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FOR FUNDAMENTAL PHYSICS

Resummation of NGLs in Isolated Photon Production

Computing a missing ingredient for NLL resummation

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Outline

- Why prompt photons are interesting and how to identify them
- Factorization: cone fragmentation function, jet function, coft function
- What LL and NLL accuracy require, and what I compute: the quark and gluon jet functions at order $\alpha\alpha_s$
- Resummation of non-global logarithms
- Status and outlook

Prompt photons

- Electroweak final states are precision probes of the hard interaction.
- Photons produced during the collision (prompt) carry information about physics at short distances.
- Large background from hadronic sources, e.g. decay of a neutral pion.
- The detector cannot tell the two apart.
- Experiments have reached few percent accuracy, unmatched by theory predictions.

We are only interested in prompt photons. We need some way to suppress the photons from hadronic sources.

Isolation cones

$$E_{T,\mathrm{tot}}^{\mathrm{cone}}(r) < \epsilon_\gamma E_{T,\gamma} \left(\frac{1-\cos r}{1-\cos R} \right)^n \qquad \text{for all } r < R$$

Smooth cone (n ≥ 1/2)

What most NNLO calculations use

- The veto tightens towards the centre, removing the collinear region entirely.
- No fragmentation function needed.
- It cannot be imposed on a real detector because of the finite resolution.

Fixed cone (n = 0)

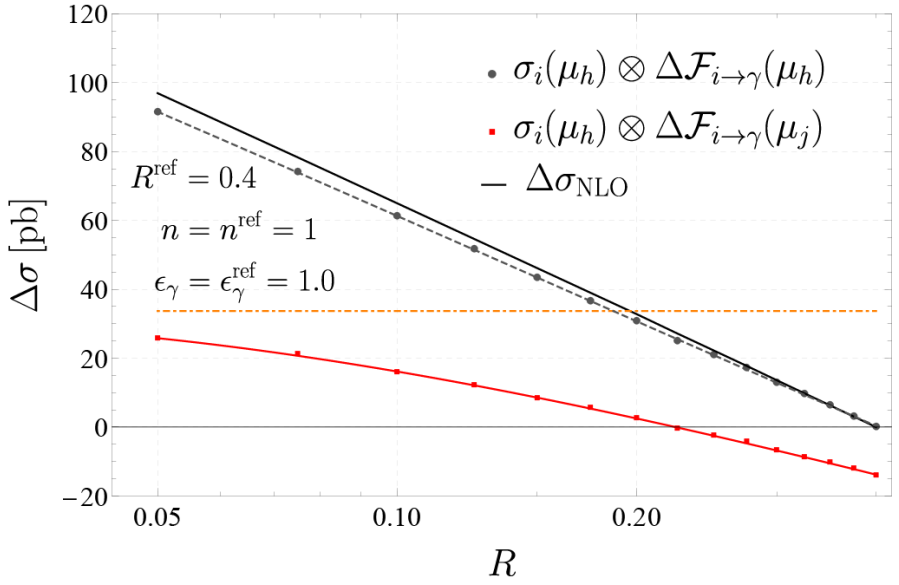
What every LHC measurement uses

- The collinear region survives, so a non-perturbative fragmentation function is needed.
- Typical parameters at LHC:
  $E_{T,\gamma} \geq 125$ GeV, $\mathrm{R} = 0.4$, $\epsilon_\gamma = 0.04$

There is a mismatch between measurement and theoretical prediction, leading to a source of uncertainty.

Why fixed order is not enough

- For small cones and isolation energies the fixed-order calculation breaks down because of large logarithms in the perturbative expansion.
- We deal with it by separating the scales of the problem.



At NLO the isolated cross section exceeds the inclusive one for R < 0.2.

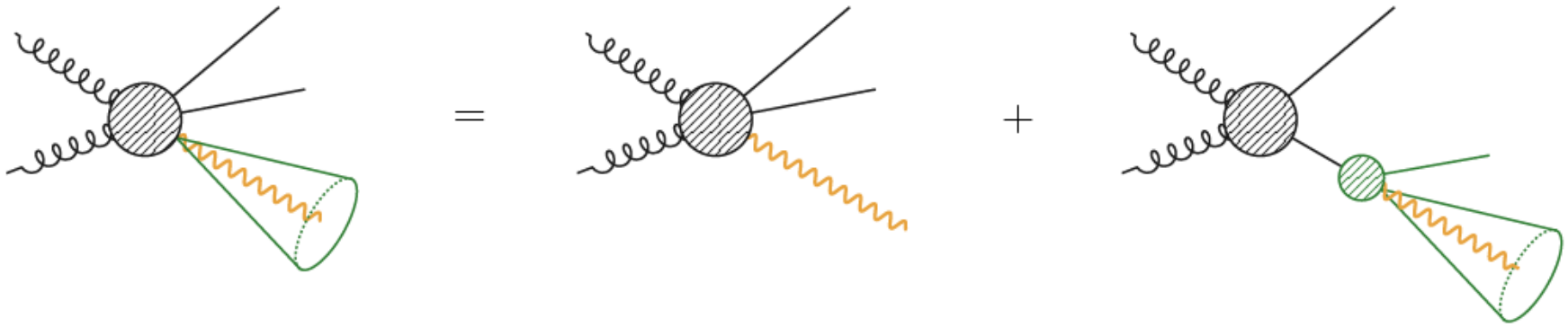
Becher, Favrod, Xu, arXiv:2208.01554 · pathology first shown at NLO in Catani, Fontannaz, Guillet, Pilon, hep-ph/0204023

Factorization for small R

- For small cone radius, all isolation effects are captured by a cone fragmentation function F: a parton going to a photon plus the accompanying constrained radiation.
- F is largely perturbative, since it encodes the isolation, but it still contains the poorly-known non-perturbative photon fragmentation function.

$$\frac{d\sigma(E_0,R)}{dE_\gamma} = \frac{d\sigma^{\text{dir}}_{\gamma+X}}{dE_\gamma} + \sum_{i=q,\bar{q},g} \int dz \, \frac{d\sigma_{i+X}}{dE_i} \, F_{i\to\gamma}\left(z, E_\gamma, E_0, R\right) + \mathcal{O}(R)$$

$(n\cdot p, \bar{n}\cdot p, p_\perp)$: **hard** $p \sim E_\gamma\,(1,1,1)$ **collinear** $p \sim E_\gamma\left(R^2, 1, R\right) \longrightarrow$ transverse scale RE_γ



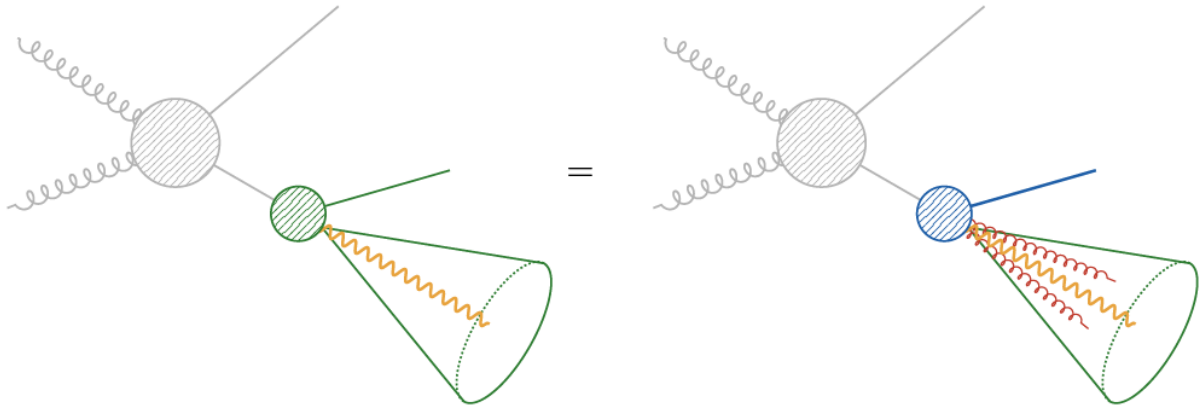
Becher, Favrod, Xu, arXiv:2208.01554

Factorization for small isolation energy

- A second hierarchy arises when the isolation energy is small.
- F factorizes again: a jet function for the energetic partons outside the cone, and a coft function for the soft radiation they emit back into it.

$$F_{i\to\gamma}\left(z, RE_\gamma, RE_0, \mu\right) = \sum_{l=1}^{\infty} \left\langle J_{i\to\gamma+l}\left(\{n\}, RE_\gamma, z, \mu\right) \otimes U_l\left(\{n\}, RE_0, \mu\right)\right\rangle$$

collinear $p \sim E_\gamma\left(R^2, 1, R\right)$ **coft** $p \sim E_0\left(R^2, 1, R\right)$ $\longrightarrow$ same angles, different energy

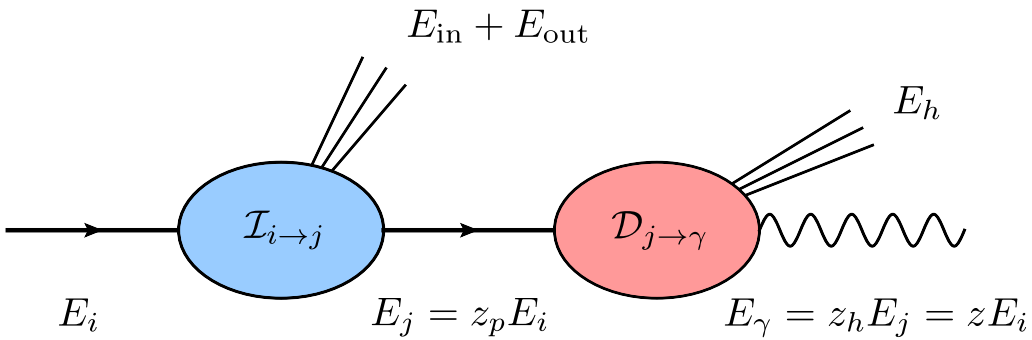


Becher, Favrod, Xu, arXiv:2208.01554

Where the non-perturbative physics goes

- The non-perturbative part of the fragmentation function describes particles which are highly collinear to the photon.
- In the limit of small isolation energy these particles are no longer allowed inside the cone and the contribution becomes trivial.

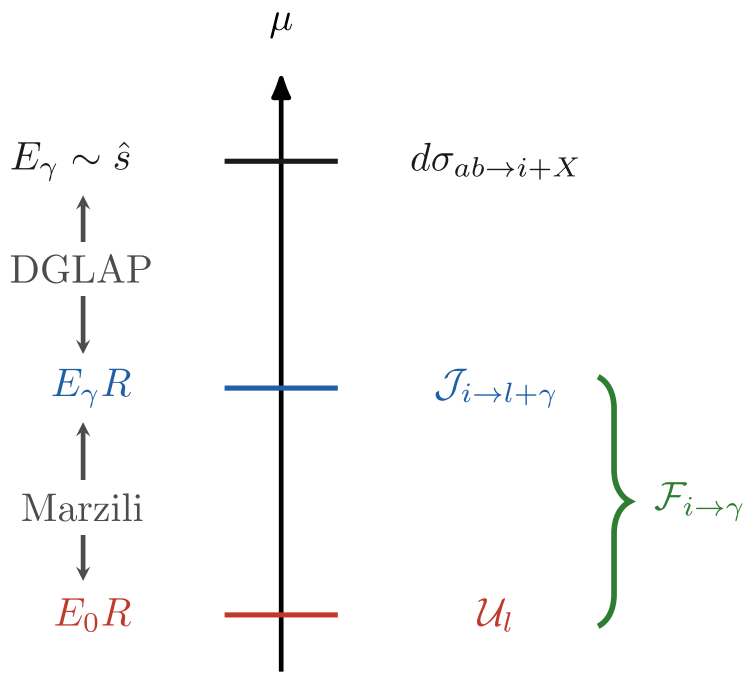
non-perturbative $p \sim E_\gamma\,(\lambda^2, 1, \lambda),\ \ \lambda = \Lambda_{QCD}/E_\gamma \ll R \quad \longrightarrow$ inside the cone, energy of order E_γ



In this limit the calculation is purely perturbative.

Becher, Favrod, Xu, arXiv:2208.01554

Three scales, three functions

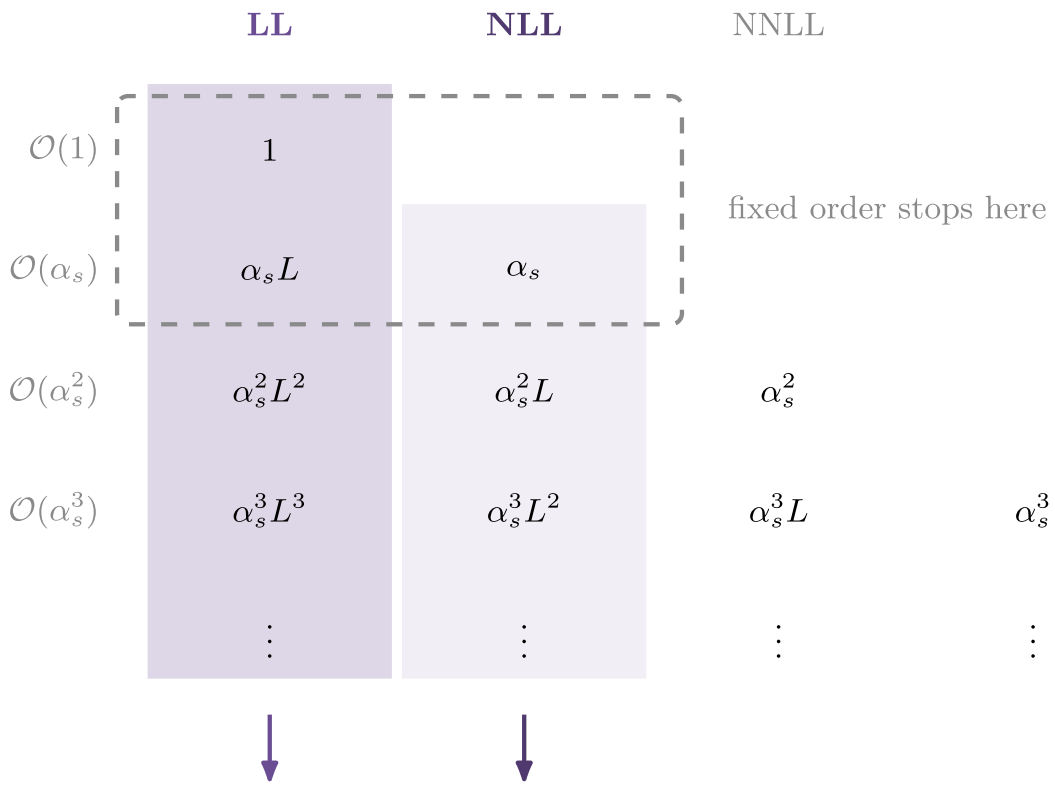


Each function is evaluated at the scale where it has no large logarithms. The evolution between the scales is the resummation.

Adapted from Becher, Favrod, Xu, arXiv:2208.01554

Logarithmic accuracy in resummation

Every power of α_s has at most one L, so the series is really an array.



The previously mentioned functions are the boundary condition; the RG evolution generates the tower of logarithms.

The NLO jet functions

The two logarithm towers are not at the same accuracy.

	ln R	ln ε_γ
LL	done	done
NLL	done	**two-loop evolution (MARZILI code) + NLO jet functions**
NNLL	needs the α_s^2 photon kernels	—

- I compute the $O(\alpha\alpha_s)$ jet functions $J(q \to \gamma + q)$, $J(q \to \gamma + q + g)$ and $J(g \to \gamma + q + \bar{q})$, for the quark and the gluon channel.

Becher, Favrod, Xu, arXiv:2208.01554

Computing $J(q \rightarrow \gamma + q + g)$

- Three-particle phase space: the jet function stays differential in the three relative angles $\theta_{q\gamma}, \theta_{g\gamma}, \phi_{qg}$ and in two momentum fractions.

- The soft and collinear singularities overlap. Using sector decomposition they can be disentangled.

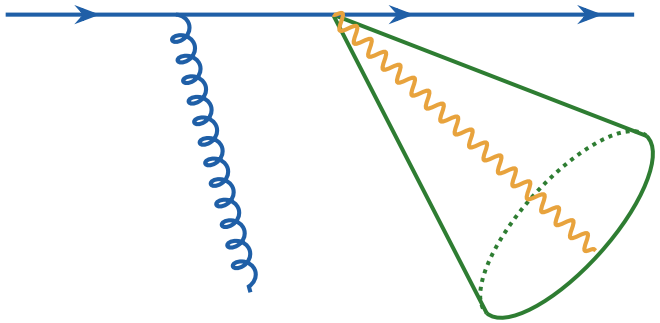
- The singularities are then made explicit by expanding in distributions:

$$x^{-1+a\,\epsilon} = \frac{1}{a\,\epsilon}\delta(x) + \sum_{n=0}^{\infty} \frac{(a\,\epsilon)^n}{n} \left[\frac{\ln^n(x)}{x}\right]_+$$

What is left after renormalisation is a distribution in the angles and in z.

RG evolution as parton shower

- The evolution down to the coft scale has no closed-form solution, so it is solved by Monte Carlo.
- Take the real-emission jet function at order $\alpha\alpha_s$: a quark and a gluon outside the cone, the photon inside.
- The MARZILI code dresses these partons with soft emissions, ordered in scale, with two-loop running.



Each emission is generated at a lower scale than the last.

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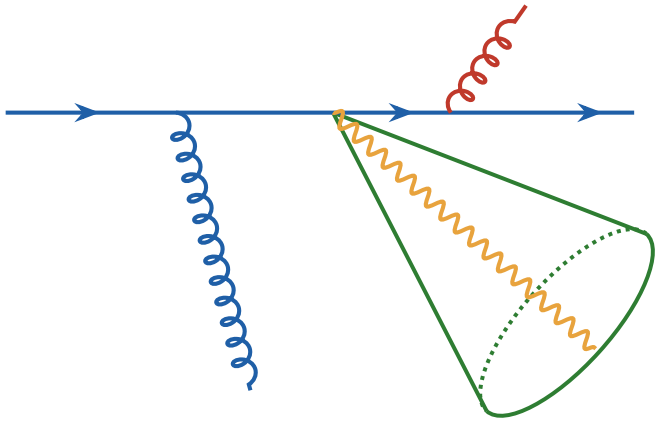

The first emission lands outside the cone, so the veto still holds.

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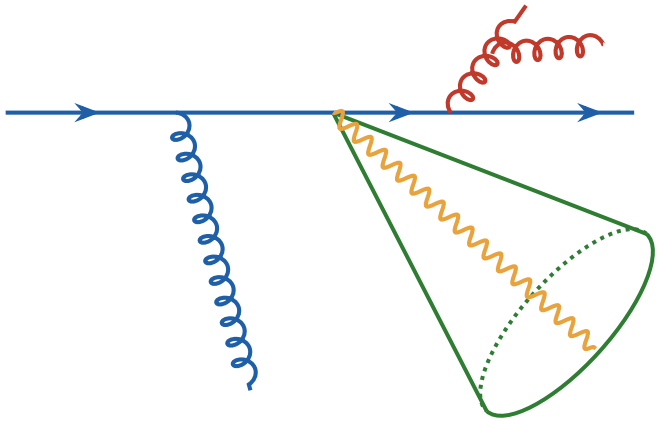

That gluon splits again — still outside, so the shower carries on.

RG evolution as parton shower

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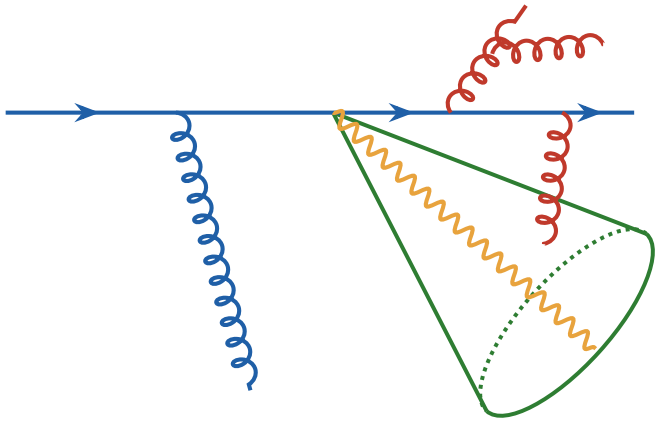


The first emission into the cone ends the shower.

Status and outlook

- The quark and gluon jet functions are computed at order $\alpha\alpha_s$, and their pole structure checked against the factorization prediction.
- Next: Cross-check of the finite part, resummation using MARZILI, and NLL predictions for isolated photon production.

Backup

Backup: why the shower solves the RG equation

- The evolution of the hard function is governed by a RG equation, which can be reinterpreted as a parton shower.

$$\frac{d}{d\ln\mu}\mathrm{J}_m(\mu) = \sum_l \mathrm{J}_l(\mu)\,\Gamma_{lm}(\mu), \qquad \Gamma = \underbrace{V_m}_{\text{virtual, } m\to m} + \underbrace{R_m}_{\text{real, } m\to m+1}$$

The solution at LL is:

$$J_m(t) = J_m(t_0)e^{(t-t_0)V_m} + \int_{t_0}^{t} dt'\, J_{m-1}(t')R_{m-1}e^{(t-t')V_m}$$

- The first term is the evolution without emitting, while the second term describes the emission at t'.
- $t = \frac{1}{2\beta_0}\ln\frac{\alpha_s(\mu_s)}{\alpha_s(\mu_j)}$ is the shower time associated with the scales.

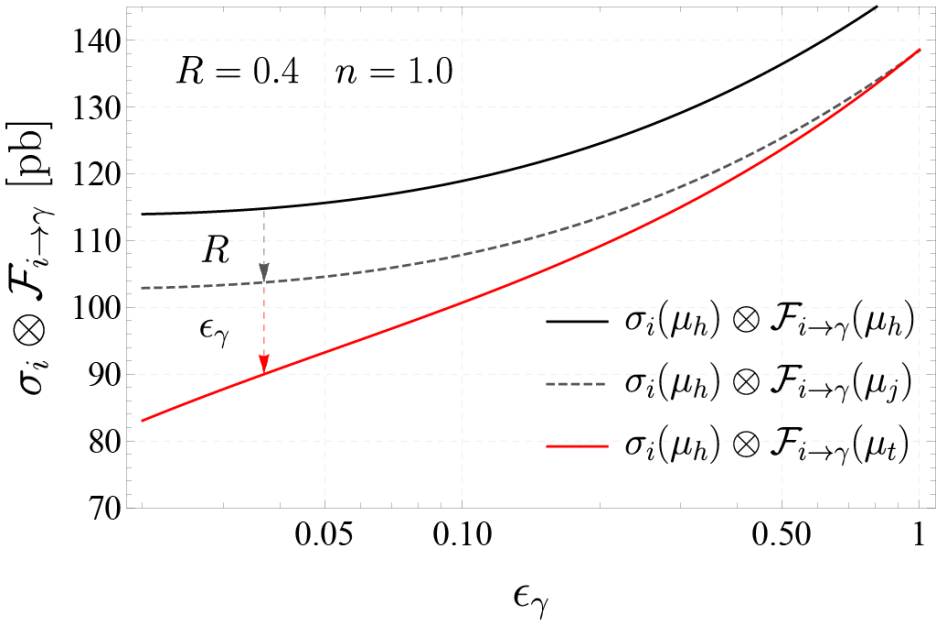
Becher, Schalch, Xu 2307.02283

Backup: the ln R resummation

$$\frac{d}{d\ln\mu}\,\mathcal{F}_{i\to\gamma}(z,Q\delta,\mu) \;=\; \sum_j \int_z^1 \frac{d\bar{z}}{\bar{z}}\;\mathcal{P}_{i\to j}(\bar{z})\;\mathcal{F}_{j\to\gamma}\left(\frac{z}{\bar{z}},\;\bar{z}\,Q\delta,\;\mu\right)$$

- $\mathcal{P}$ are the timelike parton-to-parton splitting kernels.
- Either directly solved numerically or in Melin space, where the back transformation is done numerically.

Backup: size of the resummation



- Fragmentation contribution only, at $R = 0.4$; add $\sigma^{dir} \approx 290$ pb for the full cross section.
- Dashed: $\ln R$ resummed. Red: both towers.
- ATLAS sits at $\varepsilon_\gamma \approx 0.04$.

Becher, Favrod, Xu, arXiv:2208.01554, Fig. 12 (lower panel)

Backup: definitions of the functions

Cone fragmentation function — matrix element of collinear fields

$$\frac{(\gamma^\mu n_\mu)_{\alpha\beta}}{2}\,\delta^{ab}\,\mathcal{F}_{q\to\gamma}(z,E_\gamma,E_0,R,\mu)=\sum_X\int\langle 0|\chi^a_{qa}(0)|\gamma X\rangle\left\langle\gamma X\left|\bar\chi^b_{q\beta}(0)\right|0\right\rangle\,\theta\big(2E_0-\bar n\cdot p_X^{\text{in}}\big)\,\delta\left(z-\frac{\bar n\cdot p_\gamma}{Q}\right)\cdots$$

Jet function — squared splitting amplitudes, energetic partons outside the cone

$$J_{i\to\gamma+l}(\{n\},RE_\gamma,z,\mu)=\sum_{\text{spins}}\prod_{j=1}^{l}\int\frac{dE_j\,E_j^{d-3}}{(2\pi)^{d-2}}\int\frac{dE_k\,E_k^{d-3}}{(2\pi)^{d-2}}\;\Theta_{\text{cone}}(\{p\})\;|\mathcal{M}_l\rangle\langle\mathcal{M}_l|\cdots$$

$$\text{with}\quad|\mathcal{M}_l(p_i;\{p\})\rangle=\langle k,p|\,\Phi_c^{\alpha a}(0)\,|0\rangle$$

Coft function — Wilson lines along the parton directions, plus one along $\bar n$

$$U_l(\{n\},E_0R)=\sum_{X_t}\int\left\langle 0\left|\,U_0^\dagger(\bar n)\,U_1^\dagger(n_1)\cdots U_l^\dagger(n_l)\,\right|X_t\right\rangle\langle X_t|\,U_0(\bar n)\cdots U_l(n_l)\,|0\rangle\;\theta\big(2E_0-\bar n\cdot p_{X_t}^{\text{cone}}\big)$$

$\chi_a = W^\dagger\psi$ and $A_\perp = W^\dagger\, iD_\perp\, W$ are the collinear fields with their Wilson lines. U_0 along $\bar n$ captures everything not collinear to the photon. Only the energy of the coft partons inside the cone is bounded by E_0.

Becher, Favrod, Xu, arXiv:2208.01554, Eqs. (2.5), (5.2)–(5.3) and (5.7)