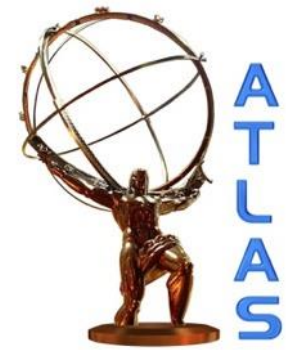
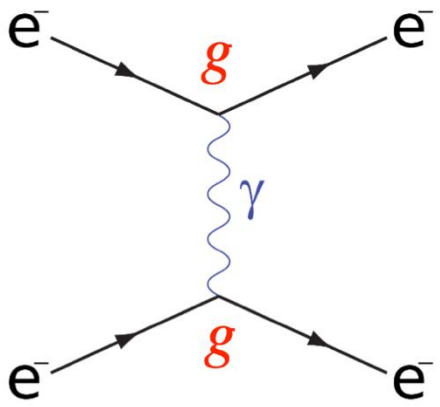
Feynman's *mathemagic*

From double-slits to silly drawings

Luis Roberto Flores Castillo
Chinese University of Hong Kong



CERN International High School Teacher Programme
July 8, 2026



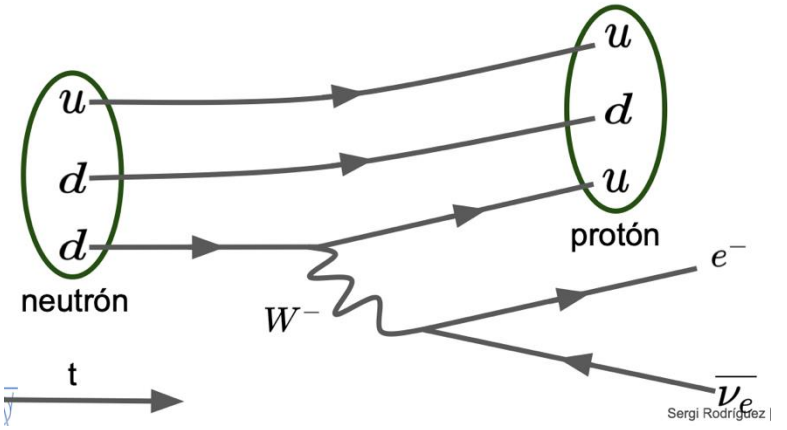
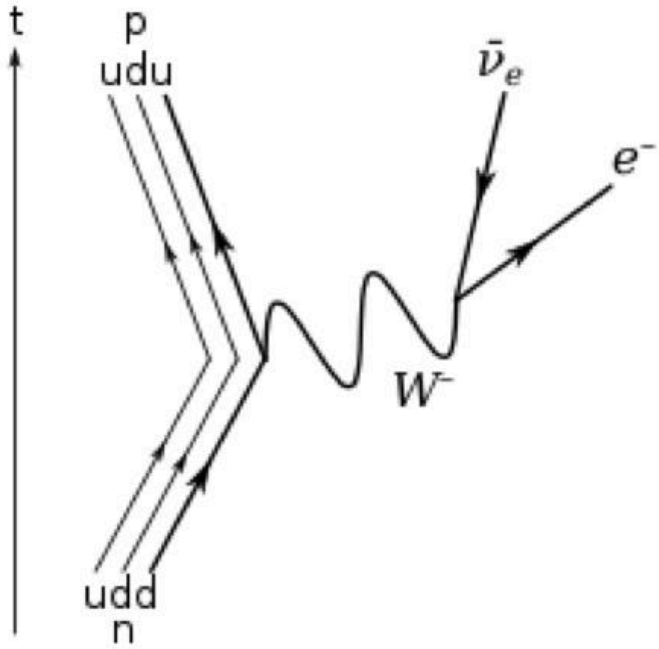


Photo: Seamus Blackley



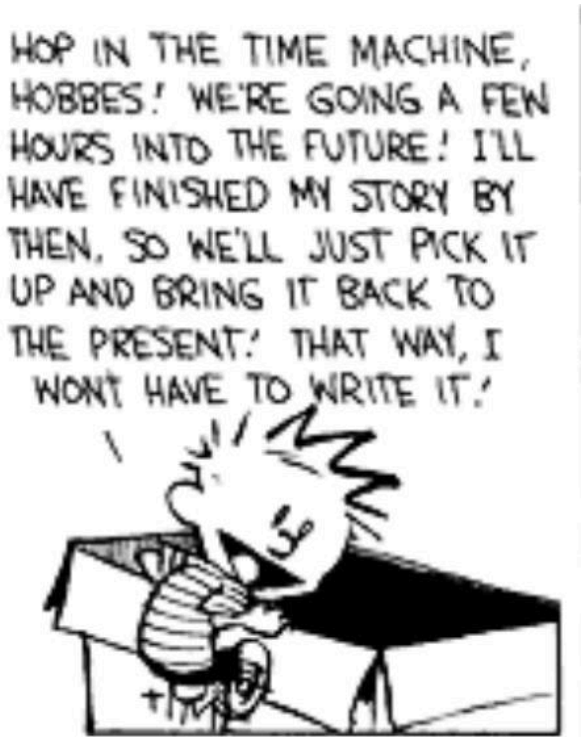
What are *Feynman diagrams*?

Preliminaries …

- *A cartoon*
- *A game*
- *A proof*

SAY, IVE GOT AN IDEA!
FOR YOUR STORY?
© 1992 Watterson/Distributed by Universal Press Syndicate

NO, I THOUGHT OF A WAY I WON'T HAVE TO WRITE ONE!
OH NO.


HOP IN THE TIME MACHINE, HOBBES! WE'RE GOING A FEW HOURS INTO THE FUTURE! I'LL HAVE FINISHED MY STORY BY THEN, SO WE'LL JUST PICK IT UP AND BRING IT BACK TO THE PRESENT! THAT WAY, I WON'T HAVE TO WRITE IT!

SOMETHING DOESN'T MAKE SENSE HERE, AND I THINK IT'S ME SITTING IN THIS BOX.
RELAX! WE'LL BE BACK AS SOON AS WE GO.
TIME MACHINE
523

$z \leftarrow z^2 + c$

c: a fixed number,

Always starting with $z = 0$

$c = -1$

$z_0 = 0$

$z_1 = 0^2 + (-1) = -1$

$z_2 = (-1)^2 + (-1) = 0$

$z_3 = 0^2 + (-1) = -1$

$z_4 = (-1)^2 + (-1) = 0$

. . .

BLACK

BOUND

$c = 1$

$z_0 = 0$

$z_1 = 0^2 + (1) = 1$

$z_2 = 1^2 + (1) = 2$

$z_3 = 2^2 + (1) = 5$

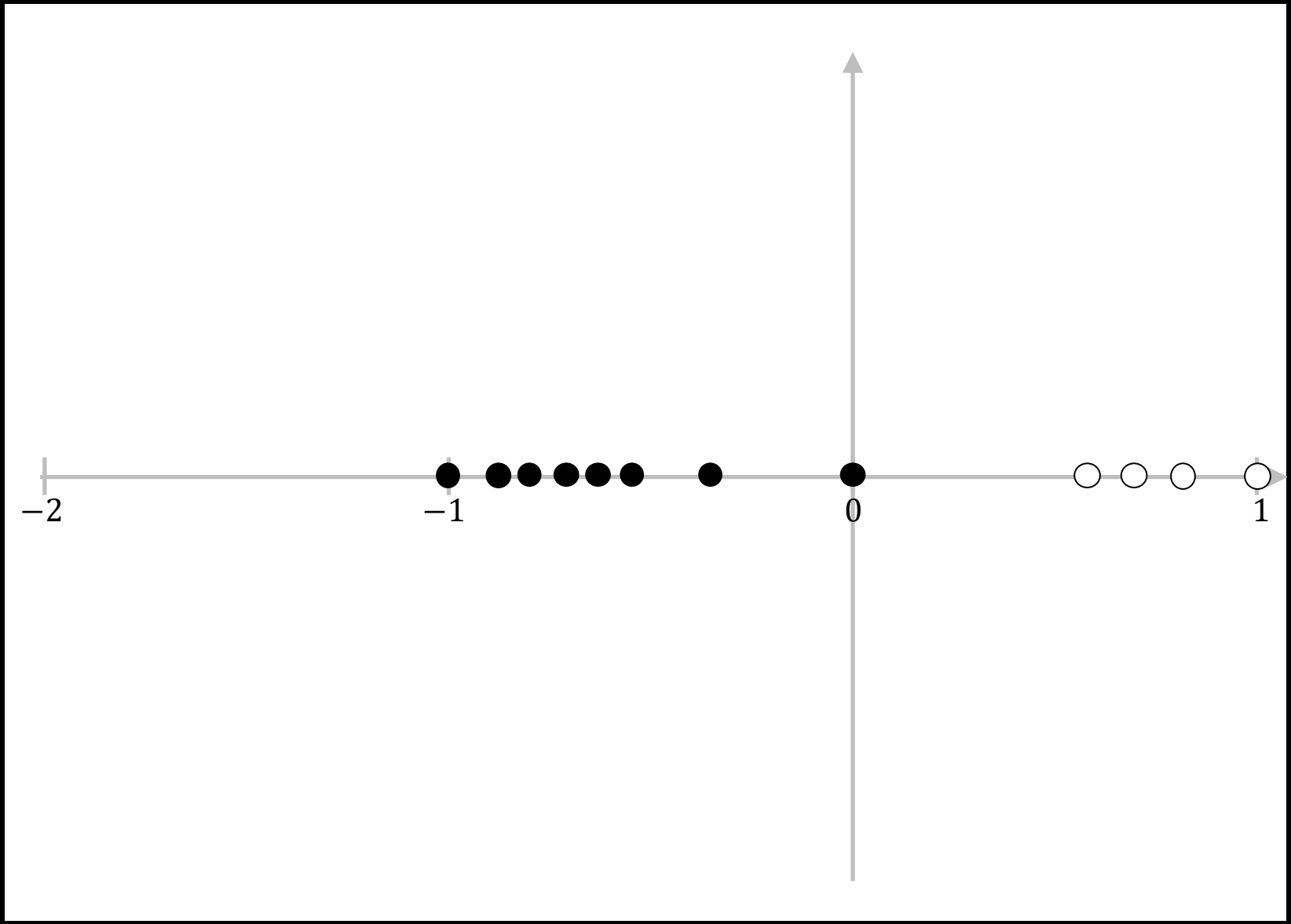
$z_4 = 5^2 + (1) = 26$

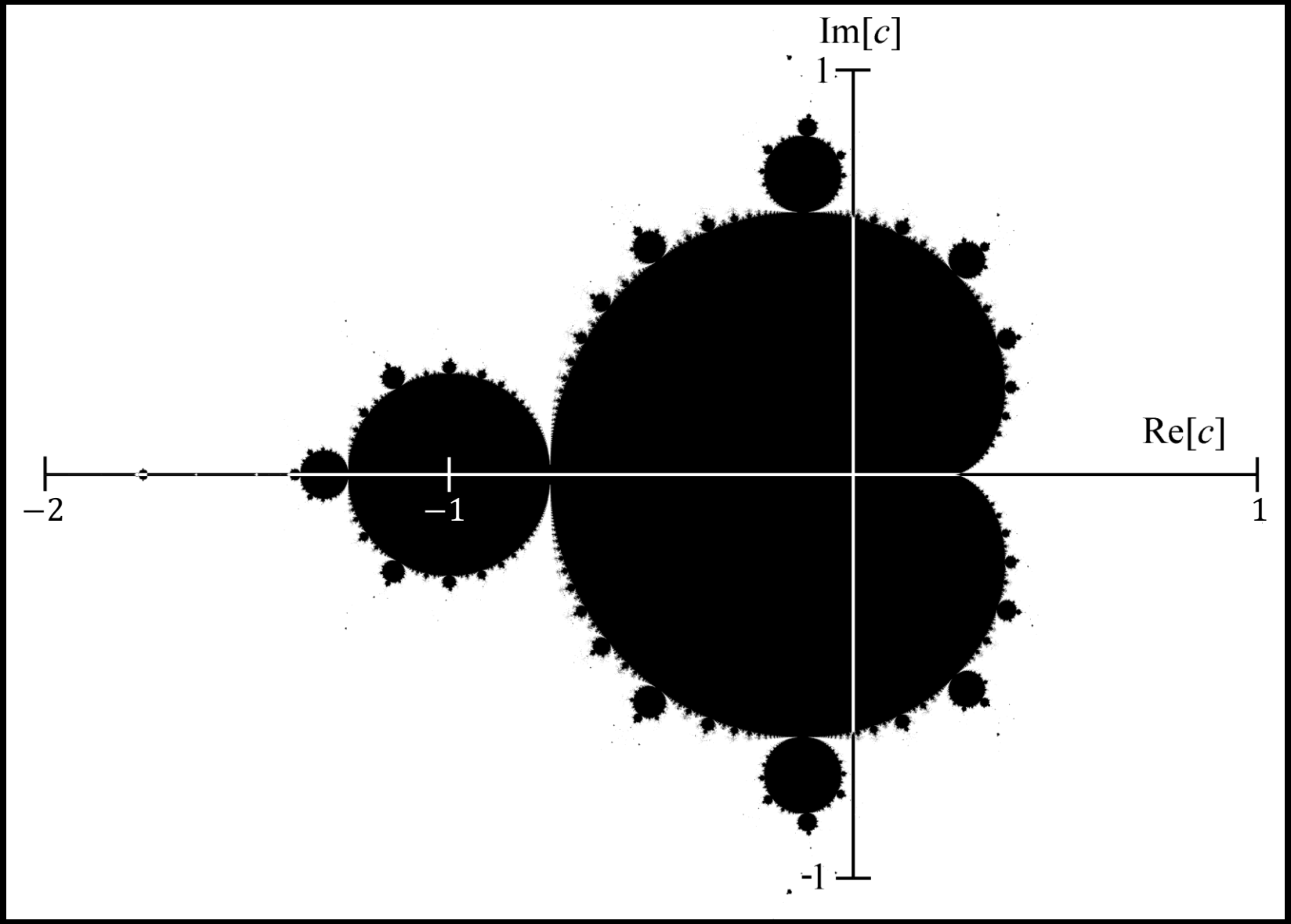
. . .

COLORS (according to divergence speed)

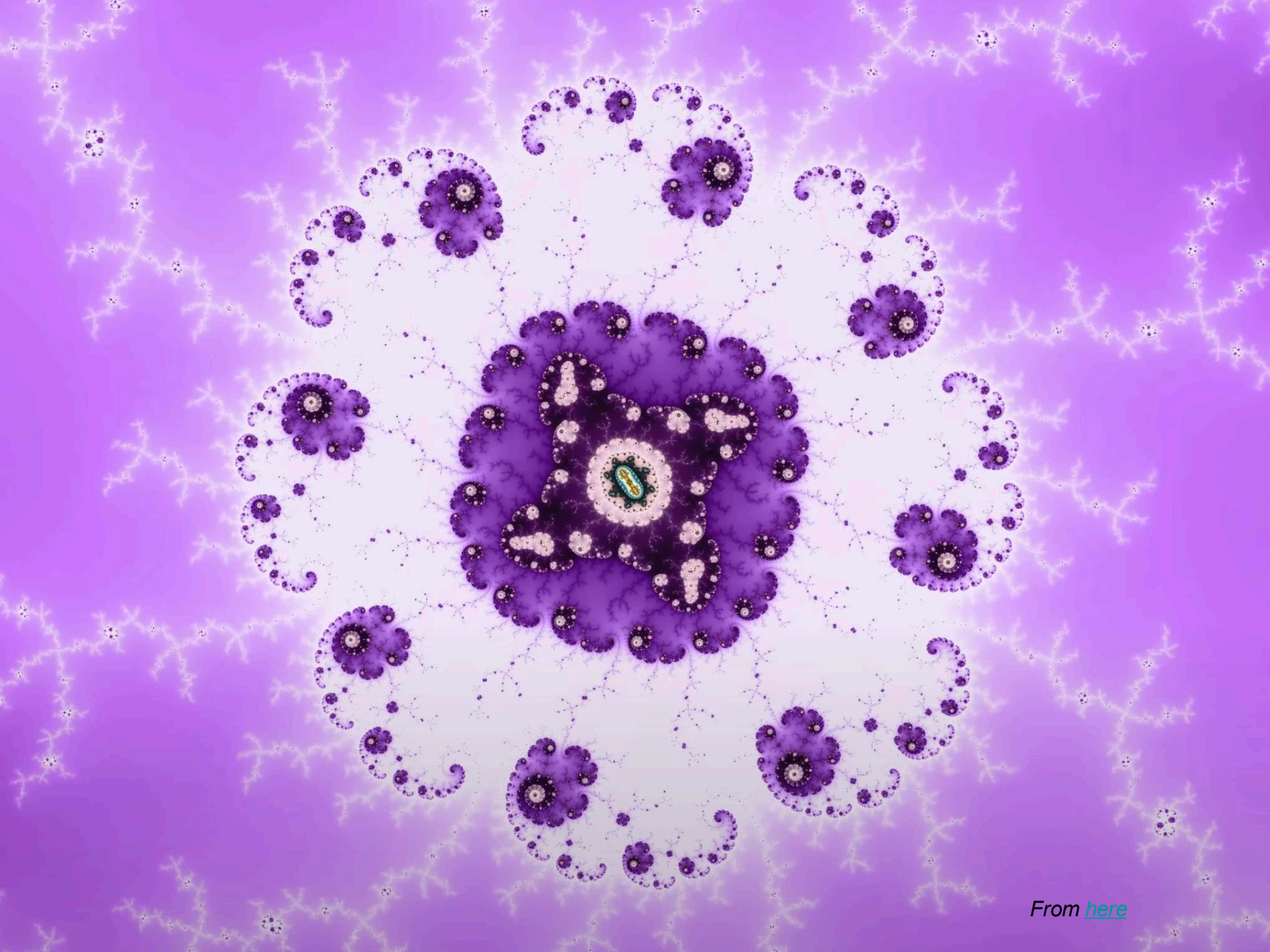
DIVERGENT

$z \leftarrow z^2 + c$





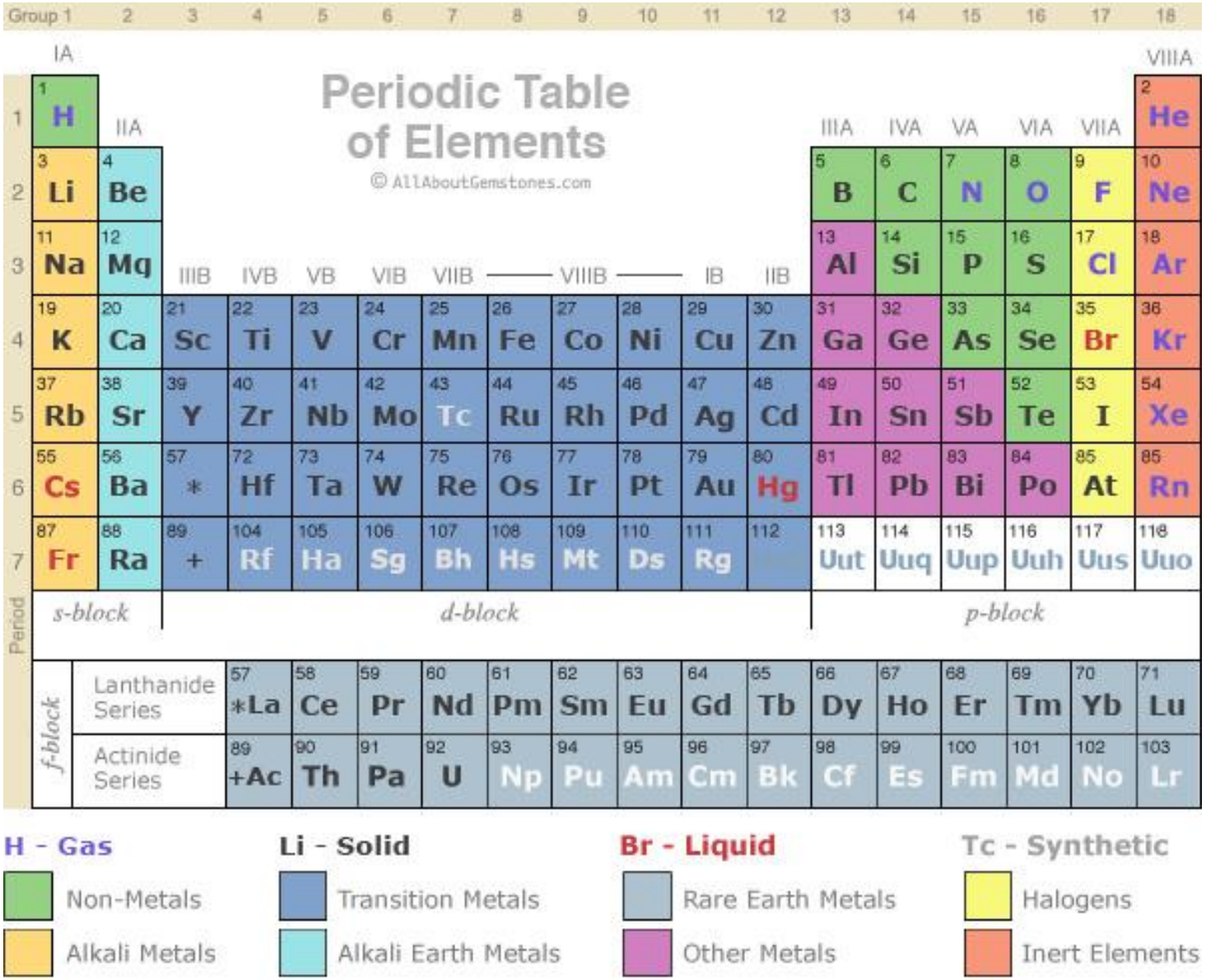
https://www.youtube.com/watch?v=pCpLWbHVNhk

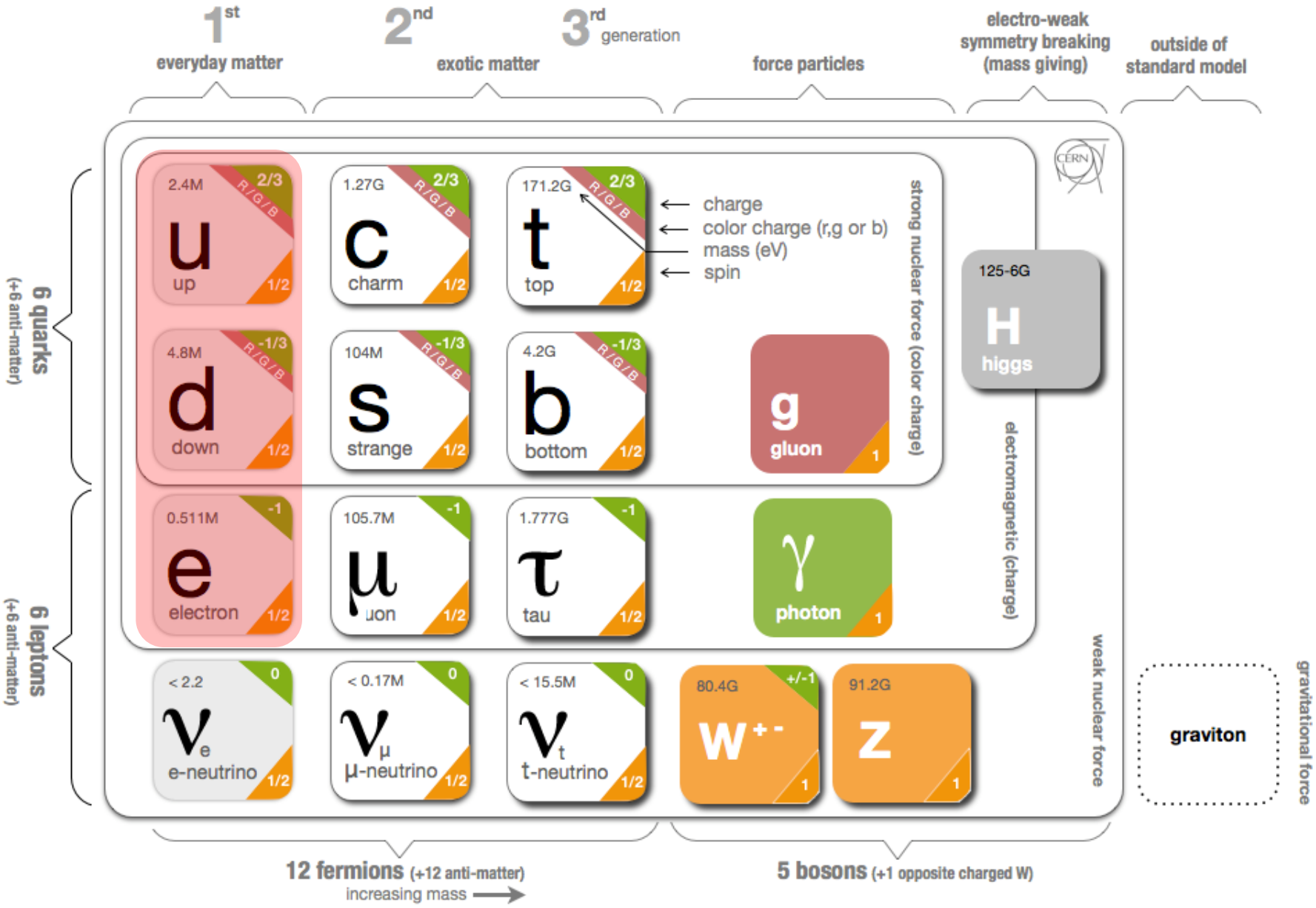


From here

A proof …

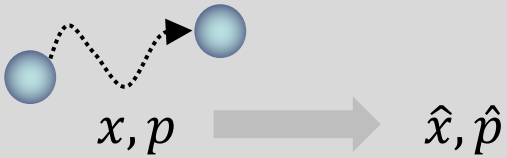
“Simplifying” our representations


Periodic Table of Elements
© AllAboutGemstones.com
Group 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
IA VIIIA
IIA IIIA IVA VA VIA VIIA
IIIB IVB VB VIB VIIB VIIIB IB IIB
Period
1 H He
2 Li Be B C N O F Ne
3 Na Mg Al Si P S Cl Ar
4 K Ca Sc Ti V Cr Mn Fe Co Ni Cu Zn Ga Ge As Se Br Kr
5 Rb Sr Y Zr Nb Mo Tc Ru Rh Pd Ag Cd In Sn Sb Te I Xe
6 Cs Ba * Hf Ta W Re Os Ir Pt Au Hg Tl Pb Bi Po At Rn
7 Fr Ra + Rf Ha Sg Bh Hs Mt Ds Rg 112 113 Uut 114 Uuq 115 Uup 116 Uuh 117 Uus 118 Uuo
s-block
d-block
p-block
f-block
Lanthanide Series
Actinide Series
57 *La 58 Ce 59 Pr 60 Nd 61 Pm 62 Sm 63 Eu 64 Gd 65 Tb 66 Dy 67 Ho 68 Er 69 Tm 70 Yb 71 Lu
89 +Ac 90 Th 91 Pa 92 U 93 Np 94 Pu 95 Am 96 Cm 97 Bk 98 Cf 99 Es 100 Fm 101 Md 102 No 103 Lr
H - Gas
Li - Solid
Br - Liquid
Tc - Synthetic
Non-Metals
Alkali Metals
Transition Metals
Alkali Earth Metals
Rare Earth Metals
Other Metals
Halogens
Inert Elements

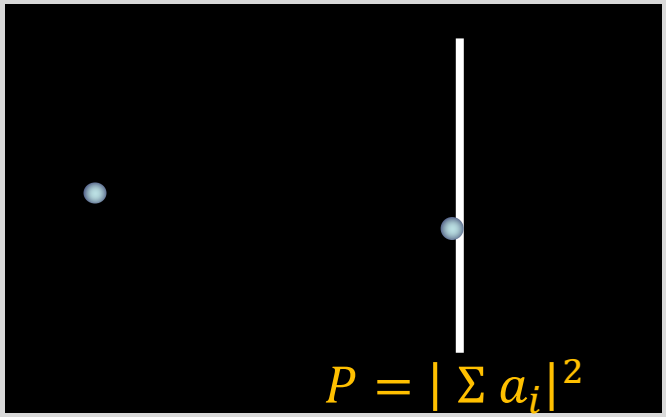

1st
everyday matter
2nd
3rd generation
exotic matter
force particles
electro-weak symmetry breaking (mass giving)
outside of standard model
6 quarks
(+6 anti-matter)
6 leptons
(+6 anti-matter)
2.4M
2/3
R/G/B
u
up
1/2
1.27G
c
charm
171.2G
t
top
charge
color charge (r,g or b)
mass (eV)
spin
4.8M
-1/3
d
down
104M
s
strange
4.2G
b
bottom
g
gluon
1
125-6G
H
higgs
strong nuclear force (color charge)
electromagnetic (charge)
0.511M
-1
e
electron
105.7M
μ
uon
1.777G
τ
tau
γ
photon
< 2.2
0
νe
e-neutrino
< 0.17M
νμ
μ-neutrino
< 15.5M
νt
t-neutrino
80.4G
+/-1
W+-
91.2G
Z
weak nuclear force
graviton
gravitational force
12 fermions (+12 anti-matter)
increasing mass
5 bosons (+1 opposite charged W)

… and then what?

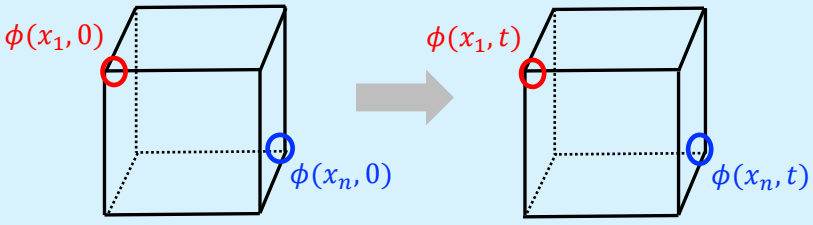
CLASSICAL MECHANICS:



QUANTUM MECHANICS:



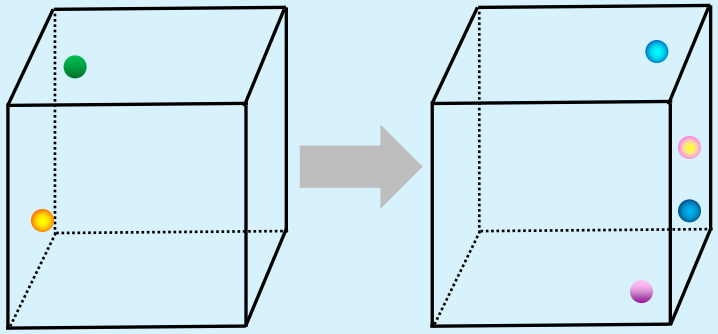
AT HIGH ENERGIES (SHORT DISTANCES):

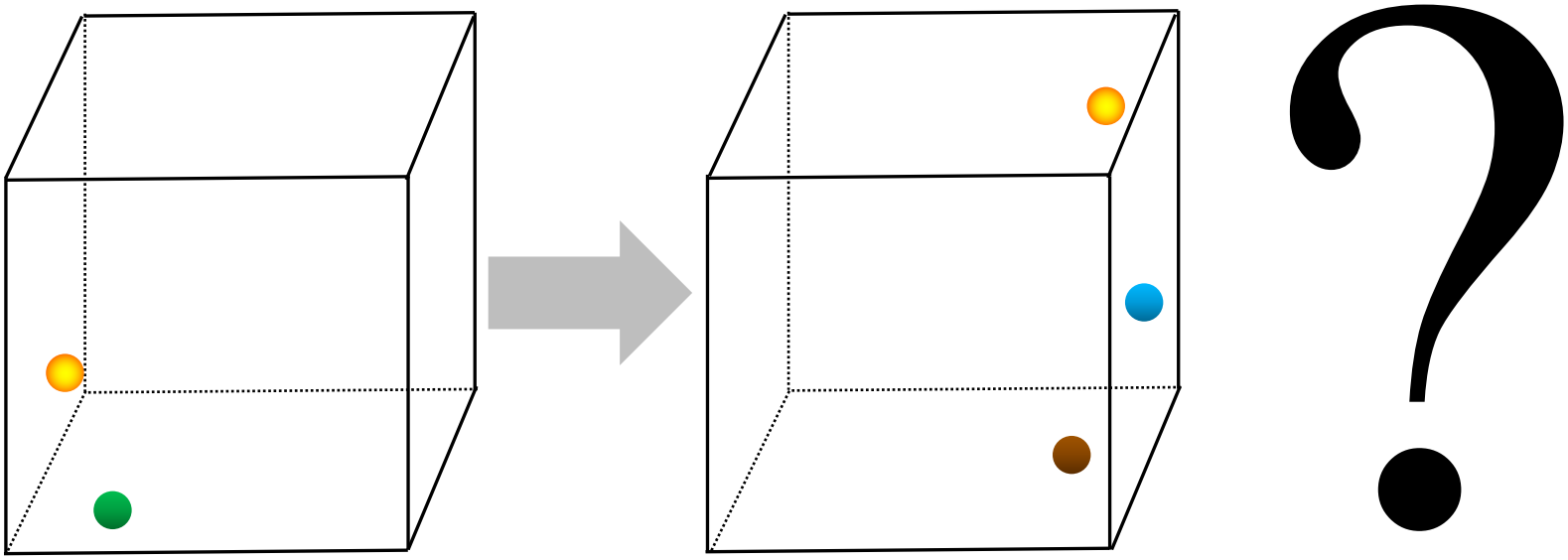


Each $\phi(x) \rightarrow \hat{\phi}(x)$

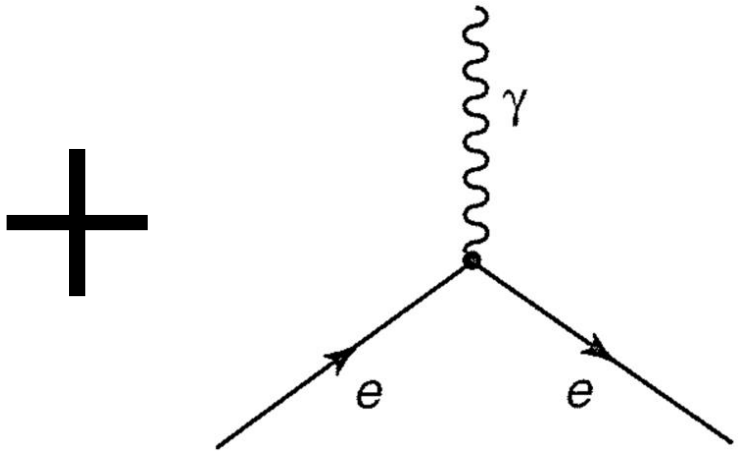
Excitations at each point: ***quanta of that field***

We cannot ignore particle production

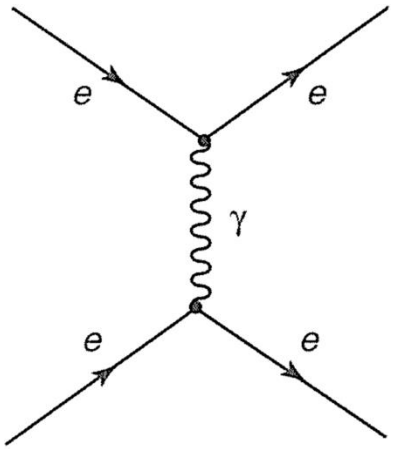


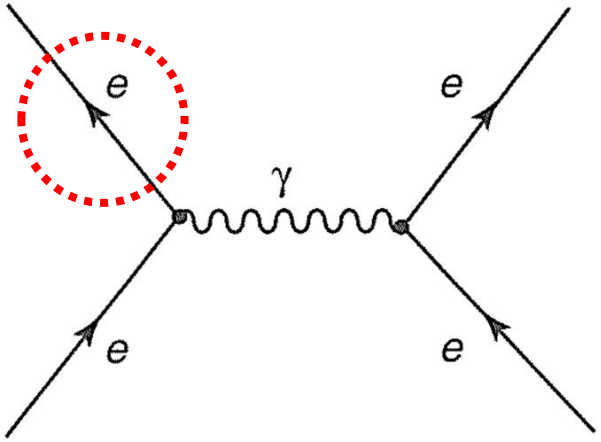


"All possible histories" are those that can be built by combining a few "basic vertices" that represent fundamental interactions.



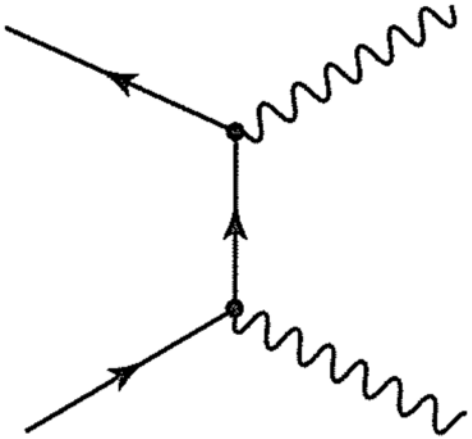
time →





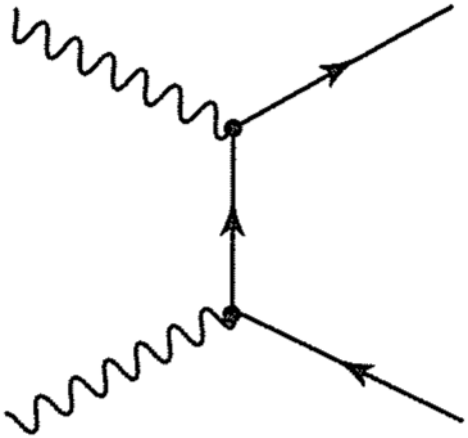
Other combinations

time



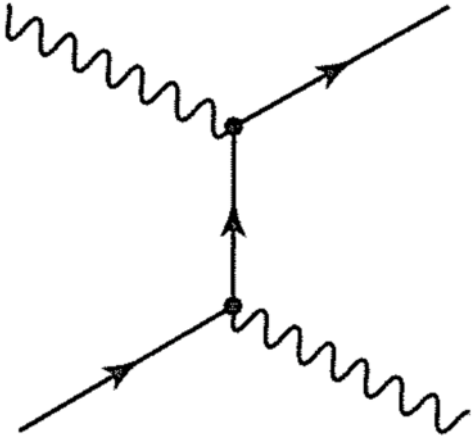
Pair annihilation

$e^+ + e^- \rightarrow \gamma + \gamma$



Pair production

$\gamma + \gamma \rightarrow e^+ + e^-$



Compton scattering

$e^- + \gamma \rightarrow e^- + \gamma$

What is $\frac{1}{0.98}$?

$$\frac{1}{1-0.02} \approx 1+0.02$$

$$\approx 1+0.02+0.02^2$$

$$\approx 1+0.02+0.02^2+0.02^3$$

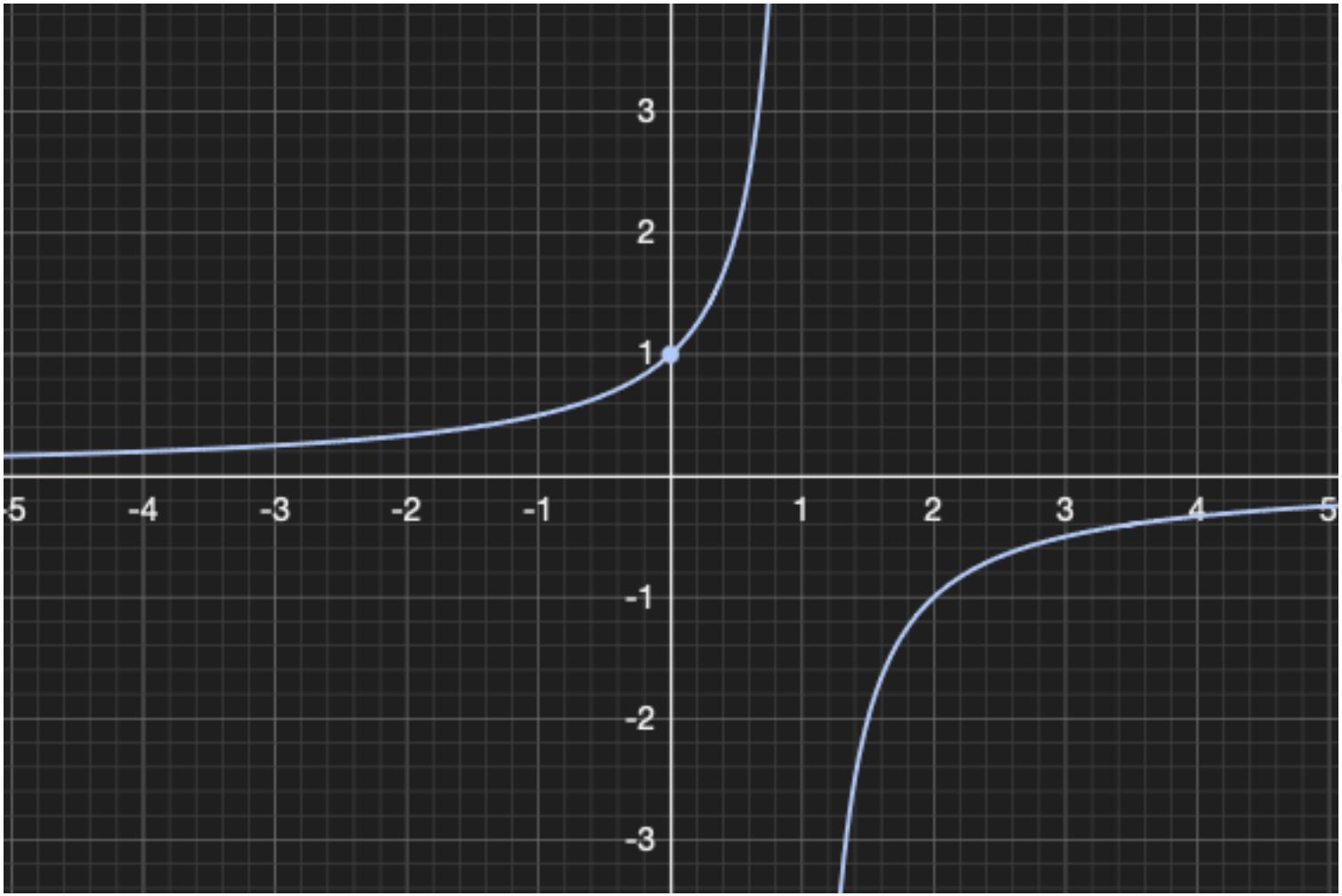
$$\cdots$$

$$\frac{1}{1-x} = 1+x+x^2+x^3+\cdots$$

"perturbation"

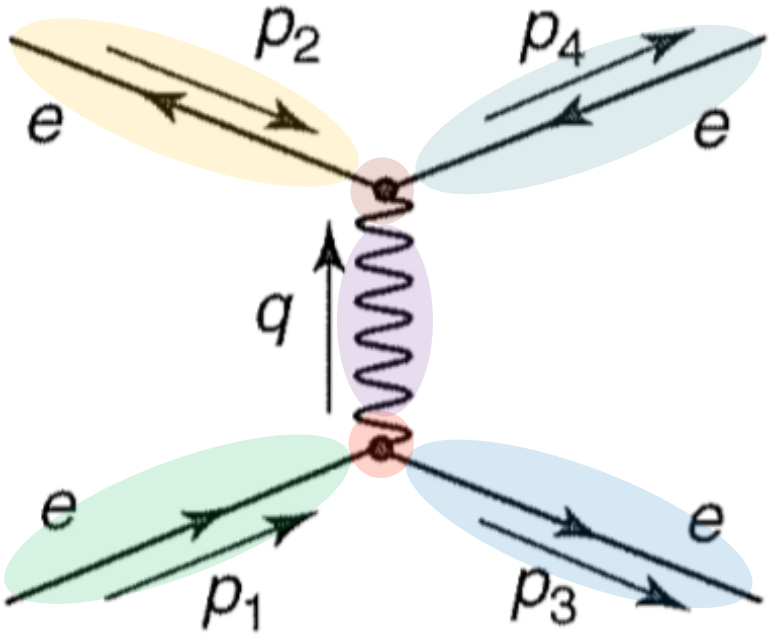
"perturbative expansion"

- It is an infinite series, but we only may need two terms.
- We know how to obtain the series, but **not "by inspection"**.
- Maybe a supersmart alien species calls this function the *"all ones"*



Imagine that we could *intuitively* determine that this can be computed as $1 + x + x^2 + x^3 + \cdots$

Each Feynman diagram stands for an *expression* that produces *one complex number*.



$$\int \bar{u}(3)\,(ig_e\gamma^{\mu})\,u(1)\;\frac{-ig_{\mu\nu}}{q^2}\;\bar{v}(2)\,(ig_e\gamma^{\nu})\,v(4)$$

$$\times\,(2\pi)^4\delta^4(p_1-p_3-q)\times\,(2\pi)^4\,\delta^4(p_2+q-p_4)\;\frac{d^4q}{(2\pi)^4}$$

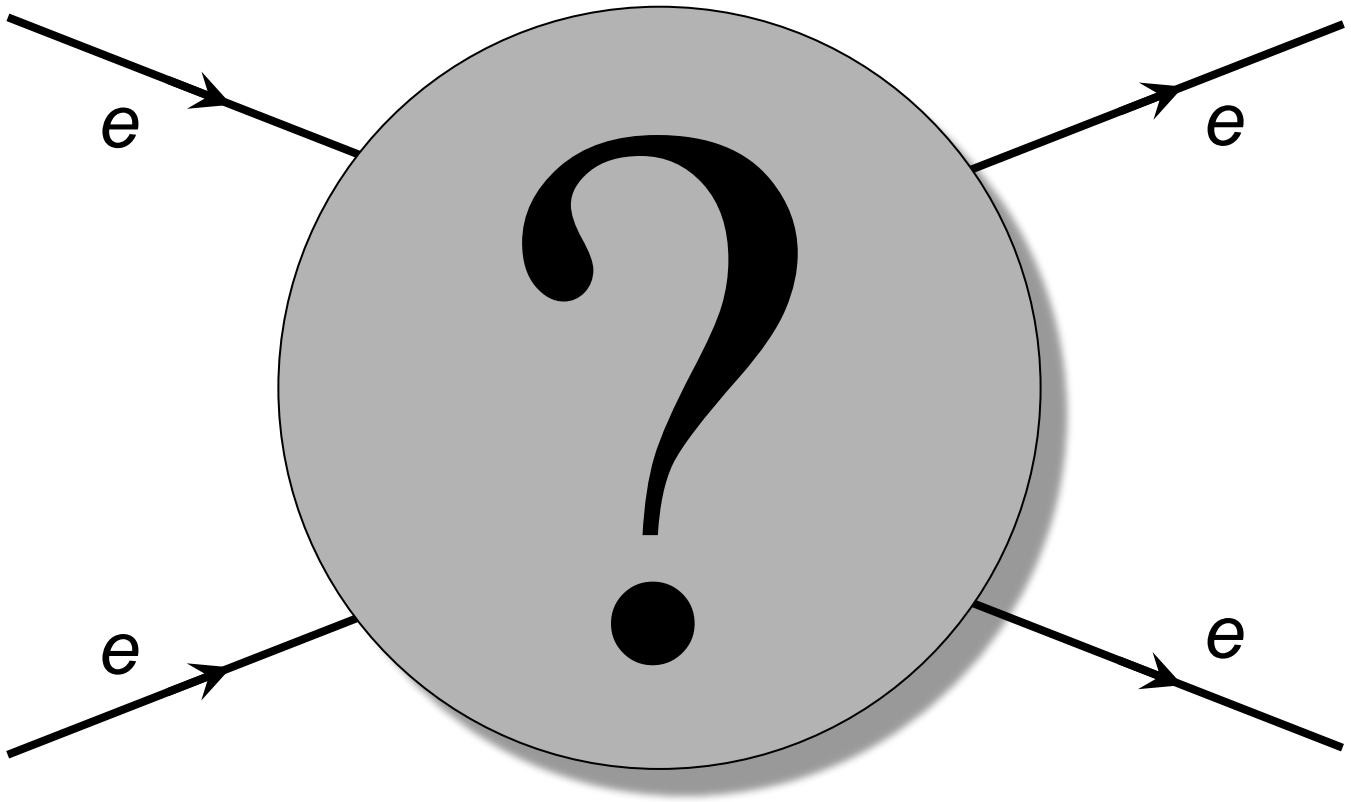
Each Feynman diagram stands for an *expression* that produces *one complex number*.

They are *terms in a series*; they are just written in an exceedingly intuition-friendly notation.

Once we know the valid vertices and what particles **enter** and **exit** a process, *we can write 'by inspection' all the terms needed!!!*

The sum of those expressions is the probability amplitude for the process.

For example,



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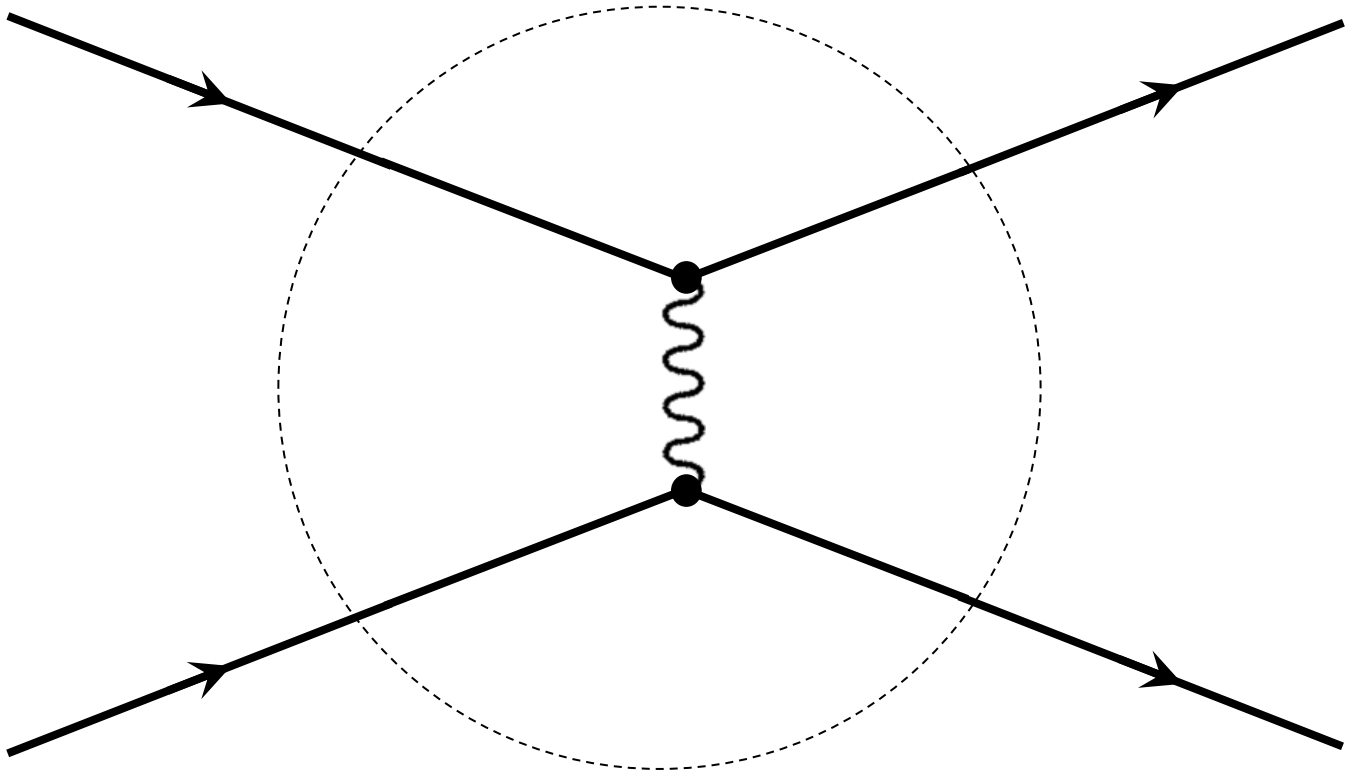
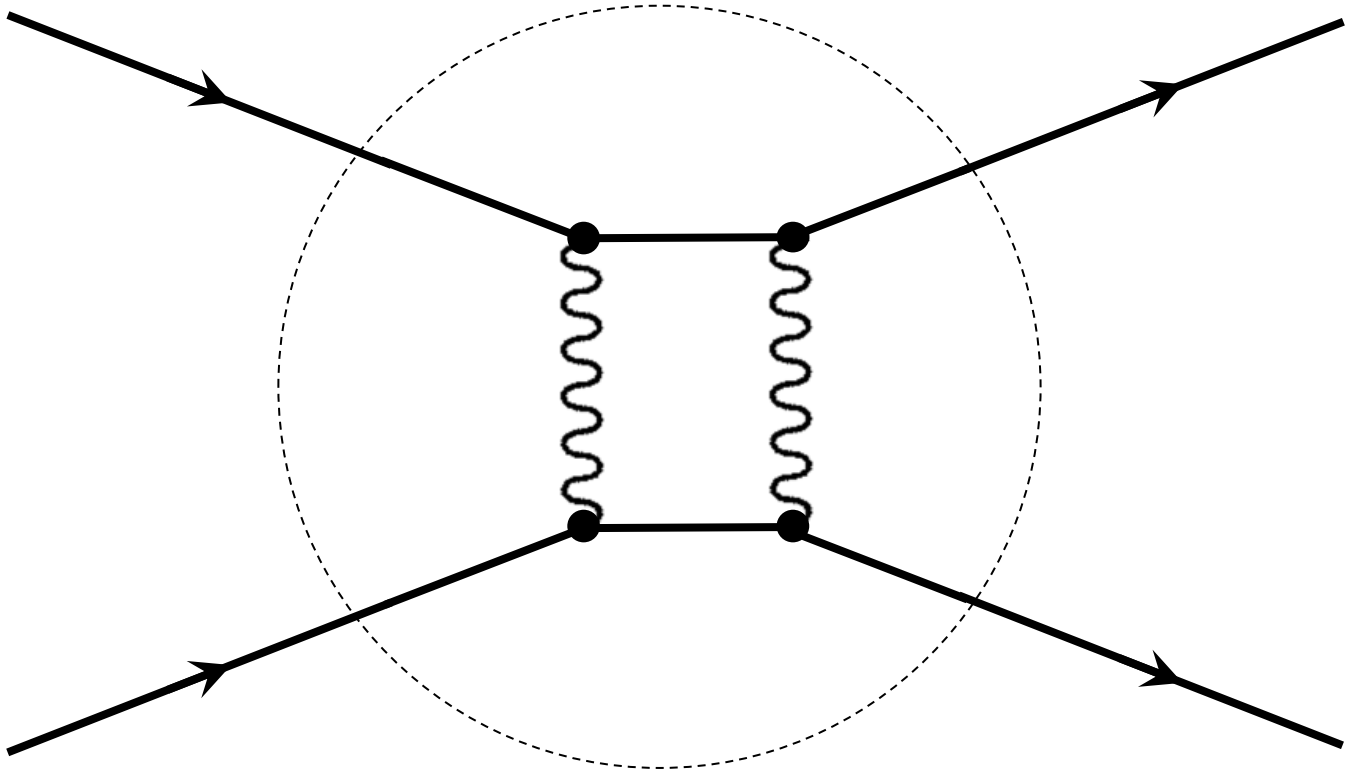
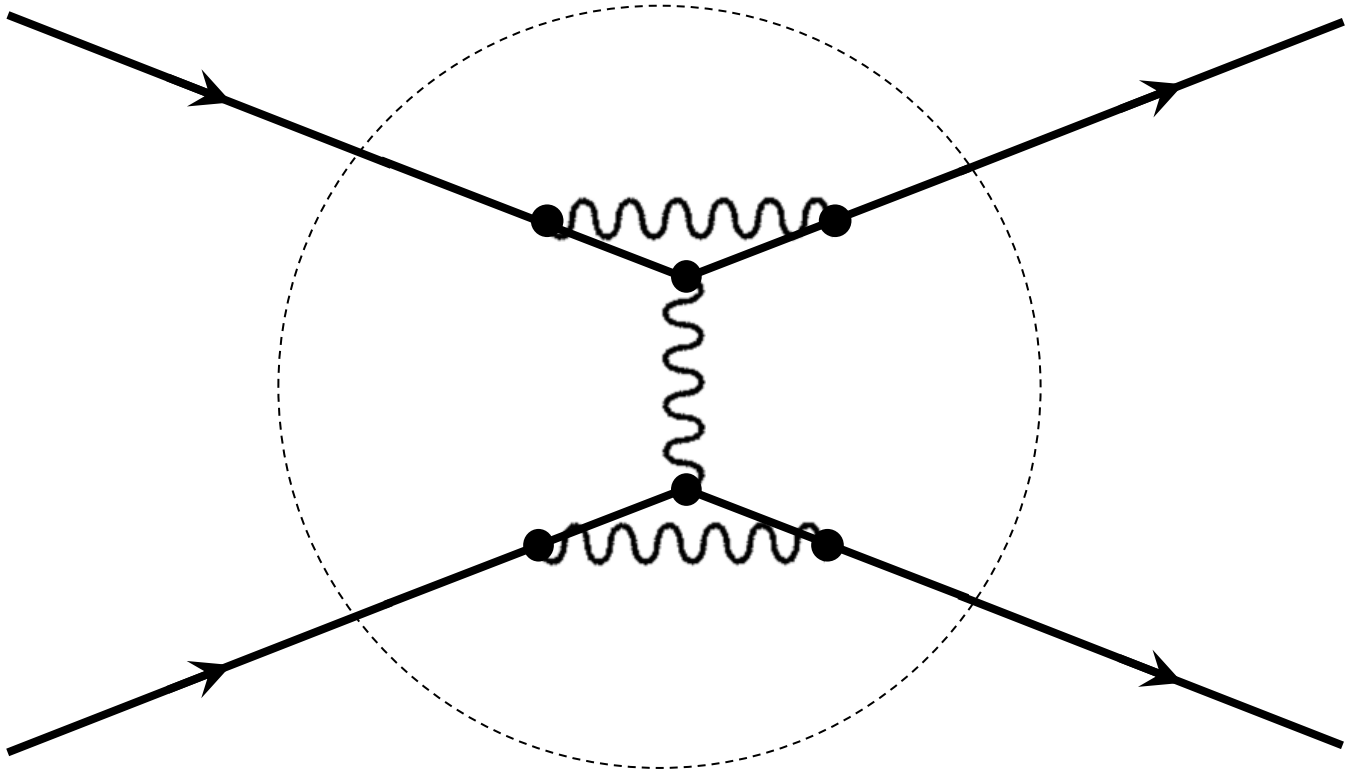


For example,

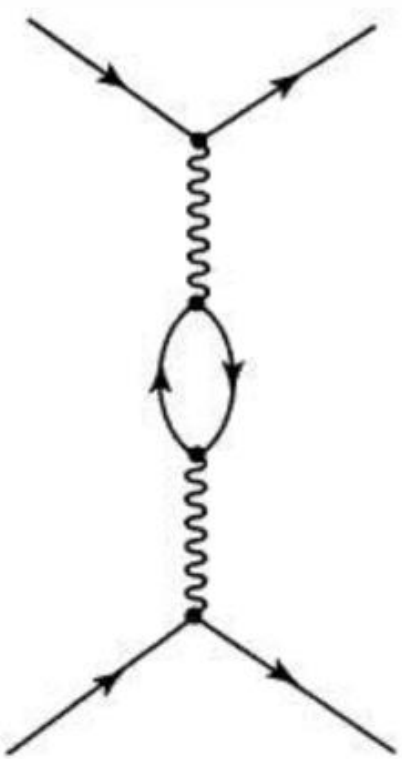


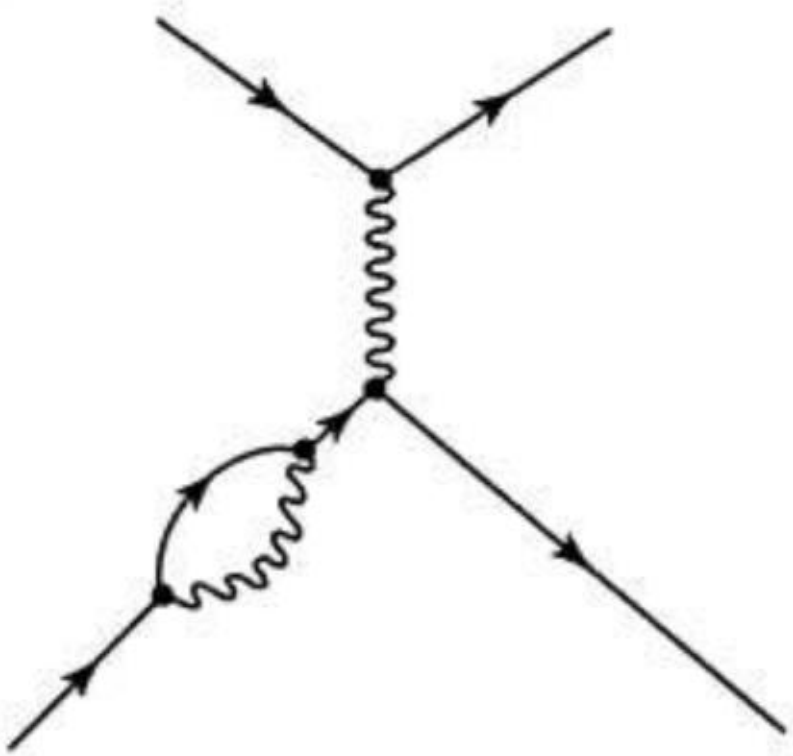
For example,

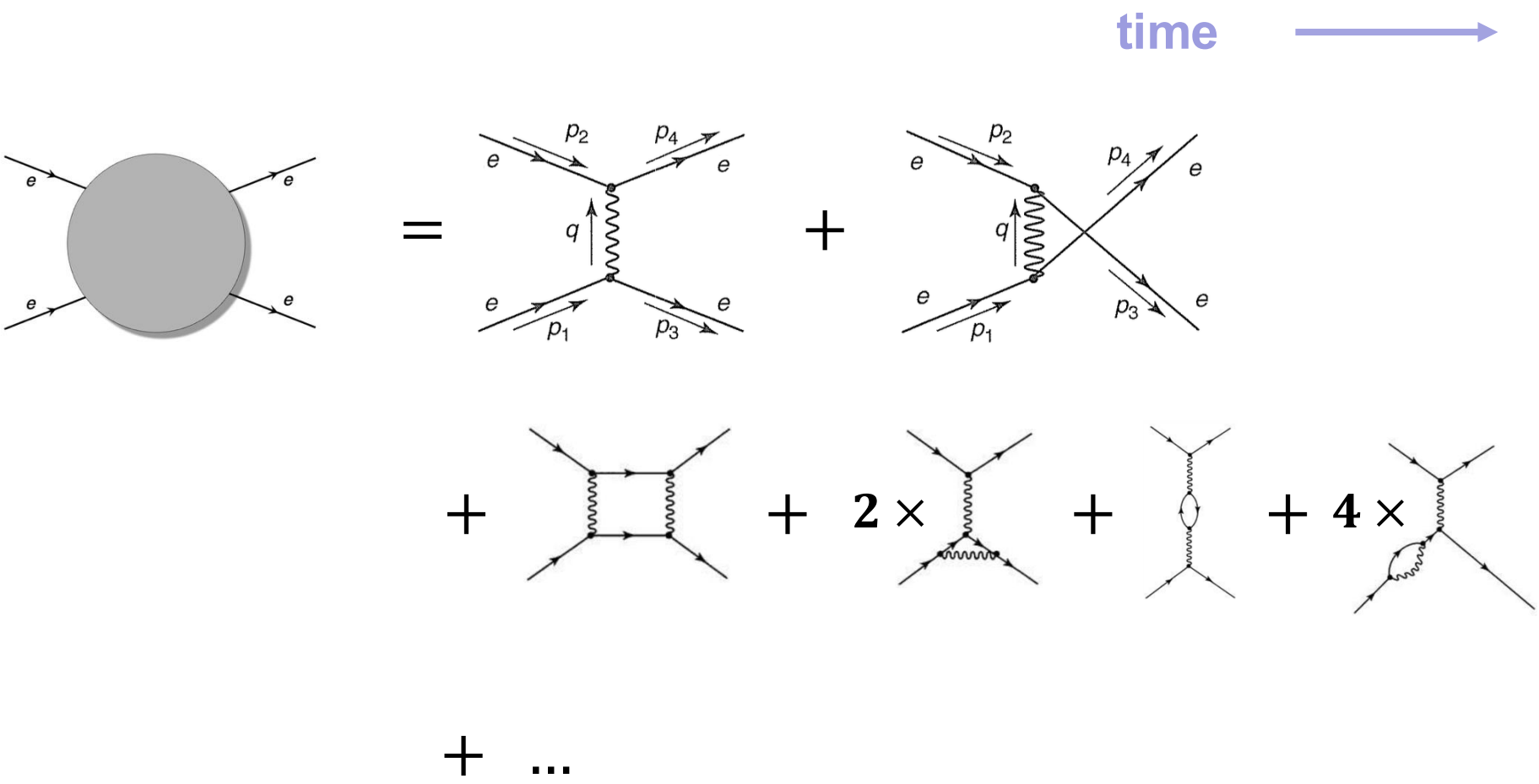


For example,

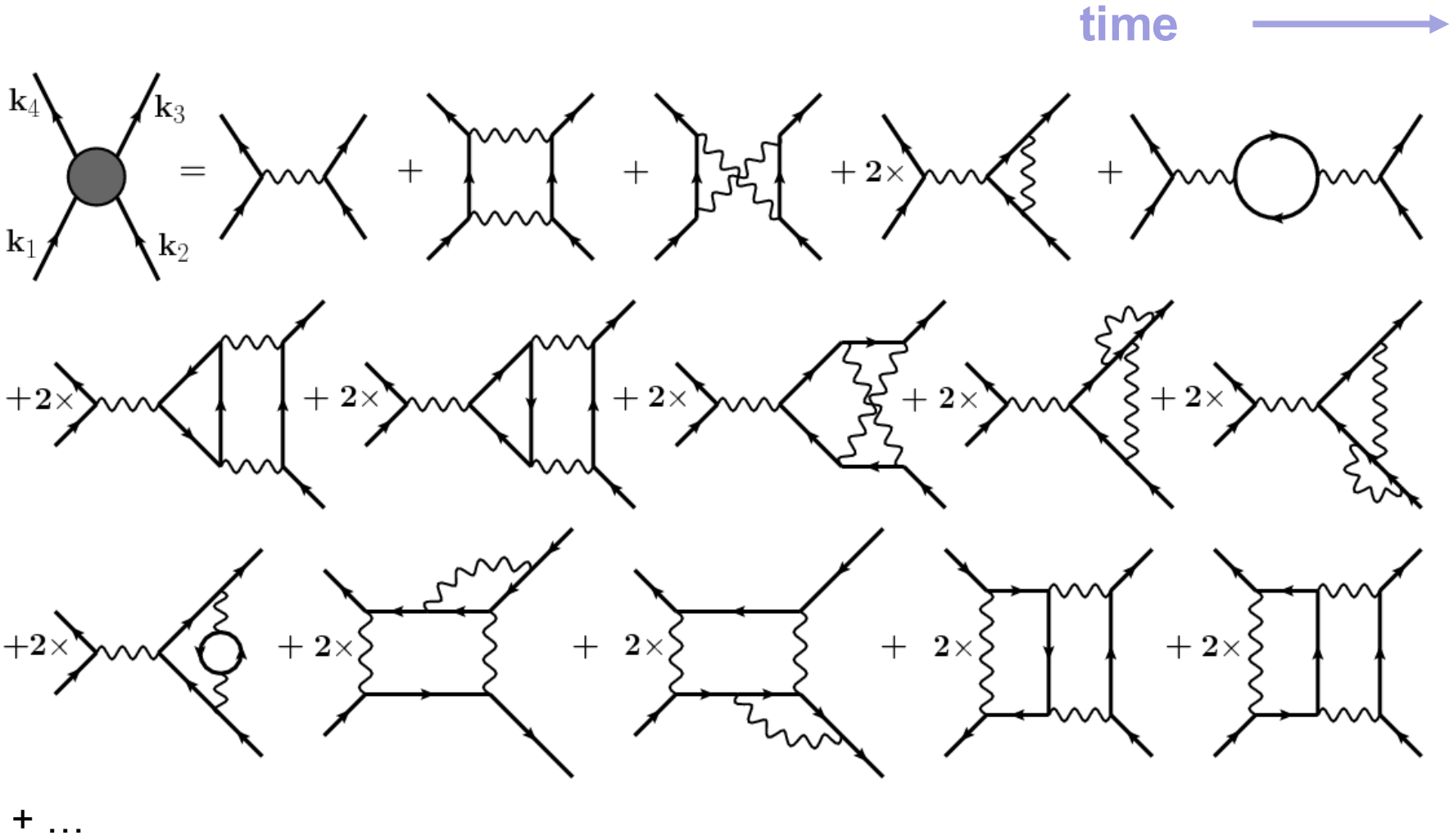
time

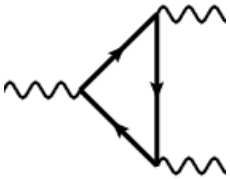


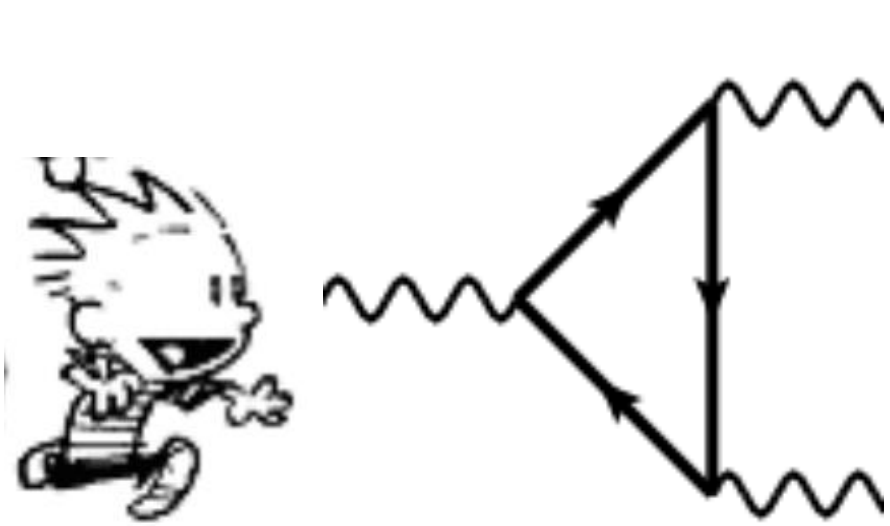


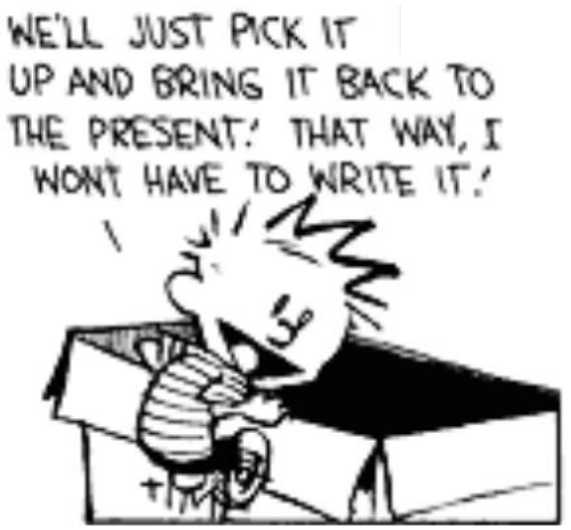


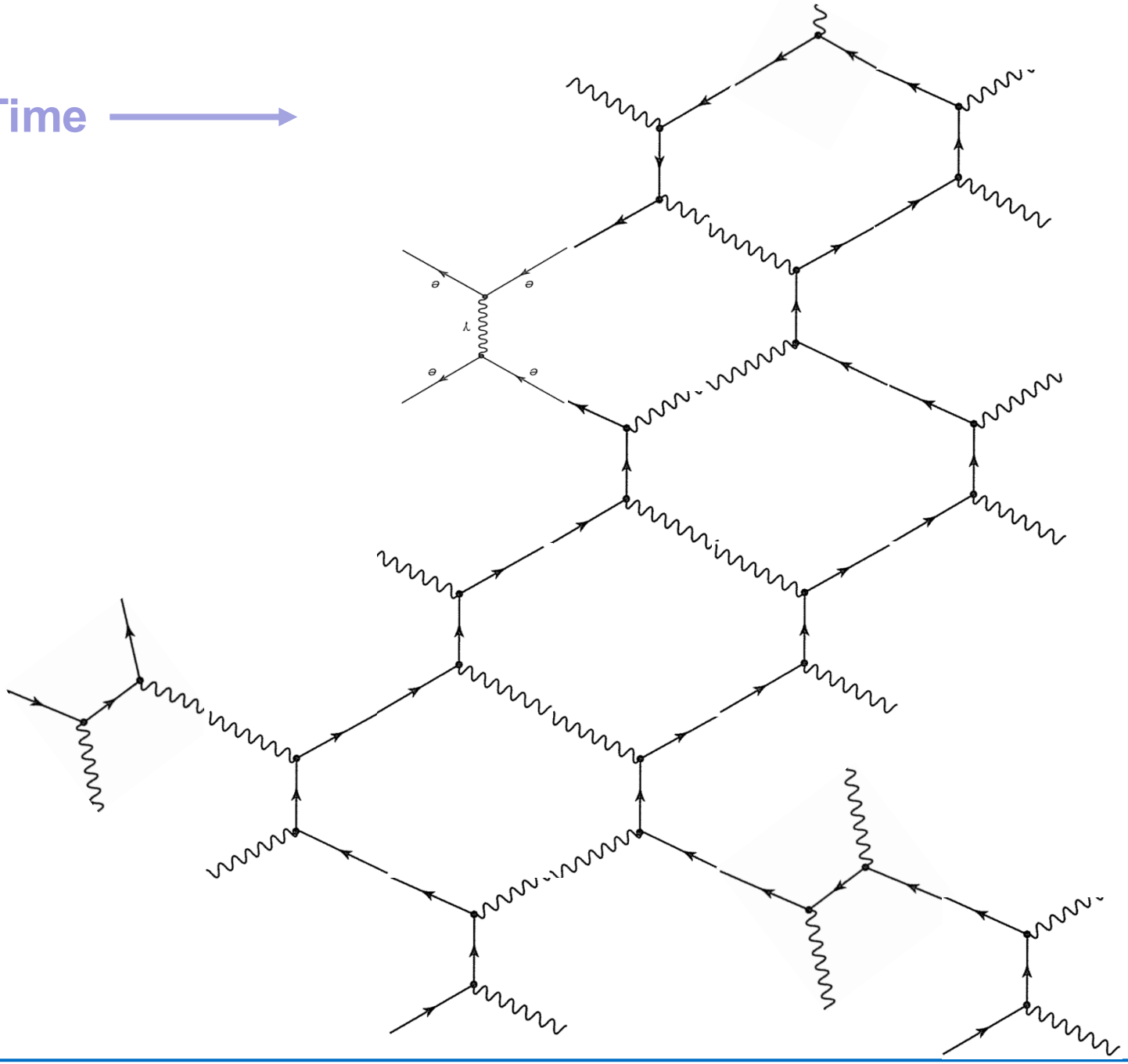
- In QED, for each diagram: **more vertices → smaller value**
- Only the connections are important (angles are irrelevant)
- For very high precision, thousands of terms may be needed


time
k_4
k_3
k_1
k_2
=
+
+
+ 2×
+
+2×
+ 2×
+ 2×
+ 2×
+ 2×
+2×
+ 2×
+
2×
+
2×
+ 2×
+ …






WE'LL JUST PICK IT UP AND BRING IT BACK TO THE PRESENT. THAT WAY, I WON'T HAVE TO WRITE IT.


Time
e
e
γ
e
e

I received a telephone call one day at the graduate college at Princeton from Professor Wheeler, in which he said,

> "Feynman, I know why all electrons have the same charge and the same mass"
>
> "Why?"
>
> "Because, they are all the same electron!"

Richard Feynman (11 December 1965).
Nobel Lecture. Nobel Foundation.

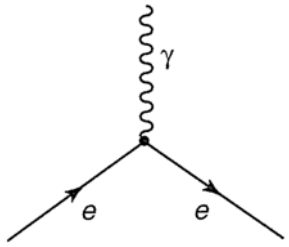
Also, remember that “proof” at the beginning?

- The two “90 degree rotations” that make it “work” are absurd because expressions and what they represent do not share rotational properties

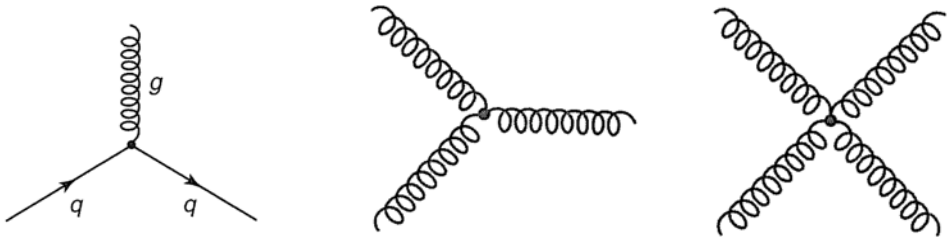
- On the other hand, when we rotate a Feynman diagram (i.e., we “rotate an expression”), we get valid expressions!

Interaction summary (w/o the Higgs boson)

QED:

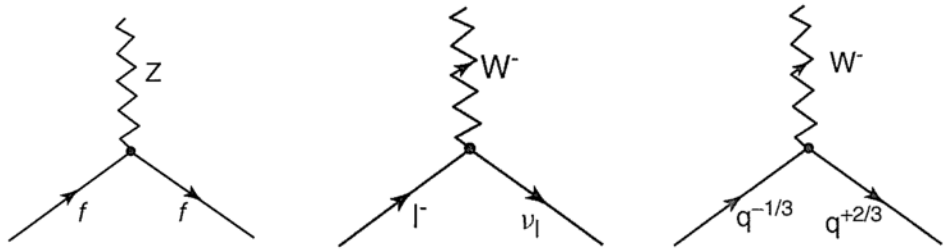


QCD:



	Fermions			Bosons	
Quarks	u up	c charm	t top	γ photon	Force carriers
	d down	s strange	b bottom	Z Z boson	
Leptons	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	
	e electron	μ muon	τ tau	g gluon	

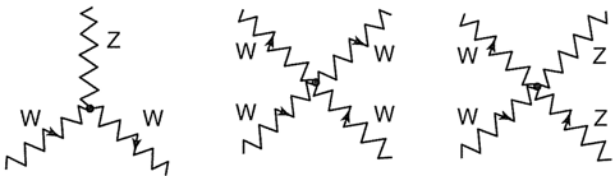
Weak:



Cabibbo-Kobayashi-Maskawa

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

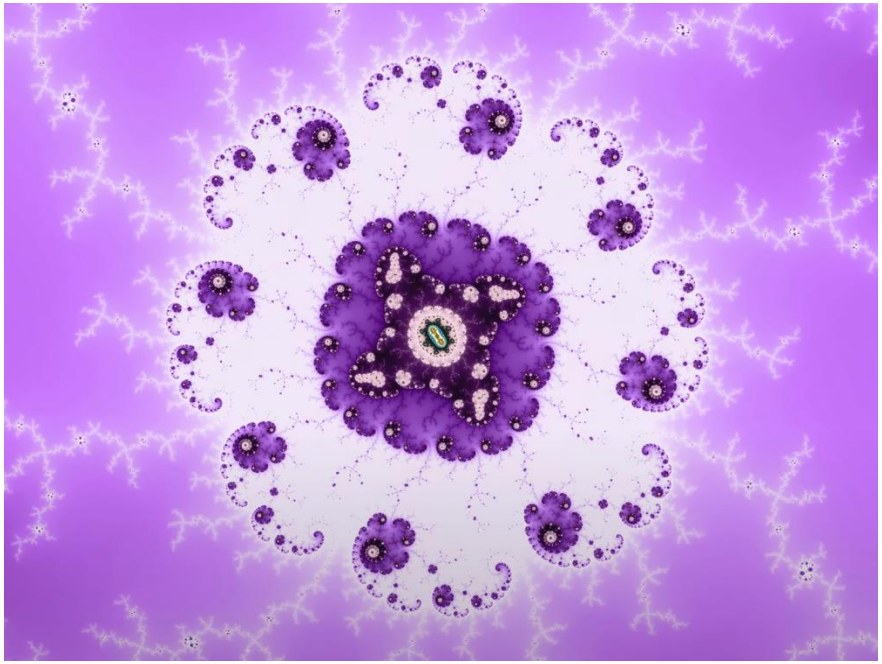
W/Z:



(W |Z)/γ:

- Each valid vertex corresponds to a term in the Standard Model Lagrangian.

- i.e., Feynman diagrams are *terms in the perturbative expansion of the probability amplitude of a process*.



$$z \leftarrow z^2 + c$$

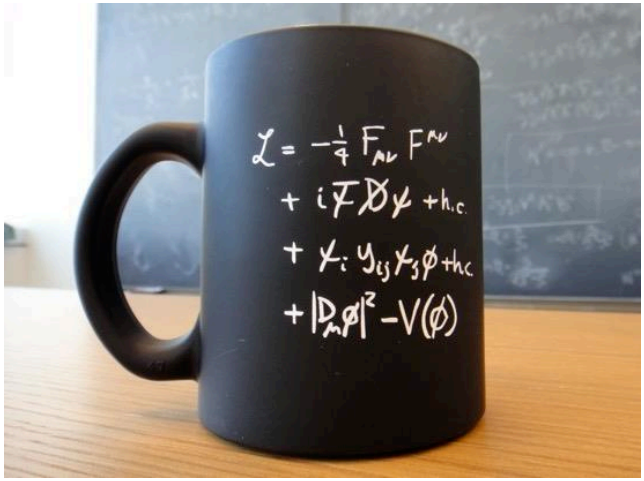


Image: NASA / STScI

Image: Vlad Gerasimov https://vlad.studio/wallpaper/higgs_boson_fluo/