Effect of Fiducial Cuts on Energy-Energy Correlators

Where did the rest of my events go?

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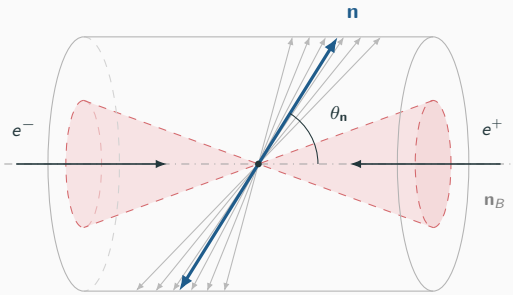
Take home message

I. We study EECs. Detector acceptance forces a cut on the event-shape axis. Current procedure: data inside the rejected region are restored using a Monte Carlo efficiency factor, but we need precision physics!

II. Decompse the angular dependence of the EEC to two structure functions. Proposal: restore the missing data from the fiducial region.

III. The axis cut is *not* the dominant acceptance effect: particle-level cuts are an order of magnitude larger.



axis inside the red cones
$\Rightarrow$ event rejected

What is an Energy–Energy Correlator (EEC)?

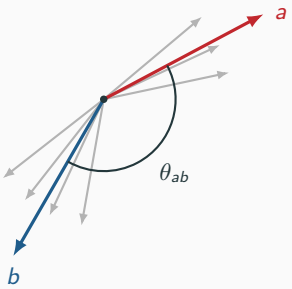
Definition:

- take one event,
- take *every ordered pair* of particles (a, b),
- weight it by $E_a E_b / Q^2$,
- histogram the "angle" between them.

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}z} \equiv \sum_{a,b} \int \mathrm{d}\sigma \, \frac{E_a E_b}{Q^2} \, \delta\left(z - \frac{1 - \cos\theta_{ab}}{2}\right)$$

$$\frac{\mathrm{dEEC}}{\mathrm{d}z} \equiv \frac{1}{\sigma} \frac{\mathrm{d}\Sigma}{\mathrm{d}z}$$

We work in the bulk, $0 < z < 1$.



$$z = \frac{1 - \cos\theta_{ab}}{2}$$

$z \to 0$	collinear region
$z \to 1$	back-to-back region
bulk	wide-angle region

Acceptance cuts

The three event axes

Thrust

$$T = \max_{|\mathbf{n}|=1} \frac{\sum_a |\mathbf{k}_a \cdot \mathbf{n}|}{\sum_a |\mathbf{k}_a|}, \qquad \text{3 partons:} \quad \mathbf{n}_T = \frac{\mathbf{k}_{\max}}{E_{\max}}$$

Sphericity

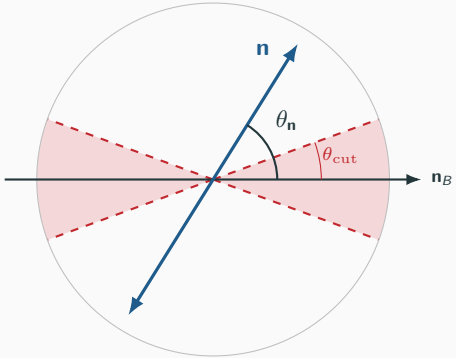
$$S^{ij} = \frac{\sum_a k_a^i k_a^j}{\sum_a |\mathbf{k}_a|^2}, \qquad \mathbf{n}_S = \text{leading eigenvector}$$

IRC-safe sphericity

$$S^{ij}_{\text{safe}} = \sum_a \frac{k_a^i k_a^j}{|\mathbf{k}_a|}, \qquad \mathbf{n}_{S^*} = \text{leading eigenvector}$$

To our knowledge $\mathbf{n}_{S^*}$ has never been used for fiducial cuts in the literature.

Detector acceptance and the event-axis cut



Coverage degrades in the forward region, so e^+e^- analyses require the oriented event-shape axis $\mathbf{n}$ to lie centrally:

$$|\cos\theta_{\mathbf{n}}| \leq \cos\theta_{\mathrm{cut}}$$

- ALEPH analysis: sphericity axis $\theta_{\mathrm{cut}} \approx 35^\circ$
- SLD analysis: thrust axis $\theta_{\mathrm{cut}} \approx 45^\circ$

The measured EEC is a fiducial quantity: the published "fully corrected" result needs an extrapolation to the full phase space.

Current practice: a Monte Carlo efficiency factor

The published inclusive result is the fiducial measurement times a bin-by-bin MC efficiency factor:

$$\frac{\mathrm{dEEC}^{\mathrm{exp}}_{\mathrm{inc}}}{\mathrm{d}z} = \frac{\mathrm{dEEC}^{\mathrm{exp}}_{\mathrm{fid}}(\theta_{\mathrm{cut}})}{\mathrm{d}z} \times \underbrace{\left[\frac{\mathrm{dEEC}^{\mathrm{MC}}_{\mathrm{inc}}}{\mathrm{d}z} \Big/ \frac{\mathrm{dEEC}^{\mathrm{MC}}_{\mathrm{fid}}(\theta_{\mathrm{cut}})}{\mathrm{d}z}\right]}_{K^{-1}(\theta_{\mathrm{cut}})}$$

- Unbiased only to the extent that the generator describes the correlation between the energy flow within the event and the orientation of the event axis *relative to the beam*.
- K^{-1} is a genuine $\mathcal{O}(\alpha_s)$ effect with a nontrivial dependence on z.
- Its *systematic* uncertainty cannot be assessed from within the generator.

Percent-level data are corrected by a factor whose accuracy is not constrained by the measurement itself.

Structure-function decomposition

Symmetry fixes the angular dependence

The EEC-weighted hadronic tensor at fixed event axis:

$$W_{\mu\nu}(q,z,\mathbf{n}) = 4\pi \sum_X \int \mathrm{d}\phi_X \; \delta^{(2)}\big(\mathbf{n},\mathbf{n}(\phi_X)\big) \sum_{a,b\in X} \frac{E_a E_b}{Q^2}\, \delta\left(z - \frac{1-\cos\theta_{ab}}{2}\right) \times \langle\Omega|J_\mu(0)|X\rangle\langle X|J_\nu(0)|\Omega\rangle$$

with

$$\int \mathrm{d}\Omega_{\mathbf{n}} \; \delta^{(2)}\big(\mathbf{n},\mathbf{n}'\big) = 1$$

Ingredients:

1. rotational invariance (unpolarised beams, no initial-state radiation),
2. the event shapes used here are invariant under $\mathbf{n} \to -\mathbf{n}$, so $W_{\mu\nu}$ is *even* in $\mathbf{n}$,
3. the leptonic tensor has no timelike components in the c.o.m. frame ($m_e = 0$), so only W_{ij} contributes.

$$W_{ij} = \frac{\mathrm{d}\Sigma_U}{\mathrm{d}z} \underbrace{3\big(\delta_{ij} - n_i n_j\big)}_{T_{ij}^U \text{ (unoriented)}} + \frac{\mathrm{d}\Sigma_O}{\mathrm{d}z} \underbrace{\tfrac{9}{2}\big(n_i n_j - \tfrac{1}{3}\delta_{ij}\big)}_{T_{ij}^O \text{ (oriented)}}$$

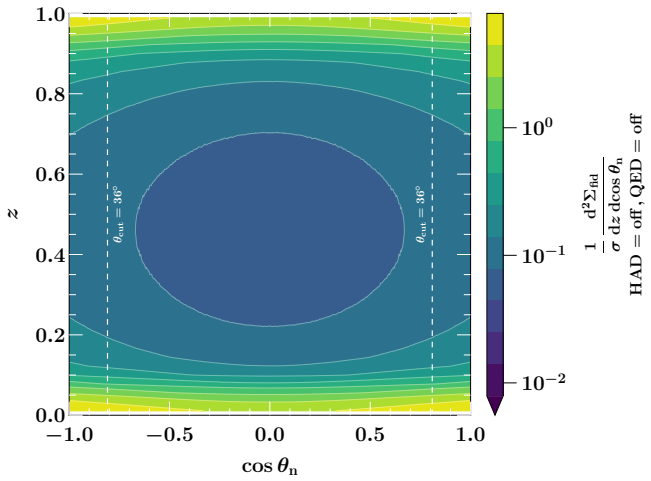
Symmetry fixes the angular dependence

$$\frac{\mathrm{d}^2\Sigma_{\mathrm{fid}}}{\mathrm{d}z\,\mathrm{d}\cos\theta_{\mathbf{n}}} = \frac{3}{8}\left(1+\cos^2\theta_{\mathbf{n}}\right)\frac{\mathrm{d}\Sigma_U(Q,z)}{\mathrm{d}z} + \frac{3}{16}\left(1-3\cos^2\theta_{\mathbf{n}}\right)\frac{\mathrm{d}\Sigma_O(Q,z)}{\mathrm{d}z}$$

- Holds to all orders in α_s, at hadron level, for any event axis. Corrections are $\mathcal{O}(\alpha^3)$.
- The $\cos\theta_{\mathbf{n}}$ dependence is *completely fixed*: only two harmonics appear.
- At each z there are exactly two unknowns function of *only* z and Q.
- Removing the cut leaves only the unoriented term: $\mathrm{d}\Sigma_{\mathrm{inc}}/\mathrm{d}z = \mathrm{d}\Sigma_U/\mathrm{d}z$.

Measuring the EEC *double differentially* in z and $\cos\theta_{\mathbf{n}}$ determines both structure functions from data inside the acceptance. The unoriented one is the full-phase-space EEC.

The double-differential distribution



$e^+e^- \to$ partons at $Q = M_Z$ (Pythia).

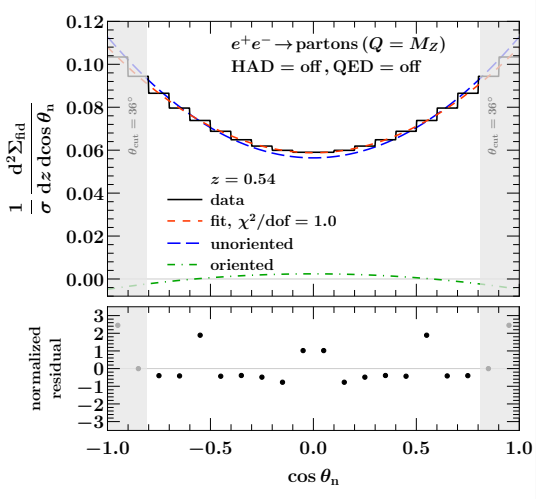
Bright bands at $z \to 0$ and $z \to 1$: the collinear and back-to-back singularities.

Dashed lines: a realistic acceptance, $\theta_{\mathrm{cut}} = 36^\circ$.

Everything outside the white lines is discarded.

By the decomposition, that region carries no information that the accepted region does not already contain.

Defiducialization on Pythia data



One slice at $z = 0.54$. The fit uses only the accepted region (white).

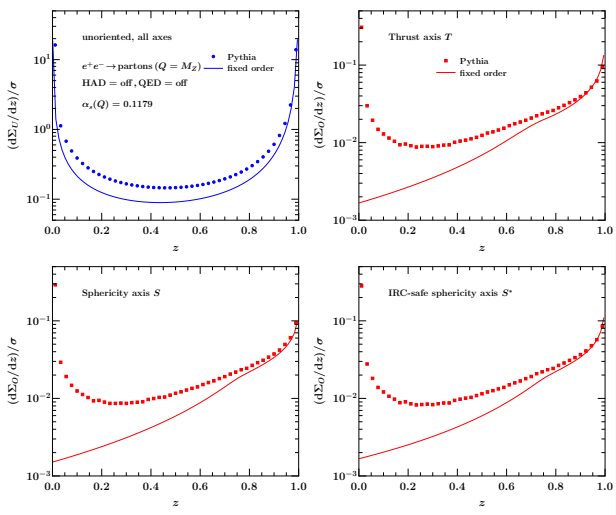
- blue: unoriented harmonic $\propto 1 + c^2$
- green: oriented harmonic $\propto 1 - 3c^2$
- red dashed: their sum, $\chi^2/\text{dof} = 1.0$

The fitted $\mathrm{d}\Sigma_U/\mathrm{d}z$ is proportional to the full-phase-space EEC, obtained without an efficiency factor.

Any additional harmonic would signal a violation of the assumptions.

The oriented EEC at $\mathcal{O}(\alpha_s)$

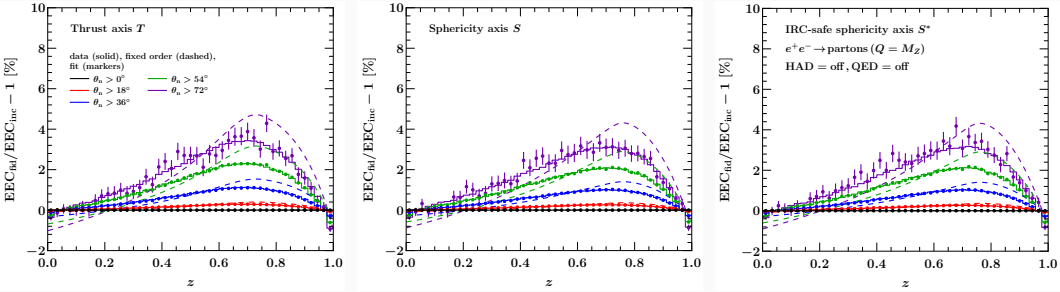
Structure functions: fixed order vs Pythia



Fit the two harmonics to Pythia at each z, and compare with the $\mathcal{O}(\alpha_s)$ result.

- Unoriented: singular at both ends.
- Oriented: at small z Pythia's oriented function reflects the shower/resummation.

Size of the fiducial correction



- Realistic $\theta_{\mathrm{cut}} = 36°$ (blue): up to 1%, peaking near $z \simeq 0.7$.
- Very similar for all three axes.

We do *not* advocate using the fixed-order curve: that is used to establish the size and shape of the effect.

Particle-level cuts

What survives with particle-level cuts

Particle-level cuts break full rotational invariance, but a residual symmetry survives:

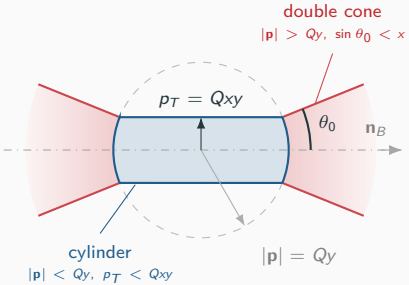
- azimuthal rotations about the beam (unpolarised beams),
- axis flip $\mathbf{n} \to -\mathbf{n}$,
- beam flip $\mathbf{n}_B \to -\mathbf{n}_B$.

The distribution is still even in $\cos\theta_{\mathbf{n}}$, but the two harmonics are promoted to a full tower:

$$\boxed{\frac{\mathrm{d}^2\Sigma_{\mathrm{fid}}}{\mathrm{d}z\,\mathrm{d}\cos\theta_{\mathbf{n}}} = \sum_{\ell\geq 0} \mathcal{D}_{2\ell}(z;\eta_{\mathrm{cut}},p_{T,\mathrm{cut}})\; P_{2\ell}(\cos\theta_{\mathbf{n}})} \qquad \left.\frac{\mathrm{d}\Sigma_{\mathrm{fid}}}{\mathrm{d}z}\right|_{\theta_{\mathrm{cut}}=0} = 2\,\mathcal{D}_0$$

- All remain accessible from *inside* the acceptance, but when can we truncate ℓ?
- $2\mathcal{D}_0$ removes the *axis* cut, but not the particle-level cuts.

Removing the particle cuts



Tie the two cuts together: $\sin\theta_0 = x$, $p_{T,\mathrm{cut}} = Q\,x\,y$.

Removed region = double cone ∪ cylinder.

ALEPH: $\eta_{\mathrm{cut}} \simeq 1.76$, $p_{T,\mathrm{cut}} = 0.2\,\mathrm{GeV}$

$\Rightarrow\ x \simeq 0.34,\ \ y = 6.4 \times 10^{-3},\ \ Qy \simeq 0.59\,\mathrm{GeV}$

We can prove that

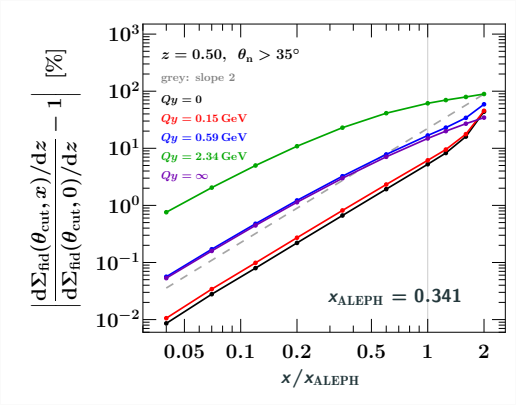
$$\frac{\mathrm{d}\Sigma_{\mathrm{fid}}(\theta_{\mathrm{cut}}, x, y)}{\mathrm{d}z} = \frac{\mathrm{d}\Sigma^{(0)}_{\mathrm{fid}}(\theta_{\mathrm{cut}})}{\mathrm{d}z} + x^2 \frac{\mathrm{d}\Sigma^{(2)}_{\mathrm{fid}}(\theta_{\mathrm{cut}}, y)}{\mathrm{d}z} + \mathcal{O}(x^3)$$

and the offset is independent of y: it is the fiducial EEC with the axis cut still imposed, but *free of particle-level cuts*.

The proposal: tighten both cuts together, measure at a few values of x, fit in x and keep the offset. The harmonic fit of the previous section then applies to it.

In Pythia, we study five different y-families.

Validation and practical limitation

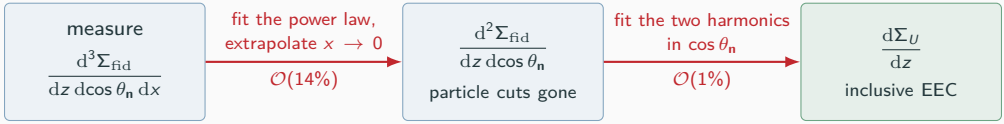


An acceptance can always be tightened, never loosened.

The extrapolation to $x = 0$ must be done from $x/x_{\mathrm{ALEPH}} \geq 1$.

Summary

Tighten both particle cuts with a single parameter: $\sin\theta_0 = x$, $p_{T,\mathrm{cut}} = Q\,x\,y$.



Every step uses only data inside the acceptance, no MC efficiency factor anywhere.

Analytic $\mathcal{O}(\alpha_s)$ results

thrust	elementary functions
(IRC-safe) sphericity	incomplete elliptics

What we would like from experiments

- publish Σ_{fid} differentially in z, $\cos\theta_{\mathbf{n}}$ *and* x,
- run the chain above.

The axis cut and the particle-level acceptance are now under control.

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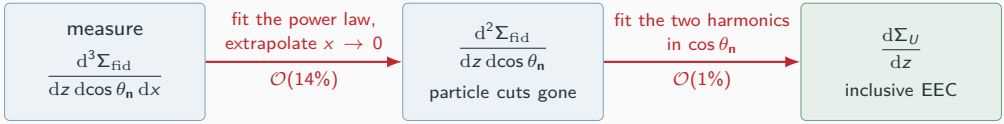

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Thank you!