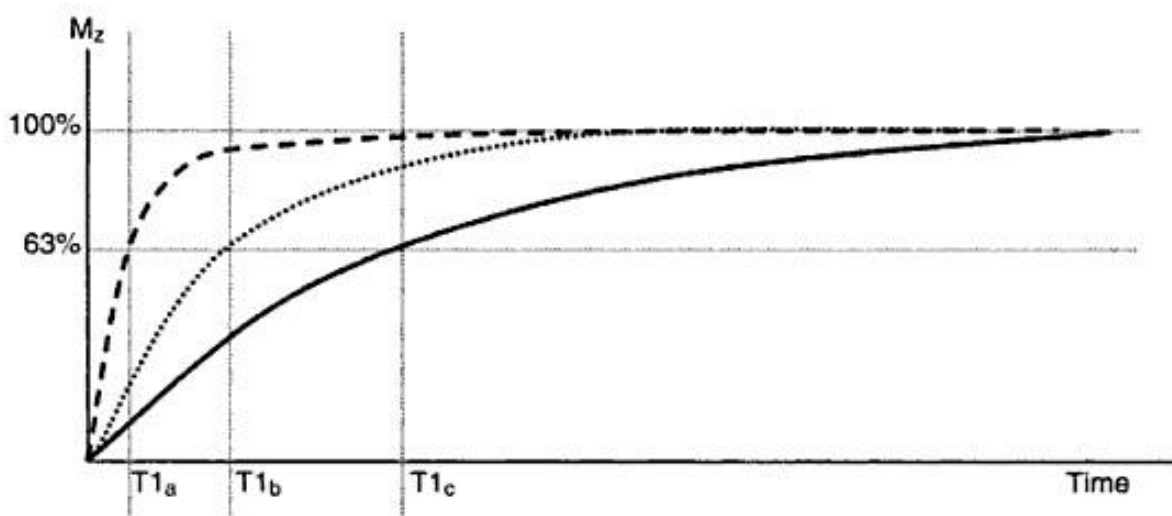

Figure 3.3.: Longitudinal relaxation T1 in different tissues. The dashed line represents fat (T1$_a$), the dotted line represents brain (T1$_b$) and the solid line represents cerebrospinal fluid (T1$_c$).

deflection of 90°, so that $M_z = 0$ and $M_t = M_0$. Once we shut the radiofrequency off, the same amount of energy will be emitted and the net magnetization vector will return to its primary position. This is called *relaxation*. There are different types of relaxation. On the one hand, there
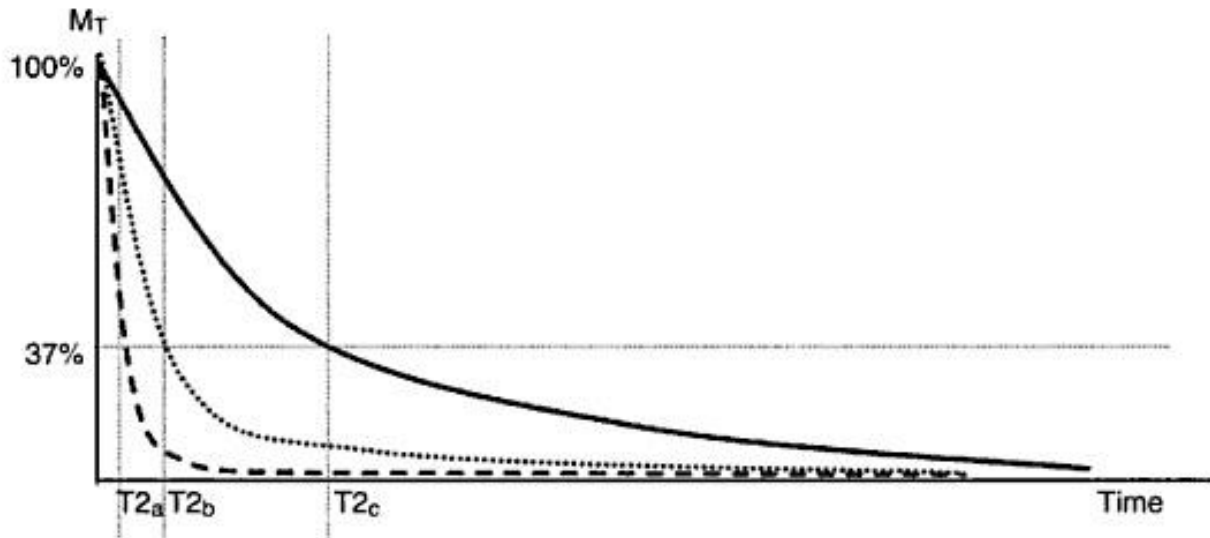

Figure 3.4.: Transverse relaxation T2. Each line represents a different tissue (see figure 3.3.). Tissues with long T1 also have long T2

is the longitudinal recovery of magnetization (T1 relaxation, see figure 3.3.) and on the other hand, there is the loss of transverse magnetization (T2 relaxation, see figure 3.4.). T1 is the time, when M_z has recovered 63% of its original value. T2 is the time, when 63% of M_t has dissipated. T1 is usually much longer (about 5 to 20 times) than T2.
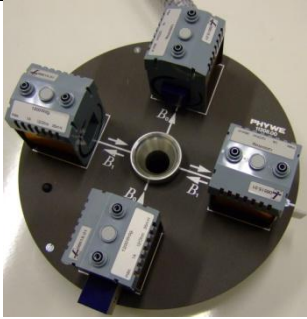
3.4. Signal localization

How can we localize the signal and distinguish different tissues? With the help of an additional linear magnetic field (gradient field) every proton in a specific volume will presses with a defined larmor frequency (remember $\omega_0 = \gamma B$). Therefore the different signals can be clearly allocated to their respective volume. The determination of the different sorts of tissue is measured with the help of the relaxation times mentioned above. Every tissue has its own characteristic relaxation time.
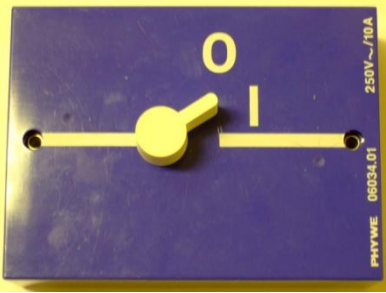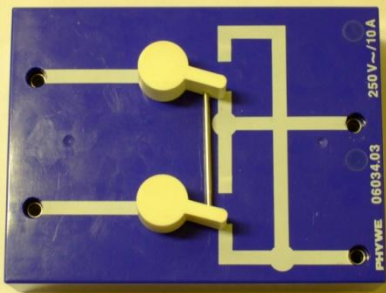
4. SETUP

4.1. Equipment

- 1 gyroscope (table + ball with a central rod magnet)

Table	Ball with magnet

- 1 pressure tube with end piece 1.5 m
- 8 connecting cables
- 2   iron cores, short, laminated
- 4 coils 1200 turns

	Notice the black curve on the coil, it denotes that the windings are anticlockwise from the connector on top of the coil facing towards you.

- Switches

On/off switch	Commutator switch

- 1 air blower

- 1 variable transformer with rectifier 25 V~ / 20 V−, 12 A

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4.2. Assembling the experiment

1. Connect the table to the air pump via the pressure tube.

2. Place the four coils on the boxes drawn on the table. align the coil such that the winding of opposite pair of coils are in the same direction as shown in Fig. 1.
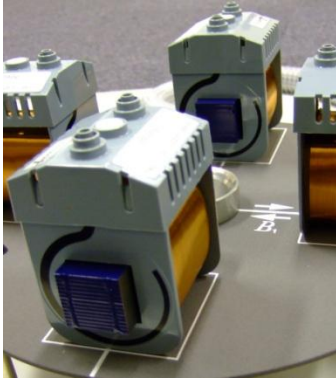
Fig. 1

3. Insert the iron cores into the two opposite coils lying on the B_o axis (see Fig. 1).

4. Connect the cables from the power supply to the two switches and the coils as shown in Fig. 2.
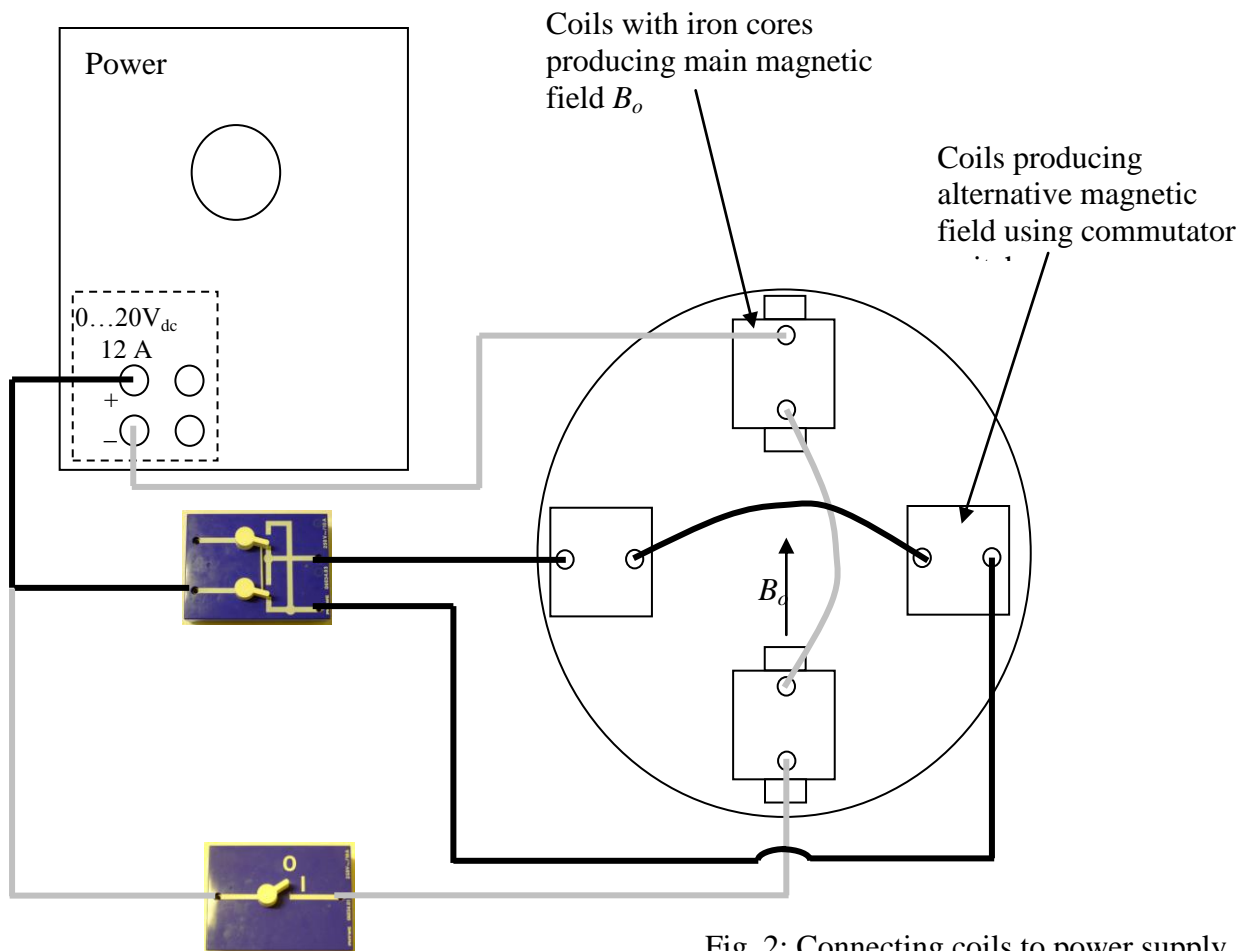

Fig. 2: Connecting coils to power supply

A complete setup is shown in Fig. 3 below.

Fig. 3: Full setup of apparatus

4.3. Safety precautions

This setup used relatively high current. Care should be taken when conducting the experiment.

4.4. Experimental procedure

1. Tilt the table by turning the knob on one of the table's leg as shown in Fig. 3.
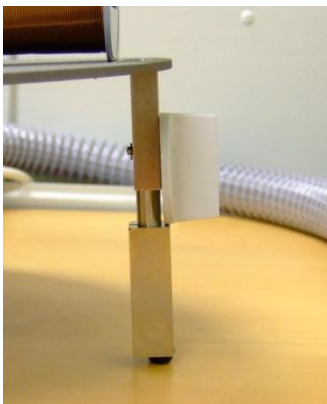

Fig. 3: Turning knob to tilt gyroscopic table

2. Place the gyroscopic ball on the blowhole such that the north pole of the central magnet (mark with a red ring) is almost parallel to the B_o field.
3. Start the air blower by turning the knob to a setting of between 3.5 to 4.

4. When the spin of the ball is steady, switch on the B_o field using the switch with a setting of between 15 to 20 V_{dc} on the power supply.

5. Once the field is switch on, the central magnet axis will start to precess[2].

6. Choose two reference points on the precession cycle that are on opposite sides, flip the commutator switch[3] from one side to another when the north pole of the magnet reaches these reference points.

7. If your timing is good, the pole of the magnet will switch direction[4] in about two minutes (less with practice).

8. A movieclip demonstration of the experiment comes together with this document.

5. OBSERVATIONS AND DRAWING PARALLELS TO QUANTUM IDEAS

5.1. Precession of central magnet axis

When the main magnetic field B_o is applied, the magnetic axis of the ball will undergo precession (refer to the theory section on Larmor precession). Since the precession frequency is proportional to the applied field, one may want to change the voltage from the power supply to observe this effect.

5.2. Spin resonance

From quantum theory, a photon of frequency (i.e., energy) matching the energy difference of an electron in a uniform field with spin up or down can cause its spin to flip. This experiment provides learners the opportunity to reconcile the quantum effect of

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[2] If the magnetic axis of the ball is parallel to the magnetic field B_o of the coils, precession is small. You may want to start by having the magnetic axis of the ball slightly at an angle to B_o.

[3] The commutator switch controls the direction of the magnetic field that is perpendicular to the main field B_o. By alternating the current in the coil, an alternating magnetic field is produced which the magnetic dipole of the ball will feel. This alternating field is the driving force on the spinning ball.

[4] When the magnetic axis changes direction, the precession frequency is rather high and it is hard to synchronize the commutator switch to the precession. The magnetic axis will become unstable and very soon it will return to its original orientation.

electron (proton) spin resonance with classical mechanics. When the frequency of the second magnetic field (analogous to the electromagnetic radiation or photon) is close to the precession frequency (analogous to the spin of the electron in a magnetic field), the magnetic axis of the ball will flip to oppose the main magnetic field (resonance).

6. SOME GUIDANCE TO TEACHERS

1. High school students should have the pre-requisite knowledge of resonance. Hence it may be useful to recap on the concept of resonance before the experiment or demonstration.
2. You may want to discuss about gyroscopic precession (if it has not been done in classical mechanics) in general with the usual bicycle wheel and turntable demonstration.
3. When the magnetic field B_o is switch on, the torque produced by the magnetic dipole of the ball and the field causes the magnetic axis to precess. This is similar to the classical explanation of Larmor precession of an electron in a magnetic field.
4. The concept of spin in quantum theory has no equivalent in the classical world. In order to provide the link between quantum spin precession and classical spin, we need a spinning ball to go with the central magnet the ball otherwise precession is not possible. It is good to remind students that although there are similarity between quantum spin and this demonstration, electron spin is an intrinsic property which got its name due to historical reason.