Top Quark Physics
(part 1)

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Topics in This Talk

➢ Motivation for top physics

➢ Top production cross section

➢ Top quark mass)

➢ Single top production

➢ Forward-backward/charge asymmetry in **tt-bar**

➢ Top spin effects (top, anti-top polarization,top – anti/top spin correlations, quantum entanglement)

➢ Associated production of **tt-bar** and **Z,W, γ**

➢ FCNC in **tt-bar** production

Top quark – the heaviest SM elem. particle

❑ **Top quark**: discovered at Fermilab (CDF + D0) in 1995

❑ Completed the 3rd generation of SM fermions

leptons	Q	T_3	quarks	Q	T_3
$\nu_e \ \ \nu_\mu \ \ \nu_\tau$	0	1/2	$u \ \ c \ \ (t)$	2/3	1/2
$e \ \ \mu \ \ \tau$	-1	-1/2	$d \ \ s \ \ b$	-1/3	-1/2

SM fundamental fermions, $Q \equiv$ electric charge,
$T_3 \equiv$ 3rd comp. of weak isospin

❑ Top quark mass (m_{top}): 172.52 ± 0.33 GeV (35×m_b)

  ✓ Pole mass (from cross sec.) : 172.4 ± 0.7 GeV

❑ Main object of study at Fermilab

  ✓ final sample 10fb^{-1} $\Rightarrow$ 70 000 $t\bar{t}$-pairs produced

❑ A very important object of study for LHC –

  ✓ Top factory: ≈ 1.5 M, ≈10M $t\bar{t}$ per 10 fb^{-1} for 7 and 14 TeV, resp.

Top quark physics: Motivation

❑ **Very high mass: near EWSB scale** η

  Top Yukawa coupling $\lambda_t = \sqrt{2}m_{top}/\eta \approx 1$

❑ **tt-bar production X-sections: test of QCD**

  $\rightarrow$ to is produced at very small distances $1/m_t$

  $\Rightarrow \alpha_s(m_{top}) \approx 0.1$: pert. expansion converges rapidly

❑ **Top decays before hadronization**

$$\frac{1}{m_t} < \frac{1}{\Gamma_t} < \frac{1}{\Lambda_{QCD}} < \frac{m_t}{\Lambda^2}$$

production 10^{-27} s   lifetime 10^{-25} s   hadronization 10^{-24} s   spin−flip 10^{-21} s

  $\rightarrow$ study of spin characteristics (top production mechanisms)

❑ **Cross sections sensitive to new physics**

  $\rightarrow$ resonat production of $t\bar{t}$, decay: $t \rightarrow Hb$

❑ **Important background for Higgs boson studies**

Stringent tests of SM + Search for New physics

$\eta = 246$ **GeV**

$\Gamma_t = 1.32$ **GeV**

$\Lambda \approx 250$ **MeV**

Top Quark Production

Strong $t\bar{t}$ pair production   **EW single top quark production**

$$q\bar{q} \rightarrow t\bar{t}$$
(13%, 85%)

$$gg \rightarrow t\bar{t}$$
(87%, 15%)

t-channel

tW-channel   s-channel

Top-quark production cross section - theory predictions

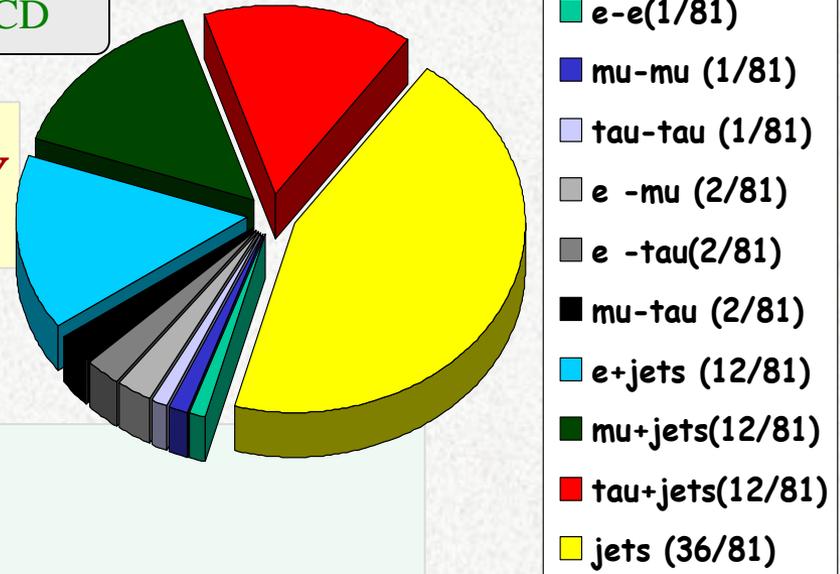|	top-quark pair production		single top-quark production	
Tevatron/1.96 TeV ≈ **7 pb**		5%	≈ **3 pb** (precision)	
LHC /7TeV:	≈ **177 pb**	5.8%	≈ **85 pb**	
/8TeV:	≈ **253 pb**	5.5%	≈ **116 pb**	6%
/13TeV	≈ **832 pb**	5.2%	≈ **300 pb**	
/14TeV:	≈ **954 pb**	4.1%	≈ **320 pb**	

Top Quark Decay

SM: by far dominant $t \to bW$
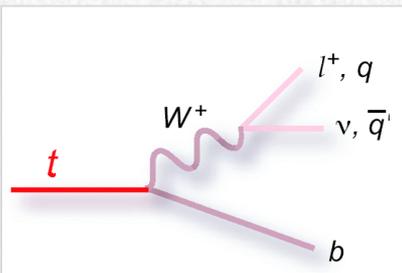
$$\frac{\Gamma(t \to bW)}{|V_{tb}|^2} \approx 0.807 \times \frac{G_F m_t^3}{8\pi\sqrt{2}} = 1.32 \pm 0.02\, GeV$$

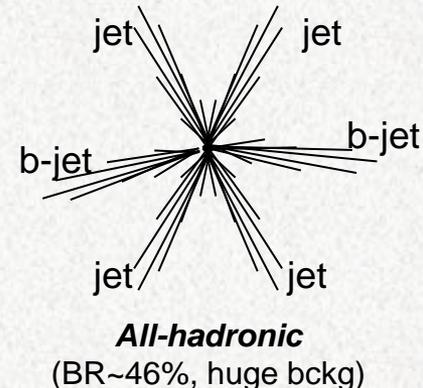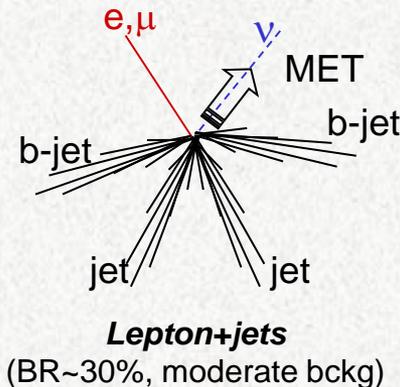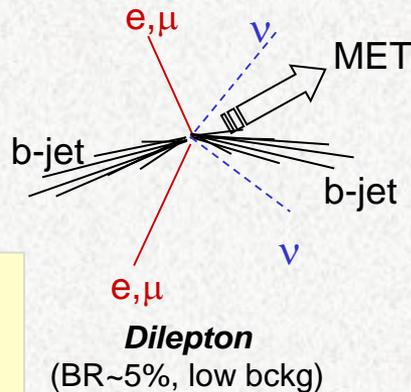$$\tau_{top} \approx 5 \times 10^{-25}\, sec \ll \tau_{hadr} \,(10^{-23} sec)$$

N³LO QCD

Top decays before hadronization !!!

- **No tt-bar bound states (**gluon exchange **)**
- t, W **helicity from SM V-A** (no depolarization via hadronization)

Pie chart legend:
- e-e(1/81)
- mu-mu (1/81)
- tau-tau (1/81)
- e -mu (2/81)
- e -tau(2/81)
- mu-tau (2/81)
- e+jets (12/81)
- mu+jets(12/81)
- tau+jets(12/81)
- jets (36/81)

tt-bar samples via W decays

Dilepton
(BR~5%, low bckg)

Lepton+jets
(BR~30%, moderate bckg)

All-hadronic
(BR~46%, huge bckg)
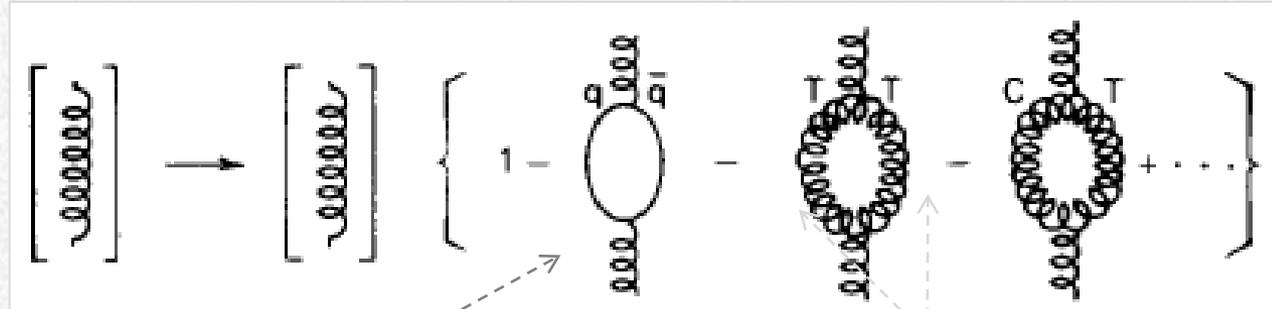
Strong interaction coupling constant

If at gluon exchange we take into account also radiation correction (production of virtual qq and gluon pairs in propagator loops):

Coupling constant α_S $(= g_S^2/4\pi)$ in one-loop approximation at high $Q^2 = |q^2|$:

$$\alpha_S(Q^2) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)\ln\frac{Q^2}{\Lambda_{QCD}^2}}$$

$$\Lambda_{\overline{MS}}^{(6)} = (89 \pm 6)\ \text{MeV},$$
$$\Lambda_{\overline{MS}}^{(5)} = (210 \pm 14)\ \text{MeV},$$
$$\Lambda_{\overline{MS}}^{(4)} = (292 \pm 16)\ \text{MeV},$$
$$\Lambda_{\overline{MS}}^{(3)} = (332 \pm 17)\ \text{MeV},$$

Index $(n) = n_f \equiv$ Nr. of quarks involved (1)

$\alpha_S(Q^2)$ decreases with increasing Q^2 → becomes small at small distances:

$$Q^2 \to \infty \quad \Rightarrow \quad \alpha_S(Q^2) \to 0 \qquad \text{(asymptotic freedom)}$$

For small Q^2 coupling constant, α_S is big and perturbative used approach cannot be → confinement of quarks for $Q \to \Lambda$.

Top-quark processes: $Q^2 \approx m_t^2 \Rightarrow \alpha_S(m_t^2) \approx 0.1$ perturbative series converges quickly.

Cross Section of Top Quark production

Top quark production cross section

Factorization theorem:

theory

$$\sigma = \sum_{i,j} \int dx_1 dx_2 \, F_i^{(1)}(x_1, \mu_F) F_j^{(2)}(x_2, \mu_F) \hat{\sigma}_{ij}(s; \mu_F, \mu_R) \qquad (2)$$
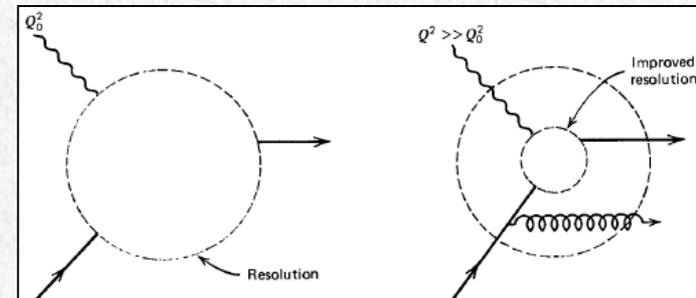
experiment

Parton Distribution Functions (PDFs)

$s = \left(p_i + p_j \right)^2 \equiv$ square of parton collision energy,

$F_i^{(\lambda)}\left(x_\lambda, \mu_F \right) \equiv$ probability density to observe a parton i with longitudinal momentum fraction x_λ in incoming hadron λ, when probed at a scale μ_F,

$\mu_F \equiv$ factorization scale (a free parameter) - it determines the proton structure if probed (by virtual photon or gluon) with $q^2 = -\mu_F^2$,

$\mu_R \equiv$ renormalization scale – defines size of strong coupling constant.

Usual choice: $\mu_F = \mu_R = \mu \in (m_t/2, \, 2m_t)$

The LO top quark pairs cross section (Born term):

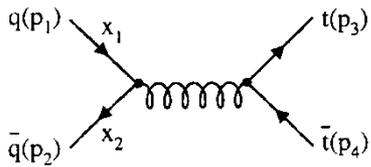**Predicted X-sec.**

$$d\hat{\sigma} = \frac{1}{2(p_1 + p_2)^2} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} \delta(p_1 + p_2 - p_3 - p_4) \overline{|M|^2} \qquad (3)$$

calculated

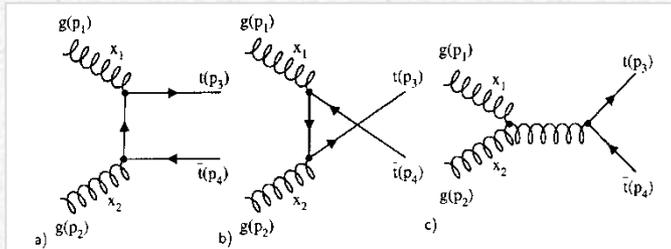Output particles´ phase space

Quark-antiquark annihilation

$$\overline{|M|^2}(q\bar{q} \to t\bar{t}) = (4\pi\alpha_s)^2 \frac{8}{9}\left(2\frac{(p_1 \cdot p_3)^2 + (p_2 \cdot p_3)^2}{(p_1 \cdot p_2)^2} + \frac{m_t^2}{(p_1 + p_2)^2}\right) \qquad (4)$$

Averaged over initial and summed over final color and spin states

Gluon fusion

Experiment:

LO tt X-section is not sufficient!

Higher orders are needed !

$$\overline{|M|^2}(gg \to t\bar{t}) = (4\pi\alpha_s)^2 \left(\frac{(p_1 + p_2)^4}{24(p_1 \cdot p_3)(p_2 \cdot p_3)} - \frac{8}{9}\right)$$

$$\times\left(4\frac{(p_1 \cdot p_3)^2 + (p_2 \cdot p_3)^2}{(p_1 \cdot p_2)^4} + \frac{4m_t^2}{(p_1 + p_2)^2} - \frac{m_t^4(p_1 + p_2)^4}{(p_1 \cdot p_3)^2(p_2 \cdot p_3)^2}\right) \qquad (5)$$

LO: $\sigma \sim \alpha_s^2$

Theory for top-pair production X-section is at NNLO:

Cross section is expanded into series of **strong coupling constant α_s**:
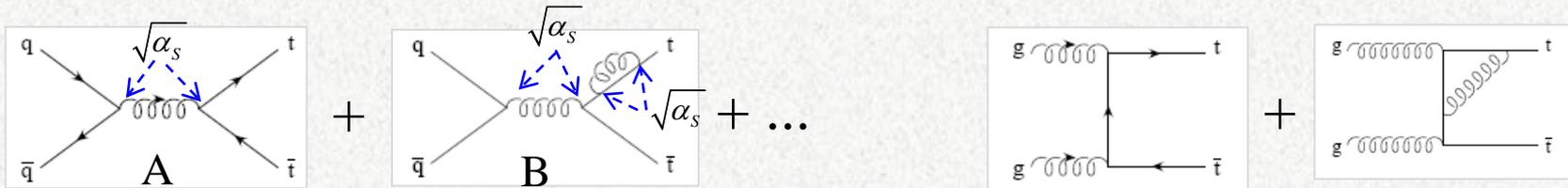
$$\sigma_{ij}\left(\beta,\frac{\mu^2}{m^2}\right)=\frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)}+\alpha_S\left[\sigma_{ij}^{(1)}+L\sigma_{ij}^{(1,1)}\right]+\alpha_S^2\left[\sigma_{ij}^{(2)}+L\sigma_{ij}^{(2,1)}+L^2\sigma_{ij}^{(2,2)}\right]+O\left(\alpha_S^3\right)\right\} \quad (6)$$

$$LO \sim \alpha_S^2, \quad NLO \sim \alpha_S^3, \quad NNLO \sim \alpha_S^4 \cdots \qquad \beta=\sqrt{1-4m^2/s}$$

$\sqrt{s} \equiv$ parton collision energy

$L \equiv \ln\beta$ is big log term

$\mu \equiv$ factorization scale

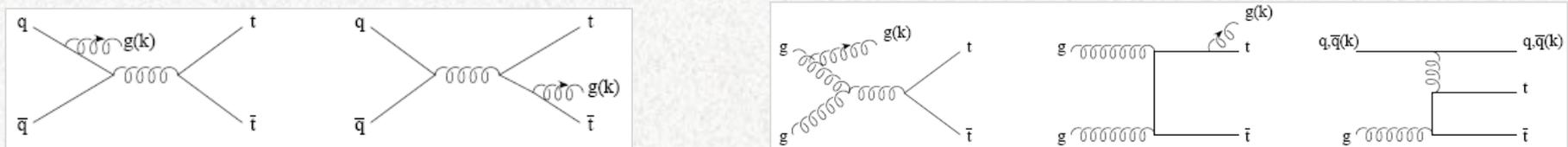**NLO**: virtual and real corrections are added to LO

Virtual corrections:

Taking $\left|A+B\right|^2=(A+B)(A^*+B^*)=...+AB^*+A^*B \sim \alpha_S^3$, amplitudes A and B are summed as they are experimentally indistinguishable.

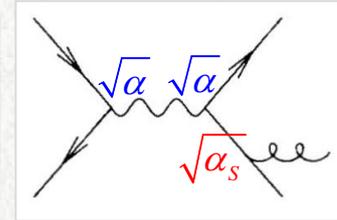**Real and virtual corrections $\rightarrow$ divergencies!**

Real corrections – with real gluons ($\sim \alpha_S^3$):

On divergencies – process $e^+e^- \to q\bar{q}g$

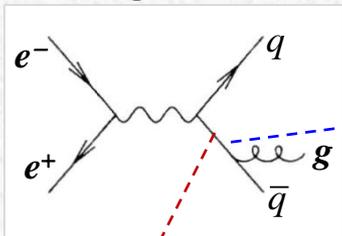Using the Feynman rules for process $e^+e^- \to q\bar{q}g$ - total cross section:

$$\sigma^{q\bar{q}g} = \sigma_0 \, 3 \sum_q Q_q^2 \int dx_1 dx_2 C_F \frac{\alpha_S}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}, \quad \sigma_0 = \frac{4\pi\alpha^2}{3s} \qquad (7)$$

Where $x_1 = 2E_q/\sqrt{s}$ and $x_2 = 2E_{\bar{q}}/\sqrt{s}$ are the energy fractions of q and $\bar{q}$

$\sigma^{q\bar{q}g}$ diverges at $x_i = 1$, since $1 - x_2 = x_1 E_g (1 - \cos\theta_{1g})/\sqrt{s}$ and $1 - x_1 = x_2 E_g (1 - \cos\theta_{2g})/\sqrt{s}$

$\sqrt{s} \equiv$ collision energy, $E_g \equiv$ gluon energy, $\theta_{ig} \equiv$ angle between gluon and quark (i=1,2)

The singularities – from soft and collinear gluon:

$$\text{propagator} \sim \frac{1}{(p+k)^2 - m^2} = \frac{1}{2E_p E_k} \cdot \frac{1}{1 - \beta_p \cos\theta}, \quad \beta_p = \sqrt{1 - m^2/E_p^2} \qquad (8)$$

$$(p+k)^2 - m^2 = p^2 + 2p\cdot k + k^2 - m^2 = 2p\cdot k = 2\left(E_p E_k - \vec{p}\cdot\vec{k}\right)$$

$$= 2\left(E_p E_k - |\vec{p}|\cdot|\vec{k}|\cos\theta\right) = 2E_p E_k \left(1 - \sqrt{1 - m^2/E_p^2}\cos\theta\right)$$

Virtual particle: $P^2 = E^2 - \vec{P}^2 \neq m^2$
is described by propagator
For final state particles $q, \bar{q}, g : p^2 = m^2$

$P = p + k \Rightarrow p$ and k are 4-momenta of quark and gluon

Dimensional regularization: go to **n = 4-2ε** dimensions (or $\varepsilon = (4\text{-}n)/2$):

$$\sigma^{q\bar{q}g}\left(\varepsilon\right) = \sigma_0 3 \sum_q Q_q^2 H(\varepsilon) \int dx_1 dx_2 C_F \frac{2\alpha_S}{3\pi} \frac{x_1^2 + x_2^2 - \varepsilon\left(2 - x_1 - x_2\right)}{\left(1 - x_1\right)^{1+\varepsilon}\left(1 - x_2\right)^{1+\varepsilon}} \qquad (9)$$

$$H(\varepsilon) = \frac{\mathbf{3(1-\varepsilon)^2}}{\mathbf{(3-2\varepsilon)\Gamma(2-2\varepsilon)}} = \mathbf{1} + O(\varepsilon)$$

For total cross section, after integration of eq. 9:

Real gluon emission:

$$\sigma^{q\bar{q}g}\left(\varepsilon\right) = \sigma_0 3 \sum_q Q_q^2 H(\varepsilon)\left[\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + \frac{19}{2} + O(\varepsilon)\right] \qquad (10)$$

Virtual gluon contribution - dimensional regularization controls infra-red divergences:

$$\sigma^{q\bar{q}(g)}\left(\varepsilon\right) = \sigma_0 3 \sum_q Q_q^2 H(\varepsilon)\left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + O(\varepsilon)\right] \qquad (11)$$

Adding (10) and (11) the result is finite in the limit $\varepsilon \to 0$:

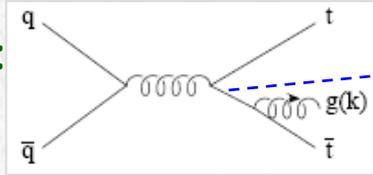$$R = \frac{\sigma^{q\bar{q}g} + \sigma^{q\bar{q}(g)}}{\sigma\left(ee \to \mu\mu\right)} = 3 \sum_q Q_q^2 \left\{1 + \frac{\alpha_S}{\pi} + O(\alpha_S^2)\right\} \qquad (12)$$

Summing the real and virtual corrections the divergences are canceled out !
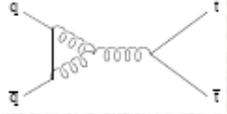
A few top Cross Section issues

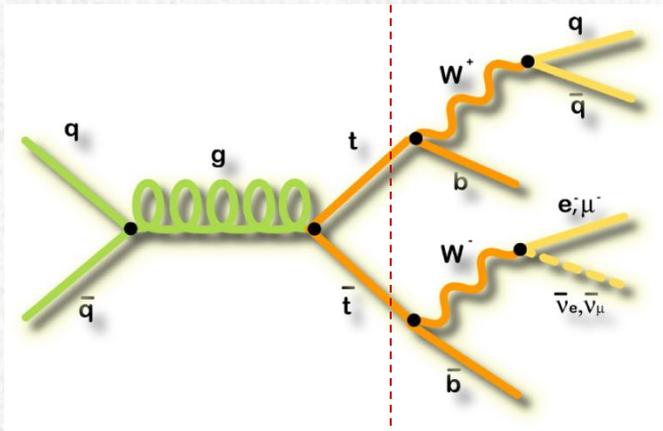Higher order real and virtual corrections exhibit IR and UV divergences:

Example:

$$\text{propagator} = \frac{1}{(p+k)^2} = \frac{1}{2E_p E_k} \cdot \frac{1}{1-\beta_p \cos\theta}, \quad \beta_p = \sqrt{1 - m^2/E_p^2}$$

✓ IR singularity: $E_k \to 0$ and $1-\beta_p \cos\theta \to 0 \Rightarrow$ cancelled when *X-sec* of virtual and real emission are summed also mass singularities are cancelled $\Rightarrow$ Cancelation is not full $\Rightarrow$ presence of big logs (*L*) in X-sec terms !

✓ UV singularities in loops ( ) are handled by renormalization.

In real we observe $t\bar{t}$ decay products not $t\bar{t}$ pair Factorization is used based on the narrow width approximation:

✓ polarized top quarks are produced on mass shell
✓ polarized on-shell top quarks decay

Narrow width app. vs direct $pp \to WWbb$:

Offshelness +$WWbb$ can go not only through $t\bar{t}$ intermediate state!

Top cross section - measurement

Selection criteria: trigger + offline selection $\Rightarrow$ candidate events

❑ Depend on the analysed channel: $t\bar{t}$ production

- lepton+jets (LJ), dilepton (DL) and all hadronic mode (AH)

- *lν2b2j*   *2(lν)2b*   *2b4j*   all: *+1j, 2j…*

- LJ: single lepton high-p_T (E_T) trigger applied + Reconstructed level:

  ✓ 1 high-p_T lepton + $\geq$4 high-p_T jets (1-2b-tagged) + high $\not{E}_T$

  ✓ Restricted on pseudo-rapidity, p_T (E_T) > 20 GeV, $\not{E}_T$ > 20 GeV

- What are selection criteria for DL and AH?

❑ Background processes – non $t\bar{t}$ events also pass *Selection criteria*:
- Basic bkgd processes for LJ channel:
  - ✓ W+jets, Z+jets, diboson, single top quark, QCD multijets
- Bkgd processes: studied using MC + data driven techniques

$$\sigma_{t\bar{t}} = \frac{N_{\text{obs}} - N_{\text{bkg}}}{A \cdot \varepsilon \int L dt}$$

$N_{\text{obs}}(N_{\text{bkg}}) \equiv$ observed (expected bkgd) events
$A \equiv$ acceptance, $\varepsilon \equiv$ trigger efficiency, $L \equiv$ luminosity

Top pair QCD NNLO cross section calculation complemented with NNLL resummation

$$\text{NNLO}_{\text{appr}} \xrightarrow{5\%} \text{NNLO} \xrightarrow{2.8\%} \text{NNLO} + \text{NNLL}$$

Experiment, LHC, $t\bar{t}$ cross section:

5 TeV: $67.5 \pm 0.9_{\text{stat}} \pm 2.3 \pm 1.1$ pb, **3.9%** ATL

theory: $\sigma_{\text{theo}}^{t\bar{t}}(5\,\text{TeV}) = 68.2 \pm 4.8_{-2.3}^{+1.9}$ pb

7 TeV: 178 ± 4.7 pb, **2.5%** ATLAS+CMS

8 TeV: 243 ± 5.9 pb, **2.5%** ATLAS+CMS

13 TeV: $829 \pm 13_{\text{stat+syst}} \pm 8_{\text{lumi}}$ pb, **1.8%** ATL

The first results at 13.6 TeV:

13.6 TeV: $887 \pm 42_{\text{stat+syst}} \pm 53_{\text{lumi}}$ pb, **7.6%** CMS
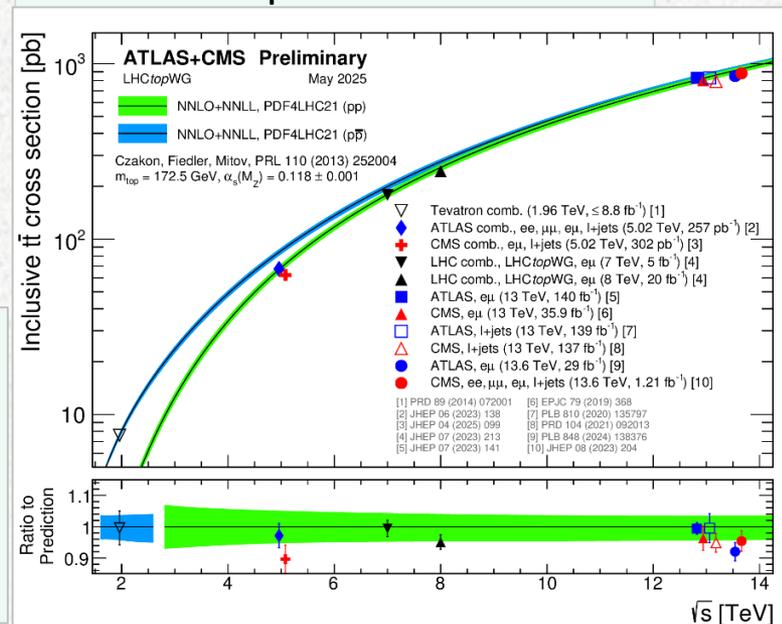
$850 \pm 18_{\text{stat+syst}} \pm 20_{\text{lumi}}$ pb, **3.2%** ATLAS

Theory: $\sigma_{\text{theo}}^{t\bar{t}}(13.6\,\text{TeV}) = 921_{-40}^{+32}$ pb

NNLO+NNLL	σ_{tot} [pb]	scales [pb]	pdf [pb]
Tevatron	7.164	+0.110 (1.5%) / −0.200 (2.8%)	+0.169 (2.4%) / −0.122 (1.7%)
LHC 7 TeV	172.0	+4.4 (2.6%) / −5.8 (3.4%)	+4.7 (2.7%) / −4.8 (2.8%)
LHC 8 TeV	245.8	+6.2 (2.5%) / −8.4 (3.4%)	+6.2 (2.5%) / −6.4 (2.6%)
LHC 14 TeV	953.6	+22.7 (2.4%) / −33.9 (3.6%)	+16.2 (1.7%) / −18.8 (1.9%)

PRL 110, 252004 (2013)
Theoretical uncertainties
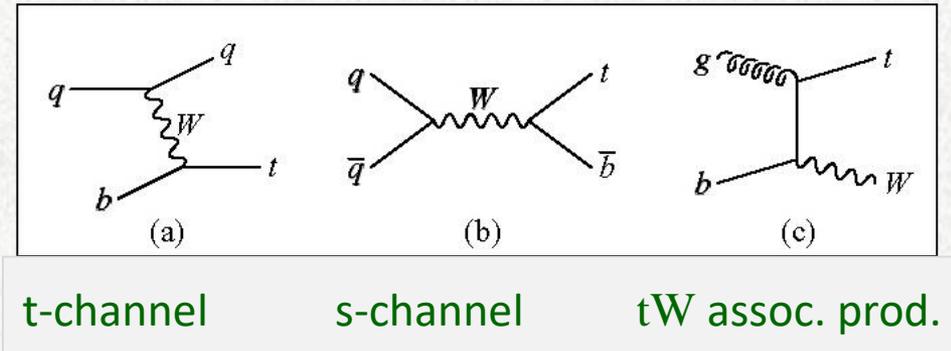
Assumed top mass: 172.5 GeV
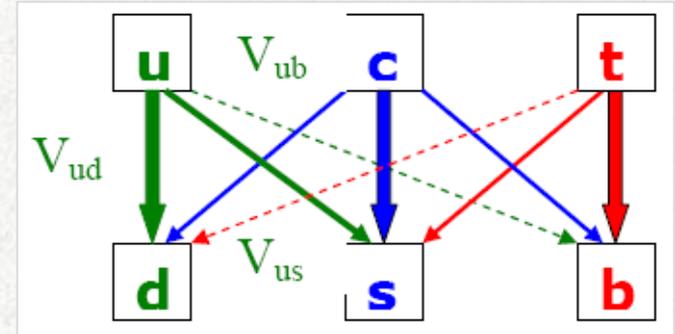
Single Top Quark production

Single top quark production

Production via weak forces

- X-section ~ $\left| V_{tb} \right|^2$ (direct measurement of V_{tb})

- Single top –test of V-A structure of EW

- Significant bckgd to Higgs signal

- Possible new physics

| t-channel | s-channel | tW assoc. prod. |

Cabbibo–Kobayashi–Maskawa matrix → weak charge current transitions change quark flavor
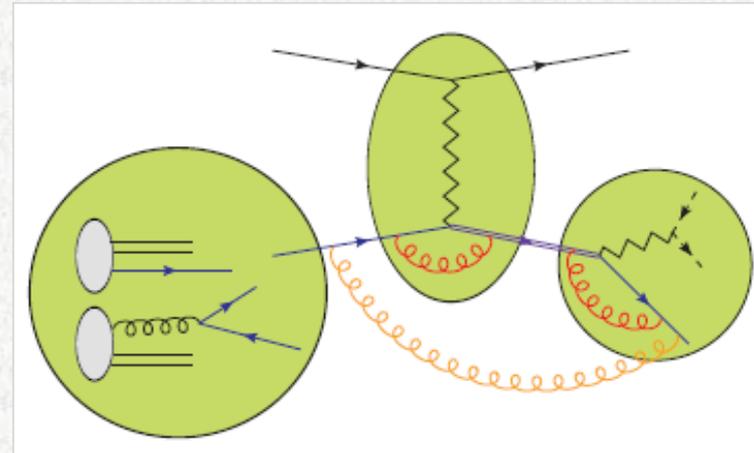

Event characteristics

✓ Only 1 isolated high p_T lepton (e or μ): $p_T >\sim$ 20 GeV (t- and s- channel)

✓ 2 leptons $\ell^+\ell^-$ (usually $e^\pm\mu^\mp$) with high p_T (tW channel)

✓ High miss-p_T (E_T) ($p_T >\sim$ 25 GeV)

✓ 2 or 3 high p_T jets ($p_T > \sim$20 GeV)

✓ $\geq$ 1 b-tagged jet

Experiment: leptonic decays taken to cope with huge background !

Single top quark cross section

Present status:

- ✓ Production and decay are factorized
- ✓ NLO corrections in production
- ✓ resummation of soft logs
- ✓ top decay, at LO/NLO, spin correlations
- ✓ off-shell effects / non-factorizable corrections
- ✓ b quark issues (m_b mass) ...

Variety of theoretical approches:
Go to NNLO taking big logs

Theory: the cross section summary

$\sqrt{s}$ [TeV]	$\sigma_{\text{t-ch}}^{\text{theo}}$ [pb]	$\sigma_{tW}^{\text{theo}}$ [pb]	$\sigma_{\text{s-ch}}^{\text{theo}}$ [pb]
7	$63.89_{-0.5-0.7}^{+0.9+1.1}$	$17.1_{-0.3-0.7}^{+0.4+0.7}$	$3.00_{-0.03}^{+0.03}$
8	$84.3_{-0.7-0.9}^{+1.1+1.4}$	$24.4_{-0.5-0.9}^{+0.6+0.9}$	$3.61_{-0.03}^{+0.03}$
13	$214.2_{-1.7-2.0}^{+2.4+3.2}$	$79.3_{-1.8-2.2}^{+1.9+2.2}$	$6.84_{-0.03}^{+0.06}$
13.6	$232.2_{-1.7-2.2}^{+2.6+3.4}$	$87.9_{-2.1-2.4}^{+2.0+2.3}$	$7.25_{-0.04}^{+0.06}$

Single top: experimental analysis

Single top quark production first observed by D0 and CDF in 2009

Main problem in experiment:

huge background – an example from
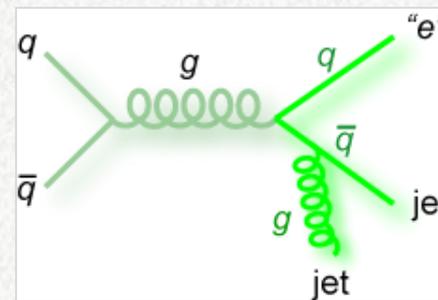
CDF: a single top anlysis at L=3.2 fb^{-1}

Single top	145.7 ±	21.4
Total background	2119.3 ±	350.9
Total prediction	2265.0 ±	375.4
Observed	2229	

Main Backgrounds

W + jets	top-antitop pairs	QCD multijet production

To cope with backgrounds: multivariete techniques are used!

Multivariate techniques

To cope with background Multivariate techniques (MVT) are used:
- ✓ Neural Networks (NN)
- ✓ Boosted Decision Tree (BDT)
- ✓ Matrix Element (ME)
- ✓ Likelihood Discriminants (LD)

Basic idea: a set of different kinematic variables ($M_{l\nu b}$, H_{T}, M_{jj}, M_{T}...) is used as input for MVT which employ them to optimize Signal vs Background.

Output of MVT: output discriminat – a variable in (0,1) or (-1, 1)

ATLAS

single top t-channel
NN output
(17 kin. variables)

13 TeV

CMS

single top Wt-channel
BDT output
(15 kin. variables)

X-sec of single top quark production in the **t-channel**, L=140 fb^{-1}, pp collision data at √s = 13 TeV

SM expectation: $\sigma(tq + \bar{t}q) = 214.2^{+2.4+3.2}_{-1.7-2.0}$ pb

Event selection: exactly one charged lepton (e or μ) with $p_T(\ell) \geq 28$ GeV, exactly two jets $p_T \geq 30$ GeV $\rightarrow 1$ b-tagged, and $|\eta| < 4.5$, $\not{E}_T > 30\,\text{GeV}$, $m_T(W) > 50\,\text{GeV}$[1]

Process	SR plus	SR minus
tq	$169\,000 \pm 6000$	150 ± 150
$\bar{t}q$	90 ± 90	$109\,000 \pm 4000$
$tW + \bar{t}W, t\bar{b} + \bar{t}b$	$51\,000 \pm 4000$	$49\,000 \pm 4000$
$t\bar{t}$	$265\,000 \pm 14\,000$	$265\,000 \pm 14\,000$
$W + b\bar{b}$	$198\,000 \pm 21\,000$	$159\,000 \pm 17\,000$
$W + c(\bar{c})$	$60\,000 \pm 13\,000$	$49\,000 \pm 11\,000$
Z+jets, diboson	$21\,000 \pm 4000$	$19\,000 \pm 4000$
Multijet	$50\,000 \pm 10\,000$	$50\,000 \pm 10\,000$
Total	$814\,000 \pm 2100$	$698\,800 \pm 2000$
Observed	$814\,185$	$698\,845$

Post-fit event yields in the two SRs

NN discriminant:

17 input variables in the jet data set:

$m(jb) \equiv$ mass of b and untagged jet
$\eta(j_u)| \equiv$ absolute value of untagged jet η
$|\Delta p_T(W, jb)| \equiv$ diff, in p_T between W and jet pair, $m(t) \equiv$ top mass,…

$\sigma(tq) = 137^{+8}_{-8}$ pb and $\sigma(\bar{t}q) = 84^{+6}_{-5}$ pb

$\sigma(tq + \bar{t}q) = 221^{+13}_{-13}$ pb

$R_t = \sigma(tq)/\sigma(\bar{t}q) = 1.636^{+0.036}_{-0.034}$ pb

[1] $m_T(W) = \sqrt{2 p_T(\ell)\,\not{E}_T\,(1 - \cos\Delta\phi(\vec{p}_T, \ell))}$

CMS: single top quark Wt-channel

SM prediction: $\sigma_{tW}^{\text{SM}} = 71.7 \pm 1.8\left(\text{scale}\right) \pm 3.4\left(\text{PDF}\right)\text{pb}$

Selection:

- $e^{\pm}\mu^{\mp}$ events in final state
- $1e$ with $p_{\mathrm{T}} > 12$ GeV + 1μ with $p_{\mathrm{T}} > 23$ GeV or 1μ with $p_{\mathrm{T}} > 8$ GeV + $1e$ with $p_{\mathrm{T}} > 23$ GeV
- ≥ 1jet with $p_{\mathrm{T}} > 30$ GeV and $|\eta| < 2.4$, ≥ 1 b-jet

Most significant region: 1j1b (1 b-tagged jet in event)

Measured Xsec in the tW-channel: inclusive + differential
BDT used to separate signal vs bkgd.

JHEP07(2023)046

Process	1j1b	2j1b	2j2b
tW	$31\,600 \pm 600$	$16\,600 \pm 500$	$5\,500 \pm 200$
$t\bar{t}$	$131\,200 \pm 500$	$160\,300 \pm 600$	$141\,100 \pm 400$
Drell–Yan	$3\,990 \pm 190$	$1\,630 \pm 100$	260 ± 20
VV+$t\bar{t}$V	$2\,800 \pm 300$	$3\,300 \pm 500$	$1\,700 \pm 400$
Non-W/Z	$1\,140 \pm 150$	$3\,700 \pm 700$	470 ± 120
Total	$170\,800 \pm 300$	$185\,400 \pm 400$	$149\,100 \pm 300$
Data	$170\,900 \pm 400$	$185\,400 \pm 400$	$148\,900 \pm 400$

$\sigma_{tW} = 79.2 \pm 0.9\left(\text{stat}\right)^{+7.7}_{-8.0}\left(\text{syst}\right) \pm 1.2\left(\text{lumi}\right)\text{pb}$

Differential tW x-sec vs jet p_{T}

ATLAS: single top quark, s-channel

SM prediction, NLO in QCD: $\sigma_s^{SM} = 10.32^{+0.40}_{-0.36}$ pb

JHEP 06 (2023) 191

- exactly one lepton with $p_T > 30$ GeV, ≥ 2 jets with $p_T > 25$ GeV & $|\eta| < 2.5$
- $E_T^{miss} > 35$ GeV and $m_T^W > 30$ GeV to suppress multijet
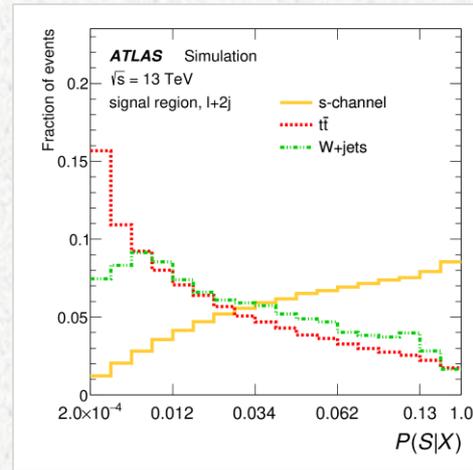
Matrix element method (MEM):

Transfer function

Probab. that H_{proc} leads to observed event X

$$\mathcal{P}\left(X|H_{proc}\right) = \int d\Phi \frac{1}{\sigma_{H_{proc}}} \frac{d\sigma_{H_{proc}}}{d\Phi} \cdot T_{H_{proc}}\left(X|\Phi\right)$$

$$P\left(S|X\right) = \frac{\sum_i P(S_i)\mathcal{P}\left(X|S_i\right)}{\sum_i P(S_i)\mathcal{P}\left(X|S_i\right) + \sum_j P(B_j)\mathcal{P}\left(X|B_j\right)}$$
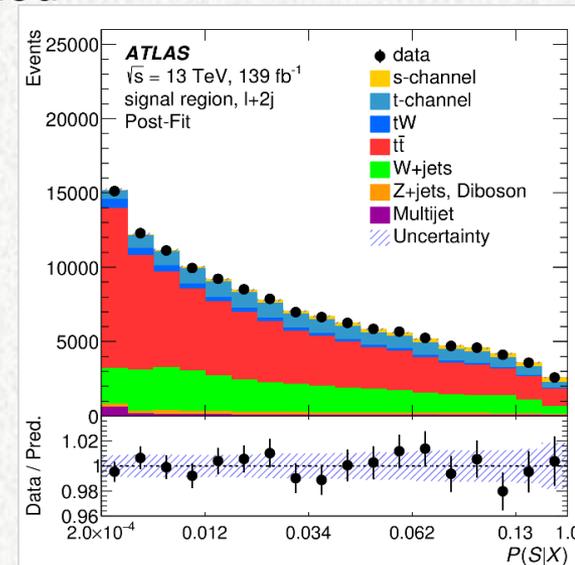
S_i (B_j) ≡ signal (bkgd) processes considered

Single-top s-channel X-sect. is measured by means of a binned profile maximum-likelihood fit of the MEM discriminant in SR.

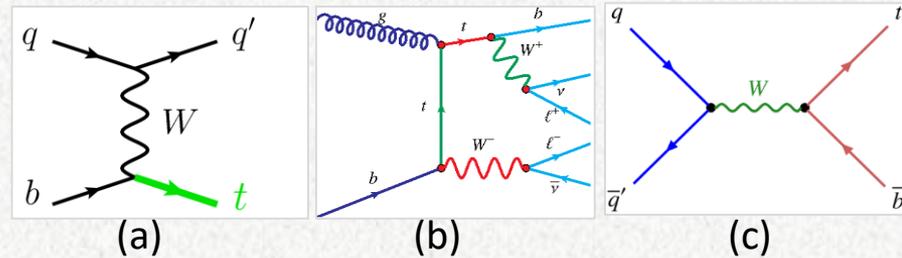| Process	Event yield	
	Pre-fit	Post-fit
s-channel	$4\,200 \pm 710$	$3\,700 \pm 1\,100$
t-channel	$13\,000 \pm 2\,000$	$15\,000 \pm 2\,300$
tW	$3\,680 \pm 970$	$4\,250 \pm 1\,100$
$t\bar{t}$	$76\,000 \pm 12\,000$	$70\,600 \pm 4\,200$
$W+$jets	$21\,500 \pm 2\,900$	$32\,200 \pm 5\,000$
$Z+$jets, VV	$2\,400 \pm 1\,400$	$2\,900 \pm 1\,600$
Multijet	$2\,150 \pm 650$	$1\,700 \pm 540$
Total	$123\,000 \pm 17\,000$	$130\,310 \pm 620$
Data		$130\,310$

Experiment:

$$\sigma_{s\text{-}ch} = 8.2^{+3.5}_{-2.9}\,\text{pb}$$

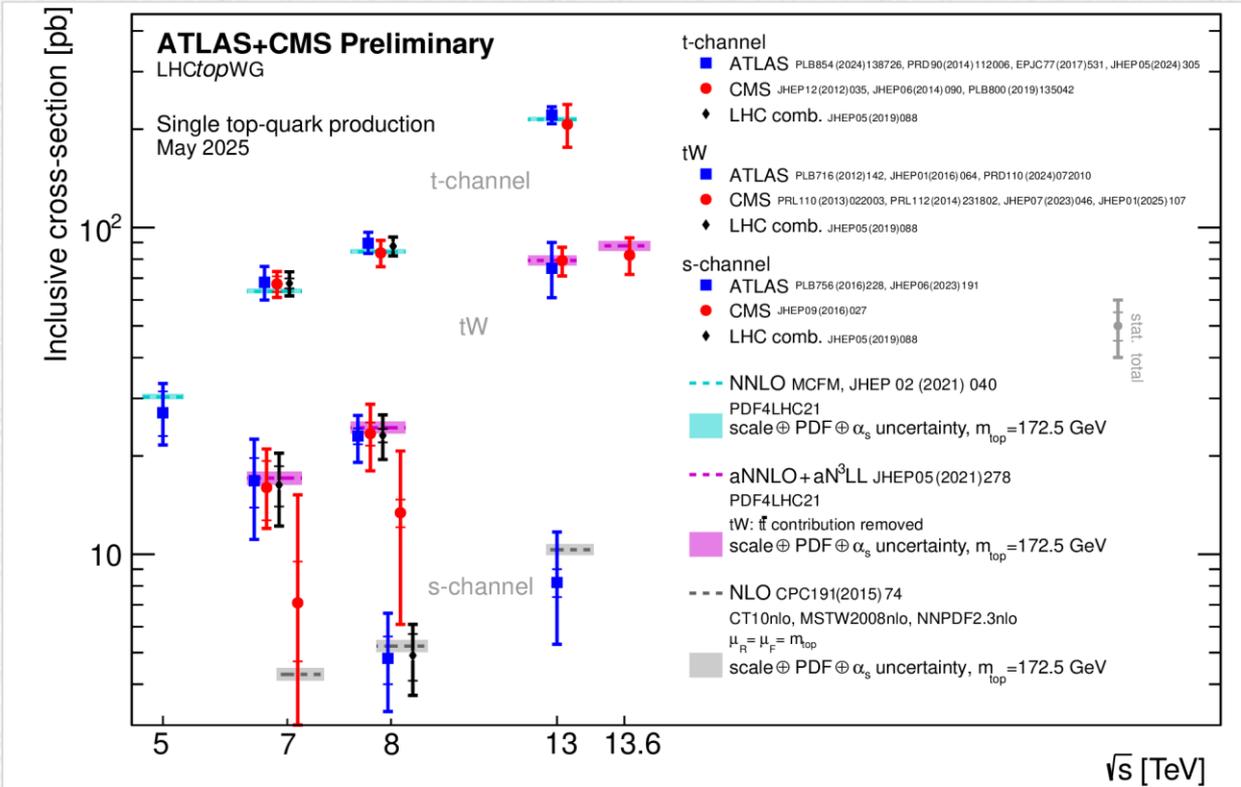Observed (expected) signal significance: 3.3 (3.9) S.D.

Theory vs Experiment: EW single-top production

Single top quark – EW interaction
t-channel (a), tW-channel (b), s-channel (c)

(a)   (b)   (c)

Experimental results vs Theory predictions:

CMS first Run3 results on tW-channel at 13.6 TeV, L = 37.4 fb^{-1}:

$$\sigma_{tW} = 82.3 \pm 2.1\,(\text{stat})^{+9.9}_{-9.7}\,(\text{syst}) \pm 3.3\,(\text{lumi})\,\text{pb}$$

$|V_{tb}| > 0.97$ at 95% CL

ATLAS:
$|V_{tb}| > 0.95$ at 95% CL
$\sqrt{s} = 13$ TeV

New results on tW-channel (13 TeV):

ATLAS: $\sigma_{tW} = 75 \pm 1\,(\text{stat})^{+15}_{-14}\,(\text{syst}) \pm 1\,(\text{lumi})\,\text{pb}$

CMS: $\sigma_{tW} = 79.2 \pm 0.9\,(\text{stat})^{+7.7}_{-8.0}\,(\text{syst}) \pm 1.2\,(\text{lumi})\,\text{pb}$
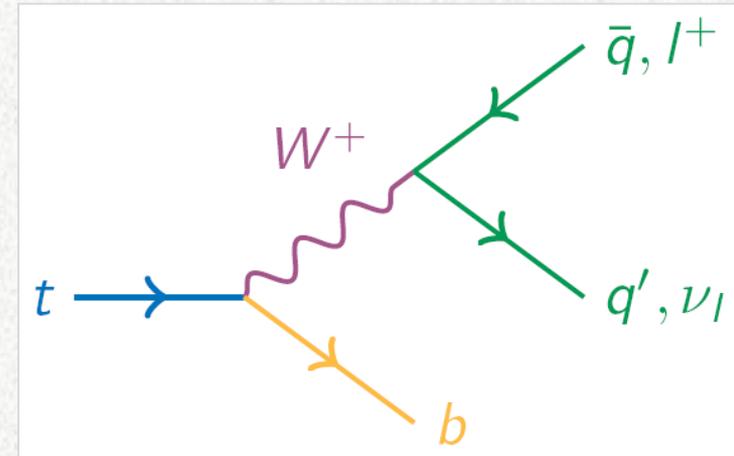
Top Quark Mass

Top quark mass is very important parameter of the SM, in addition:

- ✓ Its role in consistency test of the SM.
- ✓ Electroweak vacuum stability.
- ✓ What is the top quark mass and how to measure it.

Top quark mass: Motivation

❑ The top quark mass (m_{top}) is one of the fundametal SM parameters.

❑ Its precise value provides a key input to global EW fit $\Rightarrow$ test of internal consistency of the SM.

❑ Its value leads to a significant constraints on stability of the EW vacuum.

❑ it has a significant impact on cosmological models with inflation-

❑ It looks the top quark mass can be easily determined using the top quark decay products – it is sufficient to measure **4-momentum of b quark** and 4-momenta of **W boson decay producs:**

$$\longrightarrow \quad m_{top}^2 = \left(E_W + E_b\right)^2 - \left(\vec{p}_W + \vec{p}_b\right)^2$$

… but there are some important ambiguities…

- Measured values of m_t, m_W & m_H can be compared to EW fit predictions to check validity of SM $\rightarrow$ higher order corrections to W/Z propagator leads to

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F}\left(1+\Delta r\right),\quad \Delta r=\Delta\alpha+\frac{s_W}{c_W}\Delta\rho+\left(\Delta r\right)_{nl}$$

http://project-gfitter.web.cern.ch/project-gfitter/

$\Delta\rho \sim m_t^2$

$\left(\Delta r\right)_{nl} \propto \ln M_H$

A set of N_{exp} precisely measured observables described by N_{exp} theor. expressions with N_{mod} parameters

Contours of 68% and 95% CL obtained from scans of fits with fixed variable pairs M_W vs m_t - Gfitter package used

Present status of the SM: high level of internal consistency !

LHC: **Higgs boson is firmly confirmed** $\Rightarrow$ under SM, vacuum has **non-zero component of Higgs field** (it is a Higgs condensate)

Higgs potential: $V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2, \quad \phi = \dfrac{\phi_1 + i\phi_2}{\sqrt{2}}$
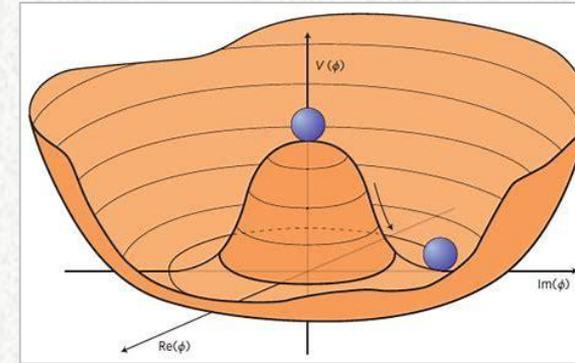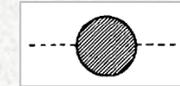
for $\mu^2 < 0$ and $\lambda > 0$

What can be said about its stability?

Higgs boson is a dynamic object $\rightarrow$ quantum fluctuations $\Rightarrow \lambda$ is running constant depending on an energy scale :

$\lambda(\mu_R) = \dfrac{G_F M_H}{\sqrt{2}} + \Delta\lambda(\mu_R)$

present status: $T_{\text{vac}} \gg T_{\text{age}}$

✓ SM: $\Delta\lambda$ we can calculate it if there is no new physics
✓ What does it mean $\lambda < 0$?

Vacuum is metastable $\rightarrow$ tunneling from a local to absolute minimum!

3σ bands in
$M_t = 173.1 \pm 0.6$ GeV (gray)
$\alpha_3(M_Z) = 0.1184 \pm 0.0007$ (red)
$M_h = 125.7 \pm 0.3$ GeV (blue)

Higgs quartic coupling λ

$M_t = 171.3$ GeV

$\alpha_s(M_Z) = 0.1205$

$\alpha_s(M_Z) = 0.1163$

$M_t = 174.9$ GeV

RGE scale μ in GeV

Instability

Top pole mass M_t in GeV

Meta–stability

$1,2,3\,\sigma$

Stability

Higgs pole mass M_h in GeV

Precise values of M_{top}, M_H and α_S needed!

Degrassi et al., JHEP 1208 (2012) 098

Particle pole mass: corresponds to pole in propagator of „free" particle $\sim 1/(q^2 - m^2)$

The top quark pole mass (m^t_{pole}): top quark is a color object – due to confinement the ambiguity of pole top mass is $\Delta m^t_{pole} \sim \Lambda_{QCD}$

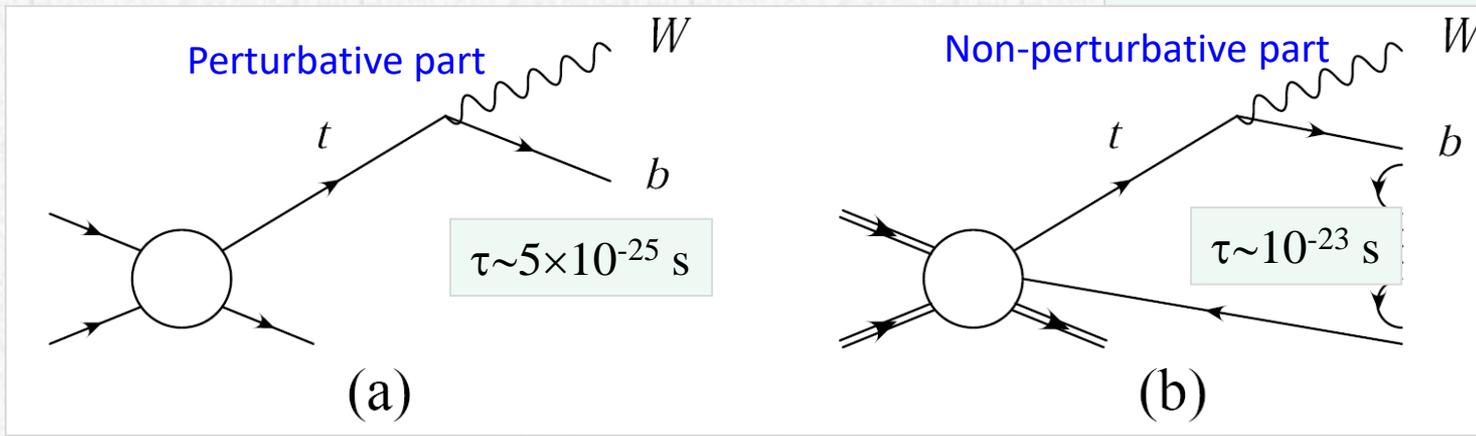$$\left(\alpha_s(Q^2) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)\ln\left(Q^2/\Lambda^2_{QCD}\right)} \right)$$

Perturbative part · W

t · b

$\tau \sim 5 \times 10^{-25}$ s

(a)

Non-perturbative part · W

t · b

$\tau \sim 10^{-23}$ s

(b)

Top quark pole mass is close to invariant mass of the top decay products.

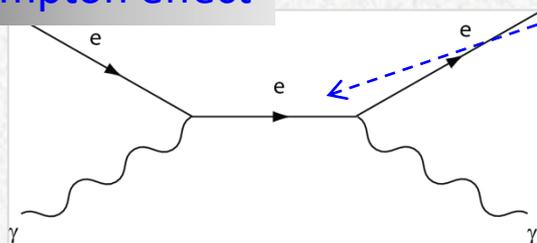Ambiguities are from: hadronization, color reconnection and extra radiation :

✓ at least one quark **not coming from top decay** is trapped by b-quark (non perturbative part).

✓ **Color reconnection**: the colored top quark decay product will interact with the colored rest of proton.

✓ **Extra radiation** from top decay products not included in reconstruction.

Particle mass – preliminary words

Let us start with electron

Compton effect

Electron propagator $\equiv \dfrac{i(\not{p}+m)}{p^2-m^2+i\varepsilon}$, $\quad \not{p}=p_\mu\gamma^\mu$

(1)

Electron mass = pole in electron propagator
$\rightarrow$ it is free-field propagator

divergent renormalization

Propagator correction:

Electron self energy: 1PI $\equiv$

Propagator in general:

Characterized by loop momentum μ or size $1/\mu$

$$\frac{i}{\not{p}-m_{pole}}=\frac{i}{\not{p}-m_0}+\frac{i}{\not{p}-m_0}\left(\frac{\Sigma(\not{p})}{\not{p}-m_0}\right)+\frac{i}{\not{p}-m_0}\left(\frac{\Sigma(\not{p})}{\not{p}-m_0}\right)^2\cdots=\frac{i}{\not{p}-m_0-\Sigma(\not{p})}$$ (2)

bare mass

Other expansion: m_0 + all bubbles with size < $1/\mu_R$ are absorbed into mass ⟶

$$\frac{i}{\not{p}-m_{pole}}=\frac{i}{\not{p}-m(\mu_R)-\Sigma(\mu_R,\not{p})}$$ (3)

$m(\mu_R) \equiv$ short distance mass, $\Sigma \equiv$ self-energy: contribution of interactions to mass

Particle mass: top quark vs electron

Difference top quark *vs.* electron:

- ✓ top is unstable – pole is complex: $m_{top} + i\Gamma_{top}$
- ✓ Top is colored object - due to confinement its mass uncertainty $\sim \Lambda_{QCD}$

Top quark is not asymptotically free particle!

Top quark self energy Σ in 1 loop approx.:

$$\Sigma^{(1)} = \quad \overset{q}{\underset{p \quad t \quad p\text{-}q}{}} \quad + \quad \overset{W}{\underset{b}{}}$$

Loops need to be integrated over q (4-momentum conservation law: $p = q + (p\text{-}q)$ valid for all q).

$\Sigma^{(1)} = \int dq$... diverges for $|q| \to \infty$ (UV divergence) $\Rightarrow$ removes by renormalization

for $|q| \to 0$ (IR divergence) $\Rightarrow$ top quark non-perturbativeness problems

Does not exist for electron

If loop size reaches a value $d \sim 1/\text{GeV}$, the coupling α_S gets big!

What is the Top Quark Mass ?

Introducing top quark short range mass, the top pole mass is

$$m_t^{\text{pole}} = m_t^{\text{MSR}}(R) + \Sigma^{\text{fin}}(R, \mu) \qquad (7)$$

Absorbs bubbles with size > 1/R (problematic region) it contains perturbative and non-perturbative part

Absorbs all self energy bubbles

Absorbs self energy bubbles with size <1/R (perturb. region)

Relation between the top-quark pole and short range mass, $R \sim 1$ GeV   PRL 117, 232001 (2016)

$$m_t^{\text{pole}} - m_t^{\text{MSR}}(R) = R\left(c_1 a + c_2 a^2 + c_3 a^3 + \ldots\right), a \equiv \alpha_S^{(5)}(R)\big/4\pi, c_1 = 5.33, c_2 = 131.79, c_3 = 4699.7$$

Advantage of short range mass: well defined mass from renormalization view point

✓ more stable in **perturbation theory** $\Rightarrow m_t^{\text{MSR}}(R=1$ GeV$)$ - close to the notion of kinematic mass, but no renormalon problems.

✓ A special case is $\overline{\text{MS}}$ mass: $m_{t,\text{MSR}}\left(R = \bar{m}(\bar{m})\right) = \bar{m}(\bar{m})$ $\Rightarrow$ it absorbs the self energy corrections $> \bar{m} \Rightarrow$ sensitive to only the short distance aspects of QCD.

Pole mass vs $\overline{\text{MS}}$ mass perturbative expansion (4 loops approximation):

arXiv:1502.01030

$$m_t^{\text{pole}} = \bar{m}(\bar{m})\left(1 + 0.4244\,\alpha_s + 0.8345\,\alpha_s^2 + 2.8345\,\alpha_s^3 + (8.49 \pm 0.25)\,\alpha_s^4 + \ldots\right) \qquad (8)$$

$$= 163.643 + 7.557 + 1.617 + 0.501 + 0.195 \pm 0.005 \text{ GeV}$$

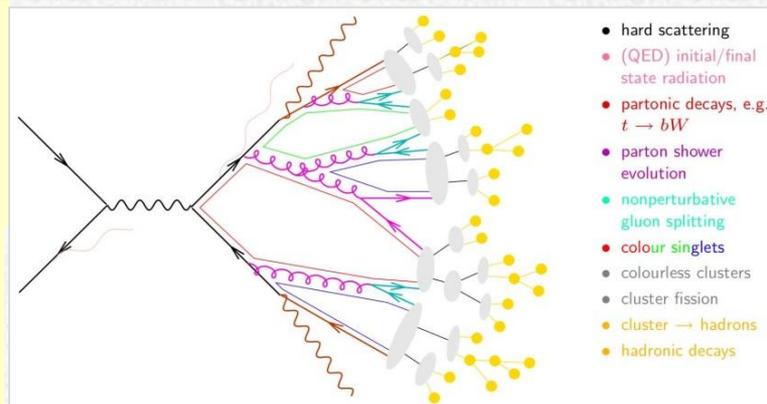$$\alpha_s \equiv \alpha_s^{(6)}(\bar{m}) = 0.1088$$

Short range mass vs reconstructed Top Quark Mass ?

How to relate the reconstructed top quark mass to short range or pole top quark mass?

MC event generators simulate:

Hard matrix element:

Initial parton annihilation and top production + additional hard partons from pQCD.

- hard scattering
- (QED) initial/final state radiation
- partonic decays, e.g. $t \to bW$
- parton shower evolution
- nonperturbative gluon splitting
- colour singlets
- colourless clusters
- cluster fission
- cluster → hadrons
- hadronic decays

Parton shower evolution:

parton shower evolution describes the top decay products and continued splitting into higher multiplicity partonic states having subsequently lower virtualities.

Splitting probabilities from $p\mathrm{QCD}$ (valid to leading order for the top decay to an approximate leading logarithmic accuracy for soft-collinear splitting).

Can be viewed as a way to sum **dominant perturbative corrections** down to shower cut $\Lambda_s \approx 1$ GeV.

Hadronization model:

Turns partons into hadrons (**non-perturbative process**) - dependent on parton shower implementation.

Description of data (frequently) much better than the conceptual (LL) precision of parton evolution part.

How to measure top mass?

Top quark mass can be reconstructed in all $t\bar{t}$ topologies (LJ, DL AH)

Two basic mass determination approaches:

Direct measurements

- ✓ Extraction from total or partial kinematic reconstruction of invariant mass of top-quark decay products.
- ✓ Comparison with MC calculations of the "invariant mass" variable – MC templates.

  The result of comparison: m_t^{MC}.

Indirect measurements

- ✓ From cross-sections (inclusive or differential).

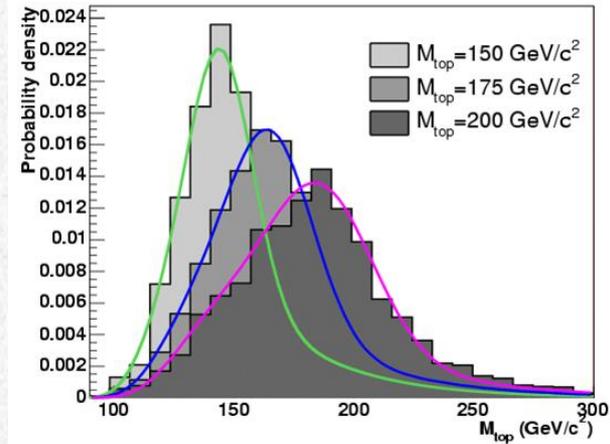- ✓ Measuring observable(s) with a strong dependence on m_t with data unfolding m_t^{pole}

Template method:

PRD 63, 032003(2001)

- ✓ uses **observable(s) sensitive to top quark mass** (e.g. invariant mass, m_t, of decay products).
- ✓ More than 1 observable can be used (to m_t also JES).
- ✓ Signal templates (for different m_{top}) + Bkgd one created by MC.
- ✓ Likelihood fit to data based on templates for S + B.

B-tagged signal templates

M_{top}=150 GeV/c²
M_{top}=175 GeV/c²
M_{top}=200 GeV/c²

Matrix element method:

K. Kondo, J. Phys. Soc. Jap. 57, 4126 (1988).

evaluates **event-by-event probability** based on the full event kinematics

- ✓ signal probability:

measured 4-momenta   Matrix el.   partonic final-state 4-momenta

$$P_{t\bar{t}}\left(\vec{x}; m_{top}\right) = \frac{1}{\sigma_{t\bar{t}}\left(m_{top}\right)} \sum_{\text{flavors}} \int dq_1 dq_2 d\vec{y} \, \frac{d\sigma\left(p\bar{p} \to t\bar{t} \to \vec{y}\right)}{d\vec{y}} \cdot f(q_1) f(q_2) \cdot W(\vec{x}, \vec{y})$$
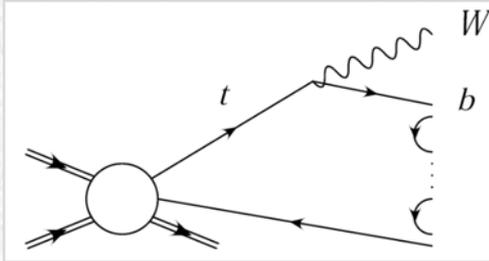
PDFs   resolution

- ✓ Top mass extracted from global likelihood fit to data based on P_{tt} **and** P_{bkg}

$$L\left(\vec{x}_1 \cdots \vec{x}_N; m_{top} \cdots\right) = \prod_{i=1}^{N}\left[f_t P_{tt}\left(\vec{x}_i; m_{top}\right) + \left(1 - f_t\right) P_{bkg}\left(\vec{x}_i\right) \right]$$

Other methods: combination of Template and Matrix element methods (ideogram m.)

Top is a colored fermion – it decays before hadronization, the b quark from its decay hadronizes – it captures at least 1 quark from neighborhood.

Effect of b-hadronization on m_{top}:

$$\Delta m_{\text{top}} \sim \Lambda_{\text{QCD}} \approx 0.2\,\text{GeV}$$
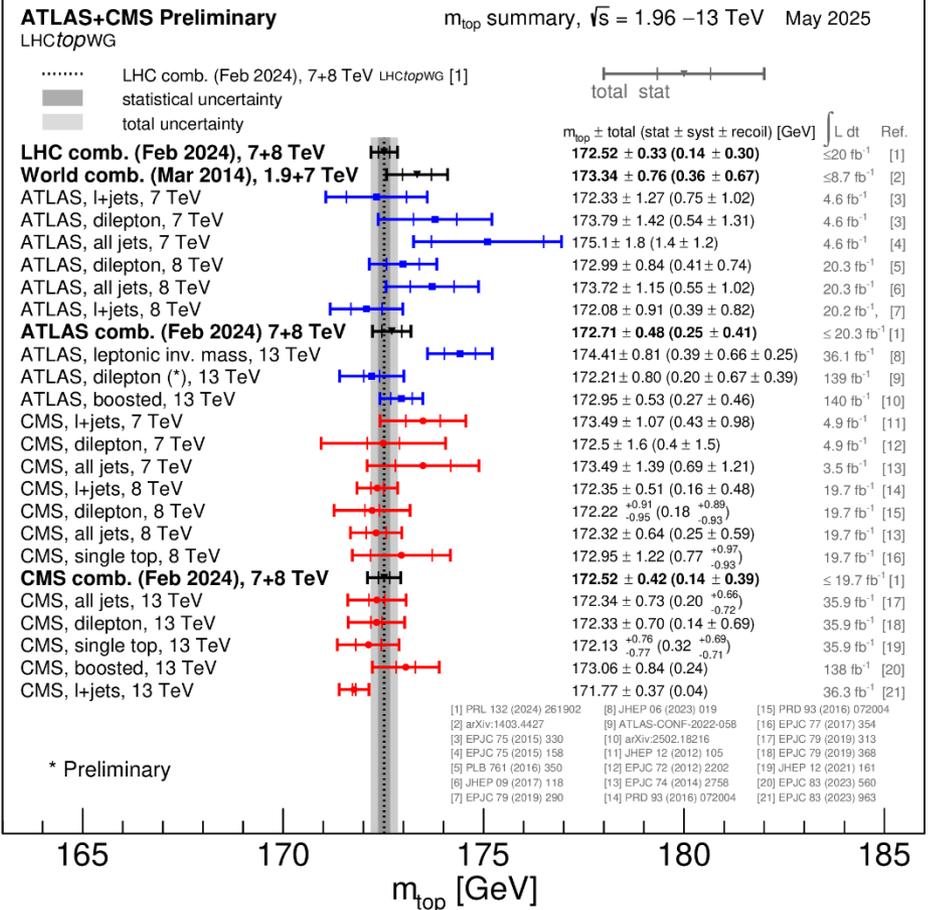
ATLAS analysis (ATL-PHYS-PUB-2021-034):
Kinematic top mass vs MSR mass

$$m_t^{\text{MC}} = m_t^{\text{MSR}}\left(1\,\text{GeV}\right) + 80_{-410}^{+350}\,\text{MeV}$$

Relation between m_t^{MSR} and m_t^{pole} (top pole mass) in 4 loop approx.

$$\Rightarrow \quad m_t^{\text{MC}} = m_t^{\text{pole}} + 350_{-360}^{+300}\,\text{MeV}$$
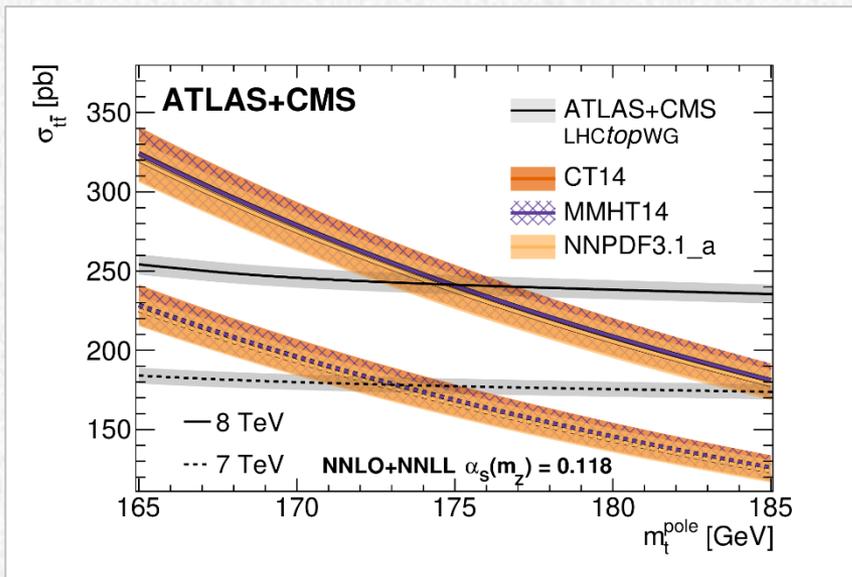

PRL 132(2024)261902

$$m_t^{\text{MC}} = 172.52 \pm 0.14(\text{stat.}) \pm 0.30(\text{syst.})\ \text{GeV} = 172.52 \pm 0.33\ \text{GeV}$$
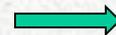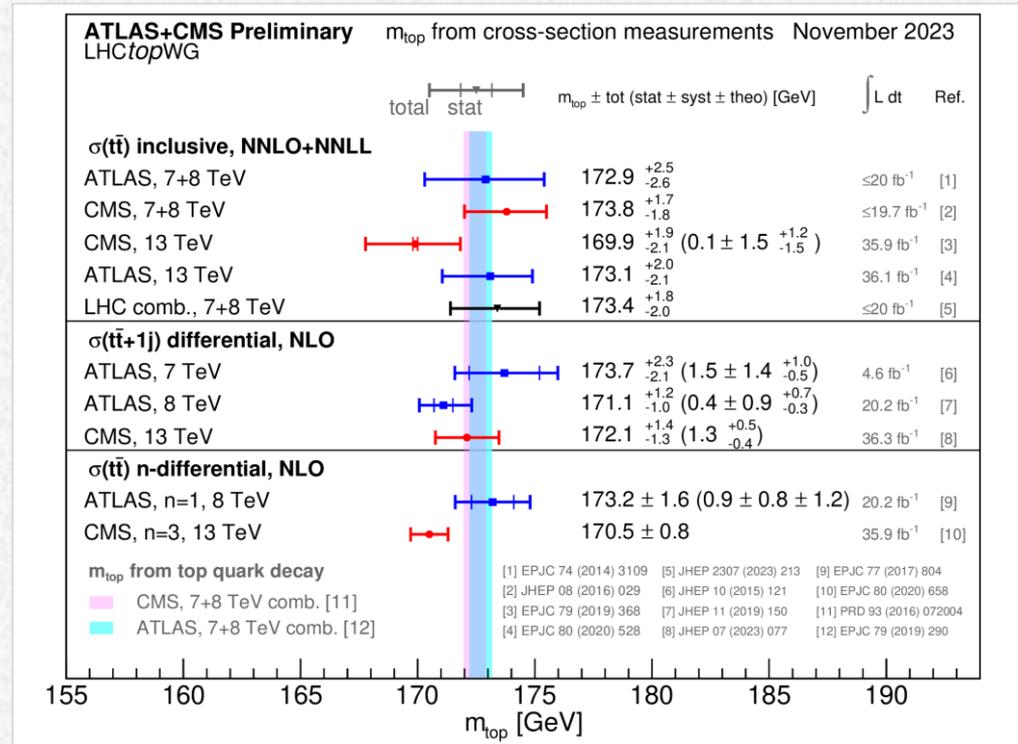
Extraction of top quark pole mass

Measured σ_{tt} can be used to extract pole mass m_t^{pole}, assuming a value of α_S (or vice versa - assume m_t^{pole} and extract α_S).

σ_{tt} results depend on assumed top mass as kinematics/acceptance depend on m_t.

Precision of ~2 GeV on m_t^{pole} limited by PDF and scale uncertainties on pred[n]

Simultaneous χ^2 fits to 7+8 TeV σ_{tt} provides very precise α_S extraction

PDF set	m_t^{pole}	$\alpha_s(m_Z)$
	($\alpha_s = 0.118 \pm 0.001$)	($m_t = 172.5 \pm 1.0$ GeV)
CT14	$174.0_{-2.3}^{+2.3}$ GeV	$0.1161_{-0.0033}^{+0.0030}$
MMHT2014	$174.0_{-2.3}^{+2.1}$ GeV	$0.1160_{-0.0030}^{+0.0031}$
NNPDF3.1_a	$173.4_{-2.0}^{+1.8}$ GeV	$0.1170_{-0.0018}^{+0.0021}$

Thank you!

Top-quark pole mass from tt-bar+1 jet

The m_t^{pole} dependence of the $t\bar{t}+1$-jet cross section ($\sigma_{t\bar{t}+1\,\text{jet}}$) is enhanced:

$$\frac{\Delta\sigma_{t\bar{t}+1\text{-jet}+X}}{\sigma_{t\bar{t}+1\text{-jet}+X}} \approx -5\frac{\Delta m_t^{pole}}{m_t^{pole}} \quad \Rightarrow \text{ from NLO calculations } \text{ [JHEP 10 (2015) 121]}$$

The pole mass can be extracted from: the normalized differential distribution

$$R\left(m_t^{pole},\rho_s\right) = \frac{1}{\sigma_{t\bar{t}+1\text{-jet}+X}}\frac{d\sigma_{t\bar{t}+1\text{-jet}+X}}{d\rho_s}\left(m_t^{pole},\rho_s\right), \quad \rho_s = \frac{2m_0}{\sqrt{s_{t\bar{t}j}}} \dashrightarrow \boxed{170 \text{ GeV}}$$

A template technique is used to extract m_t^{pole}.

ATLAS: measured top-quark pole mass (7 TeV, L=4.6fb^{-1}):

> Selection: ℓ+jets ($\ell = e$ or μ) with two **b**-tags
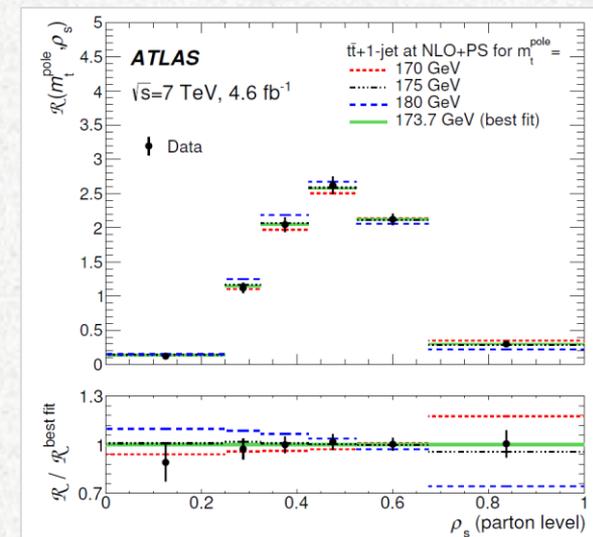> Background: Single top, W/Z+jet, fake leptons,...

$$m_t^{\text{pole}} = 173.7 \pm 1.5\left(\text{stat.}\right) \pm 1.4\left(\text{syst.}\right){}_{-0.5}^{+1.0}\left(\text{theory}\right) \text{GeV}$$

Systematics: μ_R, μ_F variation, JES, ISR/FSR, PDF

CMS: similar analysis based on observable ρ_s in *dilepton channel* at 8 TeV, L=19.7fb^{-1} [TOP-13 -006]:

$$m_t^{\text{pole}} = 169.9 \pm 1.1\left(\text{stat.}\right){}_{-3.1}^{+2.5}\left(\text{syst.}\right){}_{-1.6}^{+3.6}\left(\text{theory}\right) \text{GeV}$$

Systematics: μ_R, μ_F variation, jet-parton matching, hadronization, color reconnection

Top quark mass template method

Basic idea of the template method - ℓ+jets topology :

✓ to find invariant mass of top decay products: $t \to bq\bar{q}, \bar{t} \to \bar{b}l\nu, \; t \leftrightarrow \bar{t}$

✓ Using reconstructed objects of candidate events a kinematic fitter is used to find 4-momenta of top decays products .

✓ Kinematic fitter minimizes χ^2 function, e.g.:

$$\chi^2 = \sum_{i=l,4\,jets} \frac{\left(p_T^{i,fit} - p_T^{i,meas}\right)^2}{\sigma_i^2} + \sum_{j=1.2} \frac{\left(U_j^{fit} - U_j^{meas}\right)^2}{\sigma_j^2} + \frac{\left(M_{jj} - M_W\right)^2}{\Gamma_W^2} + \frac{\left(M_{l\nu} - M_W\right)^2}{\Gamma_W^2} + \frac{\left(M_{bjj} - m_t^{rec}\right)^2}{\Gamma_t^2} + \frac{\left(M_{bl\nu} - m_t^{rec}\right)^2}{\Gamma_t^2}$$

Problem: for candidate event we can have several event configurations – connected with different assignments of jets to quarks - without b-tagging: 12 configurations per a LJ event (and for 1 or 2-btags?)

✓ The χ^2 fit is applied to all the event configurations

✓ KF gives for each event combination m_t^{rec} and χ^2 - the correct $m_t^{rec} \Leftrightarrow$ minimal χ^2
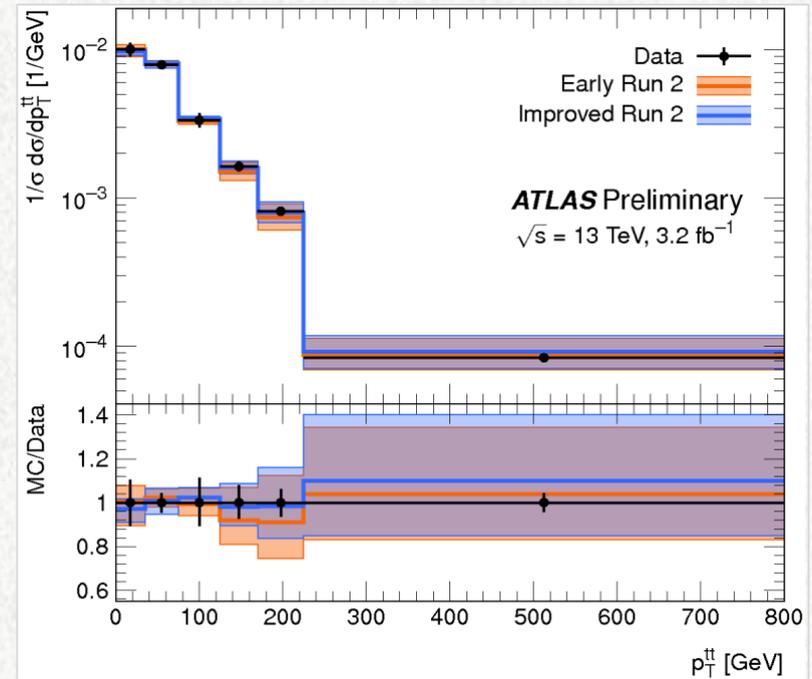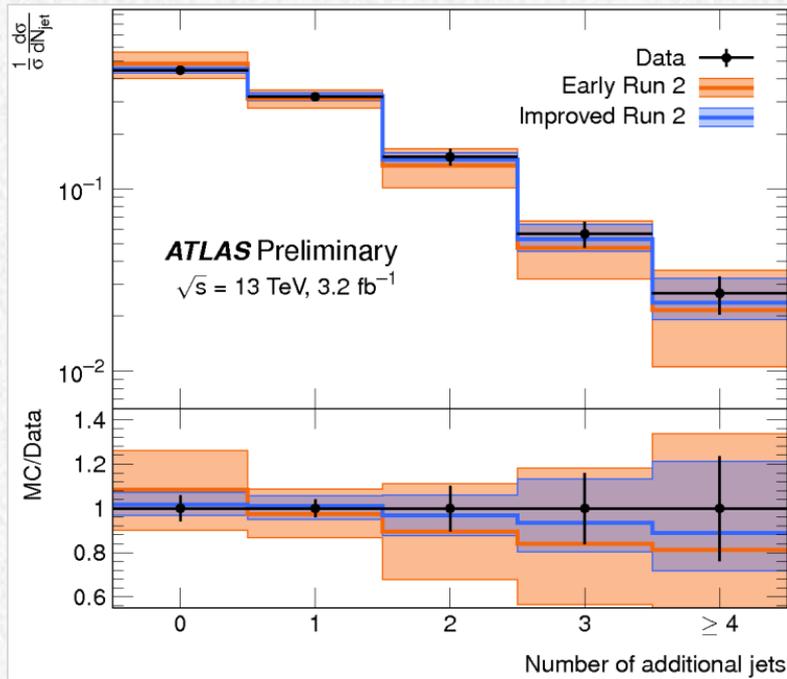
Using MC for a given input top mass - expected rec. mass distribution (template) can be found – data mass distr. is compared with mass templates

Top Quark Mass and renormalization schemes

From top quark view point:

- Pole mass absorbs all self energy corrections

  ✓ Close to the notion of the quark rest mass (kinematic mass).

  ✓ Renormalon problem: infrared-sensitive contributions from < 1 GeV.

  ✓ Has perturbative instabilities due to sensitivity to momenta < 1 GeV.

  ✓ Precision of pole mass determination is limited $\sim \Lambda_{\text{QCD}}$.

- Short range mass: like top pole mass but the self-energy corrections with momentum scale $< R$ (or with loop size $>1/R$) are not absorbed into mass :

  ✓ Advantage of short range mass: more stable in perturbation theory.

  ✓ $M_{t,\text{MSR}}$(R=1 GeV) - close to kinematic mass, but no renormalon problem.

  ✓ Precision is not limited (c.f. Appex)

Source of Uncertainty	Samples	Procedure
Nominal	Powheg+Pythia8	N/A
NLO+PS matching	Powheg+Pythia8 vs. MadGraph5_aMC@NLO+Pythia8	$\pm|\Delta|$
Parton Shower and Hadronization Model	Powheg+Pythia8 vs. Powheg+Herwig7	$\pm|\Delta|$
Additional Radiation	Powheg+Pythia8 A14 (Var. 3c up, $\mu_{R,F} = 0.5, h_{\mathrm{damp}} = 3.0\ m_{\mathrm{top}}$ vs. Var. 3c down, $\mu_{R,F} = 2.0, h_{\mathrm{damp}} = 1.5\ m_{\mathrm{top}}$)	Δ